PARTIAL REFERENCE INDEX

NOTE.—The attention of those who are not especially familiar with concrete construction is called to Chapter I, page 1, in which many of the essentials of concrete construction are pointed out and the reader is warned against the serious errors that have frequently been made in this field Chapter II gives elementary directions for concreting.

Frontispiece.

Massachusetts Institute of Technology

A TREATISE

ON

CONCRETE

PLAIN AND REINFORCED

MATERIALS, CONSTRUCTION, AND DESIGN OF
CONCRETE AND REINFORCED CONCRETE

WITH CHAPTERS BY

R. FERET, WILLIAM B. FULLER, FRANK P. McKIBBEN AND
SPENCER B. NEWBERRY

COMPLEMENTARY BOOKS BY THE SAME AUTHORS
CONCRETE COSTS
AND
IN PREPARATION
EARTHWORK BY HAND AND MACHINE

BY

FREDERICK W. TAYLOR, M.E., Sc.D.

AND

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PREFACE TO FIRST EDITION

This treatise is designed for practicing engineers and contractors, and also for a text and reference book on concrete for engineering students.

To broaden the scope of the work and avoid personal inaccuracies, each chapter has been submitted for criticism to at least one, and, in some cases, to three or four specialists in the particular line treated. We have aimed to refer by name to all authorities quoted, and where the data is taken from books or periodicals, to give the original publication, so that each subject may be investigated further. Proof clippings have also been submitted for approval to those whose names are mentioned. Numerous cross references will be found as well as many repetitions, inserted for the purpose of emphasizing important facts.

The chapters are arranged for convenience in reference, and therefore are not always in logical order.

The Concrete Data in Chapter I presents a list of definitions of words and terms relating distinctly to cement and concrete; a summary of the most important facts and conclusions, with references to the pages discussing them; data on concrete labor, and conversion ratios.

The Elementary Outline of the Process of Concreting, Chapter II, is designed, not for the civil engineer, but for those seeking simple directions as to the exact procedure in laying a small quantity of concrete. Most of the subjects there treated are discussed at length in subsequent chapters.

The Specifications for Cement in Chapter III include the latest recommendations of committees of our national societies, with incidental changes to adapt them for direct use in purchase specifications. The Concrete Specifications have been prepared by the authors to represent standard practice. Specifications for First-class or High Steel, drawn up by Mr. Taylor, are, we believe, the first recommendations which have been made to safely adapt this important material to reinforced concrete construction.

In Chapter IV the Choice of Cement is considered in an elementary fashion, which will serve as a guide to the constructor. Classification of Cements, Chapter V, distinguishes the various cements and limes manufactured in the United States and Europe.

Mr. Spencer B. Newberry, an international authority on the subject treated, has very kindly written for us Chapter VI on the Chemistry of Hydraulic Cement, discussing this complex subject in such a clear and practical manner that it will be of interest not only to the scientist, but also to the general reader and to the cement manufacturer. Mr. Newberry has also criticised Chapter V.

Chapters VII and VIII give the latest information on the testing of cement. Chapter IX presents practical rules for selecting sand for mortar, and the effect of different sands and of foreign ingredients upon its quality. Characteristics of the Aggregate are further treated, and practical data in regard to it are given in Chapter X.

The subject of Proportioning Concrete has been treated, at our request, by Mr. William B. Fuller, the concrete expert, and his practical use of mechanical analysis is fully discussed.

The tables of Quantities of Materials for Concrete and Mortar, in Chapter XII, and the diagram of curves, will be found useful in estimating materials.

The Strength of Concrete, Chapter XX, is taken up from a practical standpoint so that the data may be directly employed in design.

The theory and design of reinforced concrete are as yet in an elementary stage, but the rules and tables in Chapter XXI represent the most advanced knowledge on the subject.

Practical methods of Mixing and Laying Concrete are treated in Chapters XIII, XIV and XV.

Mr. René Feret, of Boulogne-sur-Mer, France, whose extended researches enable him to speak with authority, has kindly written for us Chapter XVI, entitled The Effect of Sea Water.

Chapters XVII, XVIII and XIX, on Freezing, Fire and Rust Protection, and Water-Tightness are of practical interest to the contracting engineer.

Plain and Reinforced Concrete Structures are treated in as much detail as space permits in Chapters XXIII to XXVIII inclusive. The designs are taken mostly from original drawings redrawn by the authors. They have been selected, not as extraordinary productions, but because the data in regard to them may be of use in designing similar structures.

Methods of Cement Manufacture in its modern types are described in detail in Chapter XXX.

The References in Chapter XXXI will be found especially valuable to one pursuing more extended investigations than can be presented in a volume of this size.

They have been selected from the large number contained in the authors' index, as those which it may be to the advantage of the reader to consult.

NOTE: The chapter numbers have been changed to agree with the Second Edition.

The articles are usually described by their subject-matter rather than by their titles verbatim.

Appendix I gives the method of chemically analyzing cement and cement materials according to the recommendations of the American Chemical Socie ;

Additional formulas for reinforced concrete beams, too complicated for insertion in the body of the book, are given in Appendix II, these having been kindly compiled by Prof. Frank P. McKibben for this treatise.

The authors desire to express their sincere appreciation of the various kindnesses extended to them while compiling the work. It has been necessary, because of the lack of authoritative information on many fundamental questions, not only to conduct numerous original investigations, but also to correspond with the most prominient engineers in this country, and with experts in England, France, and Austria.

Mr. Feret, besides writing the chapter on The Effect of Sea Water, has kindly criticised Chapter IX, and made numerous suggestions which have been incorporated.

Mr. Fuller has examined and criticised all the chapters on practical construction, and Prof. McKibben has rendered material assistance in the line of investigations and criticisms relating to the theories of reinforced concrete.

The authors are indebted to many gentlemen for careful criticism of chapters or portions of chapters, for drawings, or for replies to questions, and take this opportunity to express their sincere appreciation of all such assistance. Among those to whom especial acknowledgment is due are the following:

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Cuts have kindly been furnished by Allis-Chalmers Co., Austin Manufacturing Co., Automatic Weighing Machine Co., Bonnot Co., Bradley Pulverizer Co., Clyde Iron Works, Contractors Plant Co., Drake Standard

Machine Works, Fairbanks Co., Falkenau-Sinclair Machine Co., Farrel Foundry and Machine Co., Iroquois Iron Works, Kent Mill Co., Link-Belt Engineering Co., McKelvey Concrete Machinery Co., W. F. Mosher & Son, Tinius Olsen and Co., Philadelphia Pneumatic Tool Co , Thos. Prosser and Son, Ransome Concrete Machinery Co., Riehlé Bros. Testing Machine Co., Robins Conveying Belt Co., Sherburne and Co., T. L. Smith, Henry Troemner, Tucker and Vinton.

FREDERICK W. TAYLOR.
SANFORD E. THOMPSON.

February, 1905.

The writer wishes to state that the investigation and study necessary for the writing of this book were done by his colleague, Mr. Thompson, and desires that full credit for this should be given to him.

FREDERICK W. TAYLOR.

PREFACE TO THIRD EDITION

Developments in reinforced concrete as a result of tests and experience since the issue of the Second Edition have made it necessary to greatly enlarge the treatment of design and construction.

As in previous editions, the aim has been to give to the construction engineer, the architect, and the contractor, data for design and for building, and to the student a comprehensive and practical text and reference book useful not merely for the study of theory at college but to preserve and use in practice. The comprehensive treatment of both plain and reinforced concrete in a single volume—for reinforced concrete cannot be made satisfactorily unless the laws and best practice in plain concrete are followed—give it a peculiar value for the practicing engineer.

The entire volume has been revised and largely rewritten. The most important changes, as indicated, are in the portion of the book treating of reinforced concrete. Instead of a single chapter, three chapters are presented: Chapter XX on Theory, giving the derivation of formulas; Chapter XXI on Tests, selected from experiments in this country and abroad that give a definite basis for theory and practice; and Chapter XXII on Design, with working formulas and methods of design.

As important features of reinforced concrete, Chapter XXIII on Building Construction has been rewritten and enlarged, giving, as illustrations, drawings of typical structures and many details showing methods of handling the design in the drafting rooms of the architect and the engineer; an entirely new chapter, XXV, on Beam Bridges has been written with examples worked out for different types of design; Chapters XXVII to XXXI have been revised.

The Arch chapter kindly prepared by Prof. Frank P. McKibben for the second edition remains substantially the same; the material on stress distribution has been rewritten and transferred to Chapter XX.

Prominent among the additions in the first part of the book are the Specifications for Reinforced Concrete written by the author for the construction of the Massachusetts Institute of Technology buildings; the new cement specifications presented by the Joint Conferences and adopted in 1916; and the revised chapter on Chemistry of Hydraulic Cements by Mr. Spencer B. Newberry. Chapters XIII and XIV, Mixing and Depositing, have been rewritten.

Chapter XV on the Effect of Sea Water has been submitted to the author, Mr. R. Feret, who has approved it with certain revisions.

The effect of various agencies upon concrete is treated in Chapters XVI and XVII. Water-tightness, Chapter XVIII, and Strength of Plain Concrete, Chapter XIX, particularly the latter, have been changed so as to give results of recent tests and the conclusions derived from them.

In the Preface to the Second Edition acknowledgment was made to Messrs. E. D. Boyer, R. D. Bradbury, William B. Fuller, Frank P. McKibben, Spencer B. Newberry, George F. Swain, Arthur N. Talbot, Joseph R. Worcester, and Edward Smulski.

In the Third Edition thanks are due Messrs. Duff A. Abrams, Earnest Ashton, P. H. Bates, Edward D. Boyer, Lewis R. Ferguson, William B. Fuller, Robert W. Lesley, Spencer B. Newbury, Henry H. Quimby, Henry J. Seaman, Henry S. Spackman, Charles M. Spofford, Stone & Webster Engineering Corporation, George F. Swain, Arthur N. Talbot, George S. Webster, Rudolph J. Wig, Joseph R. Worcester.

Special acknowledgment is due to Mr. Edward Smulski for his most valuable assistance and collaboration in the preparation of Chapters XX, XXI and XXII on Reinforced Concrete; to Mr. Royall D. Bradbury for assistance in connection with Chapter XXV on Beam Bridges; and to Mr. Harold M. Davis for thorough work of analysis of material and in preparing the book for publication.

Acknowledgment for cuts is made to Aberthaw Construction Co., Ambursen Co., American Concrete Institute, American Luxfer-Prism Company, Atlas Portland Cement Co., Austin Manufacturing Co., Burchartz Fireproofing Co., Inc., Condron Co., Dayton Malleable Iron Co., Engineering Record, Fairbanks Co., Farrell Foundry & Machine Co., Howard & Morse, Lakewood Engineering Co., Special Committees on Cement Testing, Tinius Olsen Co., Ohio State Highway Department, Ransome Concrete Machinery Co., Raymond Concrete Pile Co., Simpson Bros. Corp., Sterling Wheelbarrow Co., Stone & Webster Engineering Corporation, G. L. Stuebner Iron Works, Trussed Concrete Steel Co., U. S. Bureau of Standards, University of Illinois.

Since the issue of the Second Edition a great loss has been suffered in the death of the senior author, Dr. Frederick W. Taylor, whose counsel and definite requirements were so instrumental in making the book an accepted authority. In the present revision an endeavor has been made to follow the principles laid down by our beloved colleague.

SANFORD E. THOMPSON.

Boston, November, 1916.

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A Treatise on Concrete

CHAPTER I

ESSENTIAL ELEMENTS IN CONCRETE CONSTRUCTION

The forming of concrete structures is essentially a manufacturing operation, and requires more close attention to detail both in the design and the building than most other classes of construction. For the benefit of those who are not thoroughly experienced, a number of the most essential elements are recorded below with references to pages upon which more detailed information may be obtained.

An outline of the process of concreting, in elementary form, is given in Chapter II, page 11.

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Large load of broken stone or gravel for iron wheelbarrow on short haul in concrete work......................	3.0 cu. ft.
Large load of sand for iron wheelbarrow on short haul in concrete work................................	3.5 " "
Average load of ordinary concrete* for iron wheelbarrow..	1.9 " "
Large " " " " " " " ...	2.2 " "
Number of shovelfuls of concrete per barrow in average load	13
" " " " " " " " large "	15
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Average quantity concrete* mixed, wheeled 50 ft., and rammed, per man, per day of 10 hours†.............	2.2 cu. yd.
Large quantity concrete* mixed, wheeled 50 ft. and rammed, per man, per day of 10 hours†......................	3 " "

* All measurements of concrete are reduced to terms of quantity in place after ramming.

† Note that the leveling and ramming, but not the labor on forms, are included in this item.

Average quantity concrete* laid as above with a gang of 15 men per day of 10 hours†	33	cu. yd.
Large quantity concrete* laid as above with a gang of 15 men per day of 10 hours†	47	" "
Approximate average quantity of concrete* leveled and rammed in 6-inch layers, per man, per day of 10 hours	11	" "
Approximate large quantity of concrete* leveled and rammed in 6-inch layers, per man, per day of 10 hours	16	" "
Approximate average surface of rough braced plank form built and removed by one carpenter per day of 10 hours	25	sq. "

LABOR COSTS OF HAND MIXING.‡

Values apply to 1:2:4, 1:2½:5, and 1:3:6 proportions, and approximately to other ordinary proportions. Wages of labor 20 cents per hour with allowance included for foremen, superintendence, miscellaneous job expenses, and small tools. Liability insurance, home office expense, and profit are not included.

		Labor Cost per Cu. Yd. of Concrete. Wages @ 20c. per Hour.
Mixing and placing:§		
Sand and cement spread dry on stone		$1.05
Stone dumped on sand and cement		1.03
Sand-cement mortar spread on stone		1.08
Add to base cost per cubic yard:		
For wheeling sand each additional 50 feet	$0.01	
" " stone " " "	0.02	
" loading and hauling sand and gravel 100 feet	0.41	
Add for each 100 feet, up to one mile 0.01		
" loading and hauling sand and gravel one or more miles, per mile	1.03	
" screening gravel to separate sand	0.36	
" carrying concrete on shovels 14 feet	0.16	
" wheeling " 100 feet	0.16	
Add for each 100 feet 0.07		
" wheeling very wet concrete 100 feet	0.22	
Add for each 100 feet 0.10		
" hauling concrete in single carts 100 feet	0.18	
Add for each 100 feet 0.03		

*All measurements of concrete are reduced to terms of quantity in place after ramming.

†Note that the leveling and ramming, but not the labor on forms are included in this item.

‡Summarized from Concrete Costs, Table 56, page 318.

§ Includes measuring aggregates (and wheeling 25 feet if measured in barrows), wetting stone, getting and emptying cement, wetting and mixing, shoveling to place or to barrows or buckets, leveling and tamping and miscellaneous work. Number of turns: dry sand and cement, 3 turns; sand-cement mortar, 2 turns dry and 2 turns wet; concrete 3 turns.

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Natural Cement " " " 94 " " "	282 "	82
Empty Cement Barrel weighs from 15 to 30 lb. average..	20 "	
Portland Cement, weight per cubic foot,		
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DEFINITIONS

Aggregate is the inert material, such as sand, broken stone, etc., with which the cement or other adhesive material is mixed to form concrete or mortar. The term is sometimes erroneously applied to the coarse material, such as broken stone, only.

Beton is the French word for concrete.

Beton-Coignet is a mixture of hydraulic lime, cement, and sand... 45

Concrete† is an artificial stone made by mixing cement, or some similar material—which after mixing with water will set or

* Loose unrammed concrete is 5% to 25% lighter than in place, according to the consistency.

† Also applied to mixtures of an aggregate with a material such as asphalt—which liquifies on application of heat.

PAGE

harden so as to adhere to inert material,—and an aggregate composed of hard, inert particles of varying size, such as a combination of sand or broken stone screenings, with gravel, broken stone, cinders, broken brick, or other coarse material.

Concrete Rubble is masonry of large stones, usually of derrick size, with joints of concrete instead of mortar................ 213, 769

Density represents the ratio of the sum of the volumes or mass of the particles, or absolutely solid substance, of a material contained in a measured unit volume to the total measured unit volume.. 148

Mortar is a mixture of cement or lime and sand or other fine aggregate having water added so as to make it like a paste

Natural Cement is made from natural rock containing the required constituents in approximately uniform proportions........... 43

Paste is a mixture of neat, *i.e.*, pure, cement or lime with water.

Portland Cement is made from an artificial mixture of materials containing lime and clay................................ 41

Reinforced Concrete* is concrete in which steel is imbedded to increase its strength...................................... 349

Rubble Concrete is concrete in which large stones are placed.. 213, 769

Sand Cement or **Silica Cement** is a mechanical mixture of Portland cement and fine sand................................. 41

*A more complete definition of reinforced concrete is given on page 349.

CHAPTER II

ELEMENTARY OUTLINE OF THE PROCESS OF CONCRETING

This chapter is not written for experienced civil engineers and contractors, nor for those who desire to make a scientific study of methods and principles. On the contrary, it is merely an elementary outline, indicating to the inexperienced the various steps which must be taken with this class of masonry. In subsequent chapters the various divisions of the subject are treated in detail.

The question as to whether concrete is preferable to some other form of masonry may often resolve itself into a question of cost. The cost, in turn, is dependent upon the character of the structure, the rate of labor and the price of the various materials entering into the work. Portland cement concrete has been laid in large masses at as low a price as $3 per cubic yard, while for thin walls built under disadvantageous conditions the cost of constructing molds may cause it to run as high as $30 per cubic yard, and in the case of ornamental work even above this. Before estimating the cost in any case, the materials must be chosen and the relative proportions of the ingredients determined from a consideration of the design of the structure.

WHERE CONCRETE MAY BE USED

By far the largest proportion of Portland cement concrete is laid in heavy foundation work and in other structures, such as tunnels and subways, below the surface of the ground. It is peculiarly adapted for foundations of engines or machinery, heavy walls, piers, etc. In the former the concrete is often carried all the way up to the base of the engine or machine, instead of being topped with brick or stone. It is widely used for sidewalks or floors upon the ground level, and for suspended floors. When suitably reinforced with steel, it furnishes probably the most economical and effective material for fire-proof construction. Its use for walls of buildings is largely increasing, but on account of the very indefinite time required in the building and moving of forms the cost may largely exceed the original estimate unless the builder is experienced in this class of work. Under favorable conditions, however, a 6-inch wall of concrete will cost no more, and usually less, than a 12-inch wall of brick work, and will be

stronger, more durable, and fire-proof. The strength of concrete columns and beams is readily calculated by means of formulas.

Concrete is destined to be used to a large extent in the construction of tanks and vats for holding various liquids which attack wood and iron. Their construction is comparatively simple, but the work must be carefully performed if the result is to be permanent and satisfactory. Concrete is especially suitable for all kinds of arches, because the stresses therein are chiefly compressive. Other classes of work for which concrete is largely employed are dams, retaining walls, penstocks, bridges, abutments, sewer and water conduits, and reservoirs. For ornamental work developments are constantly being made, and it is noteworthy that concrete or mortar can be cast in molds in a somewhat similar manner to that in which plaster of Paris is run for interior decoration.

SELECTION OF MATERIALS

Concrete is ordinarily composed of cement, sand, gravel or crushed stone, or both, and water. The selection of each of these materials is largely dependent upon local conditions, and no unalterable rule can be laid down in regard to it, but certain general conditions may serve as a guide to the inexperienced.

Cement. It is a wise rule to use Portland cement for nearly all classes of concrete, and the remarks in this chapter are based entirely upon this material. Portland cement is more uniform and therefore more reliable, while its strength is so much higher than Natural cement that by mixing it with larger proportions of sand and stone, properly graded, it will usually yield better results at less cost than Natural cement.

If the job is small and unimportant, it is generally safe to select in the market a brand of Portland cement of American manufacture which has a first-class reputation, and to use it without testing. As a precaution, however, it is usually advisable that samples from a few of the packages of every shipment be tested for soundness. This can be done after a little practice with scarcely any apparatus. (See p. 72.) For very important concrete construction complete specifications should be prepared before purchasing the cement, and a small laboratory established for conducting tests to determine whether it is fulfilling the requirements. (See p. 61.)

Aggregate. The sand and broken stone or gravel are termed the aggregate. The sand should be clean. One may obtain some idea of its cleanliness by placing it in the palm of one hand and rubbing it with the fingers of the other. If the sand is dirty, it will badly discolor the palm.

Unless from a bank of known quality, a sand should be tested for tensile strength of mortar (see p. 115) before using. Preference should be given to sand containing a mixture of coarse and fine grains. Extremely fine sand even if clean makes a weak mortar and should never be used unless with a large excess of cement.

Either crushed stone or clean gravel, or both, is suitable for the coarse aggregate. It is chiefly a question of cost. If the gravel is chosen, greater uniformity is attained by screening it over, say, a $\frac{3}{8}$-inch mesh screen, and then remixing the sand which falls through the screen with the coarser gravel in definite proportions, than by taking the run of the bank. If the gravel is dirty or clayey it should be washed with a hose, a little at a time, before it is shoveled on to the mixing platform.

Broken stone, if selected, may be used unscreened as it comes from the crusher, although it is preferable to screen out the dust and to use the latter as a portion of the sand. The maximum size is usually limited to $2\frac{1}{2}$ inches for heavy construction. A smaller size than this, say one inch, will give, with less care, a finer surface and should be used for reinforced concrete. In a thick wall large sound stones may be placed by hand or derrick without detriment to the work, providing the consistency of the concrete is thin enough to properly imbed them.

PROPORTIONS

Accurate methods of proportioning the cement and aggregate in concrete are discussed in chapter X, page 175, and if a large or very important mass is under consideration, or if the work must be water-tight, the correct proportioning requires more careful consideration than can be given it in this chapter. The method often adopted of pouring water into the coarser material to determine the percentage of voids, and thus finding the quantity of sand to use for filling them, is apt to be misleading, because so much depends upon the compactness of the stone, due to the method of handling it — that is, whether placed quietly, dropped, thrown, or shaken down — and because in the majority of cases the sand contains many grains so large that they will not enter the smaller voids of the coarser material. In a small job it is sufficiently accurate to select the proportion of cement to sand which will give the required strength to the concrete, and then use twice as much gravel or broken stone as sand. In figuring the capacities of the measures for the sand and stone it must be remembered that a barrel of Portland cement weighs 376 pounds, not including the barrel, and a bag of Portland cement 94 pounds, and we may assume for convenience

that a cement barrel holds 4.0 cubic feet. This is a fair average measurement of a heaped barrel, or a barrel with both heads removed—a convenient measure for sand.

As a rough guide to the selection of materials for various classes of work, we may take four proportions which differ from each other simply in the relative quantity of cement:

(a) **A Rich Mixture** for columns and other structural parts subjected to high stresses or requiring exceptional water-tightness: Proportions 1: 1½: 3; that is, one barrel (4 bags) packed Portland cement to 1½ barrels (6 cubic feet) loose sand to 3 barrels (12 cubic feet) loose gravel or broken stone.

(b) **A Standard Mixture** for reinforced floors, beams and columns, for arches, for reinforced engine or machine foundations subject to vibrations, for tanks, sewers, conduits, and other water-tight work: Proportions 1: 2: 4; that is, one barrel (4 bags) packed Portland cement to 2 bbl. (8 cu. ft.) loose sand to 4 barrels (16 cu. ft.) loose gravel or broken stone.

(c) **A Medium Mixture** for ordinary machine foundations, retaining walls, abutments, piers, thin foundation walls, building walls, ordinary floors, sidewalks, and sewers with heavy walls: Proportions 1: 2½: 5; that is one barrel (4 bags) packed Portland cement to 2½ barrels (10 cu. ft.) loose sand to 5 barrels (20 cu. ft.) loose gravel or broken stone.

(d) **A Lean Mixture** for unimportant work in masses, for heavy walls, for large foundations supporting a stationary load, and for backing for stone masonry: Proportions 1: 3: 6; that is, one barrel (4 bags) packed Portland cement to 3 barrels (12 cu. ft.) loose sand to 6 barrels (24 cu. ft.) loose gravel or broken stone.

The above specifications are based upon fair average practice. If the aggregate is carefully graded and the proportions are scientifically fixed, smaller proportions of cement may be used for each class of work.

QUANTITIES OF MATERIAL

Inexperienced contractors have often lost money by assuming that the quantity of gravel plus the quantity of sand required will be equivalent to the volume of the finished concrete — that is, that 7½ cubic yards of concrete in the proportions of 1: 2½: 5 will require 2½ cubic yards of sand and 5 cubic yards of gravel. This is absolutely wrong, since the grains of sand fill, to a certain extent, the spaces between the larger pebbles. It is incorrect, on the other hand, to figure a quantity of gravel equal to the total

volume of the concrete, because the introduction of the mortar, which is always in excess of the actual voids, swells the bulk.

If gravel or stone having particles of uniform size is used it must be recognized that the work will cost from 5 to 10 per cent. more, on account of the additional quantity of material required to make a given volume of concrete. In measuring the gravel or stone before mixing there will be less solid matter in a measure, and consequently more sand and cement will be necessary to fill the spaces between the stones. This fact, which is often overlooked even by experienced men, is illustrated in a somewhat exaggerated fashion in Figs. 1 and 2. Here Fig. 1 illustrates

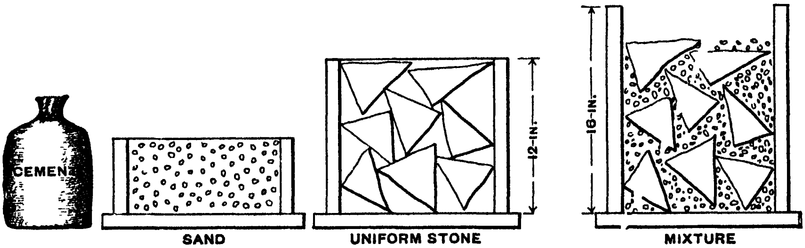


FIG. 1.—Diagram illustrating measurement of Dry Materials and the Mixture when Broken Stone is of uniform size. (*See p.* 15.)

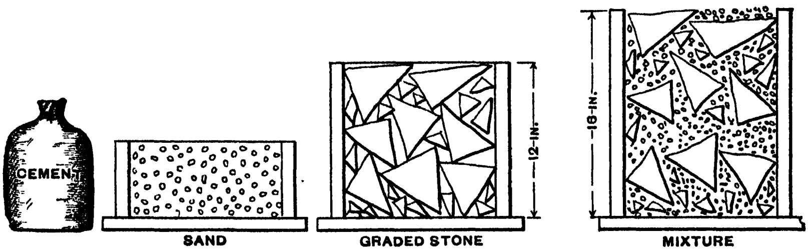


FIG. 2.—Dry Materials and Mixture when the Stone is of varying sizes. (*See p.* 15.)

the measurement of the dry materials and the mixture produced therefrom when the stone has been screened to one uniform size, while Fig. 2 shows the dry materials and the mixture when the stone is what is termed "crusher run" — that is, of varying sizes as it comes from the crusher.

It is obvious at a glance that the uniform stone measured in Fig. 1 contains less solid stone than the graded stone measured in Fig. 2. The spaces between the stones in the first case are very nearly equal to the volume of

the solid particles, and as the measure of the sand is one-half that of the stone, and the particles of cement fill the voids in the sand, this sand and cement mixes in between the stones, filling the spaces or voids, and resulting in a mixture but very slightly greater in volume than the stone alone. In the second case, Fig. 2, the spaces between the large stones in the stone measure are filled with graded smaller stones, so that there is a much smaller volume of spaces or voids. Hence, when the sand and cement, which are identical with that in Fig. 1, are mixed with it the volume of mixture becomes considerably larger than the original bulk of the stone. Consequently, if we start with definite proportions of materials, more concrete will be made with graded stone — such as "crusher run" broken stone, or gravel containing various sizes, ranging, say, from ¼ inch up to 2 inches — than if the stone has been screened to uniform size. If, on the other hand, the proportions of the materials are changed on account of the fewer voids in the mixed stone, and less sand and cement are used, a saving in these materials results.

Fuller's Rule for Quantities. The simplest rule for determining the quantities for a cubic yard of concrete is one devised by William B. Fuller. Expressed in words it is as follows:

To find the number of barrels of Portland cement per cubic yard of concrete, divide 10.5 by the sum of the parts of all the ingredients.

To find the cubic yards of sand per cubic yard of concrete, divide 1.55* by the sum of the parts of all the ingredients and multiply by the number of parts of sand.

To find the cubic yards of gravel per cubic yard of concrete, divide 1.55* by the sum of the parts of all the ingredients and multiply by the number of parts of gravel.

Let c = number of parts of cement.
s = number of parts of sand.
g = number of parts of gravel or broken stone.

Then

$\frac{10.5}{c+s+g}$ = number of barrels of Portland cement per cubic yard of concrete.

$\frac{1.55}{c+s+g}s$ = number of cubic yards of sand per cubic yard of concrete.

$\frac{1.55}{c+s+g}g$ = number of cubic yards of gravel per cubic yard of concrete.

* 1.55 is the result of multiplying 10.5 and 4, and dividing by 27, where 10.5 is empirical; 4 is the volume of a cement barrel in cubic feet, and 27 is the number of cubic feet in a cubic yard.

The following table, made up from Fuller's rule, represents fair averages of all classes of material. The first figure in each proportion represents the unit, one barrel (4 bags), of packed Portland cement (weighing 376 pounds), the second figure, the number of barrels loose sand (4 cubic feet each) per barrel of cement, and the third figure, the number of barrels loose gravel or stone (4 cubic feet each) per barrel of cement:

Materials for One Cubic Yard of Concrete

Proportions.	Cement, Barrels.	Sand, Cubic Yards.	Gravel or Stone, Cubic yards.
1 : 2 : 4	1.50	0.45	0.89
1 : 2½ : 5	1.24	0.46	0.92
1 : 3 : 6	1.05	0.47	0.93
1 : 4 : 8	0.81	0.48	0.96

If the coarse material is broken stone screened to uniform size it will, as is stated above, contain less solid matter in a given volume than an average stone, and about 5 per cent. must be added to the quantities of *all* the materials. If the coarse material contains a large variety of sizes so as to be quite dense, about 5 per cent. may be deducted from all of the quantities.

Example.—What materials will be required for six machine foundations, each 5 feet square at the bottom, 4 feet square at the top, and 8 feet high?

Answer.—Each pier contains 163 cubic feet, and the six piers therefore contain $\frac{6 \times 163}{27} = 36.2$ cubic yards. If we select proportions 1 : 2½ : 5, we find, multiplying the total volume by the quantities given in the table, that there will be required, in round numbers, 45 barrels packed cement, 16.7 cubic yards loose sand, 33.3 cubic yards loose gravel.

TOOLS AND APPARATUS REQUIRED FOR CONCRETE WORK

The quantity of tools will, of course, vary with the size of the gang. The following schedule is based upon a small gang of eight or ten men, making concrete by hand:

Eight square pointed shovels, size No. 3, and such as illustrated in Fig. 3, page 18. (If a very wet mixture is used substitute small coal scoops.)

Three iron wheelbarrows, Fig. 4, page 18.

Two rammers, Figs. 75 and 76, page 258.

One mixing platform, about 15 feet square, built so substantially that it can be moved without coming to pieces, and having a 2 by 3-inch strip around the edge to prevent waste of materials and water. On a small job this may be of 1-inch stuff, resting on joists about 3 feet apart, provided it is stiffened by being tongued and grooved.

Fig. 3.—Square Pointed Shovel. (*See p.* 17.)

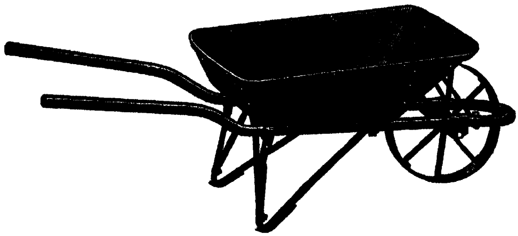


Fig. 4.—Concrete Wheelbarrow. (*See p.* 17.)

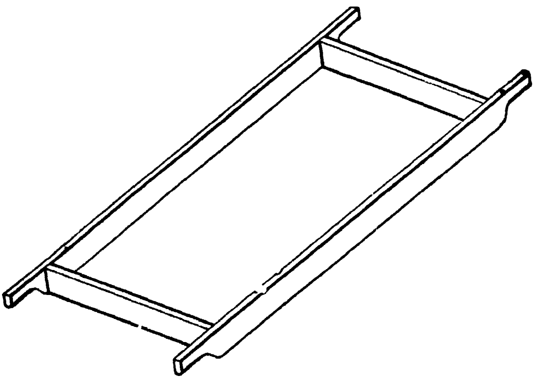


Fig. 5.—Measuring Box for Gravel. (*See p.* 18.)

One measuring box or barrel for sand, of a capacity for one batch of concrete. A convenient measure is a cement barrel, either whole or sawed in two, with both heads removed. It is filled and then lifted in such a manner as to spread the sand.

One measuring box for gravel (see Fig. 5) of a capacity for one batch of concrete.

Lumber for making and bracing forms.
Nails and, for some kinds of work, bolts, for forms.

CONSTRUCTION OF FORMS

Green spruce or fir lumber is suitable for forms. If a smooth face is required the surface of the boards or plank next to the concrete must be dressed and the edges tongued and grooved or beveled. The forms must be nearly water-tight. The sheeting, which is usually laid horizontal, may be 1 inch, 1½ inch or 2 inches thick, the distance apart of the studding being governed by the thickness selected. The studs must be placed not more than 2 feet apart for 1-inch sheeting nor more than 5 feet apart for 2-inch sheeting. They must be securely braced so as to withstand the pressure of the soft concrete and of the puddling or ramming.

The lower portion of a foundation wall in a trench excavated in earth so stiff as to stand nearly vertical may sometimes be laid with no form at all, and then narrowed in at the top to the required thickness, but if the sides of the trench are sloping it is generally cheaper to save concrete material by carrying the forms to the bottom. A thin wall may be

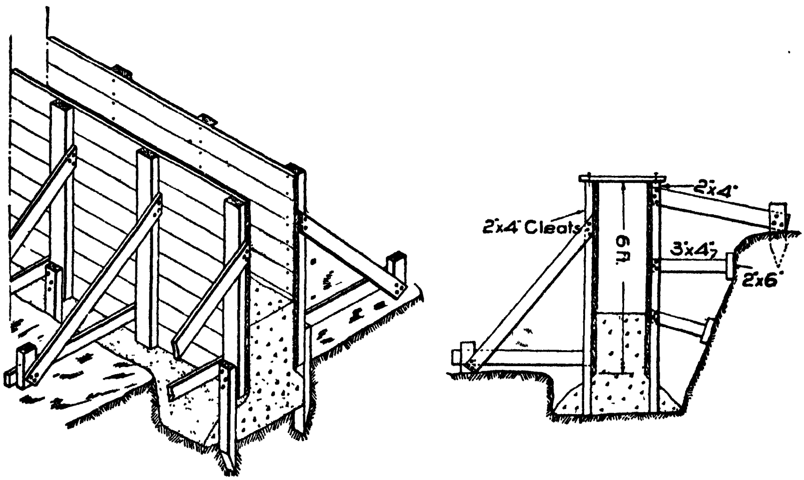


FIG. 6.—Forms for Foundation Walls. (*See p.* 19.)

greatly strengthened by spreading the base, which is readily accomplished by starting the boards or plank 6 or 8 inches above the bottom of the excavation and allowing the soft concrete to flow out under them on both sides of the wall so as to make footings, as shown in Fig. 6. If some of the studs themselves are extended to run down into the ground they should be tapered and greased so that they may be withdrawn without injury to the concrete.

For all walls under 9 or 10 inches in thickness, small steel rods $\frac{1}{4}$ or $\frac{3}{8}$ inch in diameter, spaced about 12 inches apart, will greatly increase the stiffness and add to the safety of the structure, especially while the concrete is hardening.

Forms must be left in place for three or four weeks if there is earth or water pressure against the wall. If, on the other hand, there is no strain upon it, 24 hours setting, or until the concrete will stand the pressure of the thumb without indentation, is sufficient.

Further descriptions of form construction and methods of design are given in Chapter XXIII, p. 646. Forms for special structures are described and illustrated in subsequent chapters treating of concrete design.

MIXING AND LAYING CONCRETE

The advisability of employing machinery for mixing the concrete depends chiefly upon the quantity to be laid. On a small job the first cost of mixing machinery and the running expenses, such as the labor of the engineman, which continue when the machine is idle, may bring the cost of machine-mixed concrete higher than hand-mixed. The decision may be based entirely upon the cost per cubic yard of concrete laid, provided a first-class machine is employed, since good concrete can be made either by machine or by hand. The various types of concrete mixers and the methods of employing them are discussed in Chapter XIII, p. 231.

The foreman for a gang of concrete mixers need not be of great intelligence, but must be one who will obey orders strictly, and know how to keep all of his men constantly busy. The amount of work turned out will depend to quite an extent on the arrangement of the gang, whether each man has certain definite operations to perform over and over again, and whether these operations fit into the work of the rest of the gang so that none of the men have idle moments.

A gang of at least 6 men besides the foreman is required even on small work, while as many as 23 men may be effectively employed. In addition to these, an inspector is generally necessary to watch the placing of the

concrete and see that the mixture is uniform and of proper consistency. Cheap laborers, as for instance Italians, make good men for mixing and transporting the concrete.

The materials for the concrete ought, of course, to be deposited as near the work as possible. The cement, whether it comes in bags or barrels, must be sheltered from the rain. Covering with plank is insufficient. Bags should be protected from moist atmosphere; a cellar is likely to be too damp. To keep the sand and stone as near the mixing platform as possible, it may be advantageous to haul the materials as they are required from day to day. If the sand or stone pile is at any time farther from the measuring boxes than a man can profitably throw with shovels without walking, say more than 8 or 10 feet, do not hesitate to have it loaded into wheelbarrows and dumped into the measuring boxes. Materials can be wheeled in barrows to a distance of 10 to 25 feet from the platform at about the same cost that they can be shoveled direct with a long throw.

There are many methods of mixing concrete by hand, as discussed in Chapter XIII, all of which with care produce good work. For the convenience of the inexperienced the following directions for the work of a small gang of six men with foremen may be useful. They are given merely for illustration, and must be more or less varied to suit local circumstances.

Directions for Mixing Concrete. Assume a gang of four men to wheel and mix the concrete, with two other men to look after the placing and ramming.

When starting a batch, two mixers shovel or wheel sand into the measuring box or barrel — which should have no bottom or top — level it and lift off the measure, leveling the sand still further if necessary. They then empty the cement on top of the sand, level it to a layer of even thickness, and turn the dry sand and cement with shovels three times, as described below, after which the mixture should be of uniform color.

While these two men are mixing sand and cement, the other two fill the gravel measure about half full, then the two sand men take hold with them, and complete filling it. The gravel measure is lifted, the gravel hollowed out slightly in the center, and the mixture of sand and cement shoveled on top in a layer of nearly even thickness.* A definite number of pails are filled with water, and poured directly on the top of these layers, greater uniformity being thus attained than by adding the water directly from a hose. After soaking in slightly the mass is ready for turning.

* Some engineers prefer to spread the stone on top of the sand and cement, while others prefer to mix the water with the sand and cement before adding them to the stone.

The method illustrated in Fig. 7 of turning with shovels materials which have already been spread in layers is as follows:

Two men, *a* and *b*, with square pointed shovels, stand facing each other at one end of the pile to be turned, one working right-handed and the other left-handed. Each man pushes his shovel along the platform under the pile, lifts the shovelful, turns with it, and then, turning the shovel completely over, and with a spreading motion drawing the shovel toward himself, deposits the material about 2 feet from its original position. Repetitions of this operation will form a flat ridge of the material, on a line with the pile as it originally lay, and flat enough so that the stones will not roll. As soon as, but not before, a single ridge is complete, two other men, *c* and *d*, should start upon this ridge, turning the materials for the second time, as shown in the illustration, and forming as before a flat ridge and finally a level pile which gradually replaces the last. A third mixing is accomplished in a similar way.

Fig. 7 gives the position of the piles as the concrete is being turned.

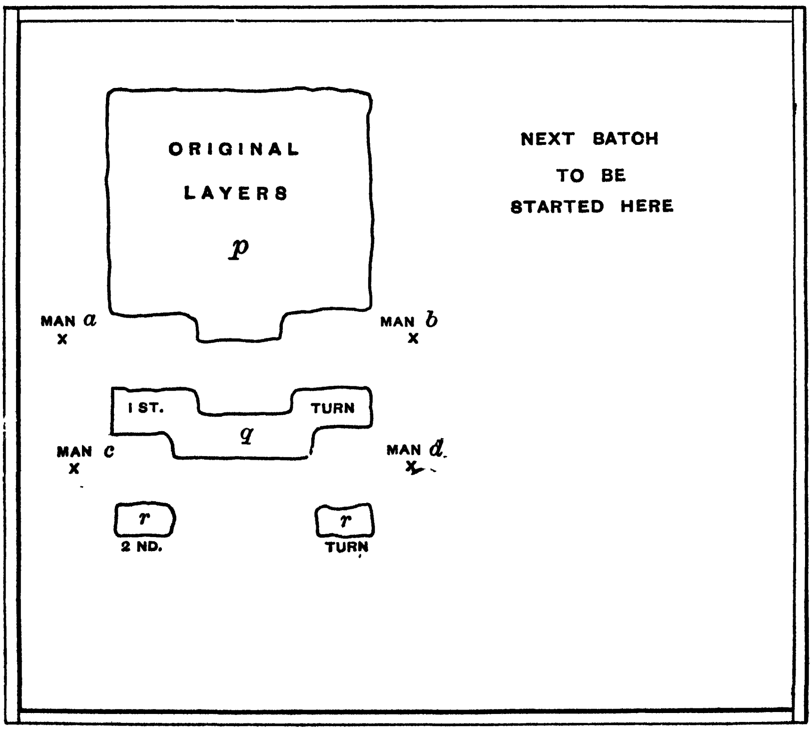


FIG. 7.—Position of Men and Concrete on Platform while Turning. (*See p.* 22.)

A portion of the original layers is shown at *p*, the ridge formed by men *a* and *b* shoveling from pile *p* is shown at *q*, and the beginning of the ridge formed by men *c* and *d* is shown at *rr*. The third turning is not shown.

The quantity of water used must be varied according to the moisture in the materials and the consistency required in the concrete. While the opinions of engineers regarding the proper consistency vary widely, it is advisable, the authors believe, for an inexperienced gang to use an excess of water. The rule may be made in hand mixing to use as much water as can be thoroughly incorporated with the materials. Concrete thus made will be so soft or "mushy" that it will fall off the shovel unless handled quickly.

After the material has been turned twice, as described, and as soon as the third turning has been commenced, two of the mixers who have finished turning may load the concrete into barrows and wheel to place. They should fill their own barrows, and after the mass has been completely turned for the third time by the other two men the latter should start filling the gravel measure for the next batch.

If the concrete is not wheeled over 50 feet, four experienced men ought to mix and wheel on the average about 10½ batches in ten hours. This figure is based on proportions 1 : 2½ : 5, and assumes that a batch consists of one barrel (four bags) Portland cement with 10 cubic feet of sand and 20 cubic feet of gravel or stone.

Assuming, as given on page 17, that 1.24 barrels of cement are required for 1 cubic yard of concrete, one barrel of cement—that is, one batch—will make 0.81 cubic yard of concrete; hence 10½ batches mixed and wheeled by four men in ten hours are equivalent to 8¼ cubic yards of concrete. This is for the very simplest kind of concreting and makes no allowance for the labor of supplying materials to the mixing platform or for building forms.

Placing Concrete. The concrete may be transported and handled by any means which will not cause the materials to separate. If mixed wet it may be dropped directly from shovels or barrows to place, or it may be run down an inclined pipe or chute provided the stones are not allowed to separate from the mortar. For a dry or a jelly-like mixture common square ended rammers are employed and the mass must be rammed until the mortar flushes to the surface. Wet concrete must be merely puddled or "joggled" to expel the air and surplus water. Before placing a fresh layer upon work which has set, the surface must be cleaned of dirt and scum and thoroughly wet.

The placing of concrete and the kinds of rammers for different classes of work are discussed more at length in Chapter XIV.

APPROXIMATE COST OF CONCRETE

The cost of concrete depends more upon the character of the construction and the conditions which govern it than upon the first cost of the materials. In a very general way, we may say that when laid in large masses or in a very heavy wall, so that the construction of the forms is relatively a small item, the cost per cubic yard in place is likely to range from \$4 to \$9. The lower figure represents contract work under favorable conditions with low prices for materials, and the higher figure small jobs and inexperienced men. Similarly, we may say that for sewers and arches, where centering is required, the price may range from \$7 to \$14 per cubic yard. Thin building walls under eight inches thick may cost from \$10 to \$20 per cubic yard, according to the character of construction and the finish which is given to the surface.

These ranges in price seem enormous for a material which is ordinarily supposed to be handled by unskilled labor, but it must be borne in mind that skilled workmen are required for constructing forms and centers, and often the labor upon these may be several times that of mixing and placing the concrete. As a rule, unless the job is a very small one or under the personal supervision of a competent engineer, it is cheaper and more satisfactory to employ an experienced contractor than day labor. Green men under an inexperienced foreman may not be counted upon to mix and lay over one-half the amount of concrete that will be handled by a skilled gang under expert superintendence.

A close estimate of cost may be reached, in cases where the conditions are known in advance, by taking up in detail and then combining the various units of the material and labor as outlined below.

Cost of Cement. As the price of Portland cement varies largely with the demand, it is necessary to obtain quotations from dealers for every purchase. It is such heavy stuff that the freight usually enters largely into the cost, and quotations should therefore be made f.o.b. the nearest point of delivery to the work. The cost of hauling by wagon may be readily estimated by assuming that a barrel of cement weighs 400 pounds (gross), and that a pair of horses will haul over an average country road a load of, say, 5 000 pounds, traveling in all a distance of 20 to 25 miles in a day, that is, 10 to 12½ miles with load. This assumes, of course, that the teams are good and properly handled.

Having found the cost of the cement per barrel, delivered, the approximate cost per cubic yard is at once obtained from the table on page 17. If, for example, the cost is \$2 per barrel and proportions 1 : 2½ : 5 are selected, the cost of the cement per cubic yard of concrete will be 1.24 × \$2.00 = \$2.48.

Cost of Sand. The cost of sand depends chiefly upon the distance hauled. With labor at 20 cents per hour, the cost of loading (including the cost of the cart waiting at pit) may be estimated, if handled in large quantities, at 24 cents per cubic yard, or on a small job at 36 cents per cubic yard. For hauling add 1⅓ cents for each 100 feet of distance from the pit. The additional cost of screening, if required, will vary with the coarseness of the material, but 20 cents per cubic yard may be called an average price for this, unless the sand is obtained by screening the gravel, when no allowance need be made. After finding the cost of one cubic yard of sand, the cost of the sand per cubic yard of concrete is readily figured from the table referred to. If, for example, the cost of sand screened, loaded and hauled 1 000 feet is 52 cents per cubic yard, the cost per cubic yard of concrete for proportions 1 : 2½ : 5 will be 0.46 × \$0.52 = \$0.24.

Cost of Gravel or Broken Stone. If broken stone is used upon a small job for the coarse aggregate, it is usually purchased by the ton or cubic yard. A 2000-pound ton of broken stone may be considered as averaging approximately 0.9 cubic yards, although differences in specific gravity cause considerable variation. A two-horse load is generally considered 1½ to 2 yards, the latter quantity requiring very high sideboards. The cost of screening gravel, if this is necessary, while a very variable item, may be estimated at 47 cents per cubic yard. The cost of loading gravel into double carts, with labor at 20 cents per hour may be estimated on a small job at 50 cents per cubic yard. If handled in large quantities, 33 cents is an average cost. The cost of loading includes loosening and also the cost of the cart waiting at the pit. Hauling costs about 1⅓ cents per cubic yard additional for each 100 feet of distance hauled under load. To illustrate, if the cost of gravel picked, screened, loaded and hauled 1 000 feet is 83 cents per cubic yard, the cost of the gravel per cubic yard of concrete for proportions 1 : 2½ : 5 will be 0.92 × \$0.83 = \$0.76½.

For distances up to 300 feet both sand and gravel can be hauled more economically by wheelbarrows than by teams. The cost of loading wheelbarrows is about half the cost of loading carts, while the cost of hauling with barrows per 100 feet is about four times greater.

Cost of Labor. With an experienced gang working at the rate of 20 cents per hour, the cost of mixing and laying concrete, if shoveled directly to place from the mixing platform, will average about $1.10 per cubic yard, in addition to the work on forms. If, as is usually the case, the concrete is wheeled in barrows, 12 cents per cubic yard must be added to the above price for the first 25 feet that the barrows are wheeled under load, and $1\frac{2}{3}$ cents for each additional 25 feet wheeled. With other rates of wages, the cost may be considered as proportional. With a green gang, the cost will be nearly double the above figures, but as the men become worked in and the organization perfected, the cost should approximate more nearly the prices given.

In building construction where the material is mixed by machinery and hoisted to place, there are numerous incidental expenses and delays, so that it is not safe to figure the cost of labor for simply mixing and laying the concrete under ordinarily good conditions at less than $1.50 to $2.00 per cubic yard. The cost of materials must be added to this, so that the cost of the concrete itself laid in place but *not* including forms nor reinforcement is apt to be about $8.00 per cubic yard. Approximate costs per cubic foot of finished concrete are given in Chapter XXIII.

Cost of Forms. The labor on forms is not included in the above. This is an extremely variable item. The cost of rough plank forms, including labor and lumber for both sides of a 3-foot wall, may be as low as 50 cents per cubic yard of concrete, with other thicknesses of wall in inverse proportion. On elaborate work the price, which is really dependent upon the face area, will reach several dollars per cubic yard of concrete, the cost of the form work, in fact, usually exceeding the cost of the concrete. In building construction, such as a factory six stories in height of symmetrical design, the cost of materials and labor on forms may be estimated at from 8 to 12 cents per square foot of surface of forms. If forms are to be used only once, or if conditions are disadvantageous, these values may be doubled. The costs vary with the price of lumber, the design of the structure, the design of the forms, the character of the supervision, and the skill of the workmen.

Cost of Steel. The cost of bending and placing steel for reinforced concrete is apt to vary from $\frac{1}{2}$ to $1\frac{1}{2}$ cents per pound. If, therefore, the cost of the steel is about $40.00 per ton or 2 cents per pound, the cost in place may be estimated at 3 cents per pound.

Detail Construction Costs. For complete treatment of costs and time of performing the various operations of concreting, reference should

be made to "Concrete Costs"* by Taylor and Thompson. In this book are given tables for finding times and estimating costs for excavating and crushing stone; handling and transporting materials; mixing concrete by hand; mixing concrete by machine; bending and placing steel reinforcement; making and erecting forms; and for the unit operations involved in concrete construction.

THE STRENGTH OF CONCRETE

The strength of concrete varies (1) with the quality of the materials; (2) with the quantity of cement contained in a cubic yard of the concrete; and (3) with the density of the mixture.

We may say that the strongest and most economical mixture consists of an aggregate comprising a large variety of sizes of particles, so graded that they fit into each other with the smallest possible volume of spaces or voids, and enough cement to slightly more than fill all of these spaces or voids between the solids of the aggregate.

On important construction the various materials to be used should be carefully tested, and specimens of the mixture selected made up in advance and subjected to test. As a guide to the loads which concrete will stand in compression—that is, under vertical loading where the height of the column or mass if not reinforced is not over 6 times the least horizontal dimension, or if properly reinforced is not over 15 times the least horizontal dimension, the following strengths are given.

The figures, compared with the results of recent experiments on long columns, allow with first-class construction a factor of safety of at least four at the age of one month, or about five and one-half at the age of six months, and are based on conservative practice.

Safe Strength of Portland Cement Concrete in Direct Compression.

Proportions.	Pounds per square inch.	Tons per square foot.
1 : 1½ : 3	500	36
1 : 2 : 4	450	32
1 : 2½ : 5	400	29
1 : 3 : 6	360	26
1 : 4 : 8	290	21

With a vibrating or pounding load, take one-half these values.

The tensile strength of concrete in beams is only about one-sixth the compressive strength. For this reason it is not safe to use concrete for beams unless reinforced with steel.

* John Wiley & Sons, Inc., Publishers.

CHAPTER III

SPECIFICATIONS FOR REINFORCED CONCRETE

The following reinforced concrete specifications are based in large measure upon the recommendation of the Joint Committee on Concrete and Reinforced Concrete, enlarged and extended where necessary, and are presented in such form that they may be used direct, making only such minor changes as are necessary for local conditions.

Specifications for Portland and for Natural cements are given on pages 62 and 82.

If sand, stone or other aggregates, are purchased on separate contracts, the proper paragraphs from the concrete specifications may be taken and enlarged upon as needed by the requirements of the job.

The requirements for steel reinforcement, based on the standard specifications of the American Society for Testing Materials are given in Chapter XXII.

SPECIFICATIONS FOR REINFORCED CONCRETE

Specifications essentially as prepared by Sanford E. Thompson, Consulting Engineer, for Massachusetts Institute of Technology and used in the design and construction of their new buildings by the Stone & Webster Engineering Corporation.

MATERIALS AND TESTS

1. CEMENT. The cement shall be a first-class Portland cement of a standard brand which has been manufactured continuously during a period of at least five years previous to its purchase. It shall be packed in strong cloth or canvas sacks and shall be free from lumps. The cement shall be stored in a building which will protect it from the weather, and the floor of which is at least 6 inches above the ground. It shall be stored so as to permit of easy access for inspection and identification of each shipment. A sufficient quantity shall be kept on hand to allow at least 8 days for inspection and necessary tests.

2. The cement shall conform to the requirements of the Standard Specifications of the American Society for Testing Materials.

3. The cement shall be tested at the mill and a partial test shall also be made after its receipt at the job. Its acceptance shall be subject to the approval of the Consulting Engineer.

4. FINE AGGREGATE. Fine aggregate shall consist of sand, crushed stone or gravel screenings, graded from fine to coarse and passing when dry a screen having ¼-inch diameter holes. It preferably shall be a silicious material and not more than 30

per cent. by weight should pass a sieve having 50 meshes per lineal inch. It shall be clean and free from soft particles, from lumps of clay, vegetable loam, and all organic matter.

5. Fine aggregates shall be of such quality that mortar composed of one part Portland cement and three parts fine aggregate by weight when made into briquets, or into prisms or cylinders, will show a tensile or compressive strength at an age not less than seven days at least equal* to the strength of 1 : 3 mortar of the same consistency made with the same cement and standard Ottawa sand. If the aggregate be of poorer quality, the proportion of cement shall be increased in the mortar to secure the desired strength. If the strength developed by the aggregate in the 1 : 3 mortar is less than 70 per cent. of the strength of the Ottawa sand mortar, the material shall be rejected. To avoid the removal of any coating on the grains, which may affect the strength, bank sands shall not be dried before being made into mortar, but shall contain natural moisture. The percentage of moisture may be determined upon a separate sample for correcting weight. From 10 to 40 per cent. more water may be required in mixing bank or artificial sands than for standard Ottawa sand to produce the same consistency.

6. Laboratory tests of fine aggregate in accordance with the above specifications shall be made so as to have the result at least of the 72-hour test before the fine aggregate is used on the work. At this age a strength 90 per cent. of the strength of standard sand mortars will be allowed. In case the sand at this age does not come up to the specifications, it shall not be used unless and until the 7-day test shows that it conforms to the specifications. Routine laboratory tests of the fine aggregates shall be made of each shipment received on the job.

7. Coarse Aggregate. The course aggregate shall consist of crushed stone, or of gravel, having particles coarser than ¼ inch in size and graded from the smallest to the largest particles. It shall be clean, hard, durable, and free from all deleterious matter. It shall not contain dust nor shall it contain soft, flat, or elongated particles.

8. The maximum size of the coarse aggregate to be used in heavy construction unreinforced, or reinforced with bars spaced not less than 4 inches apart on centers, shall be not greater than will pass a 2-inch ring. For thin reinforced walls and for the superstructure of the building, the maximum size of the coarse aggregate shall be not greater than will pass a 1¼-inch ring tested in a laboratory sieve.

9. Water. Water used in mixing concrete shall be free from oil, acid, alkalies, or organic matter.

10. Steel Reinforcement. The steel reinforcement shall be free from excessive rust, scales, or coatings of any character which tend to reduce or destroy the bond with the concrete.

11. Steel shall be of medium hardness rolled from new billets by the Bessemer or the open-hearth process. It may be plain round, or of a deformed shape approved by the Consulting Engineer. The steel shall conform to the requirements of the Standard Specifications for Billet-Steel Concrete Reinforcement Bars of the American Society for Testing Material. The steel for reinforcement shall be tested at the mill in accordance with said Specifications.

12. Structural Steel. Structural steel shall conform to the requirements of Specifications for Structural Steel.

* A slightly lower percentage of strength was required for Technology because tests of concrete made from the aggregates selected showed that this was permissible.

13. CONCRETE TESTS. Specimens of concrete composed of the cement, fine aggregate, and coarse aggregate, selected for the work shall be made in the form of cylinders 8 inches in diameter by 16 inches long, or of smaller size than this provided the proper correction is made for the relative strength due to a different shape or size, and tested for compression at the age of 28 days. For the reinforced concrete superstructure, the strength of the concrete based on said 8-inch diameter by 16-inch cylinders shall be not less than 2 000* pounds per square inch at the age of 28 days. In making these blocks, the consistency shall be such that a small specimen of concrete placed immediately after mixing in a tapering form, which is then inverted and removed, shall nearly but not quite keep its shape.

PROPORTIONS

14. The proportion of the raw material for the concrete shall be exactly determined from time to time by the Consulting Engineer in accordance with the relative coarseness of the aggregates so as to attain as dense a concrete as is practicable.

15. The proportions shall be such as shall give the following strength based on cylinders 8 inches in diameter by 16 inches long at the age of 28 days, with maximum limits as noted.

(a) Mass concrete to be subjected to stresses at least 20 per cent. less than the working stresses, given in paragraph on "Stresses," shall have an ultimate compressive strength not less than 1 600 pounds per square inch at the age of 28 days, stored under laboratory conditions in a moist atmosphere, with the further requirement that the proportions, unless specially graded, shall be not leaner than one bag of cement to $7\frac{1}{2}$ cubic feet of fine and coarse aggregates measured before mixing them.

(b) Reinforced concrete for superstructure of the buildings shall have a strength of 2 000 pounds per square inch at the age of 28 days with the further requirements that the proportions, unless specially graded, shall be not leaner than one bag of cement to 6 cubic feet of fine and coarse aggregates measured before mixing them.†

(c) Water-tight concrete shall have a strength of 2 400 pounds per square inch at the age of 28 days with the further requirement that the proportions, unless specially graded, shall be not leaner than one bag of cement to 5 cubic feet of fine and coarse aggregates measured before mixing them.

16. The relative proportions of fine to coarse aggregate in (b) shall be approximately one part of fine aggregate to two parts of coarse, but the exact relation shall be varied from time to time in accordance with tests and with the exmaination of the concrete as it is being mixed to see that the mortar is only slightly in excess of that required to fill the voids in the coarse aggregate.

17. In case the aggregates are specially graded so as to obtain an exceptionally dense mixture and the above requirements for strength are complied with, leaner mixtures than those specified may be used for each class of concrete, provided the amount of total aggregate is not increased more than 25 per cent. above the maximum proportions specified.

* See footnote page 29.

† Proportions 1:1½:3 were finally adopted for Technology superstructure because these proportions were found to be more economical and a strength of 2 400 pounds per square inch was required at the age of 28 days.

18. GRANOLITHIC FLOOR FINISH. The materials and proportions for granolithic floor finish shall be specified by the Consulting Engineer after investigations and tests.

MIXING AND PLACING CONCRETE

19. STORAGE AND MEASUREMENT OF AGGREGATES. Separate storage shall be provided for the cement and three kinds of aggregates. Methods of measurement of the proportions of the various ingredients shall be used which will secure separate and uniform measurements of each material including the water.

20. MACHINE MIXING. A machine mixer of a type which insures the uniform proportioning of the materials throughout the mass shall be used for mixing the concrete. The mixing shall continue for a minimum time of at least $\frac{3}{4}$ minute after all the ingredients are assembled in the mixer.

21. HAND MIXING. When it is necessary to mix by hand, the mixing shall be on a water-tight platform and special precautions shall be taken to mix the sand and cement separately and to turn all the ingredients together at least four times and until they are homogeneous in appearance and color.

22. CONSISTENCY. The materials shall be mixed wet enough to produce a concrete of such consistency as will flow sluggishly into the forms and about the metal reinforcement and which at the same time can be conveyed from the mixer to the forms without separation of the coarse aggregate from the mortar.

23. RE-TEMPERING. Mortar or concrete shall not be re-mixed with water after it is partly set.

24. PLACING CONCRETE. Concrete after the completion of the mixing shall be handled rapidly and in as small masses as is practicable from the place of mixing to the place of final deposit. Under no circumstances shall concrete be used that has partly set either on account of elapsed time or on account of a flash set cement. If the concrete is conveyed by inclined chute or spout, the angle of slope must be 27° with the horizontal.

25. Concrete should be deposited in such a manner as will permit a most thorough compacting such as can be obtained by working with a straight shovel or slicing tool kept moving up and down until all the ingredients have settled in their proper places by gravity, and the surplus water, or moisture, has been forced to the surface. Excessive tamping shall be avoided, special care being exercised to prevent the formation of laitance. All laitance shall be removed before placing new concrete.

26. Before placing concrete, the reinforcement shall be accurately placed in accordance with the plans, and adequate means provided to hold it in its proper position, each piece or member being so firmly fixed as to positively prevent any subsequent displacement. The forms shall be substantial and thoroughly wetted (except in freezing weather) or oiled. The space to be occupied by the concrete shall be free from shavings, sticks of wood, or other debris. When the placing of concrete is suspended, any necessary grooves for joining future work shall be made before the concrete has had time to set. When work is resumed, concrete previously placed shall be thoroughly cleaned from all dirt and scum or laitance, roughened, drenched with water and then slushed with a mortar consisting of one part of Portland cement and not more than one part fine aggregate. For water-tight work or concrete located in such a position that cracks are especially objectionable, neat cement paste instead

of mortar shall be used for slushing the old surface. Neat cement shall be used at all joints in floor members which are to have a granolithic finish.

27. The faces of concrete exposed to premature drying shall be kept wet for a period of at least 7 days.

28. FREEZING WEATHER. Concrete shall not be mixed or deposited at or near freezing temperature unless special precautions are taken to avoid the use of materials covered with ice-crystals or containing frost, and to provide means to prevent the concrete from freezing after being placed in position or until it has thoroughly hardened. Special care shall be taken to see that the coarse aggregate is heated to well above the freezing point.

29. FORMS. Forms shall be substantial and unyielding so that the concrete shall conform to the designed dimensions and contours; and they shall be tight to prevent the leakage of the mortar. The concrete shall be carefully inspected and its hardness ascertained before removing the forms. The dead and the construction loads coming upon the members also shall be taken into consideration. Forms shall not be removed from floor slabs in less than 7 days. Sides of beams may be removed at the same time as the floor slabs provided the original supports under beam and girders are left in place. Column forms shall not be removed in less than 4 days.

Original supports for all beams and girders must remain in place at least 10 days. The length of time before removal shall be increased when the temperature runs below 40° Fahr.

JOINTS

30. Joints in columns shall be made at a level flush with the bottom of the girder or the bottom of the column-head. Joints in beams and girders shall be located midway between supports, except that where a beam intercepts a girder the joint in the girder shall be offset a distance equal to twice the width of the beam. Joints in the members of a floor system shall generally be made at or near the center of the span. Joints in columns shall be perpendicular to the axis of the column, and in girders, beams, and floor slabs, perpendicular to the plane of their surfaces.

31. Girders shall never be constructed over freshly poured columns without permitting a period of at least 2 hours to elapse to allow for settlement or shrinkage in the columns. No joint shall be allowed between slab and beam or girder.

32. Footings shall be cast to their full depth at one operation.

SPLICING REINFORCEMENT

33. Tension reinforcement shall not be spliced at points receiving stress.

34. In columns, bars larger than $1\frac{1}{8}$ inch in diameter not subject to tension shall be properly squared and butted in a suitable sleeve. Smaller bars may be lapped the distance required for bond.

35. In foundations, bearing plates shall be provided for supporting the bars, or the bars may be carried into the footing a sufficient distance to transmit the stress of the steel to the concrete through bearing and bond. In no case shall the ends of the bars merely rest on the surface of the footing.

36. TEMPERATURE REINFORCEMENT. Where reinforcement is required to distribute the temperature stresses and prevent cracks which are observable, the cross-sectional area of the reinforcement must be at least 0.3 per cent. of the cross-sectional area of the concrete.

PRINCIPLES OF DESIGN OF REINFORCED CONCRETE

37. APPROVAL. The designs shall be subject to the approval of the Consulting Engineer.

38. NOTATION. The following notation is used in these specifications:

d = depth of beam to center of steel.
jd = arm of resisting couple.
b = breadth of beam or width of the stem in T-beams.
V = total shear.
v = shearing unit stress.
u = unit bond stress.
Σo = sum of circumference of all bars.
w = load per linear foot of beam.
l = length of span in feet.

39. LOADS. The dead and live load shall be added together in computing stresses. The weight of reinforced concrete shall be taken as 150 pounds per cubic foot. The live load shall include all loads and forces which are variable. If heavy concentrations are likely to occur on floors, provision shall be made for them. For members carrying cranes, 25 per cent. shall be added to the live load to provide for the effects of vibration and impact. Any allowance required for dynamic effect shall be taken into account by adding the desired amount to the live load. *Crane runways shall withstand a lateral loading of two-tenths of the lifting capacity of the crane equally divided between the four wheels of the crane.

40. LENGTHS OF BEAMS AND COLUMNS. The span length for beams and slabs shall be taken as the distance from center to center of supports, except that it shall not be taken to exceed the clear span plus the depth of beam or slab. Brackets shall not be considered as reducing the length of span.

41. The length of columns shall be taken as the maximum unsupported length.

42. FOOTINGS. The area of the footing must be made sufficient to distribute the superimposed load on the foundation or on the piles without exceeding the allowable unit pressures. For combined footing, the shape must be determined so that the resultant of the downward load will coincide with the resultant of the upward reaction. Areas of bases of footings must be proportioned to give uniform unit loading throughout the building.

43. Besides investigation or bending moment, the footings must be specially investigated as to bond stresses. The depth and the amount of steel must be made so that the maximum unit bond stress (figured according to formula given under "Bond," where V is the total shear outside of the column, and Σo, the circumference of all bars used in calculation of tensile stresses) does not exceed the allowable unit stress.

44. Independent reinforced concrete column footings and wall footings shall be computed on the assumption that the maximum bending moments are in the planes of the faces of the superimposed column or wall. For independent column footings, reinforced in two directions, the bending moment resisted by the steel in one band shall be considered as the bending moment produced by the upward loads on a trapezoid formed by the face of the column, edge of the footing, and the two diagonal lines, formed by connecting the corners of the column with the corners of the footing. In calculating the moment for footings on a foundation where the upward pressure is uniformly distributed, the load on the trapezoid may be divided into load on the

* Omit if not required.

rectangle in front of the face of the column, the moment arm for which equals half of the projection, and the load on the two remaining triangles for which the center of pressure may be considered as located at a point 0.6 of the width of the projection from the column. In figuring the moment of resistance for wall footings, all steel may be considered as effective. For rectangular or square footings with two-way reinforcement, the resisting moment at a section at the face of the column shall be calculated using the area of all bars placed within a width of the footing equal to the width of the column plus twice the thickness of the footing plus half of the remaining distance on each side to the edge of the footing. The maximum diagonal tension for independent footings shall be assumed at a section placed at a distance from the face of the column equal to the effective depth, d, of the footing. It must be figured according to the ordinary formula, $v = \frac{V}{bjd}$, where b is the circumference of a figure similar to and concentric with the column section, distant from the column face a distance equal to the effective depth of the column, and V is the upward load between this circumference and the edge of the footing.

45. Combined footings must be considered as beams supported on columns. To insure proper distribution of the pressure, the footing must be reinforced with lateral distribution bars. For combined footings, diagonal shear must be computed as for ordinary beams.

46. Floor Slabs. Floor slabs shall be designed and reinforced as continuous over the supports. Rectangular slabs may be reinforced in both directions, the distribution of the load being determined by the formula, $r = \frac{l}{b} - 0.5$, in which

r = proportion of load carried by transverse reinforcement,
l = length, and
b = breadth of slab.

If the length of the slab exceeds 1.5 times its width, the entire load shall be carried by transverse reinforcement. The total amount of reinforcement determined by this formula may be reduced 25 per cent. by gradually increasing the rod spacing from the third point to the edge of the slab.

47. T-Beams. If adequate bond and shear resistance between slab and web of beam is provided, the slab may be considered as an integral part of the beam, but its effective width shall be determined by the following rules:

(a) It shall not exceed one-fourth of the span length of the beam.
(b) Its overhanging width on either side of the web shall not exceed four times the thickness of the slab.

48. Moments in Continuous Beams and Slabs. Beams continuous over one or more intermediate supports shall be designed with due regard to the positive and negative bending moments.

49. The following bending moments shall be used in computation:

(a) For floor slabs, the bending moments at center and at support shall be taken at $\frac{wl^2}{12}$ for both live and dead loads.

(b) For continuous beams, the bending moments at center and at support for interior spans shall be taken at $\frac{wl^2}{12}$, and for end spans it shall be taken at $\frac{wl^2}{10}$ for center and for adjoining support for both dead and live loads.

(c) In the case of beams and slabs continuous for two supports only, the bending moment at central support shall be taken at $\frac{wl^2}{8}$, and near the middle of span at $\frac{wl^2}{10}$.

(d*) For beams continuous over two intermediate supports where the end spans are l and the center span $0.4l$ to $0.5l$, the bending moments shall be taken as follows:

Near middle of end spans, $\frac{wl^2}{12}$.

For interior supports, $-\frac{wl^2}{11}$.

In middle of middle span, $+\frac{wl^2}{60}$ and $-\frac{wl^2}{18}$, the design being such as will satisfy both requirements. (Note that l is the length of the end span.)

(e) For other special conditions the bending moments will be specified by the Consulting Engineer.

(f) Ends of continuous beams (unless free to move) shall be provided with at least half of the reinforcement used in the center of the span. No reliance, however, shall be placed on the fixity of the beam at that support, and it shall not be considered as reducing the bending moment in the center.

50. In designing the negative bending moment reinforcement, the point of inflection shall be considered as distant from the support one-quarter of the span, and the bending moment as changing according to a straight line from its maximum at the edge of the support to zero at the point of inflection. The positive bending moment shall be considered as changing according to a parabola. The points of zero moment shall be assumed for intermediate spans as distant 0.2 of the span from the edge of each support, and, for the end span, one point of zero moment 0.2 of the span from the middle support and the other at the end support. *For middle spans of beams under (d) the negative bending moment shall be considered as varying according to a parabola from its maximum at the support to the minimum specified for the center.

51. In placing the reinforcement, the bending moment not only at the critical section but also in the whole length of the beam must be considered. Bars may be bent up only when not required by the positive bending moment. The arrangement of reinforcement over the support must be made with due regard to bending moments. The bent portions of the bars and any steel, the strength of which is not developed by bond, must not be counted in in the cross-section of the effective steel.

52. Reinforcement for negative bending moment shall be provided preferably by bending up a portion of the positive bending moment reinforcement and carrying it a sufficient distance over the support to develop its strength by bond and also to serve as negative bending moment reinforcement in the adjoining span. If the amount of steel obtained in that way is not sufficient, additional short bars must be introduced. When short bars are used, they must extend both ways beyond the support fifty diameters for plain bars and forty diameters for deformed bars unless a greater length is required by the bending moment.

53. In T-beams, the compressive stresses at the support must not exceed the stresses recommended in Section 77 on "Stresses." The reinforcing steel placed in

* Omit if not required

the compressive part of the beam, when properly developed by bond and bearing, may be assumed to carry its proportion of stress. In such cases, computations shall be made in accordance with the ordinary formulas for beams with steel in top and bottom.

54. In girders running parallel to the main reinforcement of slabs, cross reinforcement consisting of three-eighths-inch or half-inch bars, spaced not over 12 inches apart, shall be placed in the upper portion of the slabs over girders. The bars must extend on each side of the girder at least fifty diameters for plain bars and forty diameters for deformed bars to develop their strength by bond.

55. BOND. Bond stress shall be computed by the formula, $u = \frac{V}{jd\Sigma o}$. In this formula, "Σo" refers only to the horizontal bars of the tension reinforcement at the section.

56. In footings and in short heavily loaded beams, special attention must be paid to bond stresses. The diameter of bar and the depth of concrete must be selected to conform to bond requirements.

57. In restrained and cantilever beams the bars must be anchored in the support sufficiently to develop the full tensile stress that exists at the point of support in the reinforcing bars. The anchoring by extending the bars a proper length in the concrete of the support is to be preferred. Whenever this is not feasible, anchorage may be obtained by a suitable Considère hook, or a hook consisting of a turn through 180 degrees.

58. SPACING OF BARS. Parallel bars shall be spaced not less than two and one-half diameters apart measured from center to center, and never less than one inch in the clear. The distance from the side of a beam to the center of the nearest bar shall not be less than two diameters and never less than 1½ inches in the clear. The clear spacing between two layers of bars shall not be less than ½ inch.

59. In computations for bending moment where two or more layers of bars are used, due allowance must be made for the distance of each layer from the neutral axis.

60. DIAGONAL TENSION. Vertical or horizontal shearing stresses computed by the formula, $v = \frac{V}{bjd}$, shall be considered as the measure of diagonal tension.

61. If the diagonal tension exceeds the allowable unit stress specified in paragraph 77 on "Stresses," web reinforcement must be used.

62. Web reinforcement may consist of stirrups (vertical or inclined) or of a combination of stirrups and bent-up bars. In simply supported beams, not more than one-third of the total horizontal steel must be bent up.

63. Members of web reinforcement must be connected with, looped over, or wrapped around the longitudinal tension reinforcing bars. The tension bars for the portion of beam under positive bending moment are at the bottom of the beam and for the portion under negative bending moment, at the top of the beam. To develop the tensile strength of the web reinforcement, it must be anchored in the compressive portion of the beam.

64. The longitudinal distance between stirrups, or between points of bending of adjacent bent-up bars, shall not exceed ¾ the depth of the beam. Long and important beams must have stirrups throughout their whole length even if not required by diagonal tension.

65. Fireproofing of beams and girders. All steel in beams and girders must be protected by a 1½ inch thickness of concrete below the lower surface of steel. In slabs the protective covering must be ¾ inch in thickness.

COLUMNS

66. Unsupported Length. For columns where the dead plus the live lead produces the full working stresses, as given in paragraphs which follow, the ratio of unsupported length to least width shall be limited to 15. If the working stresses in the column are limited to four-fifths of the specified working stresses, this ratio may be increased to 20. For intermediate ratios of unsupported length to least width, the stresses may be assumed as varying according to a straight line.

67. Effective Area. In columns reinforced with vertical bars and requiring special fire-proofing, at least one inch on all sides, irrespective of the location of the steel bars, shall be considered as protective covering for the column and shall not be included in the area which is figured in taking stress.

68. The effective area to use in computations of hooped columns, or of columns containing structural steel, shall be considered as that within the hooping or within the limits of the structural steel.

69. Straight Bars as Reinforcement. Vertical reinforcement in the form of straight bars shall be assumed to carry its proportion of the compressive stress based on the ratio of the modulus of elasticity of the steel to the modulus of elasticity of concrete. The longitudinal reinforcement shall be straight and have sufficient lateral support to be securely held in place until the concrete is set. The sectional area of longitudinal reinforcement shall be not less than 1 per cent and not more than 4 per cent. of the effective cross-section of the column.

70. Hooped Columns. When hooping is used, the total amount of such reinforcement shall be not less than 1 per cent. of the volume of the column enclosed by the hooping. The clear spacing of such hooping shall be not greater than one-sixth the diameter of the enclosed column nor, preferably, more than one-tenth the diameter, and in no case more than 2½ inches. Hooping is to be circular or elliptical, and the ends of the bands must be united in such a way as to develop their full strength. Adequate means must be provided to hold the bands or hoops in place so as to form a column, the core of which shall be straight and well centered.

71. Structural Shapes. Concrete columns with structural shapes, the area of which does not exceed 4 per cent. of the effective cross-section of the concrete, may be designed as reinforced concrete columns provided the shapes are latticed or tied by steel plates.

72. Concrete columns with structural steel shapes built like steel columns, where the percentage of steel exceeds 4 per cent. of the effective area of the concrete, shall be considered as equivalent in strength to that of the plain steel column* plus the strength of the concrete core. The type of the structural column shall be subject to approval by the Consulting Engineer.

73. Bending Stresses in Columns. Stresses due to eccentric loads and lateral forces must be provided for by increasing the section until the maximum stress does not exceed the values specified under the heading of "Stresses." If in any case tension is possible in the longitudinal bars, adequate connection between the ends of the

* See page 564.

bars must be provided to take this tension, and joints must be arranged at points of low tension.

74. WALL COLUMNS. Wall columns shall be designed to resist the actual bending moment transmitted from the beams as determined by the method of least work. The reinforcing steel in the roof columns shall be stopped at the lower surface of the roof beams so as to leave a free support, or else the column shall be designed for a moment of not less than $\frac{wl^2}{20}$ in which l is the span of the beam.

WORKING STRESSES

75. The following working stresses shall be used for static loads. Proper allowances for vibration and impact shall be added to live loads where necessary to produce an equivalent static load before applying the unit stresses in proportioning parts.

76. The stresses in the concrete are based on a quality of concrete having an ultimate strength of 2 000 pounds per square inch, as shown by tests of cylinders having a diameter not less than 4 times the size of the largest stone, nor less than 4 inches, and a length double their diameter. If shorter specimens are used in tests, correction shall be made by the proper ratio. The consistency of the concrete for the test specimens shall be such that when placed in a cylinder at least 3 inches in diameter, which is then immediately removed, it will just begin to lose its shape. Specimens shall be stored in moist sand until ready to cap with neat cement 24 hours before testing. In case a stronger or a weaker concrete than this is used in certain parts of the structure, the working stresses may be increased or diminished in like ratio, provided, however, that for a concrete whose strength at 28 days is greater than 2200 pounds and less than 2900 pounds per square inch, the ratio of moduli of elasticity shall be taken at 12 instead of at 15.

77. Allowable working stresses for 2 000 pound concrete shall be taken as follows:

BEARING. When compression is applied to a surface of concrete at least twice the loaded area, a compressive stress of 650 pounds per square inch.

DIRECT COMPRESSION. For columns with longitudinal reinforcement only to the extent of not less than 1 per cent. and more than 4 per cent., 450 pounds per square inch on the effective area of the concrete.

For columns reinforced with not less than 1 per cent. and not more than 4 per cent. of longitudinal bars, and with bands, hoops, or spirals, as specified above, a stress of 700 pounds per square inch, provided the ratio of the unsupported length of the column to the diameter of the hooped core is not more than 10.

For the core of the structural steel column, 350 pounds per square inch.

In computing the strength of the columns with longitudinal reinforcement, the longitudinal steel bars shall be figured as carrying their proportion of the load.

COMPRESSION IN EXTREME FIBER. For extreme fiber stress in beams, girders, and slabs, a stress of 650 pounds per square inch. Adjacent to the supports of continuous members, stresses 15 per cent. higher may be used.

SHEAR AND DIAGONAL TENSION. In calculations on beams, the maximum shearing stress in a section shall be used as the means of measuring diagonal tension stress, and the following allowable values for the maximum vertical shearing stress are recommended:

(a) For beams with horizontal bars only and without web reinforcement, calculated by the formula given in Section 60, 40 pounds per square inch.
(b) For beams thoroughly reinforced with web reinforcement, the value of the shearing stress being calculated according to the formula in Section 60, 120 pounds per square inch. The web reinforcement shall be proportioned to resist two-thirds of the shearing stresses, as computed by the formula in Section 60.
(c) For punching shear, 120 pounds per square inch.

BOND. Bond stress between concrete and plain reinforcing bars, 80 pounds per square inch; drawn wire, 40 pounds per square inch; deformed bars of mercantile types of design, 100 to 120 pounds per square inch.

REINFORCEMENT. Tensile or compressive strength for steel bars used as reinforcement, 16 000 pounds per square inch.*

MODULUS OF ELASTICITY. The ratio of modulus of elasticity of steel to modulus of elasticity of concrete shall be taken as 15, except in cases where the strength of the concrete, as determined by the strength of cylinders 8 inches in diameter by 16 inches long at the age of 28 days, exceeds 2200 pounds per square inch and is less than 2900 pounds, when it shall be taken as 12.

* At Technology, as a result of tests, twisted steel was adopted using a working stress in tension of 18 000 pounds per square inch.

CHAPTER IV

CLASSIFICATION OF CEMENTS.

From an engineering standpoint, limes and cements may be classified as

Portland cement.
Natural cement.
Puzzolan cement.
Hydraulic lime.
Common lime.

Typical analyses of each of these are presented in the following table. The composition of Natural cement, even different samples of the same brand, is so extremely variable that their analyses cannot be regarded as characteristic of locality.

Typical Analyses of Cements.

	PORTLAND CEMENT		NATURAL CEMENT							COMMON LIME	
			AMERICAN		ENGL'H	FRENCH					
	Lehigh Valley[1] (mixed rock)	Western[2] (marl and clay)	Eastern Rosendale[3]	Western Louisville[3]	Roman[4]	Vassy[5]	Grappiers[6]	Puzzolan Cement[7]	Hydraulic Lime (Le Tiel)[8]	Lime[9]	Magnesian Lime[10]
Silica SiO_2	21.31	21.93	18.38	20.42	25.48	22.60	26.5	28.95	21.70	1.03	1.12
Alumina Al_2O_3	6.89	5.98	15.20 (Alumina and Iron Oxide)	4.76	10.30	8.90	2.5	11.40	3.19	1.27 (Alumina and Iron Oxide)	0.61
Iron Oxide Fe_2O_3	2.53	2.35		3.40	7.44	5.30	1.5	0.54	0.66		
Calcium Oxide CaO	62.89	62.92	35.84	46.64	44.54	52.69	63.0	50.29	60.70	97.02	58.51
Magnesian Oxide MgO	2.64	1.10	14.02	12.00	2.92	1.15	1.0	2.96	0.85	0.68	39.6
Sulphuric Acid SO_3	1.34	1.54	0.93	2.57	2.61	3.25	0.5	1.37	0.60		
Loss on Ignition	1.39	2.91	3.73	6.75	3.68	6.11	5.0	3.39	12.20		
Other constituents	0.75		11.46	3.74	1.46			0.30	0.10		

[1]W. F. Hillebrand, Society of Chemical Industry, 1902, Vol. XXI.
[2]W. F. Hillebrand, Journal American Chemical Society, 1903, 25, 1180.
[3]Clifford Richardson, *Brickbuilder*, 1897, p. 229.
[4]Stanger & Blount, Mineral Industry, Vol. V, p. 69.
[5]Candlot, Ciments et Chaux Hydrauliques, 1898, p. 174.
[6]Le Chatelier, Annales des Mines, September and October, 1893, p. 36.
[7]Report of the Board of U. S. Army Engineers on Steel Portland Cement, 1900, p. 52.
[8]Candlot, Ciments et Chaux Hydrauliques, 1898, p. 24.
[9]Rockland-Rockport Lime Co.
[10]Western Lime and Cement Co.

PORTLAND CEMENT

Portland cement is defined by Mr. Edwin C. Eckel, formerly of the U. S. Geological Survey, as follows: "By the term Portland cement is to be understood the material obtained by finely pulverizing clinker produced by burning to semi-fusion an intimate artificial mixture of finely ground calcareous and argillaceous materials, this mixture consisting approximately of 3 parts of lime carbonate to 1 part of silica, alumina and iron oxide."

The definition is often further limited by specifying that the finished product must contain at least 1.7 times as much lime, by weight, as of silica, alumina, and iron oxide together.

The only surely distinguishing test of Portland cement is its chemical analysis and its specific gravity.

Portland cement should always be used in reinforced concrete construction and in any construction liable to shocks or vibration or stresses other than direct compression.

White Portland Cement is manufactured for surface finish and ornamental work. It contains relatively high alumina, with only 1 per cent or less of iron oxide.

Iron Portland Cement has been made with a view to its use in sea and alkaline waters. It contains relatively high iron, with less than 2 per cent of alumina. It is slow setting, with high tensile strength in long time tests.

The term **Natural Portland Cement** arose from the discovery in Boulogne-sur-Mer, France, as early as 1846, of a natural rock of suitable composition for Portland cement. A similar discovery in Pennsylvania gave rise to the same term in America, but the manufacturers soon found it necessary to add to the cement rock a small percentage of purer limestone. Since the chemical composition of Portland cement, as defined above, is substantially uniform regardless of the materials from which it is made, in the United States the terms "natural" and "artificial" are meaningless.

In France, cements intermediate between Roman and Portland are called "natural Portlands."*

Sand Cement. Sand or silica cement is a mechanical mixture of Portland cement and mineral matter, ground together very finely in a tube mill or other suitable machine. Tests by the U. S. Reclamation Service† indicate that the action of this matter on the cement when

* Candlot's Ciments et Chaux Hydrauliques, 1898, p. 164.

† Rapier R. Coghlan, *Engineering News*, June 19, 1913, p. 1270, and L. E. Sale, *Engineering Contracting*, December 3, 1913, p. 623.

mixed with water is chemical, and to produce a satisfactory sand, a rock, such as sandstone or certain igneous rocks, which contains soluble or colloidal silica, must be employed. Silica in this form appears to act as a weak acid so as to form an insoluble compound with the hydrated lime set free from the Portland cement when mixed with water. The proportion of sand that the Portland cement will carry depends on the amount of colloidal silica in the rock to be used. Usually the proportions are equal parts.

The production of sand cement is economical only in regions where the cost of Portland cement is very high because of long transportation. The U. S. Reclamation Service has used it on large dams throughout the West at a cost about two-thirds that of Portland cement, in spite of the necessity of building a mill at each job.

The properties of sand cement are satisfactory. It sets slower but its strength at all ages is as high or higher than Portland. It is, moreover, less liable to injury from the action of the alkalies that are abundant in the western soils and waters.

Tufa Cement. A cement that partakes of the nature of both sand and puzzolan cement (see p. 44) is the tufa cement employed on the Los Angeles Aqueduct and other work in California. Tufa cement as there used is a mechanical mixture in equal parts of commercial Portland cement and tufa rock formed from volcanic puzzolanic powder which occurs in stratified beds 100 feet or more in thickness along the line of the Aqueduct. The table below shows typical chemical analyses of Monolith tufa cement (Monolith Portland and ground

Typical Analyses of Portland Cement, Tufa Cement, and Tufa Rock

Constituents.	Portland Cement.		Tufa Cement.	Tufa and Similar Rocks.				
	* Lehigh Valley.	† Monolith.	† Monolith Tufa	† Monolith Tufa Rock.	† Hawaiian Tufa Rock.	† Italian Tufa Rock.	† Italian Puzzolana.	† German Trass
Silica SiO_2	21.31	25.02	35.34	69.46	51.98	42.36	45.68	68.49
Iron Oxide Fe_2O_3	2.53	2.72	11.89	2.52	2.90	28.35	30.09	2.27
Alumina Al_2O_3	6.89	5.58		11.37	15.86			9.83
Lime CaO	62.89	62.70	41.05	1.80	9.57	9.15	11.95	1.05
Magnesia MgO	2.64	1.76	3.04	2.95	5.61	0.54	3.76	0.67
Sulphuric Acid SO_3	1.34	1.14		0.43		0.56	0.56	0.06
Loss	1.39		8.52	6.28		13.68	6.30	10.25
Other constituents	0.75							

* W. F. Hillebrand, Society of Chemical Industry, 1902, Vol. XXI.

† J. B. Lippincott, Transaction American Society of Civil Engineers, Vol. LXXVI, 1913, p. 526.

tufa), a Lehigh Valley Portland cement, Monolith Portland cement, and several puzzolanic materials.

The tufa cement mortar as tested by the Los Angeles engineers* showed higher tensile strength after ten days than straight Portland cement mortar, while the leaner the mortar the greater the ratio. The growth in strength, as shown by five year tests, is steady. In compression, 1 : 2 : 4 tufa cement concrete averaged about 20 per cent. weaker than 1 : 2 : 4 Portland cement concrete at ages up to 3 months, while 1 : 3 : 6 mixes were of substantially equal strength. The slow-hardening property of tufa cement requires the forms to be left in place longer than is ordinarily the practice. Special care also is necessary in protecting the concrete from low temperatures and from drying out and cracking.

NATURAL CEMENT

Natural cement is "made by calcining natural rock at a heat below incipient fusion, and grinding the product to powder." † Natural cement contains a larger proportion of clay than hydraulic lime, and is consequently more strongly hydraulic. Its composition is extremely variable on account of the difference in the rock used in manufacture.

Natural cements in the United States in numerous instances bear the names of the localities where first manufactured. For example, Rosendale cement, a term heard in New York and New England more frequently than Natural cement, was originally manufactured in Rosendale, Ulster County, N. Y. Louisville cement first came from Louisville, Ky. The James River, Milwaukee, Utica, and Akron are other Natural cements named for localities.

In England the best known Natural cement is called Roman cement. Occasionally one hears the term Parker's Cement, so called from the name of the discoverer in England.

Natural cement may be substituted for Portland in concrete, if economy demands it, for dry unexposed work where the load in compression can never exceed, say, 75 pounds per square inch (5 tons per square foot) and will not be imposed until three months after placing.

In mortar Natural cement is adapted for ordinary brickwork not subjected to high water pressure or to contact with water until, say, one month after laying, and for ordinary stone masonry where the chief requisite is weight and mass.

Natural cement concrete or mortar should never be allowed to freeze,

* See paper by J. B. Lippincott, Transactions American Society of Civil Engineers, Vol. LXXVI, 1913, p. 520.

† Professional Papers, No. 28, U. S. Army Engineers, p. 33.

should never be laid under water, in exposed situations, in columns, beams, floors or building walls, or in marine construction.

LE CHATELIER'S CLASSIFICATION OF NATURAL CEMENTS

In France there are several classes of Natural cement. Mr. H. Le Chatelier* classifies Natural cements as those obtained "by the heating of limestone less rich in lime than the limestone for hydraulic lime. They may be divided into three classes: †

"Quick-setting cements, such as Vassy and Roman (Ciments à prise rapide, Vassy, romain);

"Slow-setting cements (Ciments à prise demi-lente);

"Grappiers cement (Ciments de grappiers).

PUZZOLAN OR SLAG CEMENT

Puzzolan cement is the product resulting from mixing and grinding together in definite proportions slaked lime and granulated blast furnace slag or natural puzzolanic matter (such as puzzolan, santorin earth, or trass obtained from volcanic tufa). The ancient Roman cements belonged to the class of Puzzolans.

Blast furnace slag is essentially an artificial puzzolana, formed by the combustion in a blast furnace, and Puzzolan or slag cements, formerly made in the United States, are ground mixtures of granulated blast furnace slag, of special composition, and slaked lime. The cement is of light lilac color, of low specific gravity (2.6 to 2.8), very finely ground, and characterized by an intense bluish green color in a fresh fracture after long submersion in water. Puzzolan cements must not be confounded with true Portlands made from blast furnace slag and lime.

Puzzolan or slag cements have been limited to certain possible uses by the engineer officers of the U. S. Army‡ as follows:

Puzzolan cement never becomes extremely hard like Portland, but Puzzolan mortars and concretes are tougher or less brittle than Portland.

The cement is well adapted for use in sea water,§ and generally in all positions where constantly exposed to moisture, such as in foundations of buildings, sewers, and drains, and underground works generally, and in the interior of heavy masses of masonry or concrete.

It is unfit for use when subjected to mechanical wear, attrition, or blows. It should never be used where it may be exposed for long periods to dry air, even after it has well set. It will turn white and disintegrate, due to the oxidation of its sulphides at the surface under such exposure.

* Procédés d'Essai des Matériaux Hydrauliques, Annales des Mines, 1893.

† For description of these classes see 2nd edition "Concrete Plain and Reinforced" pp. 49 and 50.

‡ Professional Papers No. 28.

§ See Chapter XV, by R. Feret.

Puzzolanic material has been suggested by Dr. Michaelis, of Germany, and Mr. R. Feret, of France (see Chapter XV), as a valuable addition to Portland cement designed for use in sea water.

HYDRAULIC LIME

The hydraulic properties of a lime—its ability to harden under water—are due to the presence of clay, or, more correctly, to the silica in the clay. Hydraulic lime, although not manufactured in the United States, is still used to a certain extent in Europe as a substitute for cement. The celebrated lime-of-Teil of France is a hydraulic lime. Beton-Coignet is a mixture of hydraulic lime with cement and sand. Hydraulic lime is made up artificially in localities where labor is very cheap, as in the British dependencies, by grinding together balls of clay and lime hydrated for the purpose. A large dam* was built in 1914 in Mexico, of one-man stone laid in mortar of hydraulic lime made up on the job. It set very slowly, but eventually became extremely hard. Regular tests were carried on during construction.

Mr. Edwin C. Eckel states† that "theoretically the proper composition for a hydraulic limestone should be calcium carbonate 86.8%, silica 13.2%. The hydraulic limestones in actual use, however, usually carry a much higher silica percentage, reaching at times to 25%; while alumina and iron are commonly present in quantities which may be as high as 6%. The lime content of the limestones commonly used varies from 55% to 65%."

Although the chemical composition of hydraulic lime is similar to Portland cement, its specific gravity is much lower, lying between 2.5 and 2.8.‡

In the manufacture of hydraulic lime the limestone of the required composition is burned, generally in continuous kilns, and then sufficient water is added to slake the free lime produced so as to form a powder without crushing.

COMMON LIME

Common lime is not suitable for a principal ingredient in concrete. It will not set in contact with water, sustain heavy loads, or resist wear.

The use of lime mortar, in the building laws of some cities, is limited to chimney construction in frame buildings, while other cities permit its use in walls of all except fireproof buildings. The stresses on brick laid in lime mortar should be limited to 7 tons per square foot.

* Authority of William B. Fuller.

† *American Geologist*, March, 1902, p. 152.

‡ Candlot's Ciments et Chaux Hydrauliques, 1898, p. 26.

Lime and Natural cement mortar is suitable for ordinary building brickwork, for light rubble foundations and for building walls.

Lime and Portland cement mortar is adapted for the same purposes as mortars of lime and Natural cement, but is of superior quality and strength.

The commercial lime of the United States is "quicklime," which is chiefly calcium oxide (CaO).

Lime is now manufactured by a continuous process. Limestone of a rather soft texture, so as to be as free as possible from silica, iron and alumina, is charged into the top of a kiln which may be, say, 40 ft. high by 10 ft. in diameter. The fuel is introduced into combustion chambers near the foot of the shaft, and the finished product is drawn out from time to time through another opening in the bottom of the shaft. The temperature of calcination may range from 1400° Fahr. (760° Cent.) to, at times, 2,000° Fahr. (1,090° Cent.) The product (see analysis, p. 40), in ordinary lime of the best quality, is nearly pure calcium oxide (CaO). Upon the addition of water the lime slakes, forming calcium hydrate (CaH_2O_2), and, with the continued addition of water, increases in bulk to twice to three times the original loose and dry volume of the lump lime as measured in the cask. In this plastic condition it is termed by plasterers "putty" or "paste."

The setting of lime mortar is the result of three distinct processes which, however, may all go on more or less simultaneously. First, it dries out and becomes firm. Second, during this operation, the calcic hydrate, which is in solution in the water of which the mortar is made, crystallizes and binds the mass together. Hydrate of lime is soluble in 831 parts of water at 78° Fahr; in 759 parts at 32° and in 1136 parts at 140°. Third, as the per cent. of water in the mortar is reduced and reaches five per cent., carbonic acid begins to be absorbed from the atmosphere. If the mortar contains more than five per cent. this absorption does not go on. While the mortar contains as much as 0.7 per cent. the absorption continues. The resulting carbonate probably unites with the hydrate of lime to form a sub-carbonate, which causes the mortar to attain a harder set, and this may finally be converted to carbonate. The mere drying out of mortar, our tests have shown, is sufficient to enable it to resist the pressure of masonry, while the further hardening furnishes the necessary bond.*

Magnesian Limes evolve less heat when slaking, expand less, and set more rapidly than pure lime. A typical analysis is given on page 40.

*The authors are indebted to Mr. Clifford Richardson for this paragraph.

Hydrated Lime is the powdered product formed by slaking quick lime with the requisite amount of water. The material as it comes into commerce is a very finely divided white powder, and if properly prepared contains no unhydrated particles of lime. For this reason it is preferable to common lime paste or putty for use with Portland cement, because if properly manufactured it is more thoroughly slaked and is easily handled and measured.*

* See S. Y. Brigham in *Engineering News*, Aug. 27, 1903, p. 177, and Charles Warner in *Rock Products* Feb. 1904, p. 26.

CHAPTER V

CHEMISTRY OF HYDRAULIC CEMENTS*

By Spencer B. Newberry

INTRODUCTION

Hydraulic cements are compounds consisting chiefly of lime, silica, and alumina, which have the property, when mixed with water to a paste, of hardening to a stone-like mass. They may be classified as follows:

1. **Portland cement,** made by calcining at high heat an artificial mixture of carbonate of lime with clay, shale or slag, in certain exact proportions, and grinding the resulting clinker to powder.

2. **Natural cement,** made by burning at low heat limestone containing excess of clay and usually much magnesia, and grinding the product.

3. **Hydraulic lime,** obtained by burning limestone containing a small percentage of clay, slaking by sprinkling with water, and sifting the product.

4. **Puzzolan** or **slag cement,** made by mixing and grinding slaked lime with certain kinds of volcanic scoria or blast-furnace slag.

It will be noted that Portland cement, Natural cement and hydraulic lime are all made by calcining mixtures of carbonate of lime with argillaceous material such as clay, shale or slag, the differences being in the proportions and temperatures of burning. It is well known that pure lime, while it hardens slowly in air, remains soft if kept constantly wet. Smeaton, the engineer of the Eddystone lighthouse, in 1756, experimented with various kinds of lime, and discovered that only those containing a considerable proportion of clay would harden under water. He was thus the first to show the important part played by the insoluble constituents of impure limestone in conferring hydraulic properties on the lime burned from such stone.

Limestones containing varying proportions of argillaceous matter are of frequent occurrence. It is found that if the clay substance contained is only 10 or 12 per cent., the product will heat and slake on sprinkling with water, and a *hydraulic lime* will result. On the other

* The authors are indebted to Mr. Newberry for this chapter, which has been especially prepared by him for this Treatise.

hand, if the clay substance is 30 per cent. or more, and the stone is burned at high heat, it will fuse to a slag having no hydraulic properties; but if burned at low heat, just sufficient to drive off the carbon dioxide, a soft product is obtained which does not heat or slake with water. On grinding this to powder a quick-setting *Natural cement* is produced. Between these two extremes lies a certain proportion of lime and clay substance which stands a very high heat without fusing, does not slake or expand with water, and on grinding yields a material far superior to hydraulic lime or Natural cement in strength and promptness of hardening. This is known as *Portland cement.*

CHEMICAL CONSTITUENTS OF CEMENT

The nature of the compounds formed in the burning and hardening of cement remained obscure for a long period after Portland cement had been made and used on a large scale. Le Chatelier, in his epoch-making work in 1887, showed that cement clinker consists essentially of tri-calcium silicate, $3\ CaO.SiO_2$, and tri-calcium aluminate, $3\ CaO.Al_2O_3$, and that on hardening with water the tri-calcium silicate is converted into crystalline calcium hydrate and hydrated mono-calcium silicate, as follows:

$$3CaO.SiO_2 + Aq. = 2Ca(OH)_2 + CaO.SiO_2.2\tfrac{1}{2}\ H_2O.$$

The aluminate also hydrates to hydrated tri-calcium aluminate.

Subsequent investigators denied the existence of tri-calcium silicate, as they failed to obtain this compound by fusing lime and silica, a mixture of di-calcium silicate and free lime being obtained in all cases. For many years the question of the constitution of clinker was investigated by Clifford Richardson, E. D. Campbell, and others, and especially by E. S. Shepherd and G. A. Rankin of the Geophysical Laboratory of the Carnegie Institute, and P. H. Bates, A. A. Klein and A. J. Phillips of the Bureau of Standards. These investigators found that tri-calcium silicate is readily obtained if a very small percentage of alumina or magnesia is present in the mixture of lime and silica; they thus have fully confirmed Le Chatelier's observations of twenty-eight years ago.

The whole of our present knowledge of the chemistry of cement is admirably summed up in a paper by G. A. Rankin on "The Constituents of Portland Cement Clinker" (Journal Industrial and Engineering Chemistry, June, 1915. This paper should be read by all who are interested in the subject The important conclusions stated are imperfectly summarized as follows:

The essential components of well-burned Portland cement clinker are

Tri-calcium silicate, $3\ CaO.SiO_2$

Di-calcium silicate, $2\ CaO.SiO_2$

Tri-calcium aluminate, $3\ CaO.Al_2O_3$

If the clinker is not well burned, so that practically complete equilibrium is not reached, there will be present also free lime, CaO, and an aluminate of the formula, $5\ CaO.3Al_2O_3$ The other substances present, magnesia, iron oxide and alkalies, are useful in forming a flux, from which during the burning the tri- and di-silicate and tri-aluminate crystallize out. Fineness of grinding of the raw material and temperature and time of burning are factors in reaching equilibrium. Commercial gray cement clinker, containing 6.7 per cent. of fluxes, requires a temperature of about 1425°Cent. White cement clinker, containing 2.4 per cent. fluxes, requires 1525°Cent., while a mixture of pure lime, silica and alumina requires 1650°Cent.

Tri-calcium silicate, $3\ CaO.SiO_2$, is formed by prolonged heating of the constituents at a temperature somewhat below the fusing point of the compound, which is 1900°Cent. If heated to fusion it is decomposed into a mixture of free lime and di-calcium silicate. It is this behavior which caused the existence of the compound to be so long denied.

Di-calcium silicate, $2\ CaO.SiO_2$, exists in four forms. Of these, the beta form changes to the gamma form on cooling down to 675°Cent., and in this transformation expands in volume about 10 per cent.; this causes the crystals to fall to powder, a phenomenon often seen in the cooling of overclayed clinker and known as "dusting."

Tri-calcium aluminate, $3\ CaO.Al_2O_3$, is formed by heating the constituents for a long time at about 1400°Cent. If heated to the melting point, 1535°Cent., it is largely decomposed into free lime and fusible low-lime aluminates.

In the burning of clinker, the changes which take place may be briefly stated as follows:

Carbon dioxide is driven off, and the resulting lime begins to combine with the silica, alumina, etc. present, forming first the fusible aluminate, $5\ CaO.Al_2O_3$, and di-calcium silicate. These two compounds unite with more lime to form tri-calcium silicate and tri-calcium aluminate. The formation of these substances is greatly aided by the presence of a certain amount of liquid flux, which begins to form at 1335°Cent. If held at about 1425°Cent. for some time, the 5 : 3 aluminate and the free lime will disappear, and the clinker will consist of tri-

calcium silicate, di-calcium silicate, tri-calcium aluminate and flux of indefinite composition.

Professor Edward D. Campbell has published a most interesting account of the mechanical separation of the solid and liquid constituents of clinker at the burning temperature. Discs of clinker were prepared, placed between discs of pure magnesia and exposed for 3 hours or longer to temperatures of 1475° to 1575° Cent. The fused portion of the clinker, corresponding to the "celite" of Törnebohm, was absorbed by the magnesia discs, leaving the "alite" or crystalline portion in comparatively pure condition. This alite proves to be a calcium silicate, containing 2.8 to 3 molecules of lime to one of silica, according to the basicity of the clinker, with little alumina and very little iron oxide. The celite or liquid portion is essentially a calcium aluminate, containing little silica but most of the alumina and iron oxide. These results accord with those stated by Rankin, the temperatures having been high enough to hold most of the tri-calcium aluminate in fused condition.

The writer has permission to quote the following from a letter from Mr. P. H. Bates of the Bureau of Standards, referring to certain investigations not yet published:

I am at present engaged on a paper which reports the results which we have obtained, with both neat and sand briquettes, made of the very pure constituents which are found in cement. We have been able to obtain tri-calcium silicate, containing in one case about 90 per cent. and in another 95 per cent. of this material. The di-calcium silicate has been prepared with much greater purity and also the tri-calcium aluminate in a very pure condition. We have followed the tensile specimens of these three compounds for a year, obtaining both the strengths and the amount of hydration determined by ignition loss as well as by microscopical examination. We have also mixed these three compounds in various proportions, by simply grinding together, and have obtained the same kind of data from these. The results have been very striking indeed. We found that the tri-calcium silicate of this purity, and containing only a few tenths of one per cent. of alumina, has all the properties of Portland cement, especially rate of setting and strength developed. The tri-calcium aluminate hydrates as rapidly as quick lime, and in 24 hours has as much strength as it will ever obtain, about 100 pounds per square inch. The di-calcium silicate hydrates very slowly and can hardly be removed from the molds before the end of a week. At the end of a year the neat specimens will have a strength very close to 600 pounds, but even with this strength it will have only about 5½ per cent. water of hydration as compared with the tri-calcium silicate which has a little over 13 per cent. water of hydration but not much more strength.

MATERIALS

As above stated, hydraulic lime is made by burning natural limestone containing a small amount of clay substance, while Natural cements are made by burning at low heat natural limestones containing a relatively large proportion of clay substance. Portland cement, on the other hand, is made by burning at high heat a mixture of materials of exactly correct composition, usually containing approximately 75 per cent. of carbonate of lime and 20 of anhydrous clay substance. A variation of even one per cent. in the carbonate of lime, from the percentage found correct for given materials, will injuriously affect the quality of the cement obtained. If stone containing exactly the right amount of clay and of perfectly uniform composition could be found, Portland cement could be made from it simply by burning and grinding. No deposits of such stone are however known or likely to be discovered. The only way, therefore, that the necessary exact composition can be obtained is by making an artificial mixture of materials in correct proportions.

Materials for Portland cement may be divided into two groups:

Calcareous materials, containing excess of lime over that required for the mixture, and

Argillaceous materials, containing excess of clay substance.

The calcareous materials generally used are limestone, chalk or marl. These consist chiefly of carbonate of lime, and are found abundantly in nearly all sections of the country.

The argillaceous materials in ordinary use are clay, shale, blast-furnace slag, and cement rock. The latter is a limestone containing more clay substance than is required. It occurs in extensive deposits in the Lehigh Valley region in Pennsylvania and New Jersey, and is used on a very large scale for cement manufacture, requiring the addition of only a small amount of purer limestone to give a correct mixture. At some factories in that section it is necessary only to mix different layers from the same quarry in proper proportions.

Magnesia, beyond a small percentage, has generally been considered objectionable, and liable to cause expansion and loss of strength at long periods. Authorities differ greatly in regard to the percentage of magnesia which may safely be present. This question is of great consequence in view of the wide-spread occurrence of deposits of more or less magnesian limestone in all parts of the country. The standard 1917 specifications of the United States and also the German official standard allow 5 per cent. magnesia in Portland cement. In a recent investiga-

tion by the United States Bureau of Standards, the highest strengths were obtained with cement containing 5 per cent. magnesia; cements containing 7.5 per cent. magnesia also gave strengths that were satisfactory. No bad effect in accelerated tests or normal tests for constancy of volume up to 28 days were noted with a magnesia content up to 18.98 per cent. All recent investigations indicate that magnesia in moderate amounts does not cause unsoundness as shown by ordinary tests, so that the allowance of 5 per cent. is certainly entirely safe. Cements with more than 7 or 8 per cent. magnesia, however, are inferior in strength, and appear to show increased expansion as shown by measurements of bars of concrete kept in air or water. This last point needs further exact investigation.

Clay or shale for Portland cement manufacture should be silicious, and practically free from coarse sand. It is desirable that the silica in the mixture shall be from 2.5 to 3.5 times the sum of alumina and iron oxide. This figure is often called the silica ratio. More aluminous mixtures, with silica ratio of 2.0 or less, give fusible clinker and quick-setting cement unless the lime is carried as high as is consistent with good soundness. High silica ratio permits greater latitude in proportions, and allows the lime to be carried somewhat lower without danger of quick-setting.

PROPORTION OF INGREDIENTS

Experience has shown that with given materials there is a certain definite proportion of carbonate of lime to clay substance which gives best results, and that a variation from this definite proportion, even to the amount of one-half per cent. of the carbonate of lime present, has a noticeably bad effect on the resulting product. This of course holds good only for fixed conditions of fineness of raw material and temperature and time of burning, since extreme raw fineness and thorough burning make it possible to carry the lime slightly higher, without danger of unsoundness, than would be practicable with coarser raw grinding and less perfect burning. Even with this qualification, however, the limit of proportions is exceedingly sharp and distinct, and the interval between unsoundness, due to too high lime, and low strength or quick-setting, due to excess of clay, is very narrow. Generally speaking, the higher the lime, up to the limit of soundness as shown by the boiling test, the better the quality of the resulting cement.

The recognized existence of a definite proportion which will give best results has led to many attempts to establish a definite formula by which,

from the analysis of the materials, the correct proportion could be calculated. In Germany it has been customary to so adjust the ingredients, as proposed by Michaelis, that the "hydraulic modulus," the ratio by weight of lime to silica, alumina and iron oxide, shall be from 1.8 to 2.2. These limits are of course much too far apart to be of practical use. It has also become generally recognized by cement chemists that much more lime combines with silica than with alumina or iron oxide. The "hydraulic modulus" is therefore a variable, and must be much higher in the case of silicious materials than with those high in alumina and iron.

Le Chatelier stated in 1887 that the lime and magnesia in Portland cement should not exceed a maximum,

$$\frac{CaO + MgO}{SiO_2 + Al_2O_3} \leq 3$$

nor be less than a minimum

$$\frac{CaO + MgO}{SiO_2 - Al_2O_3 - Fe_2O_3} \geq 3$$

These formulas represent chemical equivalents and not weights. The best brands of modern Portland cement approach closely to the above maximum formula, while a cement corresponding to the minimum formula would be so greatly over-clayed as to be practically useless. These limits are therefore too far apart to be of value. Another serious defect in the formulas is the inclusion of magnesia with lime. All investigations show that magnesia is practically inert so far as the proportion of lime to clay is concerned. That is, an over-clayed mixture is not corrected by the addition of magnesia, and the proportions must be calculated on the basis of lime to silica, alumina, and iron oxide, leaving the magnesia out of account.

Unfortunately, the recent scientific investigations on the constitution of cement clinker, described on preceding pages, while they identify the constituents of clinker, fail to establish the proportion in which these constituents should be present. The clinkers studied by Rankin were, in fact, much lower in proportion of lime than the best modern brands of cement, in spite of their having been burned in an electric furnace and thus not being contaminated with fuel ash. These clinkers showed tri-calcium silicate, di-calcium silicate and tri-calcium aluminate. It is an interesting question whether the di-calcium silicate would have disappeared if sufficient lime had been present to convert it into

tri-calcium silicate, and thus to bring the composition to that of Le Chatelier's maximum formula.

Practically, it is certain that sound cement corresponding to Le Chatelier's maximum formula can be made, provided the raw materials are ground sufficiently fine and burned for the necessary time at suitable temperature. There is also no doubt that cement of highest quality results from the highest practicable proportion of lime, provided the raw grinding and burning are so conducted as to develop the full benefits of this high proportion.

Some years ago the writer published a paper* containing an account of experiments based on the work of Le Chatelier. It was found that under ordinary conditions of raw grinding and burning, the maximum of lime which could be brought into combination to produce a sound cement is three equivalents for each equivalent of silica and two equivalents for each equivalent of alumina present. The composition of cement containing the maximum of lime would therefore be expressed by the formula

$$X\ (3CaO{\cdot}SiO_2) + Y\ (2CaO{\cdot}Al_2O_3)$$

It is understood that this formula is merely empirical, representing the relative proportions present, since the aluminate remains for the most part in the magma in combination with the iron oxide and part of the silica. The formula is also not in accordance with the latest researches, since these show tri-calcium aluminate instead of di-calcium aluminate, and part of the lime and silica combined as di-calcium silicate. However, we are not yet in position to construct an accurate formula based upon these researches, since we do not know how much lime it is desirable to have in the form of di-calcium silicate, or what proportions of lime and alumina are or should be present in the magma from which the crystalline silicates and aluminate separate.

If we assume that all the lime, silica and alumina are in the form of tri-calcium silicate and tri-calcium aluminate, the formula will be

$$X\ (3CaO.SiO_2) + Y\ (3CaO.Al_2O_3)$$

This is Le Chatelier's maximum formula, eliminating the magnesia from the calculation. With the ordinary temperature and time of burning this would give too much lime, and would produce unsound cement. In order to approach actual working conditions we may assume that

*Journal Society of Chemical Industry, November 30, 1897.

four-fifths of the silicate is tri-calcium and one-fifth di-calcium silicate. We shall then have

$$X\ (2.8CaO.SiO_2) + Y\ (3CaO.Al_2O_3)$$

Substituting weights for equivalents,

$$\text{Lime} = \text{silica} \times 2.6 + \text{alumina} \times 1.6$$

Applied to ordinary cement materials, this gives almost exactly the same proportions as the original formula proposed by the writer. It is to be preferred to the latter, however, as it is more nearly in accordance with the latest and most exact experiment.

It should be remembered that this formula represents the *maximum* of lime which a Portland cement, burned in the usual manner, may contain without showing unsoundness. This maximum can be reached only by extremely fine grinding of the raw material. This formula, also, by no means represents the composition of finished cement, since the ash of the fuel lowers the lime and raises the silica and alumina, above that calculated from the raw material, by at least 2 per cent.

In the laboratory, using gas as fuel, it will be found practicable to prepare sound cements corresponding to the above formula. In actual manufacture it is safer to reduce the lime slightly, to counterbalance possible defective grinding of raw material or unavoidable variations in composition. It will be found that the raw mixture at factories where the best Portland cements are made rarely falls below the composition,

$$\text{Lime} = \text{silica} \times 2.5 + \text{alumina} \times 1.6$$

This may be taken as a safe practical formula for commercial use. With fine grinding of the raw material it invariably will yield sound cements, while the use of a lower proportion of lime will be likely to produce quicksetting cement, low in tensile strength. As already explained, commercial cements are usually considerably lower in lime, owing to change in composition produced by the fuel-ash.

The writer's experiments have shown that magnesia forms with clay no products having hydraulic properties. It should be disregarded, therefore, in calculating cement mixtures, the composition of which should be calculated on the basis of the silica, alumina and lime only, without regard to the magnesia present. Iron oxide, also, in the quantities usually met with in ordinary clays, plays an insignificant part so far

as the proportions of the constituents are concerned, and may be disregarded in the calculation.

As a practical example of the use of the above formula, let us suppose that we wish to make cement from limestone and clay of the following composition.

	Limestone.	Clay.
Lime	52.6	2.2
Magnesia	0.7	1.9
Silica	3.2	65.4
Alumina	1.0	16.5
Iron Oxide	0.3	6.1
Loss on ignition, etc.	42.2	7.9
	100.0	100.0

The silica and alumina in the limestone will require

$3.2 \times 2.5 + 1.6 = 9.6$ per cent. lime, leaving $52.6 - 9.6 = 43.0$ per cent. lime available for combination with clay.

The silica and alumina in 100 parts clay will require

$65.4 \times 2.5 + 16.5 \times 1.6 = 189.9$ parts lime. Subtracting the lime contained in the clay we have

$189.9 - 2.2 = 187.7$ parts lime required for 100 parts clay.

As the 100 parts stone contain 43 parts available lime, that amount of stone will require

$$\frac{43 \times 100}{187.7} = 22.9 \text{ parts clay.}$$

The composition of the charge and of the resulting cement may be tabulated as follows:

	100 Stone.	22.9 Clay.	122.9 Mix.	78.89 Cement.	100 Cement.
Lime	52.60	0.50	53.10	53.10	67.31
Magnesia	0.70	0.43	1.13	1.13	1.46
Silica	3.20	14.98	18.18	18.18	23.04
Alumina	1.00	3.78	4.78	4.78	6.05
Iron Oxide	0.30	1.40	1.70	1.70	2.15
Loss, etc.	42.20	1.81	44.01	....	....
	100.00	22.90	122.90	78.89	100.00

As stated above the ash of the fuel will change the composition of the resulting cement materially; analysis of the product, burned with coal, will probably show about 65 per cent lime and perhaps 24 per cent silica. This fuel-ash is, however, not uniformly distributed through the product, but attaches itself chiefly to the surfaces of the clinker. It is not, therefore, found practicable to materially raise the proportion of lime to counterbalance the silica and alumina of the ash.

It will be noted that in the above calculated analysis of raw mixture and cement the

$$\frac{\text{Lime} - 1.6 \text{ alumina}}{\text{silica}} = 2.5$$

The writer proposes to call this figure the *lime factor* of the mixture. Adoption of this factor will give cements of practically maximum quality with any materials, whether silicious or aluminous, provided the mix is finely ground and properly burned. Owing to the influence of the ash of the fuel, as above explained, the factor of finished cements will usually be found about 0.2 lower than that of the raw material. Commercial cements generally show a factor of 2.3 to 2.4, though made from mixtures with a factor of 2.5 to 2.6.

The following analyses, taken from a paper by the writer in *Cement and Engineering News*, November, 1901, show the influence of the fuel-ash on the composition of the clinker. The samples of clinker were taken one hour later than those of raw material, since the passage through the kiln required about one hour.

Lehigh Portland Cement Company, Allentown, Pa.

	Mix.	Clinker, calculated from mix.	Clinker found.
SiO_2	14.33	22.18	22.96
Al_2O_3	4.32	6.68	6.78
Fe_2O_3	1.46	2.26	2.54
CaO	42.69	66.08	63.95
MgO and SO_3	1.81	2.80	2.94
Loss	35.14	....	....
	99.75	100.00	99.17
Factor $= \frac{\text{Lime}-1.6 \text{ Alumina}}{\text{silica}} =$	....	2.50	2.31

Sandusky Portland Cement Company, Syracuse, Ind.

	Mix.	Clinker calculated from mix.	Clinker found.
SiO_2	13.50	22.02	22.33
Al_2O_3	3.43	5.60	5.53
Fe_2O_3	1.27	2.07	3.28
CaO	40.76	66.49	64.40
MgO and SO_2	3.27	3.82	3.61
Loss	38.30	....	....
	100.53	100.00	99.15
Factor $= \dfrac{\text{Lime}-1.6 \text{ Alumina}}{\text{silica}} =$	......	2.61	2.48

Comparison of the above analyses of mix and clinker shows how greatly the ash of the fuel affects the composition. In commercial cement a still further reduction in the proportion of lime is caused by the addition of gypsum and the absorption of moisture and carbonic acid from the air. It will be readily seen, therefore, that analysis of finished cement gives but little indication of the true proportion of ingredients or of the quality of the product.

EFFECT OF COMPOSITION ON QUALITY

Too high proportion of lime (lime factor of mix above 2.6) will give a slow-setting cement which may fail in the steam test. If the excess of lime is great, pats of cement kept in cold water will show radial expansion cracks at the edges after a certain time, perhaps even within a few days. The same defects result from *imperfect grinding of the raw material*, and are far more often due to this cause than to excess of lime. Cement which is unsound and shows expansion from either cause may be improved and perhaps made sound by storage or by exposure to air. It is not, however, safe to rely greatly on this remedy. Lack of soundness is in all cases due to faulty manufacture, since well-burned cement made from suitably prepared raw materials will invariably pass all soundness tests when fresh from the grinding mills. Consumers are advised to accept no cement which fails to pass a reasonable steam test, as they will thus err, if at all, on the safe side, and will influence careless manufacturers to improve their methods.

Too low proportion of lime, giving an over-clayed mixture, produces a fusible clinker, liable to overburning. This is especially the case with aluminous materials. If hard-burned, such mixtures give a fused clinker

liable to fall to dust on cooling, hard to grind, and yielding slow-setting cement of poor hardening properties. If light-burned, an over-clayed mixture yields soft brownish clinker, grinding to a brownish, quick-setting cement of inferior strength.

Overburning rarely occurs except with over-clayed mixtures or in consequence of the fluxing action of the fuel-ash or the brick lining of the kiln. Properly proportioned mixtures stand a very high heat without injury.

Underburning, as stated above, in the case of an over-clayed mixture, yields quick-setting and weak cement. Normal mixtures, when underburned, usually give cement which fails in soundness tests. Light burning is generally indicated by heating of the cement on mixing with water. This behavior generally accompanies quick-setting, and may be so marked as to be quite apparent to the touch of the fingers. Some cements, though slow-setting when first made, become very quick-setting on storage. Cases are on record in which this change has taken place within a few days. After longer periods the original slow-setting quality may return.

Cements which have become quick-setting may often be restored to normal set by the addition of 1 or 2 per cent. of hydrated lime.

CHAPTER VI

SPECIFICATIONS AND TESTS OF CEMENTS

In this chapter is presented the recent report on Standard Tests of Cement together with detailed information on testing.

The tests which are regarded as most suitable for the selection and acceptance of cement for important concrete construction are as follows:

Chemical analysis.
Specific gravity.
Fineness.
Soundness or constancy of volume.
Activity, or time of setting.
Tensile or compressive strength of sand mortars.

The French Commission* in 1893, in addition to these tests, gave standard rules for testing weight, homogeneity (with the microscope), compressive strength, bending strength, yield of paste and mortar (*rendement*), porosity, permeability, decomposition, and adhesion, one or more of which tests may be desirable under certain conditions. As these are usually unimportant, only a brief description is given.

In unimportant construction it is sometimes safe to use a first-class American Portland cement without testing, and in other cases the test for soundness is the only one which need be actually made. Under almost all circumstances, however, when purchasing cement, full specifications are advisable, so that if the cement does not work satisfactorily it may be more carefully examined and unused portions rejected.

In this chapter are presented, in addition to the description of the methods of making cement tests, complete lists of apparatus for a large and a small laboratory (p. 83), formulas and tables for determining the quantity of water in cement mortars (p. 89), comparisons of American and European practice in cement testing, a discussion of the causes of unsoundness and the results of soundness tests (p. 103), curves showing the growth in strength of typical cements and cement mortars (p. 100), and other information with reference to the qualities and testing of Portland cement.

* Commission des Méthodes d'Essai des Matériaux de Construction, 1894, Vol. I, p. 235.

The following specifications and methods of tests for Portland cement were adopted in 1916 by the American Society for Testing Materials as a result of the recommendation of Committee C-1 of that Society acting in coöperation with special committees representing the Board of Direction of the American Society of Civil Engineers and the Government Departmental Committee. A few comments by the authors are inserted.

SPECIFICATIONS AND METHODS OF TESTS FOR PORTLAND CEMENT

SPECIFICATIONS

Definition. 1. Portland cement is the product obtained by finely pulverizing clinker produced by calcining to incipient fusion, an intimate and properly proportioned mixture of argillaceous and calcareous materials, with no additions subsequent to calcination excepting water and calcined or uncalcined gypsum.

I. Chemical Properties

Chemical Limits. 2. The following limits shall not be exceeded:

Loss on ignition, per cent	4.00
Insoluble residue, per cent	0.85
Sulfuric anhydride (SO_3), per cent	2.00
Magnesia (MgO), per cent	5.00

II. Physical Properties and Tests

Specific Gravity. 3. The specific gravity of cement shall be not less than 3.10 (3.07 for White Portland). Should the test of cement as received fall below this requirement, a second test may be made upon an ignited sample.

The specific gravity test will not be made unless specifically ordered.

Fineness. 4. The residue on a standard No. 200 sieve shall not exceed 22 per cent. by weight.

Soundness. 5. A pat of neat cement shall remain firm and hard, and show no signs of distortion, cracking, checking, or disintegration in the steam test for soundness.

Time of Setting. 6. The cement shall not develop initial set in less than 45 minutes when the Vicat needle is used or 60 minutes when the Gillmore needle is used. Final set shall be attained within 10 hours.

***Tensile Strength.** 7. The average tensile strength in pounds per square inch of not less than three standard mortar briquettes (see Par. 51) composed of one part cement and three parts standard sand, by weight, shall be equal to or higher than the following:

* See p. 80 for compressive strength recommendations.

Age at Test, days.	Storage of Test Pieces.	Tensile Strength, lb. per sq. in.
7	1 day in moist air, 6 days in water	200
28	1 day in moist air, 27 days in water..................	300

8. The average tensile strength of standard mortar at 28 days shall be higher than the strength at 7 days.

Strength requirements for neat cement are omitted in these specifications because the committee decided them to be unnecessary.

III. PACKAGES, MARKING AND STORAGE

9. The cement shall be delivered in suitable bags or barrels with the brand and name of the manufacturer plainly marked thereon, unless shipped in bulk. A bag shall contain 94 lb. net. A barrel shall contain 376 lb. net. **Packages and Marking.**

10. The cement shall be stored in such a manner as to permit easy access for proper inspection and identification of each shipment, and in a suitable weather-tight building which will protect the cement from dampness. **Storage.**

IV. INSPECTION

11. Every facility shall be provided the purchaser for careful sampling and inspection at either the mill or at the site of the work, as may be specified. At least 10 days from the time of sampling shall be allowed for the completion of the 7-day test, and at least 31 days shall be allowed for the completion of the 28-day test. The cement shall be tested in accordance with the methods hereinafter prescribed. The 28-day test shall be waived only when specifically ordered. **Inspection.**

V. REJECTION

12. The cement may be rejected if it fails to meet any of the requirements of these specifications. **Rejection.**

13. Cement shall not be rejected on account of failure to meet the fineness requirement if upon retest after drying at 100°C. for one hour it meets this requirement.

14. Cement failing to meet the test for soundness in steam may be accepted if it passes a retest using a new sample at any time within 28 days thereafter.

15. Packages varying more than 5 per cent. from the specified weight may be rejected; and if the average weight of packages in any shipment, as shown by weighing 50 packages taken at random, is less than that specified, the entire shipment may be rejected.

METHODS OF TESTS

VI. SAMPLING

16. Tests may be made on individual or composite samples as may be ordered. Each test sample should weigh at least (8) pounds, **Number of Samples.**

Method of Sampling. 17. (*a*) *Individual Sample.*—If sampled in cars one test sample shall be taken from each 50 bbl. or fraction thereof. If sampled in bins one sample shall be taken from each 100 bbl.

17. (*b*) *Composite Sample.*—If sampled in cars one sample shall be taken from one sack in each 40 sacks (or 1 bbl. in each 10 bbl.) and combined to form one test sample. If sampled in bins or warehouses one test sample shall represent not more than 200 bbl.

18. Cement may be sampled at the mill by any of the following methods that may be practicable, as ordered:

(*a*) *From the Conveyor Delivering to the Bin.*—At least 8 lb. of cement shall be taken from approximately each 100 bbl. passing over the conveyor.

FIG. 8. Sampling Iron.

(See p. 64.)

(*b*) *From Filled Bins by Means of Proper Sampling Tubes.*—Tubes inserted vertically may be used for sampling cement to a maximum depth of 10 ft. Tubes inserted horizontally may be used where the construction of the bin permits. Samples shall be taken from points well distributed over the face of the bin.

(*c*) *From Filled Bins at Points of Discharge.*—Sufficient cement shall be drawn from the discharge openings to obtain samples representative of the cement contained in the bin, as determined by the appearance at the discharge openings of indicators placed on the surface of the cement directly above these openings before drawing of the cement is started.

Treatment of Sample. 19. Samples preferably shall be shipped and stored in air-tight containers. Samples shall be passed through a sieve having 20 meshes per linear inch in order to thoroughly mix the sample, break up lumps and remove foreign materials.

A sampling iron is illustrated in Fig. 8.

VII. Chemical Analysis

Loss on Ignition

Method. 20. One gram of cement shall be heated in a weighed covered platinum crucible, of 20 to 25 cc. capacity, as follows, using either method (*a*) or (*b*) as ordered:

(*a*) The crucible shall be placed in a hole in an asbestos board, clamped horizontally so that about three-fifths of the crucible projects below, and blasted at a full red heat for 15 minutes with an inclined flame; the loss in weight shall be checked by a second blasting for 5 minutes. Care shall be taken to wipe off particles of asbestos that may adhere to the crucible when withdrawn from the hole in the board. Greater neatness and shortening of the time of heating are secured by making a hole to fit the crucible in a circular disk of sheet platinum and placing this disk over a somewhat larger hole in an asbestos board.

(*b*) The crucible shall be placed in a muffle at any temperature between 900 and 1000°C. for 15 minutes and the loss in weight shall be checked by a second heating for 5 minutes.

Permissible Variation. 21. A permissible variation of 0.25 per cent will be allowed and all results in excess of the specified limit but within this permissible variation shall be reported as 4 per cent.

Insoluble Residue

22. To a 1-g. sample of cement shall be added 10 cc. of water and 5 cc. of concentrated hydrochloric acid; the liquid is warmed until effervescence ceases. The solution shall be diluted to 50 cc. and digested on a steam bath or hot plate until it is evident that decomposition of the cement is complete. The residue shall be filtered, washed with cold water, and the filter paper and contents digested in about 30 cc. of a 5-per-cent solution of sodium carbonate, the liquid being held at a temperature just short of boiling for 15 minutes. The remaining residue shall be filtered, washed with cold water, then with a few drops of hot hydrochloric acid, 1:9, and finally with hot water, and then ignited at a red heat and weighed as the insoluble residue. **Method.**

23. A permissible variation of 0.15 per cent will be allowed and all results in excess of the specified limit but within this permissible variation shall be reported as 0.85 per cent. **Permissible Variation.**

Sulfuric Anhydride

24. One gram of the cement shall be dissolved in 5 cc. of concentrated hydrochloric acid diluted with 5 cc. of water, with gentle warming; when solution is complete 40 cc. of water shall be added, the solution filtered, and the residue washed thoroughly with water. The solution shall be diluted to 250 cc., heated to boiling and 10 cc. of a hot 10-per-cent solution of barium chloride shall be added slowly, drop by drop, from a pipette and the boiling continued until the precipitate is well formed. The solution shall be digested on the steam bath until the precipitate has settled. The precipitate shall be filtered, washed, and the paper and contents shall be placed in a weighed platinum crucible and the paper slowly charred and consumed without flaming. The barium sulfate shall be then ignited and weighed. The weight obtained multiplied by 34.3 gives the percentage of sulfuric anhydride. The acid filtrate obtained in the determination of the insoluble residue may be used for the estimation of sulfuric anhydride instead of using a separate sample. **Method.**

25. A permissible variation of 0.10 per cent will be allowed and all results in excess of the specified limit but within this permissible variation shall be reported as 2.00 per cent. **Permissible Variation.**

Magnesia

26. To 0.5 g. of the cement in an evaporating dish shall be added 10 cc. of water to prevent lumping and then 10 cc. of concentrated hydrochloric acid. The liquid shall be gently heated and agitated until attack is complete. The solution shall be then evaporated to complete dryness on a steam or water bath. To hasten dehydration the residue may be heated to 150 or even 200°C. for one-half to one hour. The residue shall be treated with 10 cc. of concentrated hydrochloric acid diluted with an equal amount of water. The dish shall be covered and the solution digested for ten minutes on a steam bath or water bath. The diluted solution shall be filtered and the separated silica washed thoroughly with water.* Five cubic centimeters of concentrated hydrochloric acid and sufficient bromine water to precipitate any manganese which may be present, shall be added to the filtrate (about 250 cc.). This is made alkaline with ammonium hydroxide, boiled until there is but a faint odor of ammonia and the precipitate iron and aluminum hydroxides, after settling, are washed with hot water, once by decantation and slightly on the filter. Setting aside the filtrate, the precipitate shall be transferred by a jet of hot **Method.**

* Since this procedure does not involve the determination of silica, a second evaporation is unnecessary.

water to the precipitating vessel and dissolved in 10 cc. of hot hydrochloric acid. The paper shall be extracted with acid, the solution and washings being added to the main solution. The aluminum and iron shall be then reprecipitated at boiling heat by ammonium hydroxide and bromine water in a volume of about 100 cc., and the second precipitate shall be collected and washed on the filter used in the first instance if this is still intact. To the combined filtrates from the hydroxides of iron and aluminum, reduced in volume if need be, 1 cc. of ammonium hydroxide shall be added, the solution is brought to boiling, 25 cc. of a saturated solution of boiling ammonium oxalate added, and the boiling continued until the precipitated calcium oxalate has assumed a well-defined granular form. The precipitate after one hour shall be filtered and washed, then with the filter is placed wet in a platinum crucible, and the paper burned off over a small flame of a Bunsen burner; after ignition it is redissolved in hydrochloric acid and the solution diluted to 100 cc. Ammonia shall be added in slight excess, and the liquid boiled. The lime shall be then reprecipitated by ammonium oxalate, allowed to stand until settled, filtered and washed. The combined filtrates from the calcium precipitates shall be acidified with hydrochloric acid, concentrated on the steam bath to about 150 cc., and made slightly alkaline with ammonium hydroxide, boiled and filtered (to remove a little aluminum and iron and perhaps calcium). When cool, 10 cc. of saturated solution of sodium-ammonium-hydrogen phosphate shall be added with constant stirring. When the crystallin ammonium-magnesium orthophosphate has formed, ammonia shall be added in moderate excess. The solution shall be set aside for several hours in a cool place, filtered and washed with water containing 2.5 per cent of NH_3. The precipitate shall be dissolved in a small quantity of hot hydrochloric acid, the solution diluted to about 100 cc., 1 cc. of a saturated solution of sodium-ammonium-hydrogen phosphate added, and ammonia drop by drop, with constant stirring, until the precipitate is again formed as described and the ammonia is in moderate excess. The precipitate shall be then allowed to stand about two hours, filtered and washed as before. The paper and contents shall be placed in a weighed platinum crucible, the paper slowly charred and the resulting carbon carefully burned off. The precipitate shall be then ignited to constant weight over a Meker burner, or a blast not strong enough to soften or melt the pyrophosphate. The weight of magnesium pyrophosphate obtained multiplied by 72.5 gives the percentage of magnesia. The precipitate so obtained always contains some calcium and usually small quantities iron, aluminum, and manganese as phosphates.

Permissible Variation. 27. A permissible variation of 0.4 per cent will be allowed and all results in excess of the specified limit but within this permissible variation shall be reported as 5.00 per cent.

A simple test which sometimes may determine adulteration with raw or partially burned rock, is the purity test* with muriatic acid. It does not furnish the percentage of foreign ingredients, but the black precipitation of the adulterant darkens the color of the yellow jelly to a degree depending upon the quantity of adulteration.

* Described in "Concrete Plain and Reinforced" 2nd edition, p. 4.

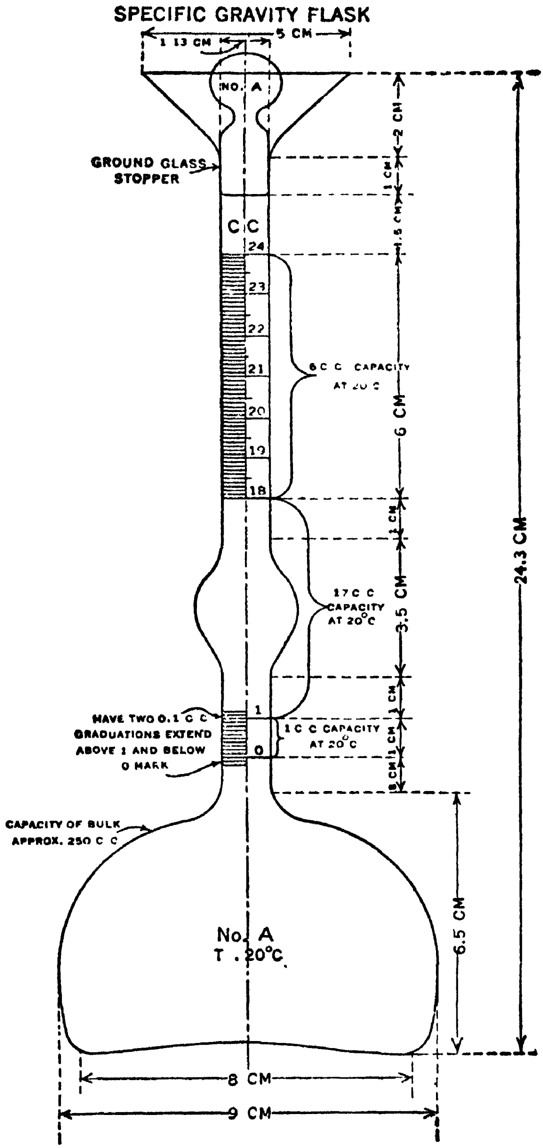


FIG. 9. Le Chatelier's Specific-Gravity Apparatus. (*See p.* 68)

VIII. Determination of Specific Gravity

Apparatus. 28. The determination of specific gravity shall be made with a standardized Le Chatelier apparatus which conforms to requirements as illustrated in figure 9. This apparatus is standardized by the United States Bureau of Standards. Kerosene free from water or benzine not lighter than 62° Baume, shall be used in making this determination.

Method. 29. The flask shall be filled with either of these liquids to a point on the stem between zero and one cc., and 64 g. of cement cooled to the temperature of the liquid shall be slowly introduced, taking care that the cement does not adhere to the inside of the flask above the liquid and to free the cement from air by rolling the flask in an inclined position. After all the cement is introduced, the level of the liquid will rise to some division of the graduated neck; the difference between readings is the volume displaced by 64 g. of the cement. The specific gravity shall be then obtained from the formula:

$$\text{Specific gravity} = \frac{\text{Weight of cement (g.)}}{\text{Displaced volume, (cc.)}}$$

30. The flask, during the operation shall be kept immersed in water, in order to avoid variations in the temperature of the liquid in the flask which shall not exceed 0.5°C. The results of repeated tests should agree within 0 01.

31. The determination of specific gravity shall be made on the cement as received; if it should fall below 3.10, a second determination shall be made after igniting the sample as described in section 20.

Mr. Daniel D. Jackson has more recently devised an apparatus with which temperature corrections can be made more readily than with the older types. This is described on page 85.

IX. Determination of Fineness

Apparatus. 32. Wire cloth for standard sieves for cement shall be woven (not twilled) from brass, bronze, or other suitable wire, and mounted without distortion on frames not less than 1½ in. below the top of the frame. The sieve frames shall be circular, approximately 8 in. in diameter, and may be provided with a pan and cover.

33. A standard No. 200 sieve is one having nominally an 0.0029-in. opening and 200 wires per in., standardized by the U. S. Bureau of Standards, and conforming to the following requirements:

The No. 200 sieve should have 200 wires per inch, and the number of wires in any whole inch shall not be outside the limits of 192 to 208. No opening between adjacent parallel wires shall be more than 0.0050 in. in width. The diameter of the wire should be 0.0021 in. and the average diameter shall not be outside the limits 0.0019 in. to 0.0023 in. The value of the sieve as determined by sieving tests made in conformity with the standard specification for these tests on a standardized cement which gives a residue of 25 to 20 per cent on the No. 200 sieve, or on other similarly graded material, shall not show a variation of more than 1.5 per cent above or below the standards maintained at the Bureau of Standards.

Method. 34. The test shall be made with 50 g. of cement. The sieve shall be thoroughly clean and dry. The cement shall be placed on the No. 200 sieve, with pan and cover

attached, if desired, and shall be held in one hand in a slightly inclined position so that the sample will be well distributed over the sieve, at the same time gently striking the side about 150 times per minute against the palm of the other hand on the up stroke. The sieve shall be turned every 25 strokes about one-sixth of a revolution in the same direction. The operation shall continue until not more than 0.05 g. passes through in one minute of continuous sieving. The fineness shall be determined from the weight of the residue on the sieve expressed as a percentage of the weight of the original sample.

35. Mechanical sieving devices may be used, but the cement shall not be rejected if it meets the fineness requirement when tested by the hand method.

36. A permissible variation of 1 per cent. is allowed, and all results in excess of the specified limit, but within this shall be reported as 22 per cent. **Permissible Variation.**

Laboratory scales for weighing the samples and the residue are illustrated in Fig. 10.

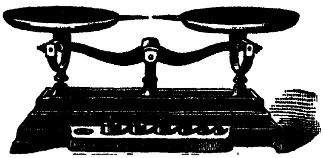


FIG. 10.—Delicate Laboratory Scales. (*See p.* 69.)

Fine grinding has a number of advantages, chief among which is the increased strength of sand mortars. This is further discussed on page 87.

It is impracticable to sift cement through a sieve finer than 200 meshes per linear inch. The particles which will just pass a No. 200 sieve are about 0.10 millimeter (0.004 inches) in diameter.* For separating the grains still finer than the No. 200 sieve, air analysis may be employed. This is briefly described on page 88.

X. MIXING CEMENT PASTES AND MORTARS

37. The quantity of dry material to be mixed at one time shall not exceed 1000 g. nor be less than 500 g. The proportions of cement or cement and sand shall be stated by weight in grams of the dry materials; the quantity of water shall be expressed in cubic centimeters (1 g. = 1 cc.). The dry materials shall be weighed, placed upon a non-absorbent surface, thoroughly mixed dry if sand be used, and a crater formed in the center, into which the proper percentage of clean water shall be poured; the material on the outer edge shall be turned into the crater by the aid of a trowel. After an interval of ½ minute for the absorption of the water the operation shall be completed by continuous, vigorous mixing, squeezing and kneading **Method.**

* Allen Hazen in Report Massachusetts State Board of Health, 1892.

with the hands for at least one minute.* During the operation of mixing, the hands should be protected by rubber gloves.

38. The temperature of the room and the mixing water shall be maintained as nearly as practicable at 21°C. (70°F.).

The apparatus required for mixing briquettes consists of a piece of 1-inch plate glass at least 24 inches square, counter scales (preferably metric system), recording from $\frac{1}{10}$ gram to $1\frac{1}{2}$ kilograms, a 250 cubic centimeter graduated measuring glass, rubber gloves, one 8-inch mason's trowel, one 4-inch pointing trowel, Fig. 11, and a thermometer.

FIG. 11. (*See p.* 70.)

European standards specify mixing five minutes instead of one minute. This difference in time is due to the methods of manipulation, in Europe the materials being mixed with a trowel or spoon. Experiments by the authors tend to show that a denser mixture can be obtained by kneading one minute than by mixing five minutes with a trowel, so that the American method is both quicker and better.

XI. NORMAL CONSISTENCY

Apparatus. 39. The Vicat apparatus consists of a frame (*A*) (Fig. 12) bearing a movable rod (*B*,) weighing 300 g., one end (*C*) being 1 cm. in diameter for a distance of 6 cm., the other having a removable needle (*D*), 1 mm. in diameter, 6 cm. long. The rod is reversible, and can be held in any desired position by a screw (*E*), and has midway between the ends a mark (*F*) which moves under a scale (graduated to millimeters) attached to the frame (*A*). The paste is held in a conical, hard-rubber ring (*G*), 7 cm. in diameter at the base, 4 cm. high, resting on a glass plate (*H*) about 10 cm. square.

Method. 40. In making the determination, 500 g. of cement, with a measured quantity of water, shall be kneaded into a paste, as described in Section 37, and quickly formed into a ball with the hands, completing the operation by tossing it six times from one hand to the other, maintained about 6 in. apart; the ball resting in the palm of one hand shall be pressed into the larger end of the rubber ring held in the other hand, completely filling the ring with paste; the excess at the larger end shall be then removed by a single movement of the palm of the hand; the ring shall be then placed on its larger end on a glass plate and the excess paste at the smaller end sliced off at the top of the ring by a single oblique stroke of a trowel held at a slight angle with the top of the ring. During these operations care shall be taken not to compress the paste. The paste confined in the ring, resting on the plate, shall be placed under the rod, the larger end of which shall be brought in contact with the surface of the paste; the scale shall be then read, and the rod quickly released. The paste shall be of normal consistency when the cylinder

* In order to secure uniformity in the results of tests for the time of setting and tensile strength the manner of mixing above described should be carefully followed. At least one minute is necessary to obtain the desired plasticity which is not appreciably affected by continuing the mixing for several minutes. The exact time necessary is dependent upon the personal equation of the operator. The error in mixing should be on the side of over mixing.

settles to a point 10 mm. below the original surface in ½ minute after being released. The apparatus shall be free from all vibrations during the test. Trial pastes shall be made with varying percentages of water until the normal consistency is obtained. The amount of water required shall be expressed in percentage by weight of the dry cement.

41. The consistency of standard mortar shall depend on the amount of water required to produce a paste of normal consistency from the same sample of cement. Having determined the normal consistency of the sample, the consistency of standard mortar made from the same sample shall be as indicated in Table I, the values being in percentage of the combined dry weights of the cement and standard sand.

TABLE I.—PERCENTAGE OF WATER FOR STANDARD MORTARS

Percentage of Water for Neat Cement Paste of Normal Consistency.	Percentage of Water for One Cement, Three Standard Ottawa Sand	Percentage of Water for Neat Cement Paste of Normal Consistency.	Percentage of Water for One Cement, Three Standard Ottawa Sand.
15	9.0	23	10.3
16	9.2	24	10.5
17	9.3	25	10.7
18	9.5	26	10.8
19	9.7	27	11.0
20	9.8	28	11.2
21	10.0	29	11.3
22	10.2	30	11.5

Formulas of Mr. Feret for determining the percentage of water for sand mortars, and a table formerly used, are presented on page 89.

The Boulogne Method for determining the proper consistency of neat paste was formerly in general use in France, and is still the best guide for determining the correct consistency of paste when the Vicat apparatus is not available. The Vicat apparatus, however, should be included in every well equipped cement laboratory, experiments by Messrs. P. Alexandre and R. Feret for the French Commission* showing that it gives much more uniform results than the Boulogne method.

The Boulogne method requires that the paste shall be firm but well bonded, shining and plastic, and shall satisfy the following conditions:

1. The consistency shall not change if it is worked 3 minutes longer than the original 5 minutes.†

2. If dropped 50 centimeters (20 in.) from a trowel, it should leave the trowel clean, and fall without losing its shape or cracking.

3. Light pressure in the hand should bring water to the surface, and the paste should not stick to the hand. If a ball thus formed falls from

* Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 49.

† The original working for the U. S. Standard tests is one minute (see paragraph 37.)

a height of about 50 centimeters (20 in.) it should retain its rounded form without showing cracks.

4. The proportion of water should be such that more or less will produce opposite effects from those of the proper consistency.

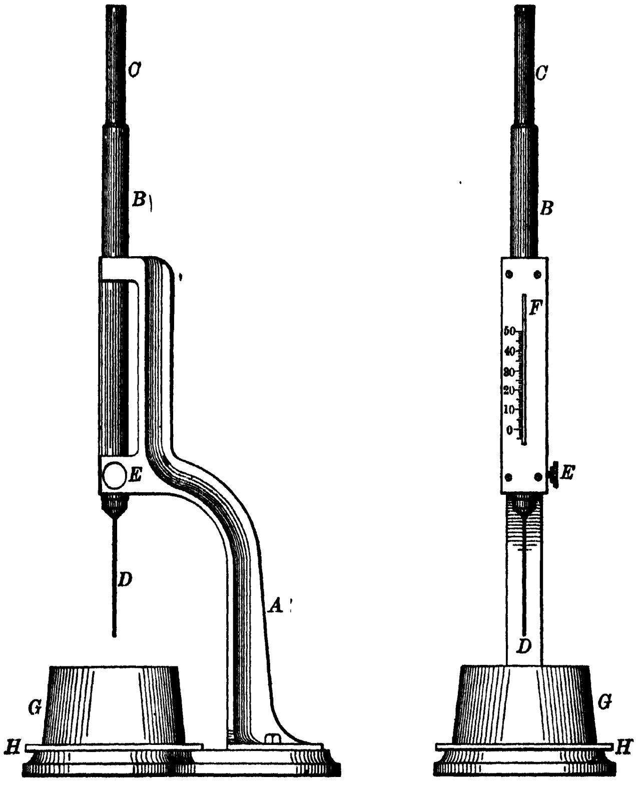


FIG. 12.—Vicat Apparatus. (*See p.* 70.)

XII. DETERMINATION OF SOUNDNESS*

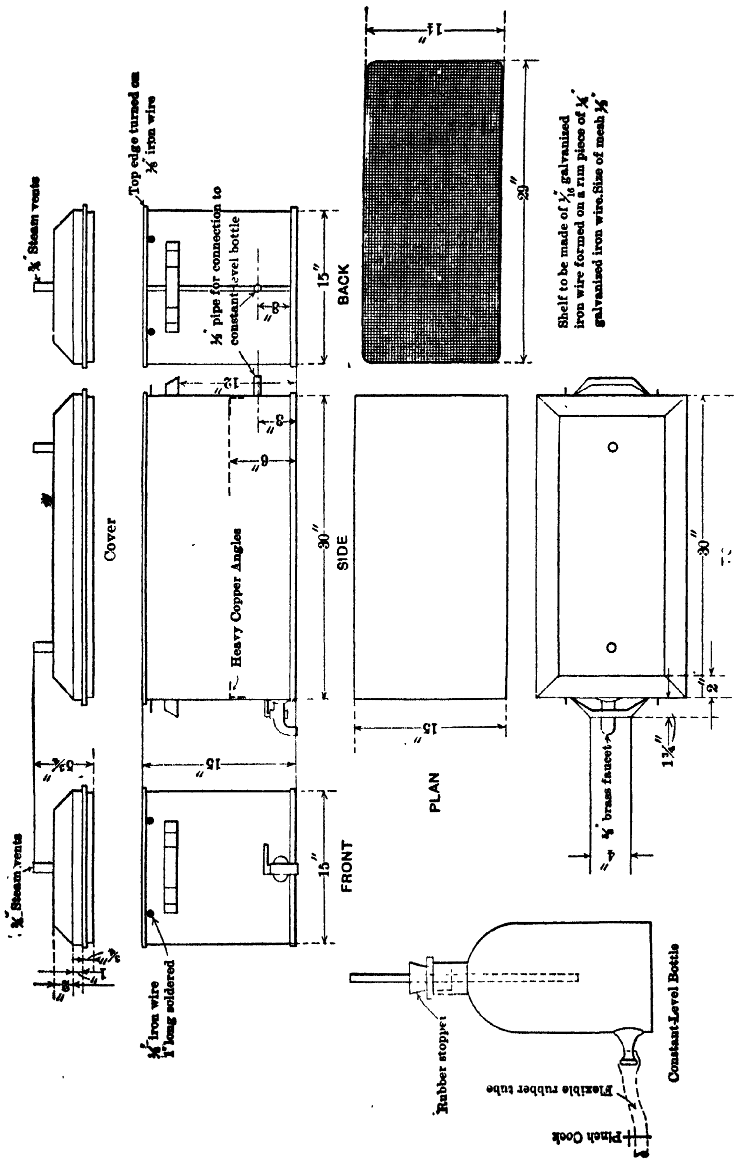
Apparatus. 42. A steam apparatus, which can be maintained at a temperature between 98 and 100°C., or one similar to that shown in Fig. 13 is recommended.† The capacity

* Unsoundness is usually manifested by change in volume which causes distortion, cracking, checking or disintegration.

Pats improperly made or exposed to drying may develop what are known as shrinkage cracks within the first 24 hours and are not an indication of unsoundness. These conditions are illustrated in Figure 14.

The failure of the pats to remain on the glass or the cracking of the glass to which the pats are attached does not necessarily indicate unsoundness.

† A loosely covered vessel gives good results. Authors.



To be made of sheet copper weighing 22 oz. per sq. ft., tinned inside. All seams to be lapped where possible. Hard solder only to be used.

FIG. 13.—Apparatus for Making Accelerated Test for Soundness of Cement. (*See p.* 72.)

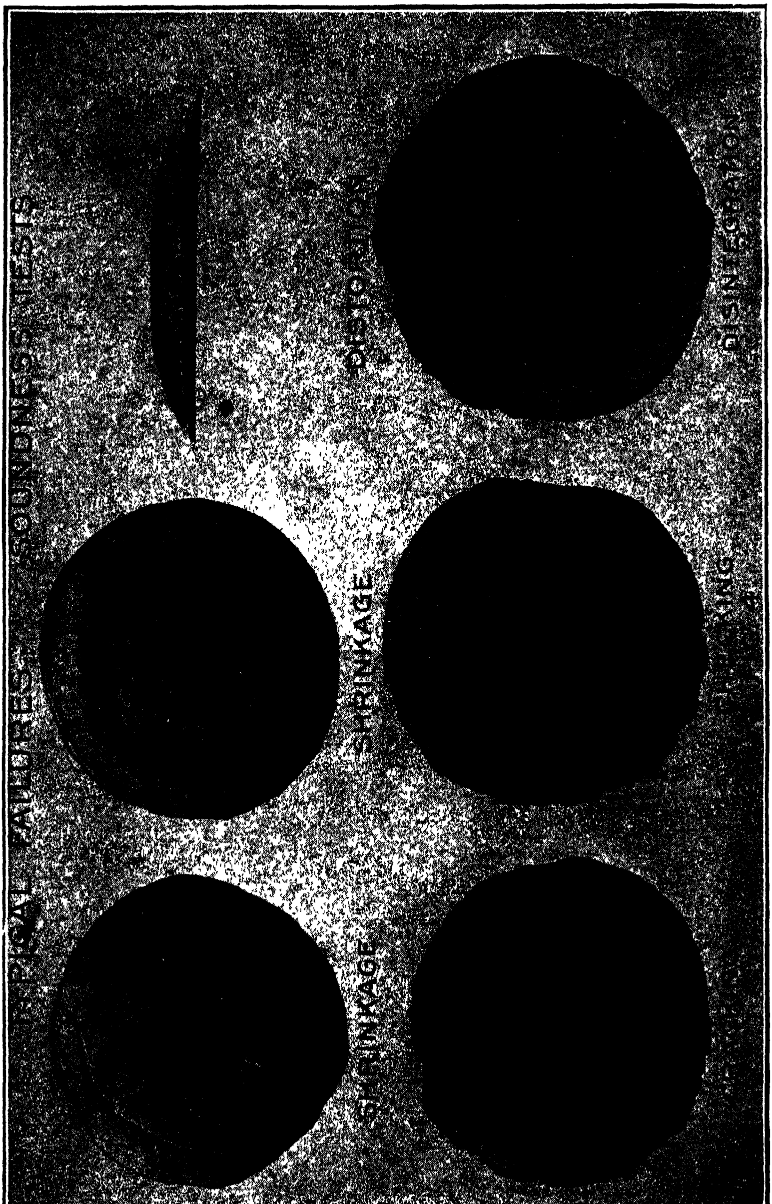


Fig. 14.—Typical Failures in Soundness Test. (*See p.* 72.)

of this apparatus may be increased by using a rack for holding the pats in a vertical or inclined position.

43. A pat from cement paste of normal consistency about 3 in. in diameter, $\frac{1}{2}$ in. thick at the center, and tapering to a thin edge, shall be made on clean glass plates about 4 in. square, and stored in moist air for 24 hours. In molding the pat, the cement paste shall first be flattened on the glass and the pat then formed by drawing the trowel from the outer edge toward the center. **Method.**

44. The pat shall then be placed in an atmosphere of steam at a temperature between 98 and 100°C. upon a suitable support 1 in. above boiling water for five hours.

45. Should the pat leave the plate, distortion may be detected best with a straight edge applied to the surface which was in contact with the plate.

XIII. Determination of Time of Setting

46. The following are alternate methods, either of which may be used as ordered:

47. The time of setting shall be determined with the Vicat apparatus described in Section 39. (See figure 12.) **Vicat Apparatus**

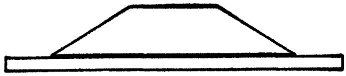
48. A paste of normal consistency shall be molded in the hard-rubber ring (*G*) as described in Section 40, and placed under the rod (*B*), the smaller end of which shall be then carefully brought in contact with the surface of the paste, and the rod quickly released. The initial set shall be said to have occurred when the needle ceases to pass a point 5 mm. above the glass plate in one-half minute after being released; and the final set, when the needle does not sink visibly into the paste. The test pieces shall be kept in moist air during the test. This may be accomplished by placing them on a rack over water contained in a pan and covered by a damp cloth, kept from contact with them by means of a wire screen; or they may be stored in a moist closet. Care shall be taken to keep the needle clean, as the collection of cement on the sides of the needle retards the penetration, while cement on the point may increase the penetration. The time of setting is affected not only by the percentage and temperature of the water used and the amount of kneading the paste receives, but by the temperature and humidity of the air, and its determination is therefore only approximate. **Vicat Method.**

49. The time of setting shall be determined by the Gillmore needles. The Gillmore needles should preferably be mounted as shown in Fig. 15 (*b*). **Gillmore Apparatus.**

50. The time of setting shall be determined as follows: A pat of neat cement paste about 3 in. in diameter and $\frac{1}{2}$ in. in thickness with a flat top (Fig. 15 (*a*)), mixed to a normal consistency, shall be kept in moist air, at a temperature maintained as nearly as practicable at 21°C. (70° F.). The cement is considered to have acquired its initial set when the pat will bear, without appreciable indentation, the Gillmore needle $\frac{1}{12}$ in. in diameter, loaded to weigh $\frac{1}{4}$ lb. The final set has been acquired when the pat will bear without appreciable indentation, the Gillmore needle $\frac{1}{24}$ in. in diameter, loaded to weigh 1 lb. In making the test, the needles should be held in a vertical position, and applied lightly to the surface of the pat.

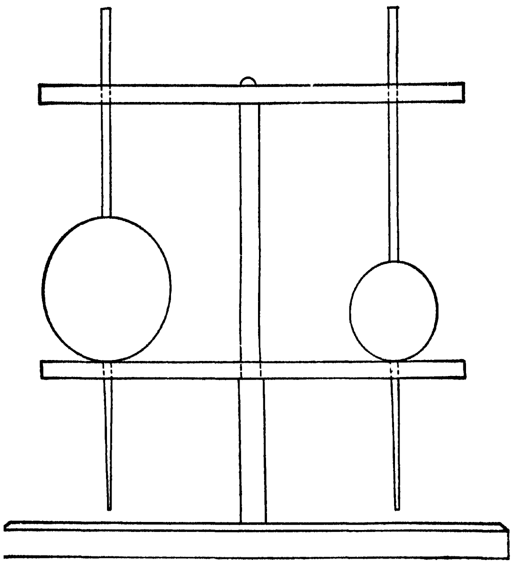
For practical purposes in ordinary construction where laboratory apparatus is unavailable, the setting qualities of a cement or mortar may often be examined by making up pats from a number of the pack-

ages and trying their hardening by pressure of the thumb. When the thumb nail fails to indent the surface the paste or mortar may be considered to have reached its final set.



Soundness Pat with Top Surface Flattened.
For Determining Time of Setting.

(*a*)



(*b*)

FIG. 15.—Gillmore Needles and Pat. (*See p.* 75.)

XIV. TENSION TESTS

Form of Test Piece. 51. The form of test piece shown in Fig. 16 shall be used. The molds shall be made of non-corroding metal and have sufficient material in the sides to prevent spreading during molding. Gang molds when used shall be of the type shown in Fig. 17. Molds shall be wiped with an oily cloth before using.

The German standard briquette is sketched on page 96.

Standard Sand. 52. The sand to be used shall be natural sand from Ottawa, Ill., screened to pass a No. 20 sieve and retained on a No. 30 sieve. This sand may be obtained from the Ottawa Silica Co., at a cost of two cents per pound, f. o. b. cars, Ottawa, Ill.

53. This sand having passed the No. 20 sieve shall be considered standard when not

more than 5 g. pass the No. 30 sieve after one minute of continuous sieving of a 500-g. sample.

54. The sieves shall conform to the following specifications:

The No. 20 sieve shall have between 19.5 and 20.5 wires per whole inch of the warp wires and between 19 and 21 wires per whole inch of the shoot wires. The diameter

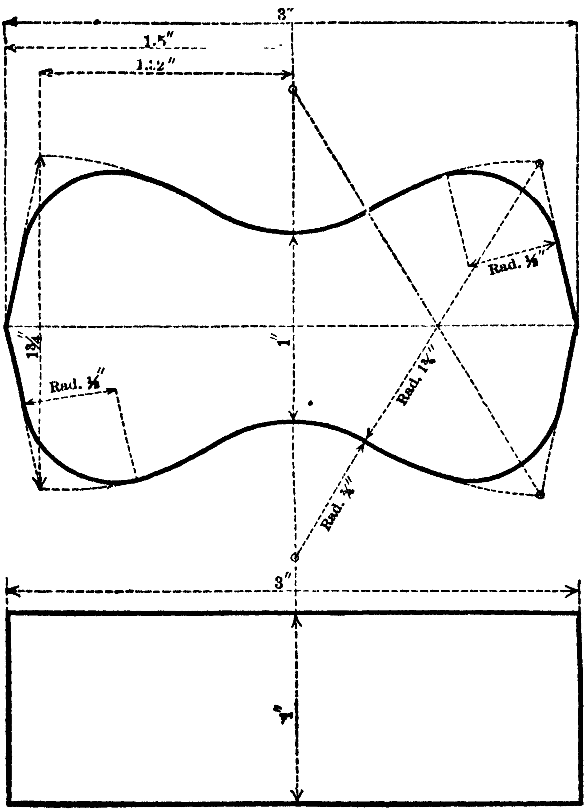


FIG. 16.—Details for Briquette. (*See p.* 76.)

of the wire should be 0.0165 in. and the average measured diameter shall not fall outside the limits of 0.0160 and 0.0170 in.

The No. 30 sieve shall have between 29.5 and 30.5 wires per whole inch of the warp wires and between 28.5 and 31.5 wires per whole inch of the shoot wires. The diameter of the wire should be 0.0110 in. and the average diameter shall not be outside the limits 0.0105 to 0.0115 in.

Photographs of the grains of Ottawa and of crushed quartz sand are shown on page 136.

European is compared with U. S. standard sand on p. 94.

Molding. 55. Immediately after mixing, the standard mortar shall be placed in the molds, pressed in firmly with the thumbs and smoothed off with a trowel without ramming. Additional mortar shall be heaped above the mold and smoothed off with a trowel; the trowel shall be drawn over the mold in such a manner as to exert a moderate pressure on the material. The mold shall then be turned over and the operation of heaping, thumbing and smoothing off repeated.

Testing. 56. Tests shall be made with any standard machine. The briquettes shall be broken as soon as they are removed from the water. The bearing surfaces of the clips and briquettes shall be free from grains of sand or dirt. The briquettes shall, be carefully centered and the load applied continuously at the rate of 600 lb. per minute.

57. Testing machines should be frequently calibrated in order to determine their accuracy.

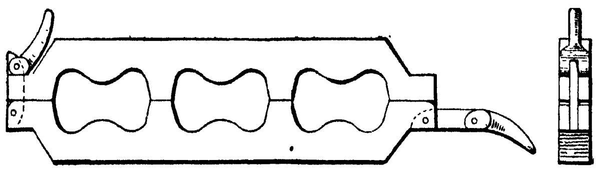


FIG. 17.—Details for Gang Molds (*See p.* 76.)

Faulty Briquettes. 58. Briquettes that are manifestly faulty, or which give strengths differing more than 15 per cent from the average value of all test pieces made from the same sample and broken at the same period shall not be considered in determining the tensile strength.

XV. Storage of Test Specimens

Apparatus. 59. A moist closet shall consist of a soapstone, slate or concrete box, or a wooden box lined with metal, the interior surface being covered with felt or broad wicking kept wet, the bottom of the box being covered with water. The interior of the closet should be provided with non-absorbent shelves on which to place the test pieces, the shelves being so arranged that they may be withdrawn readily.

Methods. 60. Unless otherwise specified all test specimens, immediately after molding, shall be placed in the moist closet for from 20 to 24 hours.

61. The briquettes shall be kept in molds on glass plates in the moist closet for at least 20 hours. After 24 hours in moist air the briquettes shall be immersed in clean water in storage tanks of non-corroding material.

62. The air and water shall be maintained as nearly as practicable at a temperature of 21°C. (70° F.).

A moist closet and storage pans designed by Mr. Richard L. Humphrey are shown in Figs. 18 and 19, page 79.

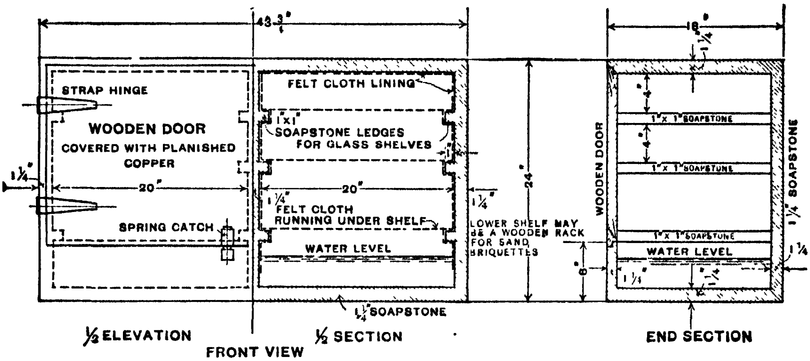


FIG. 18.—Moist Closet. (See p. 78)

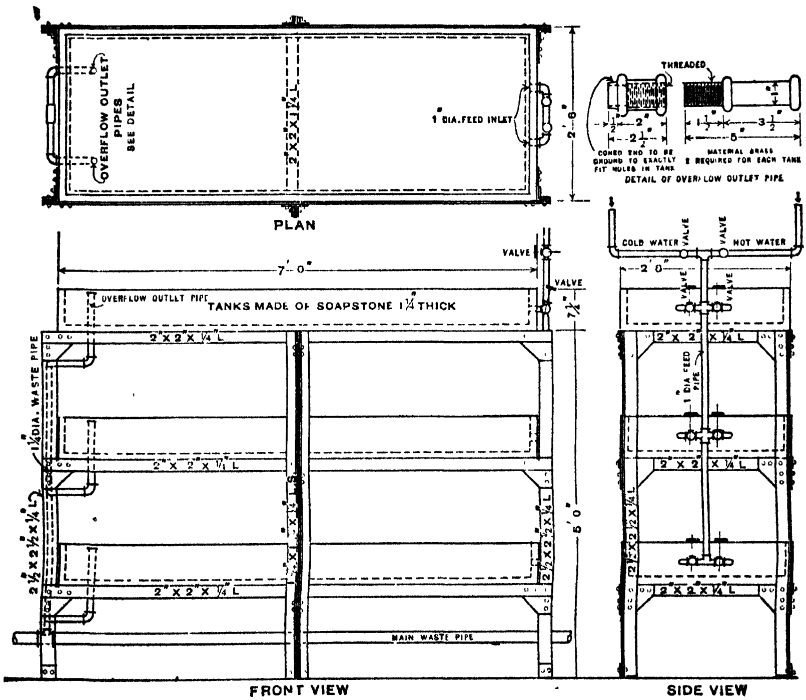


FIG. 19.—Immersion Tanks. (See p. 78)

PROPOSED TENTATIVE SPECIFICATIONS AND METHODS OF TESTS FOR COMPRESSIVE STRENGTH OF PORTLAND CEMENT MORTAR*

SPECIFICATIONS

Compressive Strength. 1. (*a*) A test piece of standard mortar composed of one part cement and three parts standard sand, by weight, shall give compressive strengths equal to or higher than the following:

Age at Test, days.	Storage of Test Pieces.	Compressive Strength, lb. per sq. in.
7	1 day in moist air, 6 days in water......	1200
28	1 day in moist air, 27 days in water.........	2000

(*b*) Each value shall be the average of the results of tests from not less than three test pieces. The compressive strength of standard mortar at the age of 28 days shall be higher than the strength determined at the age of 7 days.

METHODS OF TESTS

Mixing Standard Mortar. 2. The requirements governing the preparation of standard sand mortars for tension test pieces shall apply to compression test pieces.

Form of Test Piece. 3. A cylindrical test piece 2 in. in diameter and 4 in. in length is recommended for use in making compression tests of standard mortars. The molds shall be made of non-corroding metal. A satisfactory form of mold is shown in Fig. 20. The ends of the mold shall be parallel. The tubing used in the molds shall be of sufficient thickness to prevent appreciable distortion. The molds shall be oiled before using. During the molding of the test piece, the mold shall rest on a clean, plane surface (preferably a piece of plate glass which is allowed to remain in place until the mold is removed).

Molding. 4. The mortar† shall be placed in the mold in layers about 1 in. in thickness each layer being tamped by means of the steel tamper shown in Fig. 21. The weight of tamper is approximately $\frac{3}{4}$ lb. In finishing the test piece, the mortar shall be heaped above the mold and smoothed off with a trowel. As soon as the test pieces from one sample are molded, the top of each test piece shall be covered with a piece of glass which is brought to a firm bearing on the fresh mortar. The cover glasses shall remain in place until the molds are removed.

The compression test pieces shall be stored in the same manner as the tension test pieces.

Testing. 5. Tests of standard-mortar cylinders may be made in any testing machine which is adapted to meet the specified requirements. The test pieces shall be tested as soon as removed from the water. The ends of the test cylinders shall be smooth, plane surfaces. The metal bearing plates of the testing machine shall be

* Accepted by the American Society for Testing Materials as tentative specifications, June, 1916.

† If sufficient mortar for six 2 by 4 in. cylinders is to be mixed in a single batch, approximately 3000 g. of material will be required. In this case the mixing shall be continued for $1\frac{1}{2}$ minutes.

placed in direct contact with the ends of the test piece. During the test a spherical bearing block shall be used on top of the cylinder. In order to secure a uniform distribution of the load over the test cylinder the spherical bearing block must be accurately centered. The diameter of the spherical bearing block should be only a little greater than that of the test piece. The test piece shall be loaded continuously to failure. The moving head of the testing machine shall travel at the rate of not less than 0.05 or more than 0.10 in. per minute.

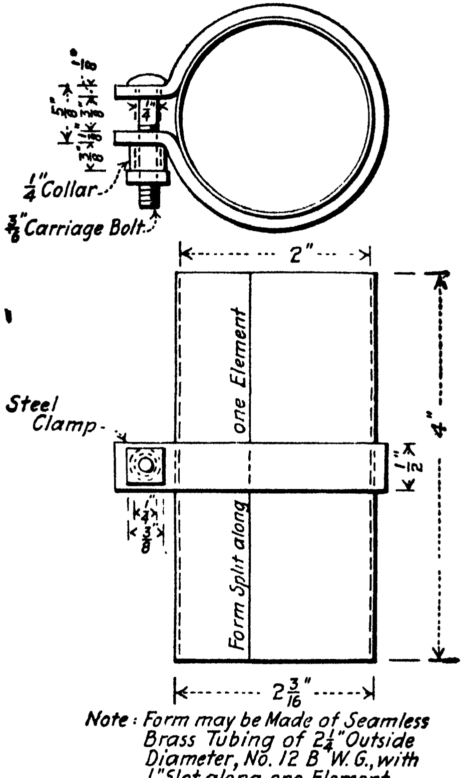


FIG. 20.—Details for 2 by 4-in. Cylinder Form (*See p.* 80.)

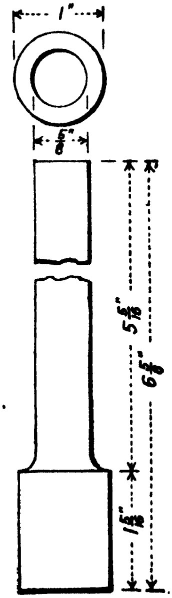


FIG. 21.—Details for Steel Tamper (*See p.* 80.)

Testing machines should be frequently calibrated in order to determine their accuracy.

Cylinders that are manifestly faulty, or which give strengths differing more than 15 per cent. from the average value of all test pieces tested at the same period and made from the same sample, shall not be considered in determining the compressive strength.

FULL SPECIFICATIONS FOR THE PURCHASE OF NATURAL CEMENT

1. **Packages.** Cement shall be packed in strong cloth or canvas sacks.† Each package shall have printed upon it the brand or the name of the manufacturer. Packages received in broken or damaged condition may be rejected or accepted as fractional packages.

2. **Weight.** Three bags shall constitute a barrel, and the average net weight of the cement contained in one bag shall be not less than 94 lb., or 282 lb. net per barrel. A cement bag may be assumed to weigh one pound. The weights of the separate packages shall be uniform.

3. **Requirements.*** Cement failing to meet the seven-day requirements may be held awaiting the results of the twenty-eight day tests before rejection.

4. **Tests.*** All tests shall be made in accordance with the methods proposed by the Committee on Uniform Tests of Cement of the American Society of Civil Engineers, presented to the Society January 21, 1903, and amended January 20, 1904, with all subsequent amendments thereto. (See Chapter VI, p. 63.)

5. **Sampling.** Samples shall be taken at random from sound packages, and the cement from each package shall be tested separately.

6.* The acceptance or rejection shall be based on the following requirements:

7. **Definition of Natural Cement.*** This term shall be applied to the finely pulverized product resulting from the calcination of an argillaceous limestone at a temperature only sufficient to drive off the carbonic acid gas.

8. **Fineness.*** It shall leave by weight a residue of not more than 10% on the No. 100, and 30% on the No. 200 sieve.

9. **Time of Setting.*** It shall not develop initial set in less than ten minutes, and shall not develop hard set in less than thirty minutes, or in more than three hours.

10. **Tensile Strength.*** The minimum requirements for tensile strength for briquettes one square inch in cross section shall be as follows, and the cement shall show no retrogression in strength within the periods specified:

*Paragraphs designated by an asterisk are quoted from the Standard Specifications of the American Society for Testing Materials.

†If the cement is to be stored in a damp place or near the sea, it must be packed in well-made wooden barrels lined with paper.

Neat Cement.

Age	Strength
24 hours in moist air	75 lb.
7 days (1 day in air, 6 days in water)	150 "
28 days (1 " " 27 " ")	250 "

One Part Cement, Three Parts Standard Ottawa Sand.

Age	Strength
7 days (1 day in air, 6 days in water)	50 lb.
28 days (1 " " 27 " ")	125 "

11. **Constancy of Volume.*** Pats of neat cement about 3 inches in diameter, one-half inch thick at the center, and tapering to a thin edge, shall be kept in moist air for a period of 24 hours.

(*a*) A pat is then kept in air at normal temperature.

(*b*) Another pat is kept in water maintained as near 70° Fahr. as practicable.

These pats are observed at intervals for at least 28 days, and, to satisfactorily pass the tests, shall remain firm and hard and show no signs of distortion, checking, cracking, or disintegrating.

APPARATUS FOR A CEMENT TESTING LABORATORY†

(The apparatus is designed for one experimenter. Where the number of pieces is not stated, their number depends upon the quantity of cement to be tested.)

*One piece plate glass, one inch thick, 24 by 24 inches square;

*Four or more gangs of 3 or 4 molds each—A. S. C. E. standard (see Fig. 17, p. 78);

*One metric counter scale recording from 10 grams to 1½ kilograms.

*One No. 200 sieve (200 meshes to the linear inch), about 8 inches in diameter, and made of woven brass wire cloth, with wires 0.0021 inch. diameter (see p. 68);

*One measuring glass graduated to 250 cubic centimeters;

*One 8-inch mason's trowel;

*One 4-inch pointing trowel (see Fig. 11, p. 70);

*One-half dozen pairs rubber gloves;

*Pieces of thick window glass 4 inches square for soundness tests;

*One tensile testing machine (see Figs. 26 and 27, pp. 97 and 98);

*Air thermometer;

*Standard sand;

*An asterisk designates the apparatus required for a temporary laboratory on construction work

†This list has been criticised and approved by Mr. Richard L. Humphrey..

Two or more gangs of 3 molds each for 2-inch cubes; or, twelve 2 by 4 inch cylinders (see Fig. 20, p. 81);
10-pound tin cans with tight covers for holding samples;
One special scale for weighing cement in ascertaining fineness (see Fig. 10, p. 69);
One pan of same diameter as the sieves and 5 centimeters (1.97 in.) deep, with cover, for holding sieve when shaking it;
One measuring glass graduated to 100 cubic centimeters;
One cement sampler 24 inches long (see Fig. 8, p. 64);
One minute sand glass;
One moist closet (see Fig. 18, p. 79);
Galvanized iron waste cans;
Apparatus for steaming and boiling specimens (see Fig. 13, p. 73);
Tanks for immersing specimens (see Fig. 19, p. 79);
Vicat needle apparatus (see Fig. 12, p. 72);
One compression testing machine (adapted also to transverse tests), capacity at least 50 000 pound (see Figs. 94 and 95, pp. 340 and 341);
Chemical thermometer;
Specific gravity apparatus (see Fig. 9, p. 67);
Microscope with 1½ inch objective;
Set of sieves, about 8-inch diameter, for analyzing sands, sizes 0.25 inch diameter holes, No. 7, 12, 20, 30, 50, 90 (the number corresponds to the number of meshes to the linear inch, see p. 118);
Mechanical shaker for sifting sand (see Fig. 54, p. 186).

SPECIFIC GRAVITY OF CEMENTS

The specific gravity test, by determining whether a cement is thoroughly burned, supplements the chemical analysis, since the latter does not indicate the degree of calcination. The specific gravity of a true Portland cement ranges from 3.05 to 3.15. The adulteration of Portland cement lowers its specific gravity, because the substances used,—powdered sand, limestone, trass or slag,—are lighter than particles of pure cement. The test will not detect a small adulteration nor adulteration with a material of high specific gravity.

Natural cement usually has a specific gravity above 2.75, ranging from this sometimes as high as 3.1,* thus overlapping the inferior limit given

*Tests of Metals, U. S. A., 1901, p. 476.

for Portland cement. Puzzolan cement usually has a specific gravity of 2.7 to 2.9.

The specific gravity of cement is lowered by exposure, because of the absorption of water and carbonic acid, hence the necessity of drying it at 100° Cent. (212° Fahr.) or igniting at low red heat (see p. 64) before determining. Even this temperature may not always be sufficient to restore old cements to their original condition.*

Jackson Specific Gravity Apparatus. Mr. Daniel D. Jackson has devised an apparatus that gives satisfactory results for cement or fine aggregate. The graduations are made to read directly in terms of specific gravity and temperature variations are corrected by a table instead of attempting to prevent such variations. The glass tube is made of much smaller bore than in the Le Chatelier apparatus so that more accurate readings may be made.

The method of finding the specific gravity which applies to cement or fine aggregate, is specified by Mr. Jackson as follows:†

1. Weigh out accurately to the tenths' place of decimals 50 grams of the dry sample of cement.

2. Fill the bulb and burette with kerosene, leaving just space enough to take the temperature by introducing a thermometer through the neck. Remove the thermometer and add sufficient kerosene to fill exactly to the mark on the neck, drawing off any excess by means of the burette.

3. Run into the unstoppered Ehrlenmeyer flask about one-half of the kerosene in the bulb. Then pour in slowly the 50 grams of cement and revolve to remove air bubbles. Run in more kerosene until any adhering cement is carefully washed from the neck of the flask, and the kerosene is just below the ground glass.

4. Place the hollow ground-glass stopper in position, and turn it to fit tightly. Run in kerosene exactly to the 200 cubic centimeter graduation on the neck, making sure that no air bubbles remain in the flask.

5. Read the specific gravity from the graduation on the burette and then the temperature of the oil in the flask, noting the difference between the temperature of the oil in the bulb before the determination and the temperature of the oil in the flask after the determination.

6. Make a temperature correction to the reading of the specific gravity by the use of the accompanying tables.

* See experiments in Tests of Metals, U. S. A., 1901, p. 476, and Dr. H. Kupfender in *Thonindustriezeitung*, translated in *Cement*, March, 1903, p. 23.

† Daniel D. Jackson in *Engineering Record*, July 16, 1904, p. 83.

Temperature Correction for Jackson Specific Gravity Flask. (*See p.* 85.)

Read temperature of oil in bulb before determination and of oil in flask after determination. Add correction if temperature of oil increases, and subtract it if it decreases.

Change in Temperature Centigrade.	Uncorrected Reading.									
	2.50 to 2.60	2.60 to 2.70	2.70 to 2.80	2.80 to 2.90	2.90 to 3.00	3.00 to 3.10	3.10 to 3.20	3.20 to 3.30	3.30 to 3.40	3.40 to 3.50
deg.										
0.2	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
0.4	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02
0.6	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
0.8	0.02	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03
1.0	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.04
1.2	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.05
1.4	0.03	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.06	0.06
1.6	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06	0.06	0.07
1.8	0.04	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07
2.0	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.08	0.08
2.2	0.05	0.06	0.06	0.06	0.07	0.07	0.08	0.08	0.09	0.09
2.4	0.06	0.06	0.06	0.07	0.07	0.08	0.08	0.09	0.10	0.10
2.6	0.06	0.07	0.07	0.07	0.08	0.08	0.09	0.09	0.10	0.11
2.8	0.07	0.07	0.08	0.08	0.09	0.09	0.10	0.10	0.11	0.12
3.0	0.07	0.08	0.08	0.09	0.09	0.10	0.10	0.11	0.12	0.12
3.2	0.07	0.08	0.09	0.09	0.10	0.10	0.11	0.12	0.13	0.13
3.4	0.08	0.09	0.09	0.10	0.10	0.11	0.12	0.12	0.13	0.14
3.6	0.08	0.09	0.10	0.10	0.11	0.12	0.12	0.13	0.14	0.15
3.8	0.09	0.10	0.10	0.11	0.12	0.12	0.13	0.14	0.15	0.16
4.0	0.09	0.10	0.11	0.12	0.12	0.13	0.14	0.14	0.16	0.17
4.2	0.10	0.11	0.11	0.12	0.13	0.14	0.14	0.15	0.17	0.17
4.4	0.10	0.11	0.12	0.13	0.13	0.14	0.15	0.16	0.17	0.18
4.6	0.11	0.12	0.12	0.13	0.14	0.15	0.16	0.17	0.18	0.19
4.8	0.11	0.12	0.13	0.14	0.15	0.16	0.16	0.17	0.19	0.20
5.0	0.12	0.13	0.14	0.14	0.15	0.16	0.17	0.18	0.20	0.21

A neat little device for dropping fine material into a specific gravity apparatus so as to prevent the entraining of air has been devised by Mr. Thomas H. Wiggin. A thin wooden board with a circular hole in it is placed above the apparatus and a paper funnel fitted into the hole and filled with dry cement. An electro-magnet, such as is used with an ordinary electric door-bell, is connected with its storage battery and arranged so that the clapper, instead of striking a bell, strikes a metal plate attached to the corner of the board. The constant tapping jars the funnel so that the grains fall slowly into the apparatus without requiring the attention of the operator.

ADVANTAGES OF FINE GRINDING

The effects of fineness of grinding upon cements are to make them,—

Stronger when tested with sand;
Weaker when tested neat;
Quicker setting;
Capable of producing a larger volume of paste;
Less affected by free lime.

Fineness is expressed by the percentage of the total weight of the cement retained on each sieve.*

A recognition of the value of extreme fineness has led to the adoption of higher standards than formerly, and manufacturers have accordingly improved the quality of their product in this respect.

Strength Affected by Fineness. With the same proportions of sand, higher tensile and compressive strength is obtained from finely ground than coarsely ground cements. Conversely, a larger porportion of sand can be used with fine ground than with coarse ground cement, with the same resulting strength.

The chief cementing value of a cement lies in the grains which are fine enough to pass a sieve having 200 meshes per linear inch. Photographs of thin sections of sand briquets several years old made by Mr. E. W. Lazell show very clearly the coarser grains of cement which have never been penetrated and chemically changed by the water.

Tested neat, a coarse cement may give higher strength than the same cement after regrinding. This is chiefly due, in the opinion of the authors, to the fact that the fine cement requires more water in gaging to produce the same consistency of paste, so that the same weight of cement produces a larger volume of paste, which therefore has less density and consequently lower strength. When sand is added, on the other hand, less influence is exerted by the water, because in any case a smaller volume of it is required in proportion to the dry materials, and besides this the very fine grains, which also have higher cementing qualities, fit more readily into the voids in the sand. The relation of the density of a mortar to its strength is discussed in Chapter IX, page 143.

The effect of the fineness of cement upon its strength was brought to general notice by Mr. John Grant† in 1880, who quotes experiments made in Germany by Dykerhoff. In 1883 Mr. I. J. Mann‡ illustrated

* Sizes of American vs. European sieves are given in Concrete Plain and Reinforced, 2nd edition, pp. 84 and 85.

† Proceedings, Institution of Civil Engineers, Vol. LXII, p. 149.

‡ Proceedings, Institution of Civil Engineers, Vol. LXXI, p. 254.

the small cementing value of the coarse particles by substituting for them grains of sand of the same size, with but little reduction in the resulting strength.

Mr. D. B. Butler* in England made extended tests to determine the value of coarse particles in cement and the effect of regrinding. The cement was reground and sand of the same size as the coarse particles in the original was substituted for them, producing a cement nearly as strong as the cement before regrinding.

The fine grinding of commercial cements has been one of the causes, by the acceleration of the setting, for the necessity of adding gypsum or plaster during manufacture.

Separating Material Passing No. 200 Mesh Sieve. The high cementing value of the grains of cement passing a No. 200 sieve necessitates, for elaborate tests, still finer apparatus. The coarsest particles passing a No. 200 sieve are approximately 0.004 inch in diameter. A device developed by the Bureau of Standards† separates this material into four parts, the limiting sizes being 0.004 inch, 0.003 inch, 0.002 inch, and 0.001 inch. Beginning with the smallest particles the successive sizes to be separated are carried off one after the other through small openings in the analyzing chamber by means of a steady stream of air under a low pressure. The residue is weighed after each separation and in this way the proportion of each size is determined.

QUANTITY OF WATER FOR NEAT PASTE AND MORTAR

The quantity of water used in gaging affects the results of tests, especially in the determination of the time of setting and of the strength. (See p. 165.) Different cements even of the same class require different proportions of water to produce the same consistency, chiefly because of differing degrees of fineness, the cement containing the largest proportion of fine particles requiring the largest percentage of water by weight.

For chemical combinations alone about 8 per cent. of water to the weight of the cement is customarilly assumed to be required, but in practice the percentage must be much greater.

Percentage of Water for Mortar of Normal Consistency. The table of percentage of water for standard mortar quoted on page 71 from the report of the American Committee is based on Mr. Feret's formula‡

* Proceedings, Institution of Civil Engineers, Vol. CXXXII, p. 343, and Butler's Portland Cement, 1899, p. 169.

† An Air Analyzer for Determining the Fineness of Cement, by J. C. Pearson and W. H. Sligh, Technologic Papers of the Bureau of Standards, No. 48, 1915.

‡ Commission des Méthodes d'Essai des Matériaux, 1895, Vol. IV, p. 103.

evolved from an interesting series of experiments.* He found that it was impracticable to determine with the Vicat needle the proper consistency of a mortar of cement and sand, and therefore based his determination upon the average judgment of several operators, plotting the consistencies designated by them upon cross-section paper.

The formula as used by the American Committee, expressing the values for convenience in percentages instead of in grams, is†

$$W = \tfrac{2}{3}\,\frac{P}{S+1} + 6.5$$

Where

W = percentage of water for mortar in terms of weight of the mixture of dry materials;

P = percentage of water required for neat cement of normal consistency;

S = parts of sand by weight to one part cement.

The following table gives percentages of water for different proportions of mortar. It must be remembered that these percentages are for standard sand only, the percentages required for natural sand varying with the coarseness of its grain.

Percentage of Water for Cement Mortars of Normal Consistency.

Percentage of water for neat cement.	Percentage of Water to Cement Plus Standard Ottawa Sand. Proportions cement to sand by weight.					Percentage of water for neat cement.	Percentage of Water to Cement Plus Standard Ottawa Sand. Proportions cement to sand by weight.				
	1:1	1:2	1:3	1:4	1:5		1:1	1:2	1:3	1:4	1:5
18	12.5	10.5	9.5	8.9	8.5	27	15.5	12.5	11.0	10.1	9.5
19	12.8	10.7	9.7	9.0	8.6	28	15.8	12.7	11.2	10.2	9.6
20	13.2	10.9	9.8	9.2	8.7	29	16.2	13.0	11.3	10.4	9.7
21	13.5	11.2	10.0	9.3	8.8	30	16.5	13.2	11.5	10.5	9.8
22	13.8	11.4	10.2	9.4	8.9	31	16.8	13.4	11.7	10.6	9.9
23	14.2	11.6	10.3	9.6	9.0	32	17.2	13.6	11.8	10.8	10.0
24	14.5	11.8	10.5	9.7	9.1	33	17.5	13.8	12.0	10.9	10.1
25	14.8	12.1	10.7	9.8	9.3	34	17.8	14.1	12.2	11.0	10.2
26	15.2	12.3	10.8	10.0	9.4	35	18.2	14.3	12.3	11.2	10.4

French Consistency of Neat Paste. The Vicat needle (see p. 72) has been adopted in England and France as well as in the United States. In France a softer consistency is adopted requiring a penetration of 34 millimeters instead of 10 millimeters.

* Methods of Mr. Feret's investigations are described and illustrated in an article by the authors on "Quantity of Water to Use in Gaging Mortars" in *Cement and Engineering News* (Chicago), November, 1903.

† Mr. Feret gives for the last term in the formula 6.0 for mortars of plastic consistency and 4.5 for mortars of dry consistency.

TESTS OF SETTING

The methods employed in mixing and depositing the mortar or concrete and the character of the construction form a guide to the necessary requirements for the time of setting of the cement.

The setting of cement is due to chemical reaction, as described by Mr. Spencer B. Newberry on page 49. The process is a gradual one, but may be arbitrarily divided into three periods:

Initial set.
Final set.
Hardening.

The dividing line between these periods is arbitrary, but the division is based upon the fact that after water is added the paste remains plastic for a certain period, and then commences to "stiffen" or crystallize. This is called the time of initial set. The setting process continues rapidly, and when a point is reached that the paste will withstand a certain pressure, arbitrarily fixed in practice, it is said to have reached its final set. The process of hardening now continues more slowly, and proceeds with increasing slowness for an indefinite period.

Those unfamiliar with cement construction must bear in mind that a cement which has reached its final "set" is not hard nor is it capable of bearing a load. Natural cement, for example, usually reaches its initial and its final set much earlier than Portland cement, but it hardens more slowly, and Natural cement masonry will not bear loading nearly so quickly as Portland cement masonry.

EUROPEAN METHODS FOR DETERMINING SET

The French and German requirements are similar to the American Vicat needle (p. 75) except that in them the commencement of the set is taken as the time when the needle can no longer penetrate entirely to the bottom of the box instead of limiting it to a penetration to a depth of 5 millimeters above the bottom surface.

For sand mortars the French Commission designate the final set as the moment when the surface of the mortar can support pressure of the thumb without indentation. As an alternate method, they use the Vicat apparatus with a needle one centimeter (0.39 in.) in diameter and weighing 5 kilograms (11.02 lb.). The preliminary reports of Mr. R. Feret and Mr. P. Alexander in Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, pp. 111 and 139, describe experiments with different apparatus.

EUROPEAN SETTING REQUIREMENTS

Country.	Time of Initial Set.	Time of Final Set.	Method.
Germany	1 hr.	Not required	Vicat
Switzerland	None required	Quick—Max. 30 m. Slow—Min. 3 hr.	Vicat
Austria	Quick—10 min. Medium—30 min.	None required None required.	Vicat
Denmark	1 hr.	Min. 2 hr. Max. 15 hr.	Vicat
England	Quick—2 to 10 min. Medium—10 to 20 min. Slow—20 min.	Min. 2 hr. Max. 12 hr.	Vicat
France	20 min.	Min. 2 hr. Max. 12 hr.	Vicat
Italy	1 hr.	Min. 5 hr. Max. 12 hr.	Vicat
Russia	20 min.	Min. 1 hr. Max. 12 hr.	Vicat

Comparison of Vicat and Gillmore Needles. The Gillmore needles were first used by General Totten in 1830.*

By these needles the initial set of neat cement is the time at which a wire one-twelfth-inch diameter, loaded to a ¼ pound, is just supported by the mass without appreciable indentation. The final set is taken as the time when a wire one-twenty-fourth-inch diameter, loaded to weigh one pound, is supported without appreciable indentation.

The diagram in Fig. 22, page 92, from experiments made at the Watertown Arsenal† upon various cements (designated by letters) shows the difference in the nominal time of setting when measured by the Gillmore needle and the Vicat needle, employing with the latter the German method. (See above.) The diagram also shows the variation in time of set of Portland cement occasioned by varying the proportion of water, and the effect of leaving out the usual "restrainer" of plaster of Paris or gypsum.

The Rate of Setting. The rate of setting of cement, that is, the process of hardening, has been studied by the French Commission‡ in France and by Prof. Edgar B. Kay in the United States. The diagram Fig.

* Gillmore's Treatise on Limes, Hydraulic Cements and Mortars, p. 80.
† Tests of metals, U. S. A., 1901, p. 492
‡ Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 111.

24, page 95, shows curves of setting made with a machine of Prof. Kay's design and the corresponding tensile strength of briquettes of the same cement. Prof. Kay calls attention to the positive change from the plastic to the granular or crystalline structure which in the cement shown occurred between the periods of 35 and 40 minutes. The elongation of the briquette when being broken gradually changed from ¾ inch at the 5-minutes period to 0.15 inch at 40 minutes, while at 200 minutes, or one hour before the initial set was completed, the elongation was not measurable.

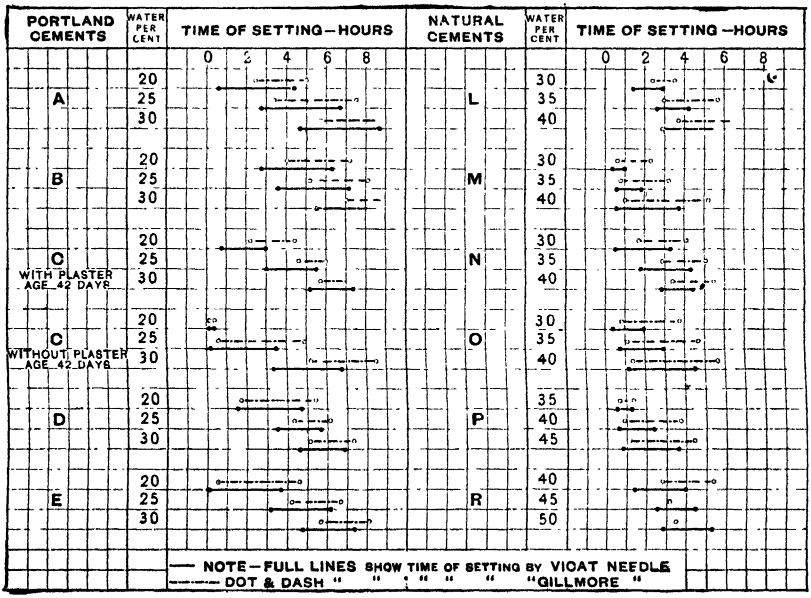


FIG. 22.—Time of Setting of Typical Cements.—and comparison of Vicat and Gillmore's Needles. (Tests of Metals, U. S. A., 1901.) (*See p.* 91.)

Flash Set. One of the most dangerous characteristics of cement is a set so quick that the concrete made from it hardens before it can be placed and rammed.

When tested in the laboratory the cement may take both initial and final set within five or ten minutes of molding, or even during the process of molding, so that it may not be distinguished by the operator. Frequently, the sample changes to slow set during transportation and may show normal set in the laboratory even although it "flashes" on the work. On the other hand, a sample exposed to the heat of the sun

or kept in a warm laboratory may change from a normal to a flash set. This very quick set appears to be caused frequently by rise in temperature of the air during transportation or storage. Concrete which hardens before it reaches its place and is not regaged with water may never attain appreciable strength. Thorough and continued soaking with water is the best treatment.

TEST OF RISE IN TEMPERATURE WHILE SETTING

The determination of the rise in temperature which takes place in a cement while setting has often been suggested as an indication of its quality, but the increase in temperature is due to so many causes that it is of slight value as a test of the cement.

Mr. Le Commandant Ribaucour* found that the temperature commenced to rise at the commencement of the setting, and the rise was generally higher with quick-setting cements.

Mr. J. E. Howard at the Watertown Arsenal† discovered that the

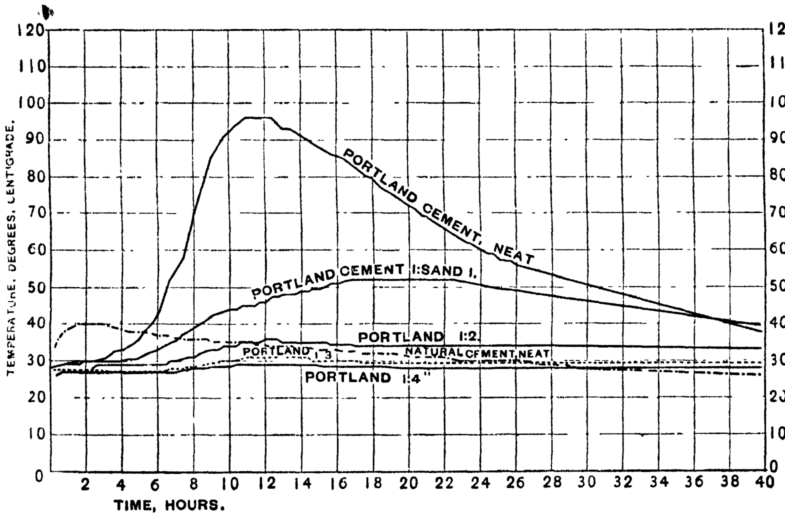


Fig. 23.—Rise in Temperature in 12-inch Cubes of Cement and Mortar. (Tests of Metals, U. S. A., 1901.) (*See p.* 93.)

temperature was largely dependent upon the size of the specimen, small cubes showing very slight increase. He therefore made a series of tests upon 12-inch cubes to determine the temperature acquired by different

* Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 133.

† Tests of Metals, U. S. A., 1901, p. 493.

brands of cement and mortars during setting, and plotted his volumes in a series of curves. The curves for a first-class brand of American Portland cement with and without sand, and for a typical Natural (Rosendale) cement, are shown in Fig. 23.

Mr. Howard found that while first-class American brands of neat Portland cement often reached a maximum temperature of 100° Cent. (212° Fahr.); the maximum temperature of the various brands of American Natural cement was generally from 35° to 40° Cent. (95° to 104° Fahr.), and was reached at a shorter time than the Portland cements. The rise in temperature of the German brands of Portland cements was in general less than that of the American Portlands.

The rise in temperature in Portland cement concrete is less than in neat Portland cement, but in the interior of a large mass like a dam may reach nearly 100° Fahrenheit.

AMERICAN AND EUROPEAN STANDARD SANDS COMPARED

The character of the sand has so great an effect upon the strength of a mortar that for comparing different brands of cement or specifying requirements of strength a sand of standard size and quality is essential.

The U. S. Standard Sand recommended by the Committee of the American Society of Civil Engineers, as specified on page 76, is a natural sand from Ottawa, Ill., screened to pass a sieve having 20 meshes per linear inch, and retained on a sieve having 30 meshes per linear inch.

The English Standard Sand is obtained from a pit at Leighton Buzzard,* and the screens are the same as in the United States.

The German Standard Sand is a natural quartz retained between sieves having respectively 20 and 28 meshes per linear inch.

The French Standard Sand, a natural sand from Leucate, France, is simple or compound. Simple standard sand must pass a screen having holes 1.5 millimeters (0.059 in.) in diameter, and be retained on a screen having holes one millimeter (0.039 in.) in diameter. Compound standard sand is made by forming a mixture of equal weights of the following:

(1) Grains passing holes of 2 mm. (0.079 in.) and retained by 1.5 mm. (0.059 in.).

(2) Grains passing holes of 1.5 mm. (0.059 in.) and retained by 1 mm. (0.039 in.)

(3) Grains passing holes of 1 mm. (0.039 in.) and retained by 0.5 mm. (0.020 in.

* Butler's Portland Cement, 1899, p. 200.

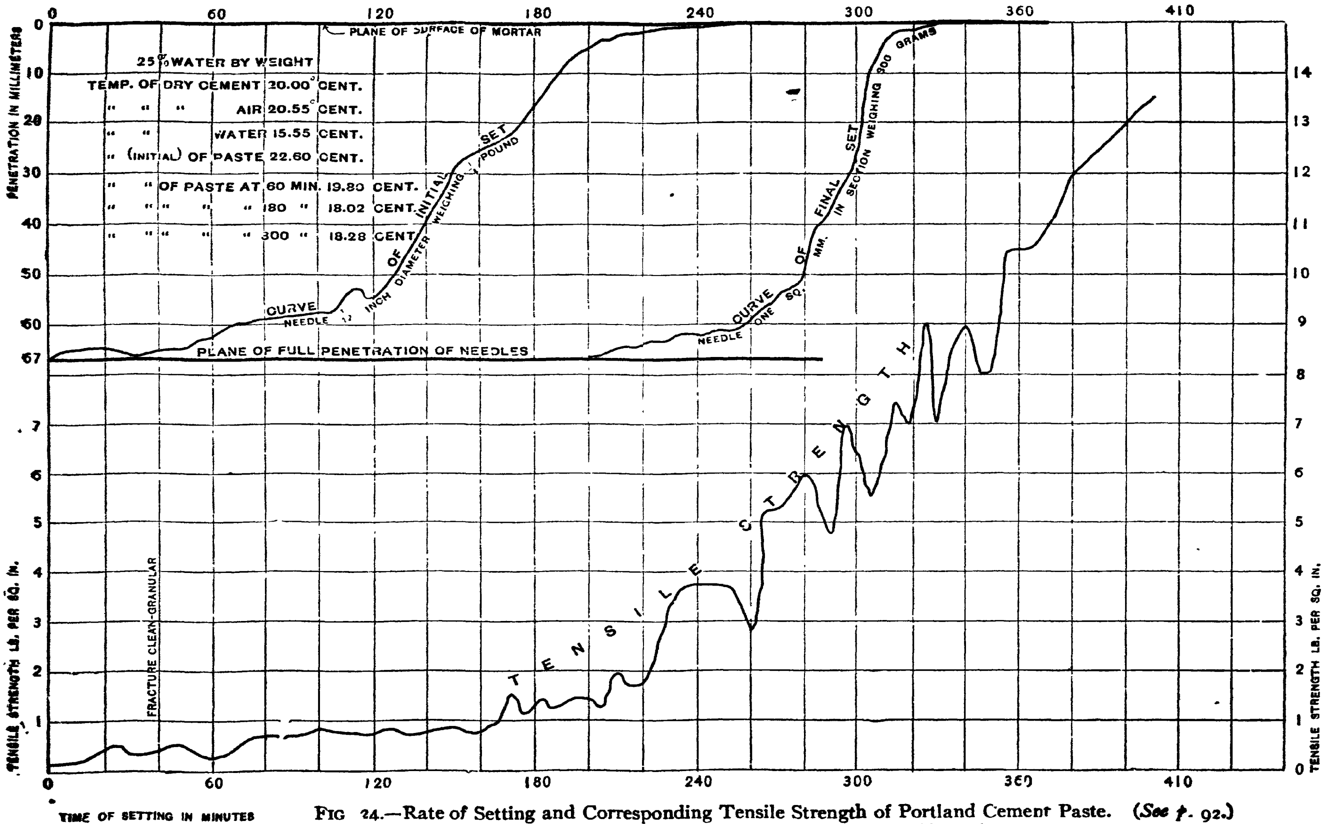


FIG 24.—Rate of Setting and Corresponding Tensile Strength of Portland Cement Paste. (*See p.* 92.)
(Especially prepared by Prof. Edward B. Kay for this treatise.)

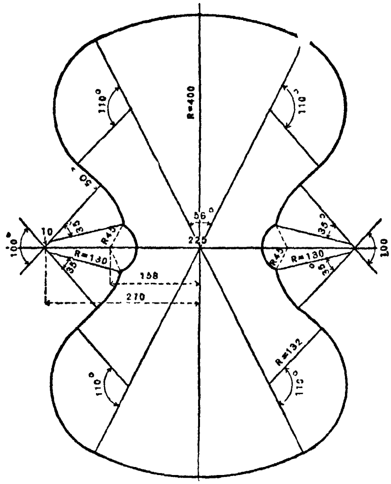


FIG. 25. The German Standard Briquette (dimensions are in millimeters). (*See p.* 96.)

THE FORM OF BRIQUETTE FOR TENSILE TESTS

Mr. John Grant in 1871* presented results of a series of experiments with different forms of briquettes and sizes of section. Ten years later† he adopted the form now used in England which is substantially the same as that recommended by the American Society of Civil Engineers in 1884, and, with a very slight alteration, in 1903. (See Fig. 16, p. 77.)

The German Standard Briquette, also adopted by the French Commission in 1893, is shown in Fig. 25. The section is 5 square centimeters (0.78 sq. in.). Results with this form of briquette are lower per unit of area than those of the American pattern. Prof. Jerome Sondericke‡ in studying the quality of strength and uniformity of breaking of different forms, found that a groove in the sides of the specimen lowered the unit strength about 13%.

M. Feret§ found that briquettes of 5 sq. cm. section gave 46% higher strength per unit of area than briquettes of 16 sq. cm., and attributed this difference to lack of homogeneity throughout the section.

MACHINES FOR TESTING TENSILE STRENGTH

A testing machine should be so designed that the strain can be applied to the briquette at a definite rate without irregularity or jar. The clips should be suspended from pivoted bearings to avoid friction, and should be stiff, so that they will not spread. The contact surfaces should hold the briquette firmly without crushing it.

The briquette must be carefully adjusted in the clips since a very small eccentricity reduces the tensile strength appreciably.

Rate of Applying Strain. The selection of the standard rate of 600 lb. per minute by the committee of the American Society of Civil Engineers (see p. 78) is based on an extensive series of tests from which it

* Proceedings Institution of Civil Engineers, Vol. XXXII, p. 282.
† Proceedings Institution of Civil Engineers, Vol. LXII, p. 137.
‡ Journal Association of Engineering Societies, January, 1899, p. 1.
§ See p. 146.

was found that the breaking load increases with the speed up to a rate of at least 800 lb. per min., but that between the rates of 400 and 600 lb. the variation is slight. Experiments of L. C. Sabin* tend to confirm this conclusion.

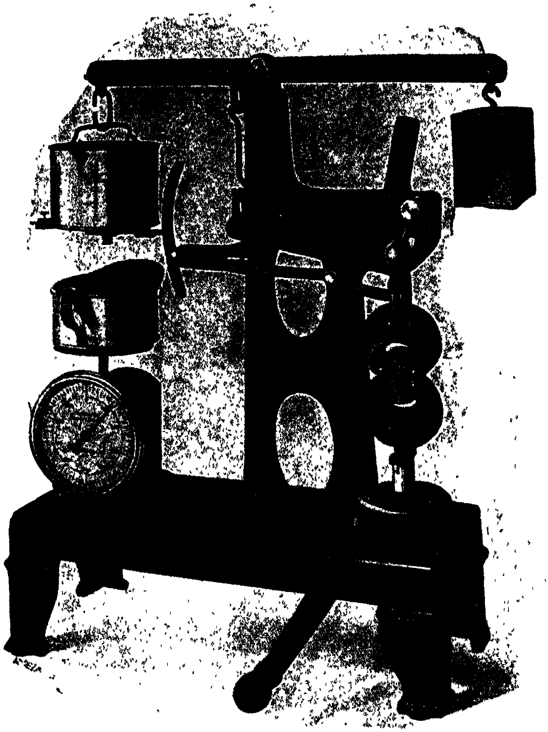


FIG. 26.—Shot Testing Machine. (*See p* 97.)

Tensile Testing Machines. The shot machine, originally designed by Dr. Michaelis, is in general use for testing cement and mortar in tension. American patterns are shown in Figs. 26 and 27. The load is applied by the discharging of a stream of shot whose flow is automatically shut off when the break occurs. The breaking load is determined from the weight of the shot.

Simple or compound lever machines which apply their load by a sliding weight operated by hand or by power were formerly used.

* Report Chief of Engineers, U. S. A., 1895, p. 2916.

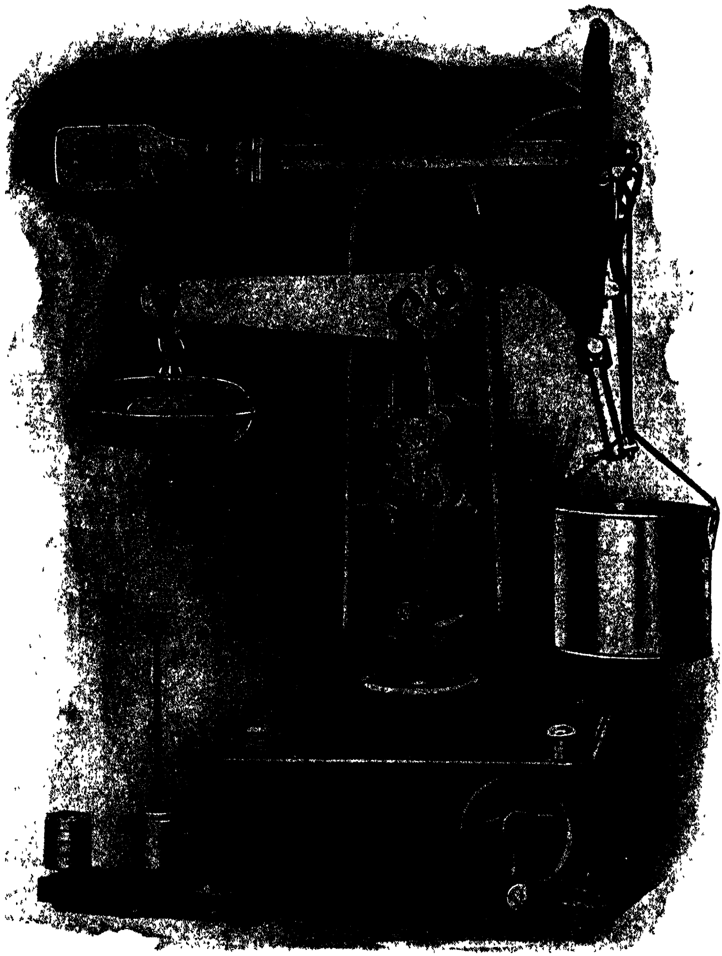


FIG. 27.—Shot Testing Machine. (See p. 97.)

TENSILE TESTS OF NEAT CEMENT AND MORTAR

Tensile tests are useful only to determine (in connection with other tests) whether the ingredients of a cement and the process of manufacture are such that the cement will give a satisfactory strength and growth in strength when used in concrete. Formerly, this test was

made by means of tests on neat cement and on 1 to 3 sand mortar briquets, but in the 1917 specifications (see p. 63) the neat test has been eliminated, the Committee deciding that sand mortar tests give all the information needed. Notwithstanding this decision the authors find the 24 hour neat test a valuable one for estimating the hardening quality of the mortar. A cement testing below the former standard of 500 lb. per sq. in. at seven days is liable to give trouble by slow hardening in cold weather.

Tensile tests on cement and mortar indicate the quality of both cement and sand, but unpublished tests show that neither of them have a clearly defined relation to the compressive strength of concrete. The effect of age in each case is so far from similar that concrete gains with age while neat cements and mortars in tensile briquets normally retrogress. So far as concrete is concerned, reliance should be placed on compression tests of concrete cylinders, 28 days old (see p. 310).

Tensile tests of mortars are usually made 7 and 28 days after mixing. For tests of sand a 3 day period is frequently convenient.

Specifications for tensile strength are given on page 63. Frequently actual strengths are considerably above these figures, which simply indicate the lowest limit acceptable. In Fig. 28, p. 100, are plotted curves averaged from results of long time tests on neat and mortar briquets made by the City of Philadelphia, U. S. Reclamation Service, and the Boston Transit Commission. In all cases the briquets remain in moist air for the first 24 hours and the balance of the time in water maintained at a temperature of 21° Cent. (70° Fahr.).

Growth in Strength. American standard specifications require the strength of mortar briquets to be higher at 28 days than at 7 days. Cements that barely pass the specified strengths usually continue to gain until they reach an ultimate strength that is fairly consistently maintained. On the other hand, cements that are much stronger than is required at 7 and 28 days may be expected to show a falling off until they reach about the strengths of the slower setting cements. Frequently, in a series of tests the cements, whether high or low at first, eventually tend to reach about the same strength.

Neat cement briquets behave in much the same way as mortars with the exception that retrogression is likely to appear between 7 and 28 days, if the cements test high at those periods.

The curves in Fig. 28, page 100, are typical of cements that give high strengths at early ages and then gradually fall off. In the second edition of this book a curve was shown of cement mortars that barely passed

the specifications at 7 and 28 days, but maintained their strength uniformly for at least a year at about 300 pounds, the figure to which the corresponding curve on page 100 (1 : 3 mortar) returns after about three years.

E. Candlot states* that the strength of all cements, excluding inferior products, is practically the same at the end of one or two years, whether the initial strength was high or not. Hence, for much work the high strength at short times is a decided advantage.

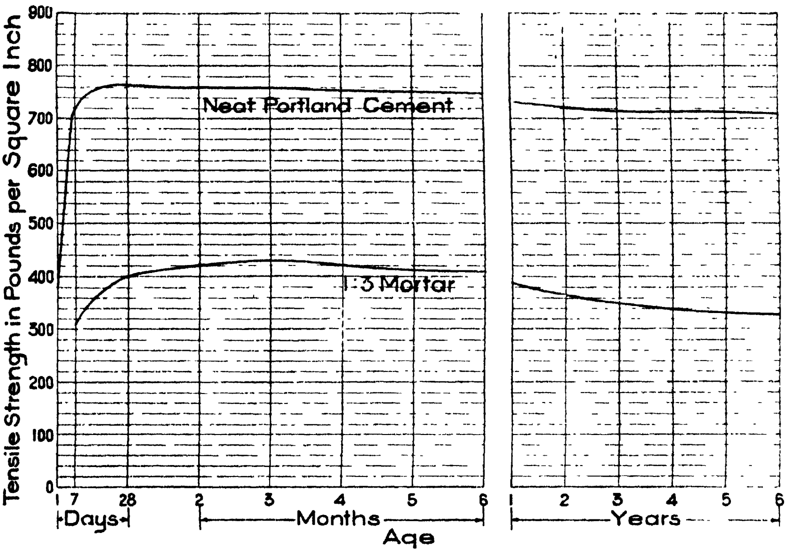


FIG. 28.—Growth in Tensile Strength of Neat Portland Cement and 1:3 Standard Sand Mortar. (*See p.* 100.)

Compiled from tests by the U. S. Reclamation Service, the City of Philadelphia, and the Boston Transit Commission.

COMPRESSIVE TESTS OF CEMENT AND MORTAR

Tensile tests can be made quicker than compressive tests and require less powerful machines, so that they are more convenient and in wider use for determining the quality of cement and sand for acceptance. Nevertheless, compression tests give a truer measure of the value of the mortar or concrete made from the materials. Compression machines are discussed and illustrated on page 340, in connection with the determination of the compressive strength of concrete.

* Proceedings International Association for Testing Materials, 1912, Second Section, XIII.

Form of Compression Specimens. Cubes 2 inches on an edge have been used commonly for cement and mortar specimens in the United States. A 2 by 4 inch cylinder agrees better with the concrete standard. In France* compression tests of halves of briquets already broken in tension are recommended. The total surface area of the U. S. Standard briquet is almost exactly 4 square inches.

A gang mold† for 2-inch cubes used in Mr. Thompson's laboratory is shown in Fig. 29. The material is cold rolled steel of commercial width and thickness. Side plates are $\frac{1}{2}$ by 2 inches, base plate $\frac{1}{2}$ by 5 inches, partition plates $\frac{3}{16}$ by 2 inches, and middle partition plate $\frac{3}{4}$ by 2 inches. Thumb nut bolts, headed on one end, are of $\frac{1}{4}$ inch steel.

The design for the 2 by 4 inch cylinder is given on page 81.

Ratio of Compressive to Tensile Strength. There appears to be no constant relation between the resistance of mortar to compression and tension. Like concrete, mortar gains compressive strength more rapidly

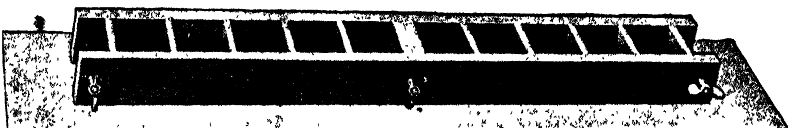


FIG. 29.—Mold for Compression Test Pieces. (*See p.* 101.)

than tensile strength, and similarly an increase in the proportion of cement increases the compressive strength in a much greater ratio than the tensile. Extended tests by Mr. R. Feret‡ and by Richard L. Humphrey|| together with more recent tests, not yet published, confirm these conclusions.

A comparison of the compressive and tensile strength of 1 : 3 mortars based upon the St. Louis tests give a formula

$$\frac{\text{Compressive strength}}{\text{Tensile Strength}} = 6.6 + 2.3 \log A$$

where

$$A = \text{age of the cement mortar in months.}$$

The ratio varies from 6.8 on a one-month test up to 10.3 on a 12-month test. The formula is in the same form but the ratios are some-

* Commission des Méthodes d'Essai des Matériaux de Construction, Vol. IV, 1895, p. 187

† Designed by Ralph E. Goodwin for the New York Public Service Commission.

‡ Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Series 5, Vol. II.

|| Bulletin No. 331, U. S. Geological Survey, 1908.

what greater than those obtained by Prof. J. B. Johnson* from Prof. Tetmajer's tests at Zurich.

TRANSVERSE TESTS OF CEMENT

Transverse, or flexion, tests of beams or prisms while very convenient for concrete are now seldom used for testing the quality of cement, although Gillmore and other of the older experimenters largely employed this form of test. Transverse tests are of value in comparing the relation between fiber stress and tension, and with proper care may give as uniform results as tension tests. As is stated below, the fiber stress bears a definite relation to the tensile strength, but since there is no fixed relation between tension and compression, there can be no fixed relation between transverse strength and compressive strength. Compression testing machines (see Figs. 94 and 95, pages 340 and 341) may be adapted for transverse tests by a suitable arrangement of supports and knife edges.

Mr. Durand-Claye, Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 211, made for the French Commission an extended series of tests by flexion or bending. He found the tensile fibre unit stress to average 1.9 times the tensile strength of the standard briquet. As a result of his report, the Commission adopted for this form of test square prisms 12 cm. (4.72 in.) long by 2 cm. (0.79 in). on a side.

Comparative tests of Mr. R. Feret in tension, flexion, and compression are shown in the table on page 146.

ADHESION TESTS OF CEMENT

Mr. E. Candlot† made a large number of tests of adhesion for the French Commission, and designed a mold, patterned like a half briquet, to be molded with the breaking section at the bottom, and this has been adopted as the French Standard.‡ He found that different brands of cement may give widely different results in adhesion of mortars; that regaging a mortar reduces its strength in adhesion about one-half even when the tensile strength is not appreciably affected; that mortars gaged dry have lower adhesion than plastic mortars; and that mortars gaged with an excess of water have in tension a resistance less than their adhesive strength.

* Johnson's Materials of Construction, 1903, p. 419.

† Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 281.

‡ This mold adapted to American standards is shown and method of using described by Sanford E. Thompson in Proceedings American Society of Civil Engineers, Apr. 1903, p. 647; Also, in the Second Edition of "Concrete, Plain and Reinforced" p. 122.

Mr. R. Feret* states that adhesion to stone increases as the stone becomes more porous. He found that irregularities of surface of the stone do not appear to affect the adhesive strength. With iron, however, roughening the surface increases the adhesion of the mortar. A dirty surface or insufficient moistening of the surface lowers the adhesion.

Tests by the authors indicate that the strength in adhesion of neat cement to a 1 : 2 mortar surface may nearly reach the tensile strength of the mortar, thus showing the value of a neat cement bond. With the introduction of sand the adhesion rapidly decreases.

The adhesion of mortar to iron or steel is discussed in connection with reinforced concrete in Chapter XXI.

SOUNDNESS OR CONSTANCY OF VOLUME

The term "soundness" is more commonly used in America and England than the expression "constancy of volume" suggested by the Committee of the American Society of Civil Engineers, or "deformation" as employed in France. The purpose of the test is to determine in advance whether a cement is in danger of disintegrating, that is, crumbling, or of expanding or contracting so as to cause distortion or cracking in the masonry.

If a cement satisfactorily passes the tests for soundness, it will in all probability withstand the effect of the elements without swelling or disintegration, and will continue to harden for an indefinite period. Failure, on the other hand, to pass the tests for soundness, especially the hot test, is not positive proof of inferiority, for a cement which fails to pass may possibly, through subsequent exposure to the air before being used, or because of mixing with sand or other aggregate, produce durable masonry.

Tests indicate that concrete in practice made with cement unsound because of freshness or high lime usually hardens satisfactorily, even although pieces of the concrete fail to pass the boiling test at early ages. Tests by Mr. P. H. Bates† of the Bureau of Standards show satisfactory results with unsound cement when the concrete is kept moist, but specimens stored in the laboratory with no special provision for supplying moisture show marked deterioration and ultimate disintegration in the course of several months' time.

We may with safety adopt the following conclusions:

If a Portland cement passes the hot test it may be used immediately with reasonable certainty of its ultimate soundness. If it fails to pass, it should be regarded with suspicion and thoroughly tested.

* Communication au Congrès de Budapest, 1901.
† Personal correspondence.

Causes of Unsoundness. Disintegration, or crumbling, of work in Portland cement properly mixed and laid, is usually due to an excess of lime in a form which can be attacked by the elements. This may come about in three entirely distinct ways, either (1) by the use of too high a proportion of lime in the raw materials from which the cement is made, (2) by under-burning the cement, or (3) by too coarse grinding.

The presence of magnesia in excess in a thoroughly burned cement may produce a gradual expansion which will disintegrate the mortar or concrete after several years. This action, brought to notice by tests of Mr. H. Le Chatelier,* is generally recognized, but opinions differ as to the limit to the percentage of magnesia which may occur in Portland cement without deleterious effect. Le Chatelier's experiments led him to consider 5% as injurious. The Association of German Cement Manufacturers first placed the limit at 3½%, and later raised it to 5%. Mr. Spencer B. Newberry states (page 53) that cements with more than 7% or 8% of magnesia will pass the boiling test but are inferior in strength and are likely to show progressive expansion. The limit of 5% recommended by the Committee of the American Society for Testing Materials in 1916 (see p. 62) is undoubtedly conservative. Natural cement, which is burned at a lower temperature, may contain a much larger quantity of free lime and of magnesia without injury.

The expansion caused by an excess of free lime is due to the hydration or slaking of the calcium oxide (CaO). This is readily understood from the expansion of common lime, which in slaking with water will produce a bulk of paste from 2 to 3 times greater than the volume of the loose powder. The presence of lime in a free or loosely combined state must not be confounded with other compounds of calcium. A thoroughly slaked lime paste, or powder, that is, one which is completely hydrated, may in fact be added to a Portland cement mortar without injurious results, to lengthen its time of setting or to produce a more water-tight mixture.

The small amount of free lime which frequently occurs and sometimes produces unsoundness in first-class Portland cement, tested when fresh, may be hydrated and rendered harmless by air-slaking after, say, two or three weeks' storage, or after spreading the cement out in the air.

Adulteration with slag may cause a cement containing an excess of free lime to pass the boiling test.

* Commission des Méthodes d'Essai des Matériaux de Construction, 1895, Vol. IV, p. 229.

Tests for Soundness. The presence of ingredients which will render a cement unsound, that is, which will cause it to expand or disintegrate, is determined by the eye, or by measuring appliances in specimens which have been exposed under conditions which as nearly as possible produce the same effect as the practical effects of time and the elements.

There is apparently no reliable method for determining the presence of free lime by chemical analysis. Mr. E. Candlot* says that "there is in fact no method for finding the percentage of free lime in the cement," and Dr. Schuman* concurs in this view in the following statement:

I do not know a method for finding out the percentage of free lime in Portland cement. I do not think there exists such a method, and I am myself of the opinion that chemists will never find out one; the solutions capable of taking away the free lime from the cement will always work in a more or less strong degree on the cement itself.

This inability to detect free lime by chemical analysis necessitates a resort to physical tests. Specimens for testing soundness are generally circular pats tapering toward the edges, so that the expansion of the mass will tend to enlarge the circumference and thus produce cracks at the edges.

Appearance of Soundness Specimens. Excellent photographs of pats showing the effects of unsoundness are illustrated in Fig. 14, p. 74. The most common condition is shown in the specimen designated as "cracking" and indicates a cement slightly unsound. A more unsound cement produces disintegration. In still more extreme cases the pat goes to pieces in boiling or steaming. An unsound cement in air or in water at the ordinary temperature will generally show defect within 28 days, although in very exceptional cases several months or even years have been known to elapse before signs of deterioration appear in specimens which have not been subjected to heat.

Cracks which appear on pats are not always caused by unsoundness. Expansion cracks, which reveal an unsound cement, should be distinguished from harmless shrinkage cracks, which may appear during setting, instead of after the cement has set, because of too quick drying out or a too wet mixture. Such shrinkage is apt to cause on the top of the specimens, hair cracks due to a large excess of mixing water which deposits a thin coating of practically decomposed cement, or else radial cracks near the center, instead of edge cracks or circumferential cracks shown in the photographs of Fig. 14. The latter are the danger marks.

* Quoted by W. W. Maclay in Transactions American Society of Civil Engineers, Vol. XXVII. p. 448.

If the pats are left exposed to dry air during setting shrinkage cracks are often developed. Ordinarily, therefore, they indicate only a lack of care in manipulation, and not dangerous properties in the cement.

Blotching is usually indicative of either adultration or under-burning. This condition in itself should not necessarily mean rejection, but should always induce an investigation of the causes producing it, which may or may not be sufficient to warrant rejection.

A peculiar condition sometimes occurs in which the pat is perfectly sound and hard, but the glass on which it is made is cracked, either in one or two places or completely. This has often been laid to chemical action, but this conclusion is doubtless erroneous. It is probably due entirely to expansion of the pat, when the adhesive strength of the cement to the glass exceeds the strength of the glass itself. It is only found in the water pats, and is not usually indicative of dangerous qualities of the cement.

Accelerated or Hot Tests. The object of all forms of hot tests is to produce in a few hours the results which at a normal temperature require several days or perhaps months. Engineers are by no means agreed as to the value of accelerated tests, the chief objection to their use being that some cements which fail in these tests prove satisfactory in construction.

An argument for the hot test lies in the fact that Portland cement manufacturers are coming to recognize it as the very best test for them to use in determining whether their own cement will fulfil the requirements of permanent construction. In a recent letter to the authors the superintendent of one of the largest factories in the United States writes, "So far as we are concerned, we consider the hot test of the greatest importance. If this shows up well, we feel quite satisfied that all other tests will show up properly." Those desiring to investigate the various opinions upon the subject are referred to References, Chapter XXXIII.

Mr. W. Purves Taylor, in a paper read before the Cement Section of the American Society for Testing Materials, at the Sixth Annual Meeting, 1903,* gives the results of a large number of accelerated tests made at the Philadelphia Testing Laboratory by boiling balls or pats (after 24 hours in moist air) for three or four hours, and the results of some of the conclusions there given are quoted as follows:

"The condition in a cement most affecting the result of an accelerated test is its age or the amount of seasoning it has undergone. Every cement,

*Proceedings American Society for Testing Materials, 1903, Vol. III, p. 374, also printed in *Engineering News*, July 23, 1903, p. 81.

no matter how well proportioned and burned, will contain at least a small amount of free or loosely combined lime, which will usually cause unsoundness if used or tested at once. This lime, however, will hydrate in a very short time on exposure to air, thus rendering it inert and preventing any expansive action. It will, therefore, be found in a large majority of cases that if a cement failing in the accelerated tests be stored for two or three weeks, this unsoundness will disappear, and the cement pass the test with ease."

This is illustrated in the following table and in Fig. 30, page 108, the first three photographs also showing various conditions which may be expected in specimens which fail to pass accelerated tests.

Effect of Age of Cement on Results of Boiling Test.

BY W. PURVES TAYLOR. (*See p.* 107.)

Age of cement when tested	TENSILE STRENGTH					NORMAL PAT TESTS		BOILING TEST
	Neat			1:3 sand				
	1 day	7 days	28 days	7 days	28 days	28 days in air	28 days in water	
1 week	550	765	762	171	225	Curled and soft.	Slightly checked.	Partly disintegrated.
2 weeks	548	67	771	170	246	Slightly curled.	Slightly curled.	Checked and cracked.
3 "	492	718	763	182	244	"O. K."	"O. K."	Slightly checked.
5 "	427	692	747	183	249	"O. K."	"O. K."	Sound.

"Coarseness of grinding is also a frequent cause of unsoundness for the reason that the larger particles are not readily susceptible to hydration, and contain for a long period of time expansive elements which very rapidly develop a disintegrating action when treated in the accelerated tests."

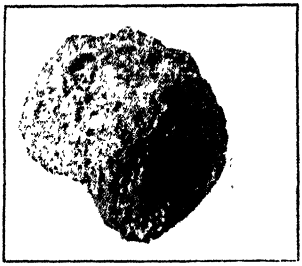
"A large number of tests on different cements were made and the time at which failure occurred was observed. In these tests it was found that of those samples which did not pass the test, 22% failed in the first half hour, 57% failed in the first hour, 85% failed in two hours, 96% in three hours, and 99% in four hours," "thus showing generally that a test piece of cement standing three or four hours of boiling will almost invariably stand a much greater length of time, and also that at least three or four hours should always be allowed for the test."

"Pats of cement allowed more than about twelve hours to harden will, if unsound, fail when tested by boiling at almost any time in the future."

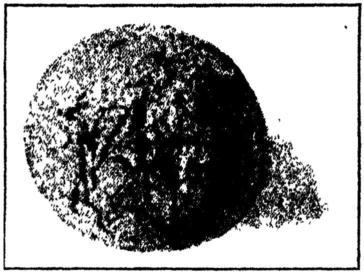
"We now come to the very important question of the relation of the boiling tests to the other tests for soundness and strength as made in the

laboratory. No one who has had much experience with the boiling test questions that, although it is by no means infallible, the results obtained from it are generally corroborated by either the tensile tests or the normal tests for soundness. The writer has recently compiled some data in regard to this point, covering over a thousand tests on many varieties of cement, with the following results:

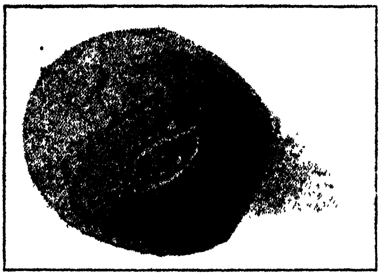
"Of all samples failing to pass the boiling test, 34% of them developed checking or curvature in the normal pats — or a loss of strength in less than twenty-eight days. Of those samples that failed in the boiling test but re-



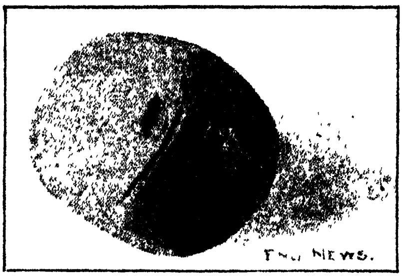
One Week Old.



Two Weeks Old.



Three Weeks Old.



Five Weeks Old.

FIG. 30.—Specimens showing the Effect of the Age of the Cement upon its Soundness. (*See p.* 107.)

mained sound at twenty-eight days, 3% of the normal pats showed checking or abnormal curvature in two months, 7% in three months, 10% in four months, 26% in six months, and 48% in one year; and of these same samples, 37% showed a falling off in tensile strength in two months, 39% in three months, 52% in four months, 63% in six months, and 71% in one year. Or, taking all these together, of all the samples that failed in the boiling test, 86% of them gave evidence in less than a year's time of possessing some injurious quality.

"On the other hand, of those cements passing the boiling test, but one-half of 1% gave signs of failure in the normal pat tests, and but 13% showed a falling off in strength in a year's time.

"This certainly makes a very strong showing in favor of the boiling test, at least considered from a laboratory standpoint.

"In order to show the great value sometimes obtained from the results of the boiling test, several examples are given in the table on page 110 of tests of cements occurring in the regular routine work of the laboratory."

The air and water pats of sample 2 of this table are shown in Fig. 31 at the age of four months. These pats were sound at twenty-eight days.

In conclusion Mr. Taylor lays special emphasis upon the fact that many cements which do not pass the boiling test will give excellent results in

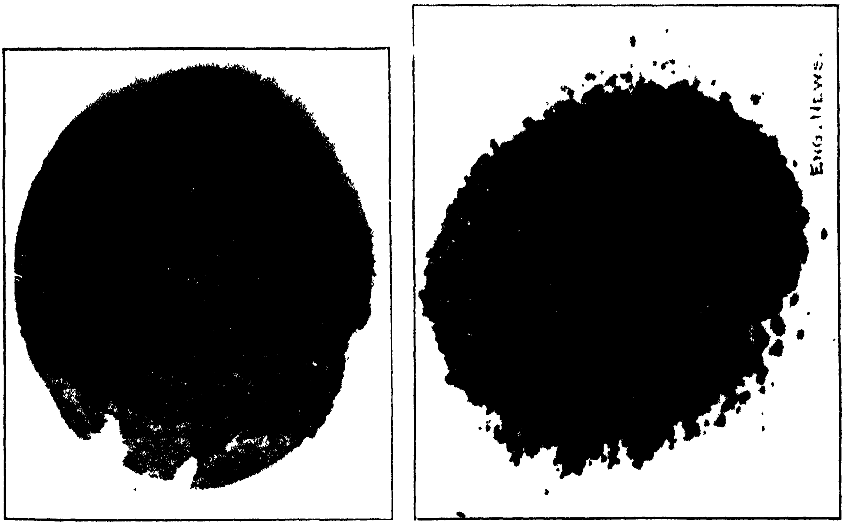


FIG. 31.—Examples of Unsound Pats at 4 months which were sound at 28 days. (*See p.* 109.)

practice. He gives as the probable reason for this that the test for soundness is generally made immediately upon the receipt of a shipment, while the cement used in construction has opportunity to season, and upon the fact "that the disintegrating action of a cement is always far greater when mixed neat than when mixed with an aggregate, and the greater the amount of the aggregate the less the tendency to unsoundness." It is often good policy before rejecting a cement which fails to pass the hot test to hold it for a week or two so that it may further season and then retest it.

Methods of Making Accelerated Tests. The methods of conducting accelerated tests are numerous, the object of all of them being to hasten

Evidences of Failure in Cement Indicated by the Boiling Test.

By W. Purves Taylor. (See p. 109.)

Tensile strength							Normal pat tests				
Neat				1:3 sand			Air		Water		Boiling test
1 day	7 days	28 days	4 months	7 days	28 days	4 months	28 days	4 months	28 days	4 months	
522	793	797	Disintegrated.	204	257	52	Very slightly curled; left glass.	Badly curled; soft and crumbly.	Left glass.	Disintegrated.	Disintegrated.
503	872	586	Disintegrated.	184	239	47	Very slightly curled; left glass.	Badly curled; soft and crumbly.	Left glass.	Disintegrated.	Disintegrated.
498	762	700	Disintegrated.	176	231	119	Very slightly curled; left glass.	Badly curled; soft and crumbly.	"O. K."	Disintegrated.	Disintegrated.
427	751	603	223	183	227	94	Very slightly curled; left glass.	Disintegrated.	"O. K."	Disintegrated.	Disintegrated.
503	827	717	177	220	252	132	Left glass.	Badly curled; soft and crumbly.	"O. K."	Disintegrated.	Disintegrated.
492	883	620	202	195	217	147	Left glass.	Badly curled; soft and crumbly.	"O. K."	Disintegrated.	Badly checked.
535	864	743	94	197	241	77	Very slightly curled; left glass.	Badly curled; soft and crumbly.	"O. K."	Badly curled and checked.	Checked and cracked.
502	829	722	320	203	247	65	Very slightly curled; left glass.	Badly curled; soft and crumbly.	"O. K."	Badly curled and checked	Checked and cracked.
Neat tests not made.				172	219	93	Very slightly curled; left glass.	Badly curled; soft and crumbly.	"O. K."	Badly curled and checked.	Checked.
Neat tests not made.				198	231	101	Left glass.	Badly curled; soft and crumbly.	"O. K."	Badly curled and checked.	Checked.

Note.—All of these cements were normal in specific gravity, time of setting, and fineness.

the hardening of the cement so as to produce in a few hours results which under ordinary conditions require weeks or months. Boiling the specimens, instead of steaming them as recommended by the Committee of the American Society of Civil Engineers, while more common, is more severe. Other methods are employed in Europe.

The Steam Test, recommended by the Committee of the American Society of Civil Engineers, requires, as already described (p. 72), that the pat after twenty-four hours in moist air shall be placed in an atmosphere of steam above boiling water.

The Boiling Test was originated by Prof. Tetmajer in Germany. After twenty-four hours in moist air, or until it is thoroughly set, the specimen is placed in cold water, which is raised to and then maintained at the boiling point for several hours. Three or four hours is the time specified by Mr. W. Purves Taylor, and often used in the United States, although some cement factories boil for twenty-four hours. Dr. Michaelis advocates six hours' boiling, and this period is specified by the French Commission.

Combined Boiling and Tensile Test. A regular test at many Portland cement factories consists in testing the tensile strength of briquettes which have been subjected to the hot test. A briquette of neat cement after twenty-four hours under a damp cloth is placed in an atmosphere of steam over boiling water for an hour or two, and then immersed in water at about the boiling point and boiled for about twenty-four hours, when it must show a certain tensile strength.

The Hot Water Test, as adopted by Mr. Henry Faija in England, and advocated there by Mr. David B. Butler, consists in subjecting a newly mixed pat to a moist heat of 100° to 105° Fahr. (38° to 40° Cent.) for six or seven hours, or until thoroughly set, and then placing it in warm water at a temperature of 115° to 120° Fahr. (46° to 49° Cent.) for the remainder of the twenty-four hours. Mr. Deval in France employed a temperature of 176° Fahr. (80° Cent.) for a period of six days.

Other Accelerated Tests which have been employed in Europe are oven tests, where the specimen is heated in an oven; glow tests, where a ball is heated over a gas flame, and Prussing disc tests, where discs are formed under heavy pressure and then exposed to hot water.

Measurement of Expansion. Appliances have been devised for testing the soundness of cement by measuring the amount of expansion or deformation which it undergoes in different periods of time. The principal of these are the long bar apparatus, devised by Messrs. Durand-Claye and

Debray, which was recommended by the French Commission, Bauschinger's caliper apparatus, and Le Chatelier's tongs.*

The Chimney Expansion Test, in which a small quantity of neat cement is solidly pressed into a straight lamp chimney with the idea that an unsound cement will break the glass, is worthless, as all first-class cements expand to a greater or less degree.

Autoclave Test. Mr. H. J. Force† has brought out in the United States an accelerated test of cement formerly used to a slight extent in Europe.‡ Three neat briquets and an "expansion bar" 1 inch square and 6 inches long are made up, stored in moist closet for 24 hours, and after the measurement of the bar placed in the autoclave apparatus, where the pressure is raised to 295 pounds in not more than one hour and maintained at this pressure for one hour longer, then gradually reduced. The specimens are taken out, placed in the moist closet for one hour, and then measured or broken in the tensile machine. The prisms must not show greater expansion than 0.5 per cent. and the briquets must break at 500 pounds or more and must show at least 25 per cent. increase in strength over the ordinary 24 hour test in order to pass the test.

To determine the real value of the autoclave test in comparison with the standard tests for unsoundness, the Bureau of Standards has in progress a comprehensive series of compression tests of concrete.§ The results indicate that the autoclave test provides no definite indication of the action of the cement when made into concrete.

COLOR OF CEMENT

The color of a cement bears but slight relation to its quality, but a variation of color in the same brand is sometimes an indication of inferiority. Natural cements made in different localities may often be distinguished from each other and from Portland cements by their color.

Portland Cement. The chemical composition of Portland cements made by different processes is so uniform that the color of different brands varies less than that of Natural cements.

The color of Portland cement is described as a cold blue gray. In England the term "foxy" is applied to a Portland cement of a brownish

* Described in Spalding's Hydraulic Cement, 1903, p. 166.

† See papers on results obtained on the autoclave test for cement, American Society for Testing Materials, Vol. XIII, 1913, p. 746.

‡ Dr. Erdmenger in Journal Society of Chemical Industries, Vol. XII, p. 927.

§ See discussion by R. J. Wig American Society for Testing Materials, Vol. XIV, 1914, p. 252.

color. According to Mr. David B. Butler* this denotes "insufficient calcination or the use of unsuitable clay or possibly excess of clay." He further states that if a Portland cement contains a large quantity of underburned particles, on account of their lower specific gravity they tend to rise to the surface on troweling, thus forming a yellowish brown film which is noticeable in the section of the briquette after fracture.

The dark color of the coarser particles of a Portland cement left as residue on a screen is due simply to the fact that cement clinker is black, and pieces which are not finely ground retain the color of the clinker.

Natural Cement. The color of Natural cement varies with the character of the rock and consequently with the locality in which it is produced. It ranges from the light écru of the Utica (Ill.) cement to the dark grayish brown of the Rosendale (N. Y.). Samples received by the authors from various manufactories show the James River cement to be a light yellowish brown, the Akron (N. Y.) cement, écru, the Milwaukee (Wis.) cement, drab, and the Louisville (Ky.) cement, a brownish gray. Certain other brands are similar in color to Portland.

Puzzolan Cement. Puzzolan cement made from slag is of a light lilac shade, much lighter than Portland. After being kept under water it assumes, when freshly fractured, a bluish green tint. This green tint, which according to Candlot† is due to sulphide of calcium present in the cement, is especially noticeable in a sample kept in sea water, and fades on exposure to dry air.

WEIGHT OF CEMENT

Weight is no indication of quality. Formerly, nearly all specifications required that a cement should reach a certain standard of weight per struck bushel or per cubic foot, on the principle that, other things being equal, a thoroughly burned cement is heavier than one which is underburned. It soon developed, however, that the degree of fineness affected the weight much more than any difference in calcination, and the test for specific gravity was substituted.

Method of Weighing Cement. The apparatus finally recommended by the French Commission, after a series of tests by Mr. P. Alexandre,‡ was a circular funnel with screen, as shown in Fig. 32. The cement placed upon the screen is stirred with a wooden spatula 4 cm. ($1\frac{5}{8}$ in.) wide, and 25 cm. (10 in.) long, and falls through the screen into the cylindrical measure of one liter capacity (61 cu. in.).

* Butler's Portland Cement, 1899, p. 255.
† Candlot's Ciments et Chaux Hydrauliques, 1898, p. 159.
‡ Commissioners des Mèthodes d'Essai des Matèriaux de Construction, 1895, Vol. IV, p. 21.

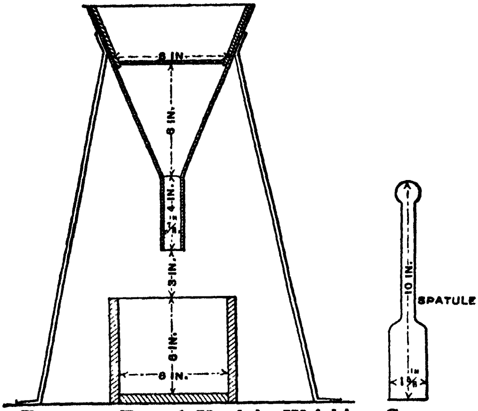


FIG. 32. Funnel Used in Weighing Cement. (*See p.* 113.)

MICROSCOPICAL EXAMINATION OF PORTLAND CEMENT CLINKER

The structure of Portland Cement clinker can be clearly discerned with the aid of the microscope and polarized light by preparing thin sections of it in the same way as those of rocks made by petrographers.

Le Chatelier, a French engineer, and Törnebohn, a Swedish petrographer, some years ago identified two essential mineral entities, and three others of less importance, as constituents of Portland cement clinker. Törnebohn denominated the two essential constituents alite and celite.

CHAPTER VII

TESTS OF AGGREGATES

It is as necessary to test the aggregate for mortar or concrete as it is to test the cement.

This is particularly true of natural bank sand, since it is frequently impossible even for the most expert engineer to determine by examination whether or not a sand is fit to use for mortar and concrete. The experience of one of the authors during the last few years in the investigation of failures of concrete structures shows that the quality of the sand is more frequently to blame than the cement.

Formerly sharpness of sand was considered its most important quality, but as discussed on page 167, it is now recognized that this has but little effect upon its use in mortar or concrete. The origin of the requirement for sharpness was probably the appearance of sand in a pile. When a sand contains a large percentage of vegetable loam, which is one of the worst impurities, the pile when dried in the sun has a dirty or "dead" appearance, while a clean sand is bright and by its glistening appearance gives the effect of sharpness even although the grains are rounded.

In this chapter are presented the important tests necessary for the acceptance of a given aggregate or for the comparison of different aggregates, also characteristics of aggregates under special conditions. These include tests of strength of mortar made from the sand in question (p. 116); mechanical analysis or gradation of grains (p. 117); test for organic impurities (p. 118); chemical tests (p. 118); color tests (p. 119); hardness and strength of particles (p. 119). Voids and characteristics of aggregates are treated in the following chapter. The effect of different characteristics and conditions on the strength of mortar, including the treatment of density and of granulometric composition, is treated in Chapter IX, on Strength and Composition of Cement Mortars.

SAMPLING AND SHIPPING SAND

To obtain a representative sample, cut into the natural bank or into the pile so as to use no sand which has fallen down from the surface. Make with the shovel a vertical face. Scrape vertically with the point

of the shovel along this vertical face so as to form at the bottom a mixed pile of sand. Repeat in another place, if this one sample does not represent a fair average, and mix with the first sample. Send 20 pounds of sand to the laboratory packed so as to prevent drying out.

TEST OF STRENGTH OF MORTAR

The most positive method of determining the quality of a given sand or comparing the relative qualities of two or more different sands is to make up specimens with cement and find the actual strength in tension or compression. Frequently this is the only test needed.

To eliminate the variation due to the quality of the cement and difference in manipulation of the specimen, the strength of the mortar from the sand in question always should be determined in comparison with that of mortar made with Standard sand from Ottawa, Ill.

The method of making up specimens should conform to standard requirements for testing cement, as given on pages 76 to 78. To avoid removal of any coating on the grains which may affect the strength, sand should not be dried before making into mortar, but should contain natural moisture, the weight of which may be corrected by determination of percentage of moisture in a separate sample. The consistency of the mortar of Standard sand should be determined by the standard method described on page 89. The percentage of water to use with the sand in question should be such as to produce the same consistency as the Standard sand mortar. The specimens may be tensile briquets of standard shapes or, for compression, 2-inch cubes or 2 by 4 inch cylinders. Compression tests are much to be preferred.

Sand Specifications. The requirement for the acceptance of fine aggregate is as follows:

Fine aggregate shall consist of sand, crushed stone or gravel screenings, graded from fine to coarse and passing when dry a screen having $\frac{1}{4}$-inch diameter holes. It preferably should be a silicious material and not more than 30 per cent. by weight should pass a sieve having 50 meshes per lineal inch. It shall be clean and free from soft particles, lumps of clay, vegetable loam, and all other organic matter. Fine aggregate shall always be tested.

Fine aggregates shall be of such quality that mortar composed of one part Portland cement and three parts fine aggregate by weight when made into briquets, or into prisms or cylinders, will show a tensile or compressive strength at an age not less than seven

days at least equal to the strength of 1 : 3 mortar of the same consistency made with the same cement and standard Ottawa sand. If the aggregate be of poorer quality, the proportion of cement shall be increased in the mortar to secure the desired strength.

If the strength developed by the aggregate in the 1 : 3 mortar is less than 70 per cent. of the strength of the Ottawa sand mortar, the material shall be rejected. To avoid the removal of any coating on the grains, which may affect the strength, bank sands shall not be dried before being made into mortar, but shall contain natural moisture. The percentage of moisture may be determined upon a separate sample for correcting weight. From 10 to 40 per cent. more water may be required in mixing bank or artificial sands than for Standard Ottawa sand to produce the same consistency.

No requirement is made as to the age of specimens at time of test. Periods most convenient in practice are 3 days, 7 days, and 28 days. A sand passing the strength requirement at the age of 3 days may be accepted without serious question, since the ratio of strength to standard sand is apt to increase with age. On the other hand, if the strength is low at 3 days, the sand may be held for the later tests.

MECHANICAL ANALYSIS

If a fine aggregate is free from organic or other impurities and is of ordinary silica composition, the strength of the mortar is governed by the size and relative sizes of its grains. A coarse sand gives a stronger mortar than a fine one, and generally a gradation of grains from fine to coarse is advantageous. The effect of the coarseness of sand upon strength of mortar is illustrated by Feret's tests on page 159, and tests by the New York Bureau of Water Supply, page 162. Mechanical analysis alone will not determine the quality of a sand because impurities may affect the strength while not appreciably affecting the analysis. The relation of mechanical analysis to granulometric composition is discussed on page 164.

The mechanical analysis of the coarse aggregate also has an important effect upon the strength of the concrete. Mechanical analysis methods are treated more fully in the chapter on Proportioning, pages 175 to 203.

Sieves for Testing Sand. For the mechanical analysis of sand the following sieves are recommended.*

* Selected from sieves in list accepted by Conference called by U. S. Bureau of Standards.

0.250 inch diameter holes.*
No. 7 mesh holes 0.111 inch width 0.032 inch wire.
No. 20 mesh holes 0.0335 inch width 0.0165 inch wire.
No. 50 mesh holes 0.0120 inch width 0.0080 inch wire.
No. 90 mesh holes 0.0059 inch width 0.0052 inch wire.

If a larger number of sieves are desired No. 12 and No. 30 may be added (see p. 187).

TEST FOR ORGANIC IMPURITIES

To determine the percentage of organic impurities in a sand, the silt can be removed from the sand by placing it in a large bottle and washing it with several waters. The wash water is evaporated, and the residue is screened through a No. 100 mesh sieve to remove coarse particles which do not affect the strength. The silt passing this sieve is weighed to obtain the percentage in the original sand, and then ignited in a platinum crucible to determine, after driving off the water, the percentage of combustible organic matter.

Although data on the subject is incomplete, tests by Mr. Thompson tend to indicate that if the silt in a sand has more than 10 per cent. organic matter, and at the same time if the organic matter amounts to over 0.1 percent. of the total sand, the use of the sand may be dangerous.† However, this is by no means a conclusive test, since the nature of the impurities governs their effect upon the mortar or concrete. The usual source of impurities is vegetable loam mixed with the sand by improper handling or leaching down into it through the original ground. Mr. Thompson has found that this sometimes affects the sand to a depth of 10 feet or more below the surface. Vegetable matter adhering to the grains of sand can sometimes be seen by examination under a glass. The so-called "dead" appearance of sand is usually due to vegetable impurities.

Loam adhering to coarse gravel is apt to produce less serious effects. Its effect on mortar is discussed on page 168.

CHEMICAL TESTS

Complete chemical tests of sand are rarely necessary as the chemical composition does not usually affect the strength of the mortar or concrete. However it is often necessary to distinguish a calcareous or

* A No. 4 sieve, having 4 meshes per linear inch, passes approximately the same size grains as a sieve with 0.25 diameter holes.

† See page 168.

limestone sand. Limestone composition is determined by tests with dilute hydrochloric acid. If the material effervesces to a marked degree, it is either limestone or magnesium composition and the percentage may be obtained by quantitative analysis. The effect of limestone composition upon the strength of mortar is shown on page 166.

COLOR TESTS

The depth of color produced by digesting sand with a 3 per cent. solution of sodium hydroxide has been found* to bear a relation to the compressive strength of mortar made with the sand. Eliminating other variables, tests showed that sands giving high strengths in mortar gave a light colored solution when mixed with the hydroxide. Although not a test that can be used alone to indicate quality, it is likely to develop into an inexpensive test that can be employed to eliminate poor sands which are contaminated with vegetable impurities.

HARDNESS AND STRENGTH OF PARTICLES

The effect of aggregates of different hardness is shown in the table of concrete with different coarse aggregates on page 316. In general, it has been found that the harder the stone from which the concrete is made, the stronger is the concrete. The hardness of the grains of fine aggregate has less effect upon the strength. In natural sand the strength of the particles seldom needs to be considered because if the grains are strong enough to have withstood the pulverizing effect of the elements, without becoming too fine for use, they are satisfactory for concrete. Furthermore, if the sand is tested for strength of mortar (see p. 116) any defect in strength will be apparent.

Specific Gravity of Sand by Jackson Apparatus. Sand may be tested for specific gravity most accurately by a specific gravity apparatus. The Jackson flask when properly calibrated is a most convenient apparatus. (See p. 85). Further data on specific gravity of aggregates and methods of determining are treated in the following chapter.

* 1916 Report of Committee C-9 of the American Society for Testing Materials.

CHAPTER VIII

VOIDS AND OTHER CHARACTERISTICS OF CONCRETE AGGREGATES

In this chapter are given tables of the specific gravities and voids of different materials, and the method of determining them, also laws relating to the voids in concrete aggregates, and the effect of compacting such materials.

LAWS OF VOLUMES AND VOIDS

The most important of the general laws relating to volumes of different materials, and to their voids, may be stated as follows:

(1) A mass of equal spheres, if symmetrically piled in the theoretically most compact manner, would have 26% voids whatever the size of the spheres, but by experiment it is found that it is practically impossible to get below 44% voids. (See p. 129.)

(2) If a dry material having grains of uniform shape be separated by screens into grains of uniform dimensions, the separated sizes (except when finer than will pass a No. 74 screen) will contain approximately equal percentages of voids; in other words, a dry substance consisting of large particles, all of similar size and shape, will contain practically the same percentage of voids as a substance having grains of the same shape but of uniformly smaller size. (See p. 131.)

(3) In any material the largest percentage of voids occur with grains of uniform size, and the smallest percentage of voids with a mixture of sizes so graded that the voids of each size are filled with the largest particles that will enter them. (See p. 132.)

(4) An aggregate consisting of a mixture of coarse stones and sand has greater density—that is, contains a smaller percentage of voids—than the sand alone. (See p. 133.)

(5) By Fuller and Thompson's experiments, perfect gradation of sizes of the aggregate appears to occur when the percentages of the mixed aggregate passing different sizes of sieves are defined by a curve which approaches a combination of an ellipse and straight line. (See p. 192.)

(6) Materials with round grains, such as gravel, contain fewer voids than materials with angular grains, such as broken stone, even though

the particles in both may have passed through and been caught by the same screens. (See p. 135.)

(7) The mixture of a small amount of water with dry sand increases its bulk. In the case of most bank sands the maximum volume—and hence the smallest amount of solid matter per unit of volume, that is, the largest percentage of absolute voids—being reached with from 5% to 8% of water. (See p. 137.)

CLASSIFICATION OF BROKEN STONE.*

Rocks which are commonly employed for concrete or for road making are commercially classified as (*a*) traps, (*b*) granites, (*c*) limestones, (*d*) conglomerates, and (*e*) sandstones.

The trade term "trap" includes dark green to black, heavy, close textured, tough rocks of igneous origin, thus covering a variety of rock whose mineralogical names are diabase, norite, gabbro, etc. As shown in the table below, the traps usually range in specific gravity from 2.80 to 3.05.

Granites, commercially so called, include the lighter colored, less dense rock, such as not only true granite, but syenite, diorite, gneiss, mica schist, and several other groups. Their specific gravities range from about 2.65 to 2.85, averaging close to 2.70. Although, as road metal, the traps are usually far superior to granites, for concrete there appears to be no great difference in the value of the two classes. The distinction, however, is worth keeping because a concrete stone is often purchased from road metal quarries.

Limestones of normal type range in specific gravity from 2.47 to 2.76, averaging about 2.60, although the very soft stones, which are not suitable for high class concrete, may fall below 2.0.

Conglomerate, or pudding stone as it is often termed, is essentially a very coarse grained sandstone, ranging in specific gravity from 2.50 to 2.80. It makes a good concrete aggregate.

Sandstones of compact texture, such as the Potsdam and Medina sandstones, and the Hudson River bluestone, may run as high in specific gravity as 2.75, while the looser textured, more porous sandstones may fall as low as 2.10, a fair average being about 2.40.

Shale and slate make poor concrete aggregates, because their crushing and shearing strength is low.

*The authors are indebted to Mr. Edwin C. Eckel for the material under this heading, which has been especially prepared by him for this Treatise.

Specific Gravity of Stone from Different Localities.
COMPILED BY EDWIN C. ECKEL.

TRAP.

Locality.	Specific Gravity.
MASSACHUSETTS	
Boston	2.78
MINNESOTA	
Duluth	3.00
Duluth	2.80
Taylors Falls	3.00
NEW JERSEY	
Jersey City Heights	3.03
Little Falls	2.99
NEW YORK	
Staten Island	2.86

GRANITE.

Locality.	Specific Gravity.
CALIFORNIA	
Penrhyn	2.77
Rocklin	2.68
CONNECTICUT	
Greenwich	2.84
New London	2.66
GEORGIA	
Stone Mt.	2.69
MAINE	
Hallowell	2.66
MARYLAND	
Port Deposit	2.72
MASSACHUSETTS	
Quincy	2.70
NEW HAMPSHIRE	
Keene	2.66
NEW YORK	
Ausable Forks	2.76
RHODE ISLAND	
Westerly	2.67
VERMONT	
Barre	2.65
WISCONSIN	
Amberg	2.71
Montello	2.64

LIMESTONE.

Locality.	Specific Gravity.
ILLINOIS	
Joliet	2.56
Lemont	2.51
Quincy	2.57
INDIANA	
Bedford	2.48
Salem	2.51
MINNESOTA	
Frontenac	2.63
Winona	2.67
NEW YORK	
Canajoharie	2.68
Glens Falls	2.70
Kingston	2.69
Prospect	2.72
Sandy Hill	2.76
Williamsville	2.71

Soft Limestone

Locality.	Specific Gravity.
FRANCE	
Caen	1.84

SANDSTONE.

Locality.	Specific Gravity.
COLORADO	
Ft. Collins	2.43
Trinidad	2.34
CONNECTICUT	
Portland[1]	2.64
MASSACHUSETTS	
Longmeadow[1]	2.48
MINNESOTA	
Fond du Lac	2.24
NEW JERSEY	
Belleville[1]	2.26
NEW YORK	
Albion[2]	2.60
Medina[2]	2.41
Potsdam[3]	2.60
Oxford[4]	2.71
Malden[5]	2.75
Oswego	2.42
OHIO	
Berea[6]	2.14
Cleveland	2.21
Massillon	2.11

[1]Brownstone.
[2]Medina sandstone.
[3]Potsdam sandstone.
[4]Bluestone.
[5]Hudson River Bluestone.
[6]Berea grit.

SPECIFIC GRAVITY OF SAND AND STONE

The specific gravity of a substance is the ratio of the weight of a given volume to the weight of the same volume of distilled water at a temperature of 4° Cent. (39° Fahr.). For ordinary tests of stone and sand, the water need not be distilled and may be at ordinary temperature without materially affecting the result.

A knowledge of the specific gravity of the particles of the sand and stone is important to the engineer in determining the percentages of voids in concrete aggregates.

For accurate determinations of density the specific gravity of a natural sand must be determined. For ordinary purposes, such as the determining of the percentage of voids, the specific gravity may be assumed as 2.65. This value has been determined as the usual specific gravity by experimenters in this country and abroad, except for calcareous sands which average about 2.69 by absolute determination, or about 2.55 if measured by the total volume of the particles having their pores filled with air.

Gravels also have quite uniform specific gravity. According to Mr. A. E. Schutté, who has tested gravel from more than forty localities in the United States and Canada, an average value is 2.66.

This uniformity in the specific gravity of different sands and of different gravels is very convenient for calculation. For stones there is considerable variation.

The following table gives average values of various concrete aggregates. In every case, the specific gravity is the ratio of the weight of an absolutely solid unit volume of each material to the weight of a unit volume of water. Specific gravities of stone from various localities are given on page 122.

Average Specific Gravity of Various Aggregates. (*See p.* 123.)

Material.	Range.	Specific Gravity Average.	Weight of a solid cu. ft. of rock.
Sand	2.62 to 2 68	2.65	165
Gravel		2.66	165
Conglomerate		2.6	162
Granite	2.65 to 2.85	2.7	168
Limestone	2.48 to 2.76	2.6	162
Trap	2.80 to 3.05	2.9	180
Slate		2.7	168
Sandstone	2.10 to 2.75	2.4	150
Cinders (bituminous)		1.5	95

METHOD OF DETERMINING SPECIFIC GRAVITY

The specific gravity of a sample of material is determined by dividing its weight by the weight of water which it displaces when immersed.

The size of sample necessary for the accurate determination of a sand or stone of fairly uniform texture depends chiefly upon the delicacy of the apparatus employed. If scales reading to grams, and measures reading to cubic centimeters, are employed, a sample of 250 grams should give accurate results to two decimal places. With scales reading to ¼ ounce, a sample of 4 lb. is necessary for similar accuracy. The water must be maintained at 68° Fahr. (20° Cent.).

The sample should be taken by the method of quartering described on page 344.

Before finding the specific gravity of siliceous sand, the sample should be dried in an oven at a temperature as high as 212° Fahr. (100° Cent.) until there is no further loss in weight. A porous stone, on the other hand, may be first moistened sufficiently to fill its pores, and then the surfaces of the particles dried by means of blotting paper. If this method is followed, the material should be in a similar condition when its voids are determined by the method given on page 126. The absolute specific gravity of the porous stone may be afterward found by drying in an oven and correcting for the moisture lost.

The apparent specific gravity of sand or stone may be determined with an apparatus consisting of scales reading to ¼ ounce or to 5 grams, and a tall glass vessel with a reference mark, such as a cylinder or a pharmacist's graduate. The method is as follows:

Make a mark at any convenient place on the neck of the vessel;

Fill the vessel with water at a temperature of 68° Fahr. (20° Cent.) up to this mark;

Take a known weight in grams or ounces of the material;

Pour material into vessel carefully, a few grains at a time, so that no bubbles of air are carried in with it;

Pour out the clear water displaced by the material (leaving water in the vessel up to the level of the mark), and weigh the water poured out.

Let

S = Weight of material placed in vessel.

W = Weight of water displaced.

Then

$$\text{Specific gravity of material} = \frac{S}{W} \qquad (1)$$

It is essential that the weight of water displaced be weighed to within $\pm 2\%$. If the scales are not sufficiently sensitive, more material must be taken and a larger vessel used. With balances sensitive to 1 gr. or $\frac{1}{16}$ oz. the displacement of more than 3 ounces of water is necessary.

An alternate method, recommended by Committee D 4 of the American Society for Testing Materials, for determining the apparent specific gravity of homogeneous coarse aggregates* is as follows:

The apparent specific gravity shall be determined in the following manner:

1. A properly selected sample which will pass a 2.54 cm. (1-in.) circular opening and which will not pass a 1.27-cm. ($\frac{1}{2}$-in.) circular opening, and approximately cubical or spherical in shape, shall be dried to constant weight at a temperature between 100 and 110° C. (212 and 230° F.).

2. The dried sample shall be suspended in air by a fine wire or thread from a scale or balance and weighed in air to 0.01 g., which weight shall be recorded as weight A.

3. It shall then be immersed, for not less than 10 minutes, in clear water having a temperature between 15 and 25° C. (60 and 77° F.) until no air bubbles appear on the surface of the sample.

4. After all air bubbles shall have been removed from the surface and after the scales have been balanced, the sample shall be allowed to remain immersed for 1 minute, and if any change in weight takes place, the sample shall remain in water until the balance remains constant within 0.01 g. for 1 minute. This weight shall be recorded as weight B.

5. After weight B has been obtained, the sample shall be removed immediately from the water, the surface water shall be wiped off with a towel or filter paper, and the wet sample shall be promptly weighed in air. This shall be recorded as weight C.

6. The apparent specific gravity of the sample shall be calculated by dividing the weight of the dry specimen (A) by the difference between the weights of the saturated specimen in air (C) and in water (B) as follows:

$$\text{Apparent specific gravity} = \frac{A}{C-B}$$

7. The apparent specific gravity of the material shall be the average of three determinations, made on three different samples, according to the method described above.

* Attention is called to the distinction between apparent specific gravity and true specific gravity. Apparent specific gravity includes the voids in the specimen and is therefore always less than or equal to but never greater than, the true specific gravity of a material.

DETERMINATION OF VOIDS

The voids in sand, gravel, and broken stone may be obtained directly from the tables on pages 127 and 128. Special determinations may be made as described below.

The percentage of voids in sand or fine broken stone cannot be accurately obtained by the ordinary method of placing in a measure and pouring in water, because it is physically impossible to drive out all the air. There may sometimes be enough of this held to amount to 10% of the volume of the sand, and thus cause a corresponding error in the percentage of voids.

The voids in coarse stone containing no particles under ½-inch diameter may be determined by placing in a box or pail of known volume and pouring in water, but if the specific gravity is known the voids may be determined directly from the weight. This method can be used both for fine or coarse aggregate.

The only apparatus required are scales of fair accuracy and an exact measure which contains not less than ½ cu. ft. If a cubic foot measure is not available a 16-quart pail will answer the purpose, although compactness of the sand is less easily adjusted because of the small diameter. Such a pail holds slightly over ½ cu. ft. and the exact measure is determined by weighing the pail, pouring in 31 lbs. 2 oz. of water, and marking the level of the surface. The pail up to this mark contains ½ cu. ft. of any material.

The method of determining the voids is as follows:

Weigh the measure;

Fill the measure to the required level with the material in the state in which the percentage of voids is required, that is, loose, shaken, or packed;

Take a measure holding preferably not less than ½ cubic foot, weigh and fill with the material in the state in which the percentage of voids is required, that is loose, shaken, or packed. For use in proportioning, it is the authors' practice to weigh, deduct the weight of the measure, and figure the net weight of a cubic foot of the material, W. If the material consists of, or contains, sand or fine stone, correct for moisture by taking an exact weight,—about 10 lb.,—drying in an oven at a temperature of at least 212° Fahr. (100° Cent.) until there is no further loss in weight, and after calculating the percentage of moisture in terms of the weight of the original moist sand or stone, express the percentage as a decimal, p.

In the following formula:

Let W = weight of material per cubic foot;

p = percentage of moisture;

S = Specific gravity of material;

62.3 is the weight of a cubic foot of water.

$$\text{Per cent. of absolute voids} = \left(1 - \frac{W - Wp}{62.3\,S}\right) 100 \qquad (2)$$

The air voids are determined, if desired, by deducting the volume of moisture (its weight divided by the weight of one cubic foot of water)

Percentages of Voids Corresponding to Different Weights per Cubic Foot of Sand, Gravel, and Broken Stone Containing Various Percentages of Moisture. (*See p.* 129.)

Weight of one cu. ft. of sand or gravel.*	Percentages of absolute voids in material containing moistures by weight.† 0%	2%	4%	6%	8%	Moisture by volume corresponding to 1% by weight.†	Weight of one cu. ft. of sand or gravel.*	Percentages of absolute voids in material containing moistures by weight.† 0%	2%	4%	6%	8%	Moisture by volume corresponding to 1% by weight.†
	%	%	%	%	%	%		%	%	%	%	%	%
70	57.6	58.4	59.3	60.1	61.0	1.1	98	40.6	41.8	43.0	44.2	45.3	1.6
75	54.5	55.4	56.4	57.3	58.2	1.2	99	40.0	41.2	42.4	43.6	44.8	1.6
80	51.5	52.5	53.4	54.4	55.4	1.3	100	39.4	40.6	41.8	43.0	44.2	1.6
81	50.9	51.9	52.9	53.9	54.8	1.3	101	38.8	40.0	41.2	42.5	43.7	1.6
82	50.3	51.3	52.3	53.3	54.3	1.3	102	38.2	39.4	40.7	41.9	43.1	1.6
83	49.7	50.7	51.7	52.7	53.7	1.3	103	37.6	38.8	40.1	41.3	42.5	1.6
84	49.1	50.1	51.1	52.2	53.2	1.4	104	37.0	38.2	39.5	40.8	42.0	1.7
85	48.5	49.5	50.6	51.6	52.6	1.4	105	36.4	37.6	38.9	40.2	41.4	1.7
86	47.9	48.9	50.0	51.0	52.0	1.4	106	35.8	37.0	38.3	39.6	40.9	1.7
87	47.3	48.3	49.4	50.4	51.5	1.4	107	35.2	36.4	37.7	39.0	40.3	1.7
88	46.7	47.7	48.8	49.9	50.9	1.4	108	34.6	35.9	37.2	38.5	39.7	1.7
89	46.1	47.1	48.2	49.3	50.4	1.4	109	33.9	35.3	36.6	37.9	39.2	1.7
90	45.5	46.5	47.6	48.7	49.8	1.4	110	33.3	34.7	36.0	37.3	38.7	1.8
91	44.8	45.9	47.0	48.2	49.2	1.5	115	30.3	31.7	33.1	34.5	35.9	1.8
92	44.2	45.4	46.5	47.6	48.7	1.5	120	27.3	28.7	30.2	31.6	33.1	1.9
93	43.6	44.8	45.9	47.0	48.1	1.5	125	24.2	25.8	27.3	28.8	30.3	2.0
94	43.0	44.2	45.3	46.5	47.6	1.5	130	21.2	22.8	24.4	25.9	27.5	2.1
95	42.4	43.6	44.7	45.9	47.0	1.5	135	18.2	19.8	21.4	23.1	24.7	2.2
96	41.8	43.0	44.1	45.3	46.4	1.5	140	15.2	16.8	18.5	20.2	21.9	2.2
97	41.2	42.4	43.6	44.7	45.9	1.6							

*Also applicable to broken stones such as granite, conglomerate, and limestone, whose specific gravity averages from 2.6 to 2.7. Table is based on specific gravity of 2.65.

†The per cent. of absolute voids given in the columns include the space occupied by both the air and the moisture. To determine the per cent. of air space, multiply the figure in the last column, opposite the weight of sand under consideration, by the per cent. of moisture by weight, and deduct result from the per cent. already found.

in a unit volume of the sand or stone, from the total voids. Expressed in percentages with notation same as above,

$$\text{Per cent. of air voids} = \text{Per cent. of absolute voids} - \frac{Wp}{62.3}\,100 \qquad (3)$$

Example.—Given a sand whose loose weight per cubic foot is found to be 92 lb. and its moisture 3% by weight. Find the percentage of voids in the loose sand.

Solution by formula.—Since from the example $W = 92$ and $p = 0.03$, and, from table on page 123, $S = 2.65$.

$$\text{Percentage of absolute voids} = \left(1 - \frac{92 - 0.03\ (92)}{62.3 \times 2.65}\right) 100$$
$$= 45.9\%$$

This percentage includes the space occupied by the moisture. The net percentage of voids occupied by air alone is the difference between the absolute voids and the percentage of moisture by volume. Moisture is $92 \times 0.03 = 2.76$ lb., or $\frac{2.76}{62.3} = 0.044$ cu. ft., corresponding to 4.4% voids by volume, hence air voids are $45.9\% - 4.4\% = 41.5\%$.

Percentages of Voids Corresponding to Different Weights per Cubic Foot of Dry Broken Stone of Various Specific Gravities. (*See p.* 129.)

Weight of one cu. ft. of dry broken stone	PERCENTAGE OF ABSOLUTE VOIDS CORRESPONDING TO SPECIFIC GRAVITIES OF STONE OF						
	2.4*	2.5	2.6†	2.65‡	2.7§	2.8	2.9‖
	%	%	%	%	%	%	%
70	53.2	55.0	56.8	57.6	58.4	59.9	61.3
75	49.8	51.8	53.7	54.5	55.4	57.0	58.5
80	46.5	48.6	50.6	51.5	52.4	54.1	55.7
85	43.2	45.4	47.5	48.5	49.5	51.3	53.0
90	39.8	42.2	44.5	45.5	46.5	48.4	50.2
95	36.5	39.0	41.4	42.5	43.5	45.5	47.4
100	33.1	35.8	38.3	39.4	40.6	42.7	44.7
105	29.8	32.6	35.2	36.4	37.6	39.8	41.9
110	26.4	29.4	32.1	33.4	34.6	36.9	39.1
115	23.1	26.2	29.0	30.4	31.6	34.1	36.4
120	19.8	23.0	25.9	27.3	28.7	31.2	33.6
125	16.4	19.8	22.8	24.3	25.7	28.3	30.8
130	13.1	16.6	19.8	21.2	22.7	25.5	28.1
135	9.7	13.3	16.7	18.2	19.7	22.6	25.3
140	6.4	10.1	13.6	15.2	16.7	19.7	22.5

NOTE.—Average specific gravity of bituminous coal cinders may be taken as 1.5.

* Sandstone.

† Limestone and conglomerates.

‡ Sand.

§ Granite and slates.

‖ Trap.

Solution by table (p. 127.) — Opposite 92 lb. per cu. ft., interpolating between 2% and 4% moisture, is 46.0% of absolute voids. From last column 3% by weight corresponds to 3% x 1.5 = 4.5% by volume. 46.0% — 4.5% = 41.5% air voids.

Tables of Voids. From the tables on pages 127 and 128, the voids in sand, gravel, and broken stone may thus be determined simply by weighing the material and finding the percentage of moisture contained in it, as above described. Since the percentage of moisture by volume is always greater than its percentage by weight, and the two are not proportional to each other, the final column is inserted in the first table for convenience in calculating the moisture by volume.

VOIDS AND DENSITY OF MIXTURES OF DIFFERENT SIZED MATERIALS

The term *density* as applied to mortar is defined on page 148. Similarly, in a dry material, such as a concrete aggregate, it is represented by the total volume of the solid particles entering into a unit volume of the aggregate. In dry materials the density is the complement of the voids, since a material which has, say, 40% voids will have a density of 0.60; but density is a more correct term to use than voids because it is applicable to concretes and mortars in which connection the term voids is somewhat ambiguous. The example on page 150 illustrates the method of determining the density of a concrete or mortar.

The densities of dry aggregates of uniform specific gravity, or of mixtures in uniform proportions of materials with different specific gravities, are in direct proportion to their weights. For example, the densities of different dry sands may be compared by weight; or the densities of different mixtures of sand and broken trap in proportions, say, 2 parts sand to 4 parts trap may be compared by weight; but the density of sand and the density of trap screenings cannot be compared by weights unless the differing specific gravities are taken into account.

In the following discussion of the laws formulated on page 120, both the terms *density* and *voids* are used in relation to the dry materials.

Voids in Masses of Similar Sized Particles. (1) The fact that the percentage of voids in a mass of equal spheres symmetrically piled in the theoretically most compact manner is independent of the actual diameter is simply a geometrical propositicn, evident without demonstration by inspection of Fig. 33.

In actual experiment it has been found that while the percentage of voids is uniform regardless of the size of the spheres, it is impossible to

pour spheres into a measure so that they will arrange themselves symmetrically, and the rather astonishing result has been reached by Mr. Fuller (see p. 177) that 44% is the smallest percentage of voids which can be obtained with equal perfect spheres, no matter what may be their actual diameters or the size of the receptacle.

The following simple demonstration,* which is of theoretical interest, proves that the percentage of voids in a mass of equal spheres symmetrically piled in the most compact manner is 26%, and that the radii (and consequently the diameters) of the two next smaller spheres which can

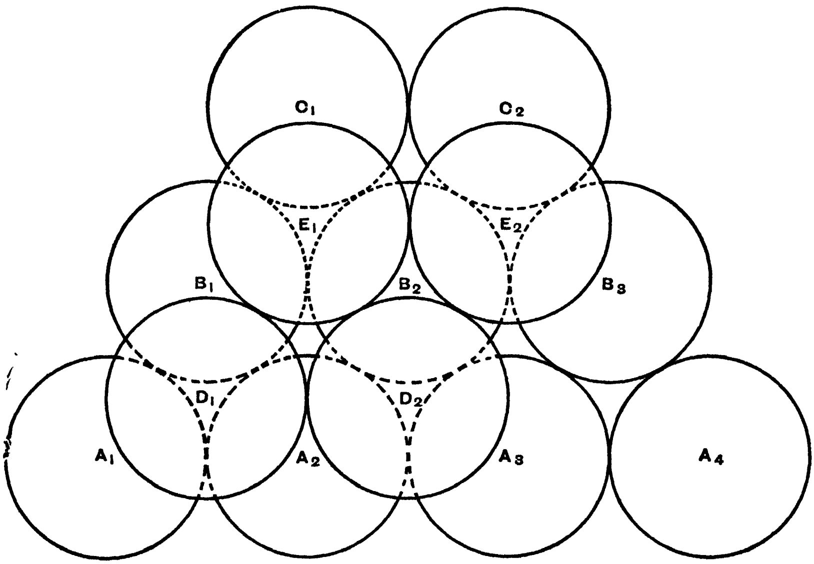


FIG. 33.—Spheres of Equal Size. (*See p.* 129.)

be inscribed between the larger ones are respectively 0.41 and 0.22 of the radius of the large spheres.

The circles in Fig. 33 represent a horizontal plan of two layers of spheres.

The centers A_1 A_2 B_1 D_1 form a regular tetrahedron.

Let edge be 2.

Altitude = difference between level of centers A, B, C, and level of centers D, E is $\frac{2}{3}\sqrt{6}$

Let number of spheres in a layer be m, number of layers n.

*For which the authors are indebted to Dr. Harry W. Tyler.

Volume of one sphere is $\frac{4\pi}{3}$

Volume of spheres in a layer, $\frac{4m\pi}{3}$

Volume of all spheres, $\frac{4mn\pi}{3}$ (approx.) $= V_1$

Cross-section of including space is $2\sqrt{3}\,m$ (approx.)

Volume of including space is $2\sqrt{3}\,m \times \frac{2}{3}\sqrt{6}\;n$ (approx.)

$= 4\sqrt{2}\,m\,n$ (approx.) $= V_2$

Ratio $\frac{V_1}{V_2} = \frac{4\,m\,n\,\pi}{3 \times 4\,m\,n\sqrt{2}} = \frac{\pi}{3\sqrt{2}} = 0.74$ (approx.) corresponding to about 26% voids.

Inscribed Spheres.

1. Sphere inscribed between spheres A_1 A_2 B_1 and D_1:
Distance from any vertex A_1 of tetrahedron to center is $\frac{1}{2}\sqrt{6}$
Radius of small sphere $= \frac{1}{2}\sqrt{6} - 1 = 0.22$ (approx.) or about $\frac{22}{100}$ of the radius of the large spheres.

2. Sphere inscribed between A_2 B_1 B_2 and D_1 D_2 E_1:
Distance from A_2 to E_1 is $2\sqrt{2}$
Radius of small sphere $= \sqrt{2} - 1 = 0.41$ (approx.) or about $\frac{41}{100}$ of the radius of the large spheres.

(2) The proposition that if a dry material such as sand, pebbles, or irregular broken stone, having grains of fairly uniform shapes, be separated by screens into grains of uniform dimensions, the separated sizes will contain approximately equal percentages of voids, is not so self-evident, but experiment proves that in portions of the same material screened to uniform sizes the percentages of voids will be substantially alike until very fine sizes are reached, such as will pass a No. 74 sieve; below this degree of fineness the particles are entangled by air. The authors have found by experiments given in the following table, that different lots of broken stone from the same quarry, each screened to uniform size, will contain substantially the same percentages of voids, but that lots of stone from different quarries screened to the same size may differ because of the structure of the rock. Published records usually show slight variations in the weight per cubic foot of different sized broken stone, but it is noticeable that some authorities give the heaviest weight,

which corresponds to the smallest percentage of voids, for the larger sizes, while others give the reverse. The variation in results is due undoubtedly to differences in methods of compacting and to the variations in the sizes of the stones of each lot.

Experiments by Mr. Feret in France, and Mr. Thomas F. Richardson in the United States, show that the percentages of voids in absolutely dry sand which has been screened to uniform size are almost identical. Mr. Feret, experimenting by shoveling dry sand loosely into a 50liter (1.8 cu. ft.) box,—a measure large enough to eliminate errors of placing,—found that fine (*F*) medium (*M*) and coarse (*G*) sands each contained about 50% voids, while mixing the sizes, which are defined on page 156, in the best proportions reduced the voids to 34%. Similar results were obtained by Mr. Richardson as a result of an extended series of tests made in connection with the construction of the Wachusett Dam in Massachusetts.

Voids and Compression of Broken Trap and Gravel. (*See p.* 131.)

Size of Stone	Class of Stone	Crusher	Size of Particles	Voids in loose stone %	Compression by light ramming or shaking %	Voids in lightly rammed or shaken stone %	Compression by heavy ramming %	Voids in heavily rammed stone %	
No. 2	HardTrap	Rotary	2½" to 1"	54.5			19.2	43.7	
No. 3		"	1" to ½"	54.5	14.3	46.9	20.5	42.8	
Nos. 2, 3, 4		"	2½" to dust*	45.0	14.5	35.7	20.8	30.6	
No. 2	Soft Trap	Jaw	2" to ¾"	51.2	11.9	44.6	17.8	40.6	Variation is due to trap breaking under rammer.
No. 3		"	¾" to ⅜"	51.2	14.3	43.1	23.0	35.0	
	Gravel		2½" to ½"	30.5	12.5†	27.4	11.5†	28.2	

Loose stone is as thrown by a laborer into a measuring box or barrel.
Material rammed in 6-inch layers.

Densest Mixture of Sand and Stone. (3) The fact that the densest mixture occurs with particles of different sizes is so evident as to require no proof, and this being recognized, it follows that the least density and hence the largest percentage of voids occurs when the grains are all of the same size. The converse of this proposition, that the smallest percentage of voids occurs in a mixture graded so that the voids of each size are filled with the largest particles which will enter them, is

* Mixed in proportions 44.4% No. 2, 33.3% No. 3, and 22.2% No. 4 (dust).

† Another gravel tested, compressed, 8.5% on shaking, and 11.2% on hard ramming.

illustrated in Figs. 34, 35, and 36, and is important in its application to the selection of materials for concrete.

(4) The fact that an aggregate consisting of a mixture of stones and sand has greater density, that is, contains fewer voids than the sand alone,

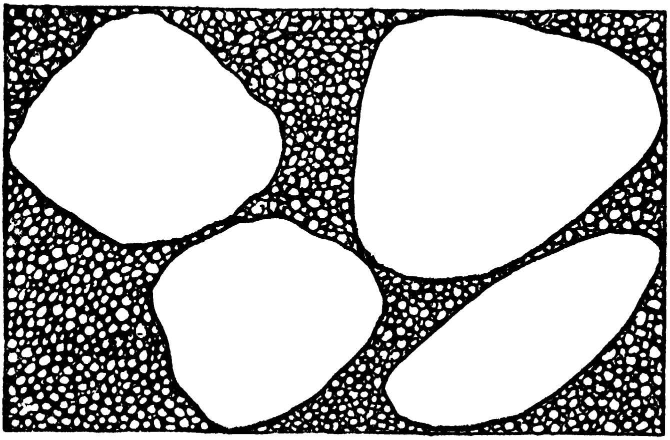


FIG. 34.—Large Stones with Voids filled with Sand. (*See p.* 133.)

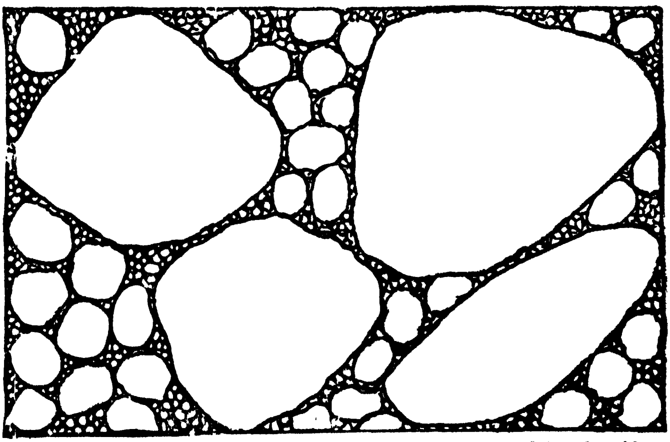


FIG. 35.—Large Stones with Voids filled with small Stones and Sand. (*See p.* 133.)

is illustrated by comparison of Figs. 34 and 36. The voids of the large stone in Fig. 34 are filled with sand, while the voids in the same large stone in Fig. 36 are filled with mixed sand and stone, and the mass of the mixture is evidently denser, that is, it contains more solid material. This

law relates directly to the difference between mortar and concrete. The substitution of stones for small masses of sand reduces the voids and consequently the quantity of cement required. Extending the principle to the fixing of proportions of sand and stone, it is evident that for maximum

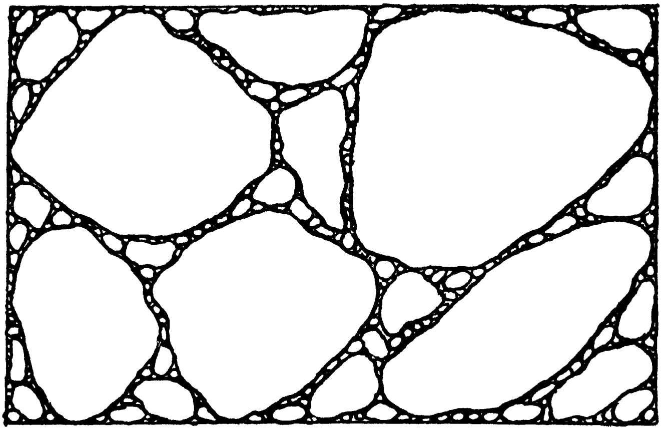


FIG. 36.—Large Stones, with Voids filled with medium sized Stones surrounded by smaller Stones and Sand so as to give Graded Mixture. (*See p.* 133.)

economy and equal strength there should be used the largest possible quantity of stone in proportion to the sand, the strength of concrete being often actually increased simply by substituting more stone for a portion of the sand. In the following table this is illustrated by tests selected from Mr. Fuller's 6-inch beam experiments, which are given in full on page 334.

Relation of Strength of Concrete to Relative Proportions of Sand and Stone. (*See p.* 134.)

Proportions by weight of cement to total aggregate.	Proportions by weight of cement to sand and broken stone.	Modulus of Rupture lb. per sq. in.
1:6	1:1:5	504
1:6	1:2:4	439
1:6	1:3:3	355
1:6	1:4:2	210
1:6	1:6:0	93

The total amount of aggregate in each case is the same, namely, one part cement to 6 parts sand and stone, but the strength varies with the relative proportions of each, from 93 lb. to 504 lb.

The fine material must be small enough to enter the voids of the coarse, else the stones are merely thrust apart. Similarly, a mixture of coarse and fine sand (see p. 159) gives a denser mix than coarse, medium and fine. See also Chapter X.

(5) The discussion of Fuller's experiments on the relation of the best practical mixture of sizes to a parabolic curve is given in Chapter X.

Effect of Shape of Grain. (6) Aggregate with round grains, such as gravel, contains fewer voids than material with angular grains, such as broken stone, even if the particles in both are the same size, as is proved from experiments in America and France. Mr. Feret* gives the following results of tests on the effect of the shape of the grain upon the density of sand, using in each case an artificial mixture of three sizes:

Effect of Character of Sand Grains upon the Volume of the Sand. (*See p.* 135.)

By R. Feret.

Nature of Sand.	Shape of Grains	Actual solid volume per liter of sand		Percentage of voids.	
		Not shaken, liter.	Shaken to refusal, liter	Not shaken.	Shaken to refusal
Quartzite crushed in jaw crusher............ ..	Laminated	0.525	0.654	47.5	34.6
Crushed shells.... ..	Flat	0.557	0.682	44.3	31.8
Ground quartzite..	Angular	0.570	0.726	43.1	27.4
Natural granitic sand	Rounded	0.651	0.744	34.9	25.6

The voids in each case are the complements of the figures given.

The conclusion to be drawn is that the density increases and the voids decrease as the sand approaches the round form.

When experimenting upon gravels and broken stone Mr. Feret† separated each into three sizes which he called respectively:

G (coarse) passing holes of 6 cm. (2.36 in.) diameter and retained by holes of 4 cm. (1.57 in.) diameter;

M (medium) passing holes of 4 cm. (1.57 in.) diameter and retained by holes of 2 cm. (0.79 in.) diameter;

F (fine) passing holes of 2 cm. (0.79 in.) diameter and retained by holes of 1 cm. (0.39 in.) diameter.

Each size of broken stone loosely measured gave about 52% voids, and each size of gravel about 40% voids. The voids in the broken stone were reduced to 47%, the lowest result obtainable, by mixing G and F in about

* Annales des Ponts et Chaussées, 1892, II, p. 32.

† Annales des Ponts et Chaussées, 1892, II, p. 153.

equal parts with no M, and in the gravel to 34% with about $3\frac{1}{2}$ parts of G to one part of F. These figures are applicable only to the materials

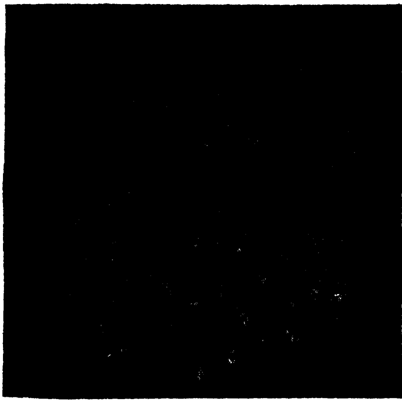
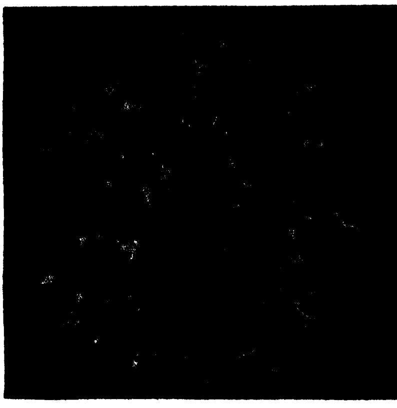


FIG. 37.—Standard Ottawa Sand, dry.* No. 20 to No. 30 Sieves. (*See p.* 136.)

FIG. 38.—Standard Ottawa Sand with 6% moisture.* No. 20 to No. 30 Sieves. (*See p.* 136.)



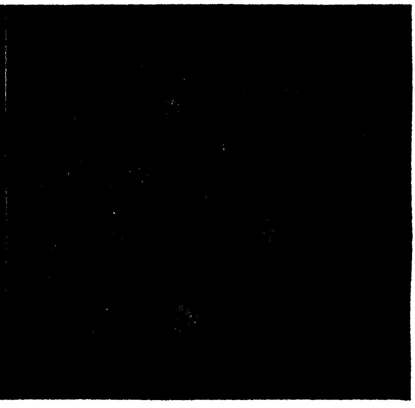


FIG. 39.—Natural Bank Sand.* No. 20 to No. 30 Sieves. (*See p.* 136.)

FIG. 40.—Crushed Quartz.* No. 20 to No. 30 Sieves. (*See p.* 136.)

studied, and do not apply to gravel or stone containing sand or dust.

Photographs of Sand.† Photographs of three types of sand are shown in Figs. 37 to 40. Figures 37 and 38 are photographs of the Ottawa,

* Magnified $10\frac{1}{2}$ diameters.

† Photographs and tests of ten sands from different localities are given in "Concrete Aggregates" by Sanford E. Thompson before the International Engineering Congress, 1915.

Illinois, bank sand screened to the size selected for the standard sand by the Committee of the American Society of Civil Engineers. They illustrate the effect of moisture upon the arrangement of the sand grains, which is more fully described below. Fig. 39 is an ordinary bank sand from Eastern Massachusetts which has passed through and been retained by the same screens as the Ottawa sand. Fig. 40 is a sample of crushed quartz sand, formerly the standard in the United States. The sands are all reduced by the same number of diameters. The Ottawa sand, Figs. 37 and 38, is apparently of finer grain than either the bank sand or the crushed quartz, but close inspection will show that its grains, very uniform in size, are of about the same diameter as the smallest grains in the other sands. In other words, all the grains correspond very closely to a No. 30 sieve, the lot of sand from which it was screened containing no larger particles.

Effect of Moisture on Sand and Screenings. (7) Moist sand occupies more space and weighs less per cubic foot than dry sand. This is directly contrary to what one would naturally suppose. Indeed, it is almost incredible that the addition of water can reduce the weight of any material. The statement is readily proved however, by shoveling a small quantity of natural sand as it comes from the bank with, say, 3% or 4% of moisture into a measure and drying it. The sand will settle, leaving the surface much below the level of the top of the measure. The explanation of this apparent anomaly lies in the fact that a film of water coats each particle of sand and separates it by surface tension from the grains surrounding it. This is illustrated in Figs. 37 and 38, page 136, the grains of the moist sand being separated from each other by the film of water. The moisture also causes the particles to adhere to each other in groups with the effect of less uniform distribution. Fine sand, having a larger number of grains, and consequently more surface area, is more increased in bulk by the addition of water than coarse sand. The volume of coarse broken stone and gravel is but slightly, if at all, changed by moisture, while small broken stone composed largely of particles of less than ½-inch diameter is affected like sand.

If a small quantity of water is poured into a vessel containing dry sand, the bulk is not increased because of the inertia of the particles, but if the sand after moistening is dumped out and then turned back into the vessel with a shovel or trowel, its bulk will be increased. On the same principle, a sand bank does not swell in bulk during a shower, but the effect of the moisture is shown in the excavated material as soon as it is loosened with the shovel, and therefore its loose measurement for concrete or mortar is affected.

The diagram in Fig. 41, plotted by Mr. Fuller* from experiments upon a single sample of natural sand mixed by weight with varying percentages of water, illustrates the effects of moisture upon the actual percentages of voids in sands loose and tamped. The volumes produced by varying degrees of compacting are located between the two curves. It is noticeable that both the loose and tamped sand increase in volume with the addition of water and reach a maximum with about 6% of water, then decrease, and finally, when saturated, return to slightly less than their original dry bulk. The same sand, it is seen, may contain from 27% to 44% of absolute voids, according to the percentage of water and the degree of compacting.

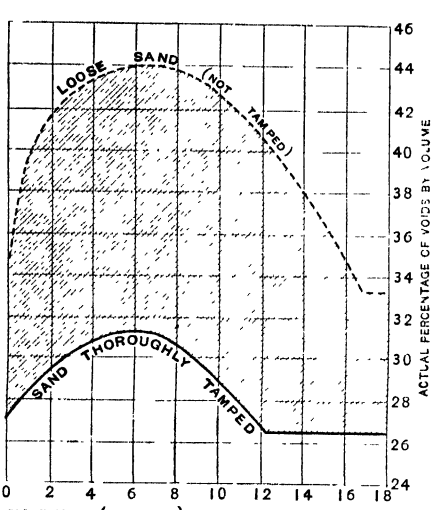


FIG. 41.—Percentage of Absolute Voids in a Natural Bank Sand containing Varying Percentages of Moisture. (*See p.* 138.)

The percentage of water by weight which will give the greatest bulk,—corresponding, of course, to the largest percentage of absolute voids,—varies with different sands from 5% to 8%.

The actual variation on different days in the percentage of moisture in a natural bank sand was found by the authors, in a series of experiments, to range from $1\frac{1}{2}$% to $5\frac{1}{4}$% of the total weight, or from $2\frac{1}{2}$% to $7\frac{1}{4}$% of the bulk of the moist sand. The sand, screened from a gravel bank in Eastern Massachusetts, ranged in coarseness from very fine to that which would pass a $\frac{3}{8}$-inch mesh screen. The moist sample was taken from the pile the day after a shower, and weighed $84\frac{1}{2}$ lb. per cubic foot, while the dryer sample, taken after a period of dry weather, weighed 107 lb. per cubic foot.

A sample of very fine sand which had been standing in a pile through the same shower contained $9\frac{1}{2}$% of moisture by weight, corresponding to 13% by volume. Ordinary gravel, on the other hand, from which the sand had been screened, was found after a heavy rain to contain only 1.8% of moisture by weight, this being apparently the maximum quantity which it would hold.

**Engineering News*, July 31, 1902, p. 81.

The maker of concrete is especially interested in the influence of moisture upon the bulk of sand and upon its voids (1) because of its effect upon the actual measurement of sand used in construction work, and (2) because of its effect upon his experimental determinations of proportions.

Rather incomplete experiments of the authors tend to show that the actual effect of moisture upon the volume of sand used in concrete and mortar may often be less than would naturally be inferred from the various experiments cited, and depends largely upon the processes of handling the sand. For example, fairly dry sand (3% moisture) shoveled by laborers from the pile into the regular sand-measuring box weighed 454 lb., while after a rain, the sand (with 5% moisture) shoveled from the pile into the same box weighed 464 lb., that is, the moist sand was slightly heavier than the dry. Further handling reversed these relations, for on weighing these two sands in a half cubic foot measure, the moist sand, as we should expect, was lighter than the dry.

The explanation of this apparent discrepancy is undoubtedly due to the fact that as the rain which affected the moisture occurred after the sand had been excavated and piled near the mixing platform, its bulk, as suggested on page 137, was not affected. The laborers handling the moist sand took large shovelfuls and the arrangement of the grains was not greatly disturbed. If the sand had been excavated after the rain, the handling with shovels and dumping from the cart probably would have rearranged the grains so that the moist sand would have weighed less than the dry in the large measure as well as in the small box.

Mr. Feret* calls attention to the fact that mortars of nominally the same proportions are richer in winter than in summer because of the greater amount of moisture in the sand, which, by increasing its bulk, reduces the absolute volume of the grains in a unit of measure. On the other hand, mortars are leaner in dry than in damp weather because the sand has greater density when dry.

In the experimental study of sand for determining the proportions of cement to be used, the effect of moisture is exceedingly important. The voids in absolutely dry sand are certainly no criterion of its qualities for mortar, while a moist sand will give entirely different results on different days. The best that can be done, if the study can be pursued no further than void determination, is to select conditions as near as possible to the average, and after determining the voids, considered as air alone and also as space occupied by the air and moisture, to use the results as a basis for judgment, bearing in mind that the volume of paste made from 100 lb.

*Annales des Ponts et Chaussées, 1892, II, p. 26.

of neat Portland cement, while varying largely with different brands, averages about 0.86 cubic feet, and that the volume of the additional water required for the sand (see pages 160 and 209) actually occupies space in the resulting mortar.

The most important conclusion to be drawn from the extreme variation in the same sand under different conditions is the impossibility of attaining results by the usual void experiments upon sand alone, which will be of accurate value in the consideration of mortar and concrete, and the practical necessity of employing methods such as are described by the authors in Chapter IX, page 149, or by Mr. Fuller in Chapter X.

In the preceding paragraphs we have referred chiefly to the variation in the condition of the same sand. The importance of studying mortars rather than the sand alone is still further emphasized by the varying effect of moisture upon sands of different sizes. This is brought out very clearly in Mr. Feret's paper.* In studying the normal consistency of mortars he finds that not only every cement but also every sand has a definite percentage of water necessary to bring it to what may be called normal consistency. This he illustrates in the triangle shown in Fig. 42 (constructed as described on page 156), giving the "proportions of water (by weight) required for ground quartz sands of all granulometric composition." It is evident from the diagram that coarse sands,† G, require 3% by weight of water, medium sands, M, 9%, and fine sands, F, 25%, while mixtures of the three sizes require intermediate percentages.

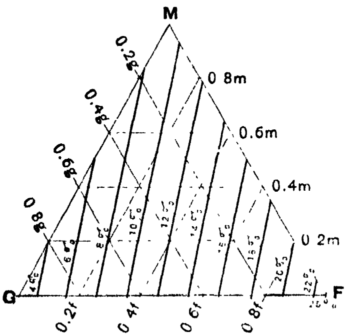


FIG. 42.—Percentages of Water Required to Gage Ground Quartz Sand of all Granulometric Compositions. (*See p.* 140.)

Compacting of Broken Stone and Gravel. Since concrete is usually compacted by ramming or lubrication of semi-liquid mortar, the density or the percentage of voids in compacted material is an important function. The statement has been made frequently that the aggregate compacts more when rammed in concrete than when rammed dry or merely moistened with water, because the mortar acts as a lubricant. Experiments by the authors indicate that broken stone under the same ram

*Annales des Ponts et Chaussées, 1892, II.

†The sizes of screens defining coarse, medium, and fine sands are given on page 156.

ming will compress on the average 1% more when it is moistened than when dry, and that an amount of mortar sufficient to lubricate without filling the voids produces no further reduction in volume. For example, a volume of broken stone mixed with 20% of mortar and rammed in 6-inch layers produced a volume exactly equal to that of the rammed broken stone which had been merely moistened.

Further experiments, partially outlined in the table on page 132, upon gravel and also upon varying sizes and mixtures of trap rock from two quarries, the one producing a soft and the other an exceedingly hard stone, lead to the conclusion that with stones of the same general structure, the percentage of reduction in volume by similar ramming in 6-inch layers is quite uniform, irrespective of the actual sizes of the particles, their relative sizes, the percentage of voids, and, within certain limits, the degree of hardness. On the other hand, the method of ramming the same stone will very largely affect the amount of compacting. Broken stone of the nature of trap, whether hard or soft, was found to compact when spread in 6 inch layers about 14% either under light ramming or shaking the measure, and about 21% under heavy ramming. In actual concrete work this large reduction of volume is of course seldom reached, because imperfect mixing and the necessary coating of the particles require a larger percentage of mortar than will just fill the voids of the rammed stone, and the bulk of concrete is usually greater than that of the original stone.

Screened gravel spread in 6-inch layers and unconfined, compacted about 12% under either light or heavy ramming.

These percentages of compacting are based upon the loose measurement of the material as thrown by a laborer into a barrel or box measure. Rehandling a material like broken stone as it comes from the crusher tends to mix particles of unequal size and therefore to compact it very slightly. In one case a screened stone fresh from the crusher compacted 1% when rehandled once, and an additional 1% when rehandled the second time.

It is interesting to note that the method of shoveling broken stone into a measure has but slight effect upon its shrinkage; for example, a lot of stone thrown with force into an inclined barrel occupied a space scarcely appreciably less than when very carefully and lightly placed. On the other hand, dropping from a considerable height does affect the volume, for Mr. Desmond Fitzgerald* states that broken stone dropped 12 feet into a car shrank to a volume 7% less than when it was measured in a box

*Transactions American Society of Civil Engineers, Vol. XXXI, p. 303.

Sand, unlike stone, is largely affected by the manner of shoveling and the size of the receptacle.

Compacting of Sand. The degree of compacting of sand is largely dependent upon the percentage of moisture which it contains. The dry sand shown in diagram in Fig. 41, page 138, when thoroughly tamped compacted from 34% to 27% voids or 9.6% in volume,* the sand with 6% moisture from 44% to 31% voids or 18.8% in volume, and the saturated sand from 33% to 26½% voids or 8.8% in volume.

Attention is called by Mr. Feret to the fact that the measurement of the weight of a given sand depends not only upon the quantity of moisture in it, but also upon the depth of the box which is used for the measure, the quantity of sand introduced at a time, — that is, the size of a shovelful, — the height from which it falls, the amount of shaking, if any, given to the box during filling, the amount of compacting given to the mass when leveling it off, and the smoothness of the surface left. As an illustration of the difference due to the method of placing in the measure, the authors found that a certain coarse sand shoveled into a pail about as a laborer would fill a measure weighed 88.9 lb. per cubic foot, while the same sand carefully poured into the pail weighed 83.3 lb. per cubic foot.

*Ratio of compacting $= \frac{0.34 - 0.27}{1.00 - 0.27} = 0.096$

CHAPTER IX

STRENGTH AND COMPOSITION OF CEMENT MORTARS

The following are the important conclusions in this chapter:

(1) The strength of a mortar depends primarily upon (*a*) percentage of cement in a unit of volume, and (*b*) density. (See p. 144.)

(2) The strongest mortar for any given proportions, by weight, of cement to dry sand, is obtained from sand which with the given cement produces the smallest volume of plastic mortar. (See p. 162.)

(3) The best sand is in general that which will produce the smallest volume of mortar of standard consistency when mixed with the given cement in the required proportions. (See pp. 144 and 163.)

(4) The density of a mortar is determined by calculating the absolute volume of its ingredients. (See p. 149.)

(5) The qualities of different sands may be studied by screening each into three sizes, and comparing their granulometric compositions with Feret's curves. (See p. 155.)

(6) Sharpness of sand grains is of slight importance. (See p. 167.)

(7) Coarse sand produces stronger mortar than fine sand. (See p.160.)

(8) Fine sand requires more water than coarse sand to produce a mortar of like consistency, and consequently its mortar is less dense. (See p. 160.)

(9) Mixed sand, *i. e.*, sand containing fine and coarse grains, in mortars leaner than 1:2, usually produces stronger and more impervious mortars than coarse sand. (See p. 159.)

(10) Screenings from broken stone usually produce stronger mortars than sand because of their greater density. The relative value of screenings or sand may often be determined by comparing the densities or the densities of mortar made from them. (See pp. 163 and 166.)

(11) Mixtures of fine and coarse sand or of sand and screenings often produce better mortar than either material alone. (See p. 163.)

(12) The variation of the sand in different portions of the same bank may be utilized by requiring the contractor to mix two sizes without exact measurement, so that material as delivered shall contain not less than a definite percentage of sand coarse enough to be retained on a certain sieve. (See p. 163.)

(13) Mineral impurities in sand, such as clay, in small quantities, may strengthen a lean mortar, and weaken a rich mortar. (See p. 168.)

(14) Organic impurities in sand, such as vegetable loam, even in minute quantities may destroy the strength of the mortar or concrete. (See p. 168.)

(15) Gaging with sea water does not affect the ultimate strength of mortars. (See p. 166.)

(16) The unit fiber stress in a cement or mortar beam is about alike for prisms 4 cm. (1.6 in.) and 2 cm. (0.8 in.) on edge. (See p. 145.)

(17) The unit fiber stress in bending is about 1.89 times the unit tensile strength of briquettes of 5 sq. cm. (See p. 145.)

(18) The unit tensile strength of specimens decreases as the breaking area is enlarged. (See p. 145.)

(19) The unit compressive strength of similar specimens of cement or mortar is not greatly affected by their size, (See p. 145.)

Laws of Strength. There are two fundamental laws of strength which apply to mortars composed of the same cement with different proportions and sizes of sand.

(1) With the same aggregate, the strongest and most impermeable mortar is that containing the largest percentage of cement in a given volume of the mortar.

(2) With the same percentage of cement in a given volume of mortar, the strongest, and usually the most impermeable, mortar is that which has the greatest density,* that is, which in a unit volume has the largest percentage of solid materials.

The first of these rules is understood by ordinary users of cement, but the second rule states a fact which is appreciated only by experts.

The value of a first-class cement is universally recognized, the effects of impurities have been studied in various ways, and the variations in strength of mortars made from different sands or broken stone screenings have been recorded, but the fundamental law of the relation of the density of a mortar to its strength, — a function nearly as important as the quality of the cement itself and explaining many of the seemingly paradoxical results of tests with different aggregates and different proportions of water, — is but vaguely comprehended by the majority of experimenters and most of the users of cement.

The application of these laws to mortar is discussed in the following pages, and to concrete in Chapter XIX.

*The meaning of *density* may be understood by referring to the figures on pp. 133 and 134.

STRENGTH OF SIMILAR MORTARS SUBJECTED TO DIFFERENT TESTS *

Mr. René Feret, Chief of the Laboratory of Bridges and Roads at Boulogne-sur-Mer, France, has made very extended tests of strength of mortars, studying his results scientifically, and in many cases formulating laws and formulas applicable to different conditions. The tests of one series in particular are of so wide a range in character and in proportions used that the authors have converted the values into English units, and reproduce the table in full on pages 146 and 147.

After plotting the strengths in various ways, Mr. Feret reaches conclusions which may be summed up as follows:

(*a*) The unit fiber stress for prisms 4 centimeters (1.6 in.) on an edge is about the same as for prisms 2 centimeters (0.8 in.) on edge.

(*b*) The tensile strength per square centimeter of prisms having a breaking area of 16 square centimeters (the strength of which he found to be similar to that of briquettes of the same section) is about two-thirds the strength per square centimeter of the normal briquettes which have an area of 5 square centimeters. This difference is attributed partly to the lack of homogeneity of the specimens, especially on their surfaces, but principally to the unequal distribution of the stress on the area of the section.

(*c*) Resistance to flexion, that is, the unit fiber stress in bending, is about 1.89 times the tensile strength per unit of area of briquettes of 5 square centimeters.

(*d*) The form and dimensions of the specimen do not greatly influence the strength per unit of area in compression when the height and width of the block are approximately equal.

(*e*) Resistances to flexion and tension are proportional to each other, and resistances to compression, shearing, and punching are proportional to one another, but there is no constant relation between the resistance to compression and the resistance to tension or flexion.

THE RELATION OF DENSITY TO STRENGTH

In the same paper from which we have quoted, Mr. Feret treats of the density and elementary volumetric composition of mortars, using in his studies the results given in the table just described. He calls particular attention to the fact that the properties of hydraulic mortar, such as durability, permeability, porosity, and ability to resist the decomposing action of sea water, depend not only upon the quality of the cement, but "in a measure greater than is generally believed, upon the granular physical

* A valuable series of tests has also been made by Messrs. Humphrey and Jordan at the U. S. Government Testing Laboratory at St. Louis, see Bulletin No. 331 U. S. Geological Survey, 1908.

Strength and Composition of Portland Cement Mortars.

BY R. FERET. (*See p.* 145.)

(Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Vol. II, p. 1593.)

SAND.*	ITEM	Approximate Proportions by Weight: Cement (1)	Approximate Proportions by Weight: Sand (2)	Composition of Mortar by Weight: Cement in 1,000 grams dry mixture ($c+s$) (3) gr.	Composition of Mortar by Weight: Water per 1000 grams dry mixture (4) gr.	Composition of Mortar by Weight: Weight of a cu. yd. of fresh mortar (5) lb.	Price of a cu. yd. of mortar (6) $	Elementary Volumetric Composition: Cement c (7)	Elementary Volumetric Composition: Sand s (8)	Elementary Volumetric Composition: Water w (9)	Elementary Volumetric Composition: Air voids v (10)	Density $c+s$ (11)	$\left(\frac{c}{1-s}\right)^2$ (12)	Strength per sq. in. after 5 months in fresh water: Flexion F_{16} (13) lb.	Flexion F_4 (14) lb.	Tension T_{16} (15) lb.	Tension T_5 (16) lb.	Compression P_{50} (17) lb.	Compression P_{16} (18) lb.	Compression P_4 (19) lb.	Average Strength: Shearing C_4 (20) lb.	Tension and flexion T (21) lb.	Compression P (22) lb.
G	(1)	1	18.6	51	62.5	3296	2.37	0.030	0.670	0.115	0.185	0.700	0.0083	134	134		65	210	340	160	170	69	240
G	(2)	1	9.9	92	70	3426	2.95	0.055	0.663	0.132	0.150	0.718	0.0266	287	256	58	152	840	980	800	570	146	870
G	(3)	1	6.9	126	76	3539	3.46	0.078	0.655	0.148	0.119	0.733	0.0511	418	370	129	220	1340	1690	1580	1070	212	1540
G	(4)	1	5.2	160	81.5	3654	4.00	0.102	0.648	0.163	0.087	0.750	0.0841	509	461	144	260	2060	2620	2360	1440	258	2350
G	(5)	1	4.1	196	88	3750	4.56	0.127	0.631	0.180	0.062	0.758	0.1190	597	568	206	326	2840	3680	3440	2000	314	3320
G	(6)	1	3.2	237	95	3814	5.17	0.155	0.605	0.196	0.044	0.760	0.1537	693	687	245	371	3750	4510	4250	2560	367	4170
G	(7)	1	2.5	287	104.5	3810	5.82	0.186	0.559	0.214	0.041	0.745	0.1772	804	807	277	407	4690	5430	5500	2790	421	5210
G	(8)	1	1.8	355	116	3792	6.64	0.226	0.499	0.234	0.041	0.725	0.2034	935	939	299	451	5720	5870	6320	3580	480	5970
G	(9)	1	1.2	449	133	3755	7.74	0.280	0.415	0.262	0.043	0.695	0.2304	1040	1060	356	502	6400	6500	7110	3930	537	6670
G	(10)	1	0.7	602	163	3694	9.35	0.359	0.287	0.306	0.048	0.646	0.2530	1070	1120	395	532	7050	6270	>7110	3640	563	>6810
S	(11)	1	12.9	72	90	3111	1.96	0.039	0.592	0.152	0.217	0.631	0.0092	141	158		85	270	360	310	256	81	310
S	(12)	1	7.0	125	96.5	3290	2.70	0.070	0.587	0.171	0.172	0.657	0.0289	330	341	54	192	970	970	940	669	182	950
S	(13)	1	5.0	167	101.5	3409	3.29	0.097	0.576	0.186	0.141	0.673	0.0524	451	438	141	249	1380	1640	1490	1040	240	1510
S	(14)	1	4.1	196	105	3485	3.74	0.116	0.569	0.196	0.119	0.685	0.0724	516	526	152	282	1780	2120	2080	1350	278	1990
S	(15)	1	3.1	241	111	3622	4.45	0.148	0.555	0.215	0.082	0.703	0.1109	612	572	212	333	2460	2870	2820	1810	320	2720
S	(16)	1	2.5	283	117.5	3645	5.02	0.173	0.525	0.227	0.075	0.698	0.1325	694	684	222	374	3130	3560	3600	2250	368	3430
S	(17)	1	2.0	333	125	3674	5.71	0.204	0.486	0.242	0.068	0.690	0.1576	821	768	246	405	4010	4440	4680	2650	415	4380
S	(18)	1	1.4	412	138	3701	6.76	0.252	0.428	0.265	0.055	0.680	0.1945	994	953	326	458	5230	5350	5730	2750	521	5440
S	(19)	1	0.9	518	157	3659	7.99	0.309	0.340	0.294	0.057	0.649	0.2190	1080	1070	307	489	5830	5920	6560	3580	541	6100
S	(20)	1	0.5	648	180	3630	9.44	0.376	0.241	0.329	0.054	0.617	0.2460	1180	1200	378	543	6600	6510	7050	3540	602	6720

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)
D	(21)	1	12.3	75	167	3178	1.49	0.038	0.564	0.269	0.129	0.602	0.0076	114	141		65	160	200	130	156	67	160
	(22)	1	5.8	148	168.5	3230	2.38	0.077	0.528	0.276	0.119	0.605	0.0269	247	238	60	122	600	550	480	370	126	540
	(23)	1	3.5	225	170	3262	3.32	0.118	0.485	0.281	0.116	0.603	0.0529	408	415	119	208	1080	1290	1320	768	214	1230
	(24)	1	2.4	299	172	3318	4.26	0.159	0.444	0.289	0.108	0.603	0.0818	576	576	196	296	1790	1910	2120	1410	302	1940
	(25)	1	1.8	363	175	3367	5.09	0.195	0.409	0.298	0.098	0.604	0.1082	703	693	233	354	2380	2970	3170	2130	364	2840
	(26)	1	1.3	429	179	3412	5.98	0.234	0.370	0.307	0.089	0.604	0.1376	814	828	233	439	3470	3570	4100	2570	436	3710
	(27)	1	1.0	500	183.5	3458	6.92	0.274	0.331	0.318	0.077	0.605	0.1681	980	963	302	502	4750	4410	5830	2750	510	5000
	(28)	1	0.7	573	190	3510	7.90	0.320	0.282	0.333	0.065	0.602	0.1998	1100	1120	373	540	5750	5450	6070	3070	574	5760
	(29)	1	0.5	659	197	3495	8.92	0.361	0.223	0.341	0.075	0.584	0.2171	1210	1280	371	623	6630	5970	6900	3570	647	6500
	(30)	1	0.3	771	208	3545	10.39	0.426	0.151	0.361	0.062	0.577	0.2530	1280	1370	401	671	7400	6830	7110	(4120)	691	>7110
M′	(31)	1	5.0	167	123	3650		0.102	0.607	0.237	0.054	0.709	0.0676	613	623	239	330	2120	2350	2570	1720	328	2350
	(32)	1	3.0	250	136	3642		0.150	0.539	0.259	0.052	0.689	0.1056	842	808	304	475	3640	3870	4510	3100	450	4010
	(33)	1	2.0	333	148	3645		0.199	0.474	0.278	0.049	0.673	0.1429	990	981	336	513	4640	4860	4920	3070	518	4810
N′	(34)	1	3.0	250	91	3620		0.156	0.557	0.179	0.108	0.713	0.1246	862		293	455	3600	3670			456	3640
C	(35)	1	0	1000	245	3552		0.534	0.000	0.414	0.052	0.534	0.2851	1280	1510	385	624	8720	7350	>7110	3680	698	8040

NOTE.—All are plastic mortars except N′.

*Description of Sands.

SAND	Nature of sand	Form of grains	Granulometric Composition† Coarse grains G	Medium grains M	Fine grains F	Approximate weights per cu. yd. lb.	Approximate price per cu. yd. $
G Sand from Gattemarre near Cherbourg	Granitic	Large and rounded	0.73	0.25	0.02	2815	1.18
S Sand from Saint-Malo	Very shelly	Varied	0.17	0.70	0.13	2537	0.59
D Sand from the dunes	Strongly siliceous	Fine and rounded	0.00	0.01	0.99	2461	0.15
M′ Ground quartzite sifted and remixed in equal parts	Quartz	Angular	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$		

N′ Ground quartzite passing a sieve of 64 meshes, and retained on one of 144 meshes per sq. cm. (20 and 31 meshes per linear inch).
C Neat Portland Cement.

†Granulometric composition is defined on p. 156.

EXPLANATION OF COLUMNS.

Col. (6) based on price and weight of given sand, on cement at 50 francs per "tonne" ($1.66 per bbl), and on labor at 3 francs per cubic meter (44 cts. per cu. yd) of mortar.

Cols. (7) to (12) are discussed on page 148.

Col.	Number of specimens of each mortar.	Size of specimens, centimeters.	Remarks.
(13)	15 prisms	4 x 4 x 16	Supports 10 cm. apart
(14)	15 "	2 x 2 x 13	" 10 " "
(15)	15 "	4 x 4 x 8±	Halves of broken prisms
(16)	25 briquettes	5 sq. cm. section	French standard briquettes
(17)	5 cubes	50 sq. cm. face	
(18)	15 prisms	4 x 4 x 8±	Halves of broken prisms
(19)	30 "	2 x 2 x 3±	Quarters of " "
(20)	15 "	2 x 2 x 6±	Halves " " "

(21) Average of cols. (13), (14) and (16) by formula

$$T = \frac{\left(\frac{F_{16} + F_4}{2}\right) + T_5}{1.89 + 1}$$

(22) Average of cols. (17), (18), (19).

composition of the mortars, that is to say, upon the dimensions and relative positions of the different elements entering into their composition."

The density (*compacité*) of a mortar is represented by the total volume of the solid particles, — exclusive of the water and the voids, — entering into a unit volume of mortar.*

The "elementary volumes" in a unit volume of fresh mortar consist of the absolute volumes of the cement, sand, water, and voids, each expressed in the form of a decimal. To illustrate, the "elementary volumetric composition" of the mortar in Item 8 of the table on page 136, which is mixed in proportions by weight of one part cement to 1⅗ parts of natural sand, is:

Cement	$(c) = 0.226$
Sand	$(s) = 0.499$
Water	$(w) = 0.234$
Air voids	$(v) = 0.041$
Total volume	$= 1.000$

Expressing this in more familiar terms, 22.6% of the unit volume of the given mortar consists of solid particles of cement, 49.9% of particles of sand, 23.4% of water, and the remaining 4.1% of air voids.

The porosity, represented by the sum of the water and air voids, is 27.5%. The term *voids* is often employed to represent the porosity, that is, the sum of the air and water.

It is obvious that

$$c + s + w + v = 1;$$

also that

$$v = 1 - (c + s + w),$$

which is equivalent to the statement that the entrained air in any volume of fresh mortar is equal to the measured volume of the mortar minus the space occupied by the cement, sand, and water.

The density of the mortar considered above is $c + s$, or, $0.226 + 0.499 = 0.725$ as given in column (11) of the table on pages 146 and 147.

A thorough understanding of the use of these symbols is essential to the study of strength of concrete and mortar, for, as will be shown further on, practical tests of strength are of small value unless the density and exact mechanical composition of the specimens are clearly defined.

*If the word density is applied to sand alone, it means the proportion of the measured volume of the sand, which is occupied by the solid sand grains; a sand, for example, having under certain conditions 40% voids, would have a density of $1.00 - 0.40 = 0.60$.

In practice, density or volumetric tests are of great value for comparing the relative values of different aggregates, and for determining the proportions for the most economical concrete. They are also useful for studying the effect of varying quantities of water. As is shown in the following pages, the density of mortars or concretes made from similar materials bears a definite relation to the strength, so that it is frequently possible to determine the best mixture as soon as the density tests are completed, instead of waiting for the tests of tensile or compressive strength. The test has been used by the authors in a practical way for comparing sands and for grading sands in special work, and also for concrete to fix on the best proportions when using merely one fine and one coarse aggregate, and in other cases to determine the proper proportions for a scientifically graded mix.*

Density of Mortars and Concrete. The density of fresh mortars of ordinary proportions, as shown by tests of the authors, averages about 0.70 (corresponding to 30% air plus water voids). Mortars of fine sands may run as low as 0.60 (40% air plus water voids), while by special grading or the use of an exceptionally good coarse sand the density may be as high as 0.75 (25% voids). The density of neat cement usually ranges between 0.50 and 0.55. The density of concrete ranges† from 0.76 to 0.88, depending upon the grading of the aggregates and the cement.

The values apply to the materials freshly mixed before setting. The chemical combination of the cement and water reduces the porosity further.

Density or Volumetric Tests of Mortar.‡ To obtain accurate results, considerable care is necessary in making the experiments. An approximate method suited to rough comparisons will be given first and this will be followed by more accurate methods advised for laboratory work.

The rough volumetric test may be made in almost any vessel or mold so long as the capacity is readily computed and its dimensions such that the depth of mortar or concrete can be measured exactly. A deep mold is more accurate than a shallow one. The volume

* See Chapter X, p. 175.

† From the "Laws of Proportioning Concrete," by Wm B. Fuller and Sanford E. Thompson, transactions American Society Civil Engineers, Vol, LIX, 1907, p. 67.

‡ The French Commission determine the "yield" of a mortar by measuring its volume green, that is, just after introduction into the molds, when an excess of water may affect the volume, and thus give misleading results with very wet mixtures.

In his Report to the French Commission, 1895, Vol. IV, p. 243, Mr. Fleet also measures the mortar wet, but he employs a vessel of known capacity,—a cylindrical measure whose height and interior diameter are each about 8 centimeters,—and uses only a portion of the mortar which he mixes, calculating his percentages by ratio of the weight of mortar made to the weight of mortar introduced into the measure to fill it exactly. This method eliminates inaccuracies in measuring the level of the surface.

of mortar and concrete of dry consistency will measure the same after setting as when green, but wet mixtures must be measured before setting, and again after they have become sufficiently hard to expel the surplus water. The measurement before setting is necessary in order to calculate the volume of air bubbles entrained in the wet mortar or concrete. The volume after setting, or partially setting, however, is the only one of real importance for studying the characteristics of strength, permeability, and cost. The sand is dried, or its moisture is determined by weighing and drying a sample of it. If stone of a porous nature is used the pores of its particles should be filled with water, but there should be no perceptible moisture on their surfaces. The quantities of dry materials for a single tube or mold are weighed in the required proportions, mixed with a known weight of water, and placed compactly in the mold, whose lateral dimensions have been exactly measured so that the volume of mortar in it may be obtained by measuring down from the top. The exact space occupied by the particles of each of the solid materials and by the water is calculated, if the metric system is employed, by dividing their total weight by the specific gravity of each, or, if English units are used, by dividing the weight times 1728 (the number of cubic inches in a cubic foot) by the specific gravity multiplied by the weight of a cubic foot of water. After partially setting, the exact depth of the mortar in the mold is measured and its volume calculated. The percentage of each of the dry materials, which really determines the density,—which is represented by the sum of the absolute volumes of the dry material,—is found by dividing the absolute volume of each material by the total volume of the set mortar or concrete.

The specific gravity of cement which has been stored for a short time may be taken at 3.10 and the specific gravity of dry sand at 2.65.

The following example from the authors' note book illustrates the method of finding the density when the measurements are in English weights and measures:

Example:—Find density of a mortar composed of Newburyport sand and Portland cement in proportions 1 : 2 by weight.

Solution:—For the mold used, it was estimated that 8 lb. cement and 16 lb. dry sand would be required. Gaging these with 3 lb., 12.6 oz. (3.79 lb.) of water, the quantity necessary for the desired consistency, the volume of the mortar was found by measurement to be 348 cu. in. when green, and 336 cu. in. after setting and pouring off the surplus

water. The absolute volumes are expressed below, first in cubic inches and finally in terms of the density $(c + s)$, of the set mortar.

$$\text{Cement} = \frac{8 \times 1728}{3.1 \times 62.3} = 71.6 \text{ cu. in.}$$

$$\text{Sand} = \frac{16 \times 1728}{2.65 \times 62.3} = 167.4 \text{ cu. in.}$$

$$\text{Water} = \frac{3.79 \times 1728}{62.3} = 105.1 \text{ cu. in.}$$

Absolute volume cement, sand and water,	344 cu. in.
Measured volume green mortar,	348 cu. in.
Volume of entrained air,	4 cu. in.
Percentage of entrained air,	1.2%

$$\text{Density of set mortar, } c + s = \frac{71.6}{336} + \frac{167.4}{336} = 0.213 + 0.498 = 0.711$$

Volumetric Tests of Mortar at Jerome Park Reservoir. The methods used by Messrs. Fuller and Thompson at Jerome Park Reservoir in tests for the New York Aqueduct Commission in 1906* have since been adopted, with slight variations, in the authors' laboratory. The procedure is indicated in the blank form used in the tests, a copy of which filled out is here reproduced on page 152. While somewhat lengthy in appearance, it is arranged to correct almost automatically for the unavoidable losses due to free water and mortar sticking to the tools. The chief object of the test is to find the density of a fresh mortar, that is, the ratio of solid material in it to the total volume, and also to determine the elementary volumes of each ingredient. In the test illustrated, for example, the density is 0.696 and the air plus water voids are therefore 30.4%.

The apparatus used for density tests of mortar are a shallow pan about 9 inches diameter, a small pointing trowel, scales to weigh to one-tenth gram, measuring glass or graduate about 1½ inches diameter and 250 cubic centimeters capacity, one or two beakers, and a stick for tamping the mortar in the glass. 300 or 400 grams of mixed cement and aggregate may be used in the tests.

It has been found that the material which sticks to the tools is either cement or similarly fine aggregate, so that the weight of the aggregate which passes a No. 100 sieve should be recorded for use in the computations.

* See paper by Messrs. Fuller and Thompson, Transactions American Society Civil Engineers, Vol. LIX, p. 67.

Volumetric test for *Reservoir* ....File *W. R*
Cement *B* Aggregates *Clean Sand*..........................Date 4–26–15.
Computed by *Brown* ....Checked by *T*.

(1) Experiment No.	152		
(2) Nominal proportions by volume	1 : 2		
(3) Proportions by weight................	1 : 1.78		
(4) Description of aggregate..............	Sand		
(5) Wt. of cement........................	150.0		
(6) Total weight of aggregate............	267.0		
(7) Wt. of the aggregate passing a No. 100 sieve........................	53.4		
(8) Wt. of vessel and water (before using) ...	287.7		
(9) " " " " " (after using)	228.7		
(10) " " water used = (8) — (9)...... ..	59.0		
(11) Percentage of water = $\frac{(10)}{(5)+(6)}$...... ..	14.2		
(12) Consistency.........................	Soft		
(13) Temperature water..................	65°F.		
(14) Total weight mixed = (5) + (6) + (10)...	476.0		
(15) Weight tray and tools (after using)......	325.8		
(16) " " " " (before using).....	322.2		
(17) Weight mix adhering = (15) - (16)	3.6		
(18) Weight measuring glass or graduate	295.4		
(19) Weight glass + mix..........	767.9		
(20) Weight glass + mix — free water......	767.9		
(21) " free water = (19 — 20)	0.0		
(22) " mix set = (14) — (17) — (21) ..	472.4		
(23) " " " = (20) — (18).........	472.5		
(24) Discrepancy = (23) — (22).....	.1		
(25) Time mixing completed	10.15 a.m.		
(26) Volume of mix, in cu. cm	210.0		
(27) Time settling.........................	2 hrs.		
(28) Final volume of mix in cu. cm........	209.5		
(29) Water left on tray = $(10)\times\frac{(17)}{(5)+(7)+(10)}$	0.8		
(30) Cement left on tray = $(5)\times\frac{(17)}{(5)+(7)+(10)}$	2.1		
(31) Aggregate left on tray = $(7)\times\frac{(17)}{(5)+(7)+10)}$	0.7		
(32) Wt. water in set mortar = (10) — (21) — (29)............................	58.2		
(33) Wt. cement in set mortar = (5) — (30)	147.9		
(34) Wt. aggregate in set mortar = (6) — (31).........	266.3		
(35) Specific gravity cement................	3.11		
(36) " " aggregate...............	2.71		
(37) Absolute volume water = $\frac{(32)}{(28)}$..........	.278		
(38) " " cement = $\frac{(33)}{(28\times(35)}$......	.227		
(39) " " aggregate = $\frac{(34)}{(28)\times(36)}$...	.469		
(40) Total absolute volume = (37) + (38) + (39)................................	.974		
(41) Density = (38) + (39)	.696		
Remarks: Fine Material on Surface ...	3 cc.		

NOTE: Weights are in grams; volumes in cubic centimeters.

The materials are carefully weighed, and enough water added,—the quantity varying with the fineness of the sand,—to produce a mortar softer than standard consistency which will scarcely hold its shape in the mixing pan. An examination of the various items in the table will show the purpose of each, the object being to correct for all losses and obtain a resulting volume corresponding to that of the mortar after setting. The figures following many of the items refer to the numbers of the other items, the fraction following item (29), for example, representing the water of the mix which adheres to the tray and tools. The weight of the water in this mortar which adheres is found from the proportion,—Mix adhering: total fine mortar = water in mix adhering : total water. Expressed in item numbers this becomes

$$\text{Item (29)} = \frac{\text{Item (17)}}{\text{Items (5)} + (7) + (10)} \times \text{Item (10)}.$$

The cement and aggregate left on tray, items (30) and (31), are similarly computed, and from these the weight of each of the materials in the set mortar is found. The absolute volumes, items (37) to (39), are then readily computed and the density determined.

Volumetric Tests of Concrete. For volumetric or density tests of concrete, molds at least 8 inches in diameter are necessary, but the process throughout is similar to that already described for the volumetric tests of mortar and a similar blank form may be readily made for records.

The density tests as made at Jerome Park Reservoir are fully described in the paper by Messrs. Fuller and Thompson already referred to† and results of the tests are there given.

Feret's Formula for Strength. For studying the relation of absolute volumes to strength, let

P = compressive strength of the mortar.
K = a constant which differs for different cements and at different ages of the same mortar.
c = absolute volume of cement.
s = absolute volume of sand.
w = absolute volume of water voids.
v = absolute volume of air voids.

The value of determining the density of mortars is made evident by the following law of Mr. Feret:*

"For any series of plastic mortars made with the same binding material

*Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Vol. II, p. 1604.
†See also Chapter X of this Treatise.

and inert sands, the resistance to compression after the same length of set under identical conditions, is solely a function of the ratio $\frac{c}{w+v}$ or $\frac{c}{1-(c+s)}$, whatever be the nature and size of the sand and the proportions of the elements, — cement, inert sand and water, — of which each is composed."

It follows from this law, as Mr. Feret says, that the strength of any

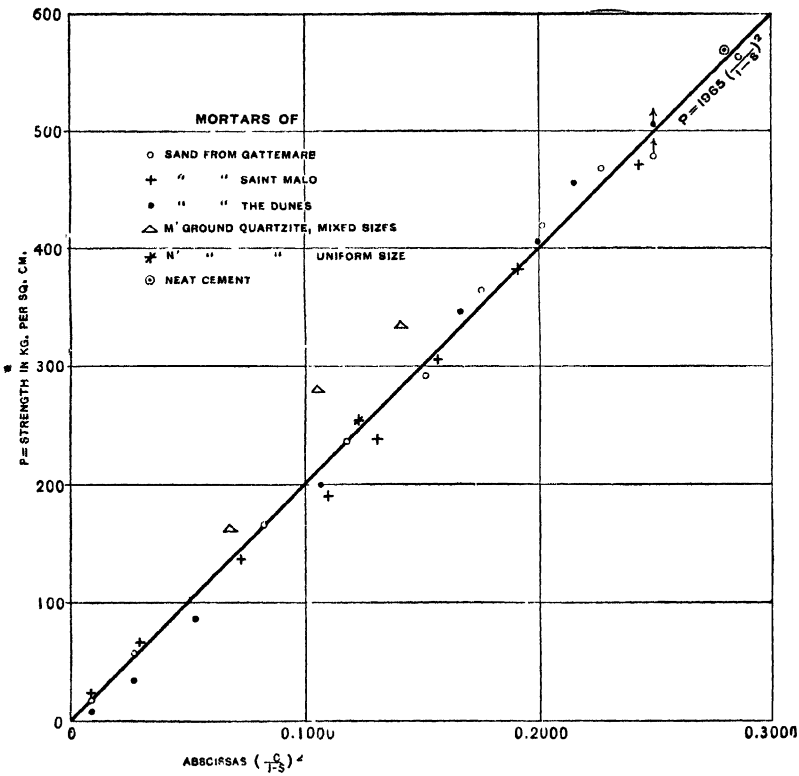


FIG. 43. — Derivation of Feret's Formula for Strength. (*See p.* 155.)
(Bulletin de la Société d'Encouragement pour l'Industrie Nationale — 1897.)

mortar increases with the absolute volume of the cement (c) in a unit volume of fresh mortar, and also with the density ($c + s$), whatever may be the relative volumes filled with water and air.

From very numerous experiments such as those tabulated on page 146 Mr. Feret evolves the approximate formula

$$P = K\left(\frac{c}{1 - s}\right)^2 \qquad (1)$$

By suitably changing the value of K the formula may be adapted to either the English or the metric system of measurement.

As a proof of this formula Mr. Feret plots on a diagram, shown in Fig. 43, values of $\left(\frac{c}{1 - s}\right)^2$ from column (12) in the table on pages 146 and 147 for abscissas, and the average compressive strengths of the various mortars, from column (22), for ordinates. Since, in formula (1), K is equal to P divided by the square of the quantity in brackets, the value of K is the tangent of the straight line passing through the points. In Fig. 43

K = 1965, if the strength is in kg. per sq. cm.

or

K = 28 000, if the strength is in lb. per sq. in.

This particular value is applicable only to the cement used by Mr. Feret in his experiments and to specimens at the age of five months, but the principles involved are of general application.

The most practical application of this formula is in the determination of the relative compressive strengths of various mortars made from the same cement, with sand in differing proportions and of different compositions. Mr. Feret calls attention also to its possible use in laboratory experiments and specifications. A cement, for example, may be required to furnish, when mixed with any sand, a definite value of K, since the value of K is independent of the choice of the sand and of the composition of the mortar.

Experiments by the authors tend to show that the formula does not apply strictly to specimens of different consistency, but that the general law of the increase of strength with the density is applicable except in extreme cases. The formula is inapplicable to tensile tests, although here, too, the general principle appears to hold good.

This subject as related to concrete is discussed on pages 312 to 314

GRANULOMETRIC COMPOSITION OF SAND

Feret's Three-Screen Method of Analyzing Sand.

The determination of the physical characteristics of the sand, which, mixed with a cement, will produce the densest mortar, has been the object

of a large number of experiments by Mr. Feret, which are recorded in Annales des Ponts et Chaussées, 1892. In America Messrs. William B. Fuller and Sanford E. Thompson have extended the researches, by a different method, to the investigation of the properties of concrete. The mechanical analysis of sand and stone is discussed in Chapter X, and the results of earlier experiments are tabulated on page 334.

Mr. Feret, in studying any sand, separates it by screening into three sizes. He then recombines these three sizes in varying proportions, so as to obtain results which are applicable to any natural or artificially mixed sand. He distinguishes sand from gravel as consisting of grains which will pass through a screen having circular holes of 5 millimeters diameter (0.20 in.). The three sizes of sand he then calls G, M, and F, representing, respectively, the large (*gros*), medium (*moyens*), and fine (*fins*) particles as defined by sifting through metallic sieves with circular holes, or wire cloth of definite mesh, as follows:

Large grains, G, passing circular holes	5 mm.	(0.20 in.)	diameter.
Retained by circular holes	2 mm.	(0.079 in.)	"
Medium grains, M, passing circular holes	2 mm.	(0.079 in.)	"
Retained by circular holes	0.5 mm.	(0.020 in.)	"
Fine grains, F, passing circular holes	0.5 mm.	(0.020 in.)	"

These sizes, Mr. Feret states, are nearly equivalent to sand screened through sieves of wire cloth as follows:

Large grains, G, passing screen of	4	meshes	per sq. cm.	(5	meshes	per	linear	inch.	)
Retained on	"	36	"	"	(15	"	"	"	)
Medium grains, M, passing	"	36	"	"	(15	"	"	"	)
Retained on a	"	324	"	"	(46	"	"	"	)
Fine grains, F, passing	"	324	"	"	(46	"	"	"	)

Sometimes, for experimental purposes, he divides each of the sands, G, M, and F, into three intermediate sizes.

The granulometric composition of any sand is represented by its relative proportions, expressed either in weights or absolute volumes, of G, M, and F. For example, a sand containing by weight 50% of the largest grains, 30% of the medium, and 20% of the fine grains, has a granulometric composition of g = 0.50, m = 0.30, f = 0.20.

The granulometric composition of a sand which has been mechanically analyzed, and plotted on a diagram similar to that shown on page 190, may be ascertained readily by drawing three ordinates corresponding respectively to screens of 5, 15, and 46 meshes per linear inch, and determining by the length or the difference in length of these ordinates the proportions which pass and which are retained by the screens of these three meshes. These three proportions or percentages represent the granulometric com-

position. An illustration of this method of transforming mechanical analysis to granulometric composition is shown in Fig. 51 on page 164.

Feret's Triangles. To simplify the tabulation of results, and arrange them so that they may be understood at a glance, Mr. Feret has used a graphical arrangement which is exceedingly ingenious. In nearly all his writings we find little triangles with the apexes labeled G, M, and F. Curves or contours in these triangles, representing the various properties of the sands or mortars, are based on a system of three instead of two

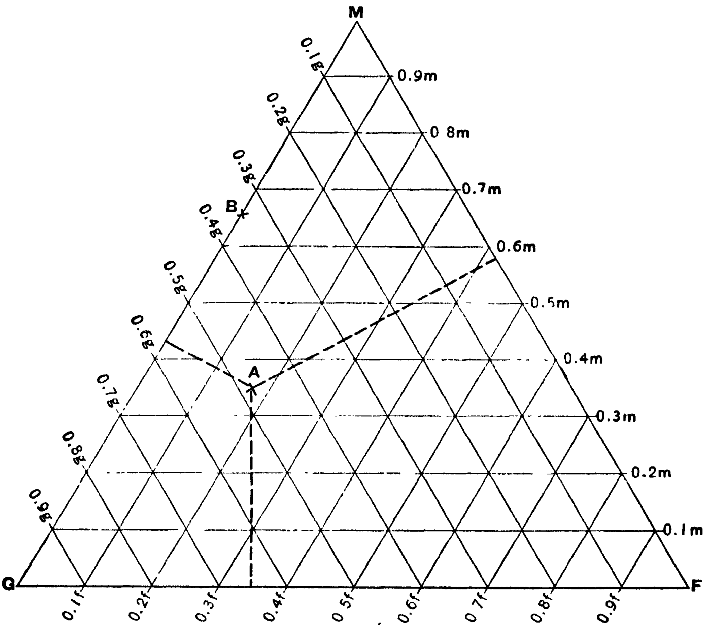


FIG. 44.—Feret's Three-Screen Method of Analyzing Sand. (*See p.* 157.)

co-ordinates, that is, each curve is the loci of points measured from 3 axes placed at angles of 60° with each other. A full discussion of the theory of this is given in his paper "Sur la Compacité des Mortiers Hydrauliques" in Annales des Ponts et Chaussées, 1892, II, but the principles may be understood by reference to Fig. 44. The apexes of the triangle are labeled G, M, and F, corresponding to the three sizes of sand described on page 156. The granulometric composition of any sand is plotted as a single point in this triangle. The proportion of each of the three sizes in the sand is represented by its perpendicular distance from the side opposite each apex.

For example, exactly at the apex G, the granulometric composition is g = 1.00, m = 0, f = 0. A sand represented by the point "*A*" in the triangle has for its granulometric composition, g = 0.48, m = 0.35, f = 0.17. Sand, *B*, whose point is on the line G M is a mixture of G and M with no fine particles. It can be readily proved by geometry that if the altitude of the triangle is 1.00, the sum of the three perpendicular distances from any given point in the triangle to the three sides equals 1.00. Also, that any combination of G, M, and F is contained in the triangle or else on one of its sides. To use Mr. Feret's language, "any sand will be represented by a point in the triangle and by one alone, and, reciprocally, one granulometric composition of sand, and only one, will correspond to a given point on the interior or sides of the triangle." If the altitude of the triangle

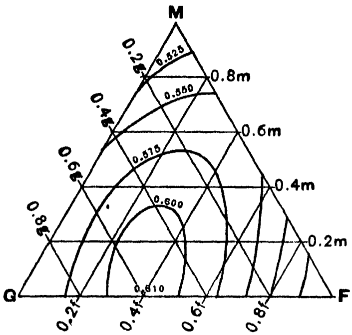


FIG. 45.—Absolute Volumes of Sand per Unit Volume of Sand not Shaken. (*See p.* 160.)

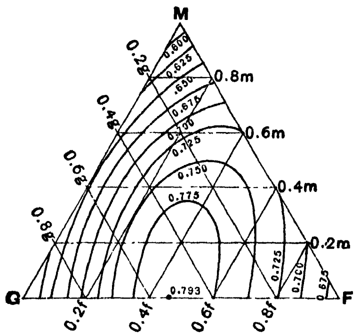


FIG. 46.—Absolute Volumes of Sand per Unit Volume of Sand Shaken to Refusal. (*See p.* 160.)

is considered 1.00, any point, *A*, in the triangle is readily plotted by locating it at perpendicular distances from each of the three sides corresponding to each component of its granulometric composition. For example, suppose that the granulometric composition of a sand, *A*, is g = 0.48, m = 0.35, f = 0.17. As the apex G represents a sand containing only coarse grains, and the line opposite to it, M F, all sands containing no coarse grains, the locus of a sand containing coarse grains (g = 0.48) will lie somewhere upon a line parallel to M F and at a distance 0.48 from M F. By similar reasoning it will also lie on a line parallel to G F and at a distance 0.35 from it. The intersection of these two lines is the locus of the sand *A*, and it will be seen that this intersection is at a perpendicular distance of 0.17 from the line M G (the side opposite F), which checks the plotting, since f = 0.17.

For comparing a special property of different sands, or of mortars com-

posed of different sands, each sand employed in the tests is plotted and labeled with its value, — which may be in units of strength, weight, or volume, — and "contour lines" are sketched in by the eye, as one would draw contours from elevations on a topographical drawing.

Any point on the same contour line represents a sand made up of the

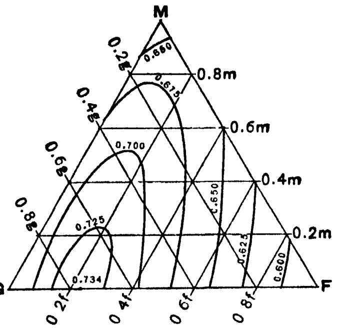


FIG. 47.—Absolute Volumes of Solid Materials (c+s) per Unit Volume of Fresh Mortar in Proportions 1:3 (by Weight). (*See p.* 160.)

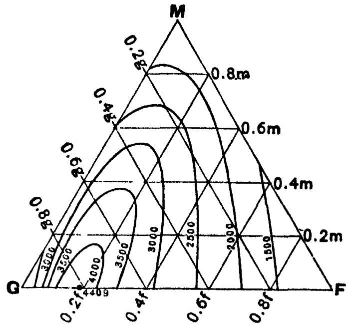


FIG. 48.—Compressive Strength in Pounds per Square Inch of 1:3 (by Weight) Mortars with Different Mixtures of Sand, after 9 Months in Air and 3 Months in Sea Water. (*See p.* 161.)

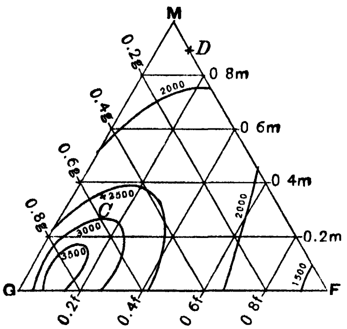


FIG. 49.—Compressive Strength in Pounds per Square Inch of Mortars with Various Mixtures of Sand, after One Year in Fresh Water. Proportions 100 lb. Portland Cement to 3.2 cu. ft. Mixed Sand. (*See p.* 161.)

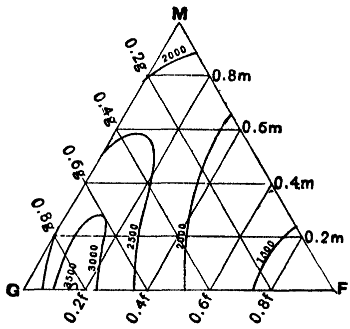


FIG. 50.—Compressive Strength in Pounds per Square Inch of Mortars with Various Mixtures of Sand, after One Year in Air. Proportions 100 lb. Portland Cement to 3.2 cu. ft. Mixed Sand. (*See p.* 161.)

different sizes, G, M, and F, in proportions corresponding to its perpendicular distances from the sides opposite each apex, but having the same strength, weight, volume, humidity, or whatever special function may be represented, as every other point on the same line.

Figs. 45 and 46, page 158, illustrate the use of the triangle for showing the volumes of sands composed of different sizes of grains. Any sand, for example, whose granulometric composition is represented by any point on the contour line labeled 0.575, in Fig. 45, has, when measured loose, 0.575 of its volume, or 57½%, of absolutely solid matter, or, taking the complement, 42½% of voids. In Fig. 45 it will be seen that the greatest solid volume of loose sand is obtained by mixing G and F in proportions 60% G and 40% F by weight. The amount of solid matter in this mixture of maximum density is 0.61 of the unit volume; in other words, the sand contains 39% voids. By interpolating between the contour lines we may see that a sand consisting of equal parts of the three sizes, which would be represented by a point at the geometrical center of the triangle, has about 0.597 solid matter, or 40.3% voids. In sands shaken to refusal, Fig. 46, the mixture of maximum density consists of sands G and F alone, in proportions about 55% G and 45% F, and the total solid matter, that is, the absolute volume of sand, in a unit volume of the shaken sand of maximum density, is 0.798, corresponding to 20.2% voids.

EFFECT OF SIZE OF SAND UPON THE STRENGTH OF MORTAR

As a matter of fact, the actual size of a sand, that is, the size of its grains, is subordinate, in its influence upon the strength and other qualities of a mortar, to the density of the mortar produced from it. One naturally would suppose that the densest sand, that is, the sand which contains, when dry, the fewest voids, when mixed with a given proportion of cement, would make, inevitably, the densest and therefore the strongest mortar. Such, however, is not necessarily the case, for the addition of both the cement and water change the mechanical composition. A mixture of fine sand and cement, for example, requires a larger percentage of water in gaging than a mixture of coarse sand and the same cement. The total volume of a mortar of plastic consistency is affected by the quantity of water used, as well as by the volumes of the dry materials. Hence, a mortar consisting of fine sand and cement will be less dense than one of coarse sand and the same cement, even though the fine and coarse sands, when weighed or measured dry, each contain the same proportions of solid matter and voids.

Fine sand has more grains in a unit measure and therefore a greater number of points of contact of the grains. The water forms a film (see Fig. 38, p. 136,) and separates the grains by surface tension.

The fact is graphically illustrated in Feret's triangle, Fig. 47, page 159.

in which the contour lines show the combined absolute volumes of the cement and sand in 1:3 mortar (proportioned by weight) made from sand of various compositions. It will be noticed that the point of maximum absolute volume, which is labeled 0.734, is much farther to the left than in Figs. 45 and 46, showing that for a mortar of maximum density, a sand is required containing more large particles, G, in proportion to the fine particles, F, than for maximum density with the same sand in its dry state.

From such experiments Mr. Feret* derives the law that:

> The plastic mortars, which, per unit of volume, contain the greatest absolute volume of solid materials (c + s), are those in which there are no medium grains, and in which coarse grains are found in a proportion double to that of fine grains, cement included.

Figs. 48, 49, and 50, page 159, show the strength in compression, converted to pounds per square inch, of mortars made from various mixtures of the three sizes of sand.

Comparing these with Fig. 47 it will be seen that the curves of strength follow the same general direction as the curves of density. This is in conformity with the general laws stated at the commencement of the chapter and with the principles upon which Feret's formula (page 155) is based.

There is one point which must be noticed when studying these and other similar triangles of Feret, namely, that his results, as shown by the curves on his triangles, apply exactly only to sands and cements, and not to mixtures of sand and coarse stone. In all the triangles, sands for maximum density are composed of a mixture of fine and coarse grains with no medium grains. It is shown on page 133 that a denser mixture can be obtained with stone and sand and cement, that is, with three sizes of materials, than with sand and cement, and it is consequently probable that Feret could have obtained greater densities by making the size of G larger (that is, employing for G gravel or broken stone) and the size of F smaller, and that with this arrangement a portion of the medium grains would have been absolutely necessary to obtain the maximum density. In this connection, however, it must be remembered that Feret's experiments were intended to cover, as far as possible, practical combinations of sizes of sand for mortar. It is noticeable, even with the sizes of sand which he uses, that the curves in Fig. 47 run sharply upward, and that mortars from mixtures of three sizes of sand are therefore very nearly as dense and strong as those made from two sizes. Furthermore, when the three sizes

*Annales des Ponts et Chaussées, 1896, II, p. 182.

G, M, and F are mixed together, a graded mixture is formed in which there are particles ranging from 0.2 inch down to fine dust.

Experiments indicate, as stated on page 196, that sand for concrete requires for best results more fine material than mortar sand.

TESTS OF DENSITY AND STRENGTH OF MORTARS OF COARSE VS. FINE SAND

The application of Mr. Feret's tests is shown in the table on pages 146 and 147, and the following tables, to illustrate its practical use in comparing the quality of different sands, are presented; the first giving the density and strength of three natural bank sands as tested by one

Compressive Strength and Elementary Volumetric Composition of 2-inch Cubes of Portland Cement and Bank Sand. (*See p.* 162.)

BY SANFORD E. THOMPSON.*

Sand	Proportions by Weight	Proportions by Volume (nominal)	Percentages Passing Sand Sieves: $\frac{1}{4}''$ Sieve	No. 8 Sieve	No. 20 Sieve	No. 50 Sieve	No. 200 Sieve	Elementary Volumes: Cement c	Sand s	Density $c+s$	$\left(\frac{c}{1-s}\right)^2$	Actual Average Compressive Strength, Age 7 days	Estimated Compressive Strength at 6 months, K = 28,000
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
Coarse	1 : 2.6	1 : 3	100	84	62	28	3	0.171	0.518	0.689	0.126	715	3530
Fine	1 : 2.6	1 : 3	100	100	84	77	6	0.154	0.466	0.620	0.083	405	2320
Very Fine .	1 : 2.6	1 : 3	100	100	92	84	27	0.149	0.451	0.600	0.074	330	2070

Tests by New York Board of Water Supply of 1:3 *Mortar Made With Sands of Different Mechanical Analysis.* (*See p.* 162.)

Percentages Passing Sieves.				Tensile Test. Lb. per sq. in.		Compression Test. Lb. per sq. in.	
No. 4.	No. 8	No. 50.	No. 100.	7 days.	90 days.	7 days.	90 days.
100	70	12	5	213	613	2690	5640
100	86	21	6	263	412	1915	4660
100	99	26	2	177	325	905	2170
100	97	28	6	178	282	1070	1500
100	94	44	12	139	228	905	1130
100	100	52	14	122	170	275	810
100	100	94	48	80	149	330	490

* From paper by Sanford E. Thompson on "Sand for Mortar and Concrete," Bulletin No. 3, Association American Portland Cement Manufacturers, 1906.

of the authors, and the second giving mechanical analyses and strengths of mortars made by the New York Board of Water Supply.

PRACTICAL APPLICATIONS OF THE LAWS OF DENSITY

(*a*) The variation of the sand in different portions of the same bank may be utilized by requiring the contractor to mix two sizes without exact measurement, so that the material as delivered shall contain not less than a certain percentage of sand coarse enough to be retained on a certain sieve.

(*b*) If two sands are available, a study of their physical characteristics will determine which is better suited to the work in hand as *the sand which produces the smallest volume of plastic mortar, when mixed with cement in the required proportions by dry weight, furnishes the strongest and least permeable mortar.*

(*c*) A good sand brought from a distance at a high price may be more economical than a poor sand from a neighboring bank.

(*d*) The relative value of crusher dust or of sand in a given locality may be determined by comparing their densities or the densities of mortars made from them.

(*e*) Frequently, a mixture of a fine and coarse sand, or of sand and crusher dust, proportioned according to their relative granulometric compositions or analyses, may be shown to produce a better mortar than either alone.

(*f*) To produce impermeable mortar or concrete, it may be economical to screen a mixed gravelly sand into different sizes, and remix these in proportions which will produce a mortar of greater density.

The use of mixed sand, as described in (*a*), was adopted by Mr. Thomas F. Richardson, Engineer, for the 1 : 2 Natural cement mortar employed in the stone masonry of the Wachusett dam of the Massachusetts Metropolitan Water Works, after an exhaustive study of the comparative tensile strength and permeability of mortars made with different sands. He required the contractors to furnish sand so coarse that at least 50% would be retained on a sieve having 30 meshes per linear inch. The sand was excavated by scrapers, and the condition was readily complied with, whenever the sand in one section was shown by samples to be running too fine, by taking scraper loads of coarse sand from another location.

Mixed or graded sands are specially advantageous when concrete is made at a central plant such as a block manufactory. By using graded screenings, instead of the fine stone as it came from the crusher, and by slightly increasing the size of the coarse aggregate, Mr. Thompson obtained a strength two and one-half times as great with the same proportions of cement and maintained equal strength with 40% less cement.

Comparative Tests of Different Sands. One of the most important applications of the laws of density is in the comparison of different sands. Void determinations of sand are valueless because of variations in moisture and compactness, but if equal dry weights of each of the sands to be compared are mixed with the same cement in the proportions required on the work, and then gaged to plastic consistency as described on page 150, the best sand, provided it does not contain vegetable loam or other impurities to affect it chemically, is that which produces the smallest volume of mortar.

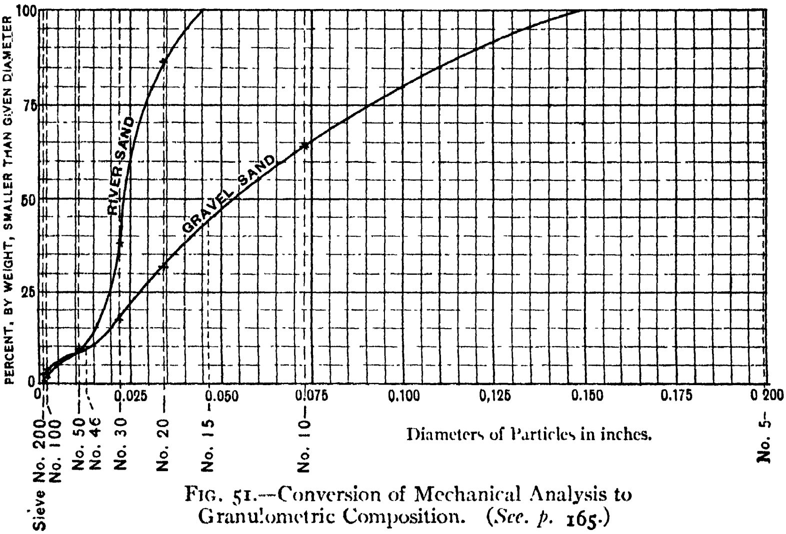


FIG. 51.—Conversion of Mechanical Analysis to Granulometric Composition. (See p. 165.)

CONVERSION OF MECHANICAL ANALYSIS TO GRANULOMETRIC COMPOSITION

As an illustration of methods of contrasting two different sands and of making practical use of Feret's researches, we may compare tests made by Mr. R. L. Humphrey* in connection with the construction of the Pennsylvania Avenue Subway, Philadelphia. He found the tensile strength at the age of one year, of 1:3 mortar made with sand screened from gravel, to be about 50% stronger than that made with sand dredged from the Delaware River. The mechanical analyses† of the two sands are plotted by

*Transactions American Society of Civil Engineers, Vol. XLVIII, p. 558.

†Mechanical Analysis Curves are described in Chapter X, page 182.

the authors in Fig. 51, page 164, from tables presented by Mr. Humphrey.

To transform these mechanical analysis curves to Feret's granulometric composition, we may draw on the diagram, ordinates corresponding to the sizes of sieves used by him, namely, No. 5, No. 15, and No. 46. (See p. 156.) From inspection of the curve it is evident that the granulometric composition of the gravel sand is g = 0.56, m = 0.35, f = 0.09, and of the river sand is g = 0.00, m = 0.89, f = 0.11. Plotting these granulometric compositions as *C* and *D* on Feret's triangle, Fig. 49, and interpolating between contours, we find the relative compressive strengths of mortars made from the two sands to be, after one year in fresh water, about as 1775 is to 2550, or as 1 : 1.44, while Mr. Humphrey's ratio of tensile strength for the two mortars at the age of one year is as 304 is to 470, or as 1 : 1.53. These ratios are remarkably similar when the differences in conditions are considered.

Numerous tests have been made in America* in proof of the general law that coarse sands are stronger than fine. Many experimenters have seemed to reach the result that coarse sand is stronger than mixed sand. In certain cases this is undoubtedly true, because of mixing the different sizes in wrong proportions, or because the mortar of coarse sand contains so large a proportion of cement that the voids are completely filled and the addition of fine sand decreases, instead of increasing, the density. Mortar, for example, as rich as 1 : 2 (*i.e.*, one part cement to two parts sand) of coarse sand is as strong as, and often stronger than, mortar of similar proportions made of almost any mixed sands, but with leaner mortars, a small admixture of from 10% to 25% of fine sand improves it. Natural sand, which in appearance is very coarse, almost invariably has a small percentage of very fine particles which, with the fine grains of cement, may assist, in the leaner mixture, in producing a dense mortar. The mechanical analysis curves of sand shown in Fig. 57, on page 190, are an illustration of the fine matter contained in all bank sands.

EFFECT OF QUANTITY OF WATER UPON THE STRENGTH OF MORTARS

An excess of water decreases the density of the mortar and therefore the strength. Fine sands require more water than coarse to produce the same consistency. Hence, the weakness of fine sand mortars. (See p. 160.) A large excess of water injures the cement. (See p. 318.) A deficiency of water may affect the permanent strength of a mortar.

*E. S. Wheeler in Report Chief of Engineers, U. S. A., 1895, p. 3013, A. S. Cooper in Journal Franklin Institute, Vol. CXL, p. 326, Ira O. Baker in Journal Western Society of Engineers, Vol. I. p. 73.

Although dry mixed mortars usually test higher than wet, because they can be more densely compacted, more uniform results, in practice as well as in experiment, can be obtained with plastic mixtures.

EFFECT OF GAGING WITH SEA WATER

Briquets gaged with sea water set much slower than those gaged with fresh water* but long time tests† show no difference in strength. Tests by the authors in 1909 on 3-inch cubes of 1:2:4 concrete 14 months old gave 4 070 pounds per square inch for the specimens mixed with sea water and 3 870 pounds per square inch for those mixed with fresh water.

LIMESTONE SAND AND SCREENINGS

Fine aggregates of limestone composition, either sand or screenings, usually produce a mortar of higher strength than common sand. Tests by the authors of natural limestone sands from Canada and northern New York show in certain cases as much as 50 per cent. to 100 per cent. greater strength than would be expected of ordinary sand of similar mechanical analysis. The gain in strength is somewhat slower than with quartz sand. The higher strength of mortar of limestone aggregates probably is due to their chemical composition. Results similar to these have been reached abroad by Mr. P. Alexandre and Mr. R. Feret.

SAND VS. BROKEN STONE SCREENINGS

The relative strength of mortars made from sand and from screenings of broken stone or crusher dust has occasioned much discussion and dispute. It is probably dependent chiefly upon the relative density of the different mortars. Usually, a mortar from screenings will show higher tests, while occasionally mortar from sand will be superior, because of the difference in size or of the relative sizes of the particles or grains composing the two materials.

In some cases the form of grain exerts an influence upon the strength of the mortar, but usually this is of less consideration than the mechanical composition.‡

*P. Alexandre in Annales des Ponts et Chaussées, 1890, II, p. 332.

†Alexandre and Feret in Commission des Méthodes d'Essai des Materiaux de Construction, 1895, Vol IV, p. 111.

‡Baumaterialienkunde, V Jahrgang (1900,) p. 21, and Annales des Ponts et Chaussées, 1892, II, p. 124.

Dusty screenings are especially bad for granolithic surfacing for side-walks, and must not be used.

SHARPNESS OF SAND

In the past all specifications have called tor clean, "sharp" sand in spite of the fact that in many parts of the country where sharp sand is not obtainable, sand with rounded grains is furnished and used with perfect satisfaction.

Comparative laboratory tests under conditions as nearly as possible identical uphold the practice of using sand with rounded grains. They indicate, as may be inferred from the previous discussion in this chapter, that the chief difference in natural sands is due to the size of the grains, and while the sharpness of grain may exert a certain influence it is of so much less importance than the size of the grain that *the requirement of sharpness for sand should be omitted from concrete specifications.*

Referring to columns (11) and (22) in the table on page 146, and to Fig. 43, page 154, it is evident that the difference in strength of nearly all the mortars made with the various sands is explained by the differing percentages of cement and densities without reference to the character of the grains. The only noticeable exception is with the artificial sand, M′, which consists of mixed sizes of crushed quartz. Mr. Feret* believes that this exception may be due to chemical action produced by the large quantity ($\frac{1}{9}$ its weight) of impalpable quartz. Sand N′, also crushed quartz, but containing none of this fine powder, produces a mortar similar in strength to like mortars of natural sand having rounded grains.

Other tests of Mr. Feret† and comparative tests, in the United States, of mortar with crushed quartz and natural sands generally confirm the above conclusion. The variation in the shape of the grains of natural sands and crushed quartz is illustrated in Figs. 37, 39, and 40, page 136.

EFFECT OF NATURAL IMPURITIES IN THE SAND UPON THE STRENGTH OF MORTAR

A clause to the effect that a sand for mortar or concrete shall be "clean" is almost universally found in masonry specifications. The necessity for this requirement is often questioned by cement experimenters, because the results of tests of mortar to which percentages of loam or clay have been added, often give higher results than those of mortar made with cement and pure sand.

*Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Vol. II.

†Annales des Ponts et Chaussées, 1892, II, p. 124.

As a matter of fact, it is impossible to make a general statement either to the effect that natural impurities in sand are beneficial or that they are detrimental. In some cases fine material may be of actual benefit, while in others the contrary is true.

The case is covered by three conditions: (1) the character of the impurities; (2) the coarseness of the sand; (3) the richness of the mortar.

Character of Impurities. If the fine material is of ordinary mineral composition, such as clay, the mortar is affected only mechanically, and the results depend upon the coarseness of the sand of which the fine material is a part and the richness of the mortar, as indicated in paragraphs which follow. One exception to this general rule is when the clay is in such condition as to "ball up" and stick together so as to remain in lumps in the finished concrete. On the other hand, a small percentage of clay well distributed may be valuable for making the concrete or mortar work smooth, and especially for increasing its water-tightness (see p. 301.)

Vegetable or Organic Impurities. When the impurities are of an organic nature, like vegetable loam, they frequently have been found to prevent the mortar or concrete from hardening or to retard the hardening for so long a period as to make the sands entirely unfit for use. A very minute quantity of vegetable matter may produce injury, so small a percentage in fact that frequently a sand which has passed careful inspection fails in practice to set properly with any brand of cement; therefore a test is absolutely necessary for any sand which has a suspicion of organic matter.

The following tests of 1 : 3 mortar made with sand satisfactory in appearance, but which nevertheless caused the fall of a concrete building, are given

Effect of Vegetable Impurities in Sand

BY SANFORD E THOMPSON, 1908. (*See p*, 168,)

Sand.	Tensile strength of 1 : 3 mortar at 7 days. Lb. per sq. inch	Tensile strength of 1 : 3 mortar at 28 days. Lb. per sq. inch.
A*..........	4	93
B†..........	43	114
B washed..........	129	201
W‡..........	165	
Standard Ottawa..........	200	300

* Poorest portion of bank; reddish and dark in appearance.

† Average sand from bank which passed inspection.

‡ A medium good sand from another bank similar to B in appearance, mechanical analysis, and chemical composition except nearly free from vegetable impurity.

in the following table. They are averaged from different series and for convenience in comparison the results are all converted to the basis of standard sand mortar, considered as 200 pounds in 7 days and 300 pounds in 28 days. The mortars were stored in air to conform to the actual conditions. Comparative tests on mortars from the same sands stored in moist air and in water corroborated the results.

The cause of the failure was traced in the expert investigation, to vegetable impurities in the sand which had washed down into the bank from the soil above. The poorest sand, *A*, showed by mechanical analysis only 4% by weight of fine material passing a No. 100 sieve and 1.61% silt by washing, but this silt was found to contain nearly 30% of vegetable matter corresponding however to only 0.5% in the total sand. The vegetable matter appeared to coat the grains of sand so as to prevent adhesion of the cement and also retarded the setting.

Effect of Fine Material in Filling Voids. Lean mortars may be improved by small admixtures of pure clay or by substituting dirty for clean sand, provided it is free from vegetable matter, because the fine material increases the density. Rich mortars, on the other hand, do not require the addition of fine material, and it may be positively detrimental, because the cement furnishes all the fine material required for maximum density. This is illustrated in experiments by Mr. Griesenauer* in which an admixture of even 2 per cent of clay (based on the weight of the sand) slightly reduced the strength of 1 : 2 mortar, while 20% of clay, added to the 2 parts of sand, reduced the strength about 30%. In 1 : 3 mortar, on the other hand, the addition of 2% slightly increased the strength, and there was no appreciable injury up to 20% addition.

In experiments by Mr. E. S. Wheeler† clay reduced the strength of neat and 1 : 1 mortars, but improved leaner mixtures.

In this connection, of course, it must be borne in mind that if the sand is composed largely of fine material, the strength of the mortar is comparatively low, as indicated in preceding pages.

EFFECT OF MICA IN THE SAND UPON THE STRENGTH OF MORTAR

The effect of mica in screenings from stone of a micaceous nature has been the subject of considerable controversy. Tests by Mr. Feret‡ in France indicated that the presence of 2% of mica has but slight influence upon the tensile strength of mortar, but a greater one upon its compressive

* *Engineering News*, April 28, 1904, p. 413.

† Report Chief of Engineers, U. S. A., 1895, p. 3004, and 1896, p. 2827.

‡ Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1897, Vol. II.

strength. More recent tests by Mr. W. N. Willis* in 1907 on mortars made with standard Ottawa sand into which mica was introduced are illustrated in Fig. 52. He found that the presence of mica increased the voids and decreased the strength. The sand used in tests, loosely shaken, contained 37% voids, but as mica was added, the voids increased rapidly until with 20% mica the voids were 67% with a corresponding decrease in weight, and three times the amount of water was required for mixing.

It is thus evident that the reduction in strength was largely due to the decrease in density and not entirely caused by the slippery character of the grains. In crushed stone screenings it is probable that the effect of the same percentage of mica in the natural state would be less marked.

Black mica, with a different crystalline form, is not injurious to mortar.

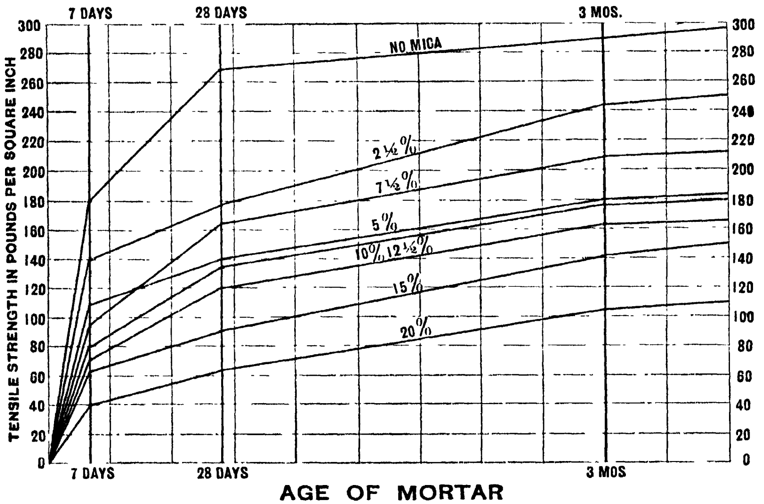


Fig. 52.—Effect of the Addition of Mica upon 1 : 3 Mortar of Standard Sand. By W. N. Willis. (See p. 170.)

EFFECT OF LIME UPON THE STRENGTH OF MORTAR

As a principal constituent of mortar in masonry construction, lime is inferior to cement in durability and strength. However, not only because of its relative cheapness, but also because a small addition of slaked or hydrated lime may increase the density of the mortar and cause it to work easier under the trowel, a limited quantity often can be used to advantage in mortar which is to be subjected to high loading.

For concrete, lime, see Chapter XVIII, is a suitable ingredient to fill the voids, rendering it more impermeable.

*Cement Age, Mar. 1907, p. 172.

Although lime mixed with neat cement is apt to decrease its strength, in combination with sand for cement mortars, a small admixture of lime may add to the strength of the mortar. The questions as to whether lime is beneficial, and as to the amount which can be used, are determined by the character of the cement, the coarseness of the sand, and the proportions in which the two are mixed. The effect of lime in cement mortar or concrete is chiefly mechanical. In a porous mortar or concrete a small quantity of it assists in filling the voids, and if it is thoroughly slaked so as to contain no quicklime, its expansion need not be feared.

Since even a neat cement paste has 35% to 45% water plus air voids, the inference might be drawn that the addition of lime would increase its density, and thus that the lime would be valuable even in very rich mortars. However, it seems to be practically impossible, except under high pressure, to replace the water which occupies the voids in neat cement paste with lime or any other fine powder. But it is evident that a lean mortar, such as a 1:4, or even a 1:3, should be improved by the addition of lime, and that this is true is illustrated in the following tests by Mr. Louis C. Sabin.* In these experiments the addition of 10% of lime—based on the weight of the cement—increases the strength of 1:3 mortar, and as shown by item (3) in the table, a 1:3⅓ mortar with 10% of lime is stronger than a 1:3 mortar with no lime. Items (4) and (5) illustrate the reduction in strength when the lime becomes more nearly a principal ingredient. Each value is an average of five briquettes.†

Effect of Lime Paste upon the Strength of Portland Cement Mortar.
BY L. C. SABIN. (*See p.* 171.)

Item	Proportions cement plus lime to sand by weight	Proportions cement to sand by weight	Cement	Lime‡	Sand	Average Tensile Strength.	
						at 28 dys.	at 3 mos
	parts	parts	grams	grams	grams	lb. per sq. in.	lb. per sq. in.
(1)	1:3	1:3	200	0	600	201	236
(2)	1:2¾	1:3	200	20	600	242	265
(3)	1:3	1:3⅓	180	20	600	238	264
(4)	1:3	1:4	150	50	600	168	171
(5)	1:3	1:6	100	100	600	57	70

*Report Chief of Engineers, U. S. A., 1896, p. 2823.

† See tests by Dr. E. W. Lazell, Transactions American Society for Testing Materials, Vol. VIII 1908, p. 418.

‡ The weight of the lime paste was 2.7 times the weights in this column.

With another brand of cement and sand of different coarseness the relative quantity of lime to produce similar results will differ, but the general principle will still hold. In determining the amount of lime to add without decreasing the strength of a certain mortar, tests should be made with the materials to be employed.

In scientific experiments by Mr. Feret* the maximum strength of 1:4 mortar of Portland cement and sand from Saint Malo† was reached with an addition of 4% or 5% by weight of hydrated lime powder. As the mortar became richer, the lime had less effect, until at proportions 1:2, the addition of lime reduced the density, and at proportions 1:1½ the strength was also lowered.

A larger number of bricks can be laid in a given time with mortar containing lime than with a lean cement mortar because the lime fills the pores in the mortar so that it spreads more readily without crumbling and adheres better to the bricks in "buttering" them.

Unslaked Lime. Unslaked lime mixed with cement either for mortar or concrete is liable to produce expansion in the masonry and it is therefore never permissible to use it under any circumstances. Builders recognize that lime, putty, or paste is much improved by standing for several days, or, better, for months, before being used, because all the small lumps are thus slaked. This thorough slaking is especially necessary when lime is to be used, even as a very small ingredient, in important concrete and masonry construction; an admixture of even 2% of ground quicklime may seriously reduce the strength of the mortar.‡

Weight and Volume of Lime. In proportioning lime to cement, the method of measurement must be clearly stated. The volume of common lime or quicklime increases in slaking to about 2½ times its volume measured loose in the lime cask, the exact increase varying with the chemical composition and the purity of the lime. The weight of lime paste is about 2½ times the weight of the same lime before slaking. Hydrated lime powder also occupies more volume than quicklime from which it is made.

GROUND TERRA-COTTA OR BRICK AS A SUBSTITUTE FOR SAND

Experiments by Mr. Louis C. Sabin§ indicated that for a mortar of light weight terra-cotta may be ground and used instead of sand. Tests

* Chimie Appliquée, 1897, p. 481.

† See p. 147.

‡ Report of Chief of Engineers, U. S. A., 1895, p. 2999.

§ Report of Chief of Engineers, U. S. A., 1896, p. 2866.

with both Portland and Natural cement mixed with the ground terra-cotta in various proportions gave at the end of three months tensile strengths which are not appreciably different from the strengths obtained with standard crushed quartz. Red brick pulverized* may also be used for the same purpose with good results.

EFFECT OF REGAGING MORTAR

Tests indicate that, up to the time of the initial set of the cement, mortar or concrete may be regaged without injury. Beyond this period the strength is reduced. The wetter the mix, the longer the time the mortar or concrete may stand without loss of strength on regaging, probably because of the slowness with which wet mortar sets and hardens. For example, tests† show that the setting time of neat ce-

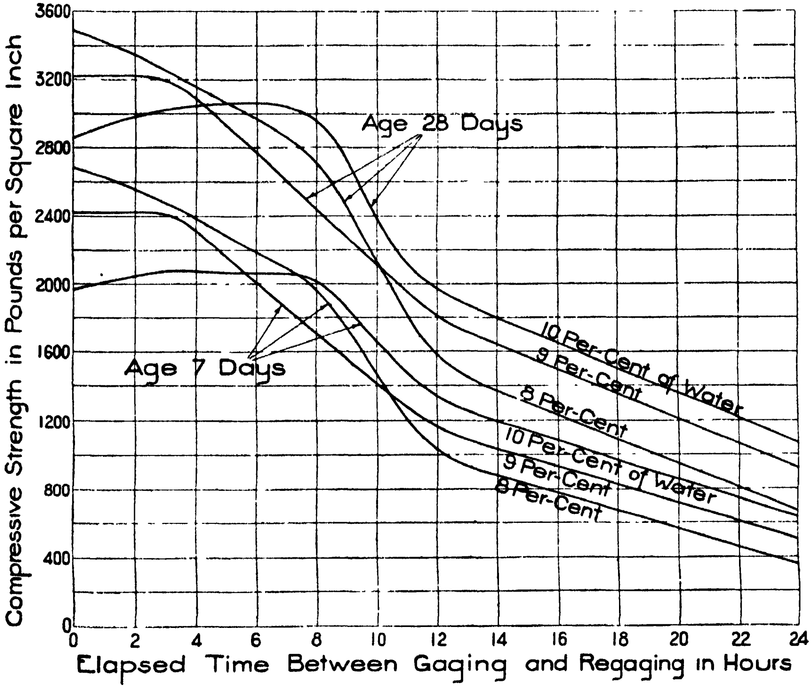


FIG. 53.—Influence of Regaging on Compressive Strength of Mortars.‡ (*See p.* 174.)

*Report Chief of Engineers, U. S. A, 1896. p. 2830.
†By Sanford E. Thompson.
‡Tests reported by H. Burchartz in Mitteilungen aus dem Koniglichen Materialprüfungsamt zu Gross Lichterfelde West 1911, p. 164.

ment paste may be increased two and one-half times by doubling the amount of water ordinarily used in testing. In the tests shown in Fig. 53, p. 173, mortar mixed with 8% of water begins to lose strength if regaged at all, whereas mortar mixed with 10% of water may stand for nearly eight hours before regaging, with no loss in strength.

In practice, regaging of mortar that has begun to stiffen should not be permitted. Under ordinary weather conditions using a medium or wet mix the time may be limited to two hours. In hot weather or when, for other reasons the cement is quick setting, a shorter time must be required.

When mortars and concretes are regaged, water must be added until normal consistency is reached. A mix, originally very wet, may require no water for perhaps six hours, while a dry mix needs additional water at all ages. The leaner the mix the less extra water required at regaging.

The tests in the figure show the effect on the strength of mortars of different consistencies regaged after various lengths of time up to 24 hours. The strength tests were made at 7 and 28 days. The following table shows the effect of different percentages of water on the setting time of the cement used.

*Setting Time of Neat Cement with Different Percentages of Water.** (*See p.* 174.)

	Percentage of water by weight.			
	27%	32%	36%	40%
Initial Set	5 hrs.	7½ hrs.	9 hrs.	9½ hrs.
Final Set	8¾ hrs.	16 hrs.	17 hrs.	18½ hrs.

The results are in general confirmed by tests in this country and abroad. Mr. Candlot†, however, found that mortars regaged after 12 and 24 hours, while showing great loss in strength at 7 and 28 days, gave only a small loss on long time tests, but this does not affect the general restrictions against the use of regaged mortar because in practice it is usually the strength up to 28 days that is critical.

* Tests reported by H. Burchartz in Mitteilungen aus dem Königlichen Materialprüfungsamt zu Gross-Lichterfelde West 1911, p. 164.

† E. Candlot, Ciments et Chaux Hydrauliques, 1898, p. 355-365.

CHAPTER X
PROPORTIONING CONCRETE

BY WILLIAM B. FULLER*

IMPORTANCE OF PROPER PROPORTIONING

The proper proportioning of concrete materials increases the strength obtainable from any given amount of cement, and also the water-tightness. Conversely, it permits, for a given requirement of strength and water-tightness, a reduction in the amount of cement, thereby reducing the cost.

Upon large or important structures it pays from an economic standpoint to make very thorough studies of the materials of the aggregates and their relative proportions. This fact has been seriously overlooked in the past, and thousands of dollars have sometimes been wasted on single jobs by neglecting laboratory studies or by errors in theory. Since cement is always the most expensive ingredient, the reduction of its quantity, which may very frequently be made by adjusting the proportions of the aggregate so as to use less cement and yet produce a concrete with the same density, strength and impermeability, is of the utmost importance.

As an example of such saving, the ordinary mixture for water-tight concrete is about 1 : 2 : 4, which requires 1.51 barrels of cement per cubic yard of concrete. By carefully grading the materials by methods of mechanical analysis the writer has obtained water-tight work with a mixture of about 1 : 3 : 7, thus using only 0.97 barrels of cement per cubic yard of concrete. This saving of 0.54 barrels is equivalent, with Portland cement at $1.60 per barrel, to $0.86 per cubic yard of concrete. The added cost of labor for proportioning and mixing the concrete because of the use of five grades of aggregate instead of two was about $0.15 per cubic yard, thus effecting a net saving of $0.71 per cubic yard. On a piece of work involving, say, 20 000 cubic yards of concrete such a saving would amount to $14 200.00, an amount well worth considerable study and effort on the part of those in responsible charge.

Proper proportioning is also important for reinforced concrete so as to give the uniformity and homogeneity which cannot be obtained without careful attention to the proportions and grading of the aggregates.

* The authors are indebted to Mr. Fuller for the material for this chapter.

METHODS OF PROPORTIONING

It is recognized generally that for maximum strength a concrete should be as dense as possible, that is, that it should have the smallest practicable percentage of voids. The various methods of aiming toward this result have been outlined as follows:*

(1) Arbitrary selection; one arbitrary rule being to use half as much sand as stone, as 1 : 2 : 4 or 1 : 3 : 6; another, to use a volume of stone equivalent to the cement plus twice the volume of the sand, such as 1 : 2 : 5 or 1 : 3 : 7.

(2) Determination of voids in the stone and in the sand, and proportioning of materials so that the volume of sand is equivalent to the volume of voids in the stone and the volume of cement slightly in excess of the voids in the sand.

(3) Determination of the voids in the stone, and, after selecting the proportions of cement to sand by test or judgment, proportioning the mortar to the stone so that the volume of mortar will be slightly in excess of the voids in the stone.

(4) Mixing the sand and stone and providing such a proportion of cement that the paste will slightly more than fill the voids.

(5) Making trial mixtures of dry materials in different proportions to determine the mixture giving the smallest percentage of voids, and then adding an arbitrary percentage of cement, or else one based on the voids in the mixed aggregate.

(6) Mixing the aggregate and cement according to a given mechanical analysis curve.

(7) Making volumetric tests or trial mixtures of concrete with a given percentage of cement and different aggregates, and selecting the mixture producing the smallest volume of concrete; then varying the proportions thus found, by inspection of the concrete in the field.

The most practical method known to the writer for accurately determining the proportions of each material is by mechanical analysis of the aggregates, as described on page 201.

Volumetric synthesis, or proportioning by trial mixtures (p. 196) is another method, sometimes useful, which produces fairly scientific results.

Since in many cases the proportions for a concrete must be selected more or less arbitrarily, after outlining the principles of proper proportioning, some of the less exact methods which are frequently used in practice will be

* From "Proportioning Concrete," by Sanford E. Thompson, Journal Association Engineering Societies, Vol. XXXVI, Apr. 1906, p. 185.

taken up before referring to the more scientific ones, and some of the causes for inaccuracies of these approximate methods discussed.

PRINCIPLES OF PROPER PROPORTIONING

The principles underlying the correct proportions of the materials of concrete are the same as those for mortar, namely, that the mass when compacted shall have the greatest possible density. In order, therefore, to obtain a knowledge of correct proportioning it will be best to first study the general conditions which are known to affect density.

Perfect spheres of equal size piled in the most compact manner theoretically possible leave but 26% voids. If the spaces between such a pile of equal-sized perfect spheres were filled with other perfect spheres of diameter just sufficient to touch the larger spheres, it would take spheres having relative diameters of 0.414 and 0.222 of the larger spheres, and the voids in the total included mass would be reduced to 20%. Using in this same manner smaller and smaller perfect spheres, it is conceivable that the voids could be reduced to so low a per cent of the total mass and to a size so small as to be only in a capillary form, and thus prevent the passage of water. This is assuming that every particle is placed exactly in its assigned place, but it is inconceivable that such an arrangement should take place under practical conditions, and in fact numerous trials by the writer with large masses of equal-sized marbles have demonstrated that they cannot be poured or tamped into a vessel so as to give less than 44% voids.

If equal quantities of spheres of, say, three sizes are mixed together, the per cent of voids in the total mass immediately increases, becoming about 65%, due probably to the smallest spheres getting between and forcing apart the largest. If, however, the containing vessel is continually shaken and the spheres stirred around, the smallest spheres will gradually all gravitate to the bottom and the largest to the top and the amount of voids in the total mass will again approach 44%. If a large number of different sized spheres are used, employing an increasingly large number of the smaller sizes so that each larger size may be said to be wholly surrounded by the next smaller size, the voids remain the same, no matter what the shaking, and will in some cases reach as low as 27%.

With ordinary stones and sands the same law holds as with perfect spheres except that they do not compact as closely, and the percentage of voids under comparable conditions is larger, varying with the degree of roughness and other features of the stones and sands used for the experiments.

When dry cement is added to a dry aggregate of stone and sand it acts

in the same manner as fine sand, and for obtaining the greatest density with dry cement, the cement must replace an equivalent amount of fine sand.

The theory of a concrete mixture is well stated by Mr. Feret* as follows:

> The problem of making the best concrete is thus reduced to the selection of a mixture of materials whose granulometric composition† corresponds to the maximum of density, since when this composition is known absolute volumes of cement may be substituted for equal absolute volumes of fine sand and vice versa, so as to vary the strength as desired while the density remains the same.

In other words, having mixed dry, inert materials in proportions necessary for greatest density, a portion of the grains of the very finest aggregate (that is, the finest particles of sand or dust) may be replaced by a corresponding quantity of cement to the extent required for the desired strength. This is not strictly true for concrete mixtures, because, when water is added to dry cement, the cement particles are separated from each other by the surface tension of the film of water, and it is no longer possible to obtain as dense a mixture as is theoretically possible with the dry mixture.

The density of concrete therefore has been found to depend upon the varying degree of roughness of the stone and sand, the relative sizes of the diameters of the stone, sand and cement, and the amount of water used.

The fineness of the cement particles and the amount of water to be used are determined by questions discussed elsewhere, and we have to deal here only with the proportioning of the sand and stone.

DETERMINATION OF THE PROPORTION OF CEMENT

The most difficult question to decide with accuracy in proportioning is the proportion of cement to use. This is to a considerable extent a matter of mature judgment, depending upon the nature of the construction, the degree of strength required within a certain limit of time, the required watertightness, the character of the aggregates, and many other matters which must be considered in direct connection with the work to be done and the available materials. An engineer experienced in concrete construction and tests can estimate approximately the strength of concrete made with certain materials, and select the proportions accordingly. The surest plan after selecting and grading the aggregates is to make up specimens of concrete and test its crushing strength, but this is usually impracticable for lack of time. The next best plan is to have the tensile strength determined of mortar made from the sand to be used and by comparing

*Chimie Appliquée 1897, p. 523.
†Proportioning of sizes.

this with the strength of the mortar of standard sand an idea can be formed of the proportion of cement to select. If a sand is fine, a richer mortar must be used, frequently instead of a 1 : 2 selecting a 1 : 1½ or even 1 : 1, and the amount of coarse aggregate also reduced to accord with this.

An experimental plan which has been followed to determine the minimum quantity of cement which will produce a concrete practically free from air voids is to mix the aggregates in the correct proportions as described in the pages which follow, compact them by ramming or hard shaking, and then determine their voids by weighing and correcting for specific gravity.* The sand should be in the natural state of moisture found in the interior of the bank, not because this is the condition in which it will be mixed in the concrete, but because it may be assumed in the natural state to contain a quantity of moisture varying with its fineness. If gravel is used it may be taken in the same way, while coarse broken stone should be dry, and dry broken stone screenings may be mixed with about 4% of water by weight. Correction must be made for this moisture after weighing the mixed material, so that the voids calculated will be simply air voids.

In determining the quantity of cement to fill these air voids it may be assumed without appreciable error that 100 lb. of cement will make 1.0 cu. ft. of neat paste. This is a larger volume than would result with ordinary plastic paste, but makes a slight allowance for the additional moisture required for the sand and stone. To the quantity of cement thus determined 10% may be added, *i. e.*, 10% of the cement, not of the total mixture, to provide for imperfect mixing.

PROPORTIONING BY ARBITRARY SELECTION OF VOLUMES

The common custom of specifying arbitrarily the proportions of cement, sand and stone in parts by volume, while convenient in construction, causes wide discrepancies in results because of different methods of measuring the materials. A concrete called a 1 : 2 : 4 mixture by one man may not contain any more cement than a concrete termed a 1 : 3 : 6 mixture by another.†

Notwithstanding this, if the units of measurement and the methods of measuring are stated definitely, arbitrary selection of proportions may give good results in practice, although necessitating a larger quantity of cement with consequently a greater net cost than more scientific proportioning would require.

The percentage of volume of sand required for ordinary gravel or broken

*See page 126.

†These variations are discussed more fully by the authors on page 206.

stone from which the finest material has been screened may be taken between the limits of 40% and 60% with an average, which is suitable under many conditions, of 50%. If the cement is taken as additional, which is not strictly correct, this ratio corresponds to proportions 1 : 1½ : 3, 1 : 2 : 4, 1 : 2½ : 5, and 1 : 3 : 6, which are suggested by the authors in Chapter II as standard mixtures for the use of those who are inexperienced in concrete work.

In cases where the coarse material contains a good many small particles, as does crusher run, broken stone or graded gravel, or the sand is so fine as to flow readily into the voids of the stone, the proportion of sand should be slightly less than half the volume of stone. Since the cement also increases the bulk of mortar and hence assists to fill the voids in the stone, it is suggested that with such aggregates the volume of the stone be made equal to the cement plus twice the volume of the sand. This would give proportions 1 : 1½ : 4, 1 : 2 : 5, 1 : 2½ : 6, and 1 : 3 : 7 for these special conditions.

Proportions adopted by various authorities and tabulated on page 202 and 203 may serve as a guide to arbitrary selection.

It is a good plan on work which will not warrant special tests and for which there is no choice of aggregates, to use at first twice as much stone or gravel as sand and then vary the relative proportions of the sand to the stone as the work progresses, governing this by the way the concrete works into place. Too much sand will be indicated by the harsh working of the concrete, while if there is too little sand, stone pockets are apt to occur on the surface of the concrete, and it will be difficult to fill the voids of the stone.

Screened vs. Unscreened Gravel or Broken Stone. Unscreened gravel is often used alone for the aggregate, but there is scarcely any case where the cost of screening and re-mixing the materials will not be less than the saving in the cement by using screened aggregates. The quantity of sand in different parts of the same gravel bank always varies greatly and the run of the bank rarely contains sufficient coarse stone to make a dense concrete. If, as is sometimes the case, the quantity of material coarser than ¼ inch is about the same as that which passes a ¼-inch sieve, then, if used without screening the same quantity of total aggregate must be used as would otherwise be specified for the coarse aggregate; that is, instead of 1 : 2 : 4 proportions, the unscreened gravel would require 1 : 4.

Broken stone as it runs from the crusher will contain considerable dust, and may sometimes be used economically by simply adding sand without screening. However, there is apt to be a separation of the coarse particles from the fine as they roll down the pile so that less homogeneous propor-

tions can be attained. Consequently the writer is in favor of separating the aggregate into as many parts as is consistent with economy for the work in hand. Even on small work he believes it preferable to screen out the sand or dust and re-mix it in the specified proportions.

PROPORTIONING BY VOID DETERMINATION

The determination of proportions by finding the volume of water which may be poured into the voids of a unit volume of stone and selecting a volume of sand equal to this volume of water is one which gives no better results in practice than arbitrary selection of the proportions, as described in the preceding paragraphs, and varying the relative proportions of sand to stone when placing. The determination of the proportion of cement to sand by void measurement is still more misleading; in fact, for reasons discussed below, it is so inaccurate that no consideration will here be given to it.

The theory of proportioning by voids is that if the stone or gravel contains, say, 40 per cent voids as measured by the contained volume of water, the required volume of sand is theoretically 40% of the volume of the stone, and supposing the ratio of cement to sand to be as 1 : 2, the relation of parts of sand to parts of the coarse aggregate would be as 2 : 5, thus making the proportions 1 : 2 : 5. Because of the inaccuracy of this method of procedure, as discussed below, it is necessary in most cases, even although the cement and water will still further increase the bulk, to take a volume of sand, say 5% to 10% in excess of the voids; that is, for gravel with 40% voids to use 45% to 50% of its volume of sand, thus making the proportions 1 : 2 : 4½. If the coarse material is screened broken stone of large size, say 1½ or 2-inch, the volume of sand may be taken equal to the volume of voids instead of in excess of them, because the particles of sand will all be small enough to fit into the voids of the stone without appreciably increasing its bulk. Such stone usually has about 45% to 50% voids, so that we should have proportions 1 : 2 : 4½ or 1 : 2 : 4, the same as for the gravel concrete.

The irregular distribution of the materials by imperfect mixing may usually be disregarded, because the volume of gaged mortar is always in excess of the volume of sand from which it is made.

Care must be exercised in any case to guard against a larger excess of sand than is absolutely necessary, because the voids in a concrete are lessened by using stone in place of sand. Take, for instance, sand having 45% voids and stone having 40% voids. With the sand just filling the voids of the stone it is easily calculated that the resultant mass has 18%

voids; but supposing an excess of 10% of sand, there would be 10% of the material having 45% voids, which means there would be 2.5% more voids in the resultant mass.*

Authorities differ as to whether the stone should be loose or shaken when determining the voids. Loose measurement is generally considered preferable because it corresponds more nearly to the final volume of the concrete, and more sand is always necessary than will just fill the voids of rammed stone, since the sand and cement separate the stones and prevent their lying close together in concrete. In determining, however, the quantity of cement required for the mixture of aggregates the materials should be compacted as described on page 201.

The chief inaccuracy of this method of basing the proportions of the finer materials of a concrete mixture upon the water contents of the voids in the larger is due to the difference in compactness of the materials under varied methods of handling, and to the fact that the actual volume of voids in a coarse material may not and usually does not correspond to the quantity of sand required to fill the voids, and that therefore the common method of proportioning by basing the volume of sand or of mortar upon the volume of water which can be poured into the broken stone leads to false conclusions. The reasons for this inaccuracy are chiefly because the grains of sand thrust apart the particles of stone, and because with most aggregates a portion of the particles of sand or fine screenings are too coarse to enter the voids of the coarsest material.

Even in a mass of stones of uniform size many of the separate voids are much smaller than the particles. If we have, then, a mass of gravel ranging from fine to coarse or a mass of crusher-run broken stone, even with the finest sand or the dust screened out of them, the individual voids are many of them so small that a large number of the particles of natural bank sand will not fit into them, but will get between the stones and increase the bulk of the mass. On account of this increase in bulk, even with thorough mixing more sand is required than the actual volume of the voids in the coarse material. The separation of the particles of stone by the sand is illustrated in the mixture shown in Fig. 2, page 15.

To illustrate this important principle, an extreme example may be cited. Suppose that we have a mixture in equal parts of 1-inch stone and $\frac{1}{8}$-inch stone. By the usual method of reasoning employed in proportioning concrete, if the 1-inch stone has 50% voids, we should require a volume of $\frac{1}{8}$-inch, equal to 50% of the volume of the 1-inch stone, in order to fill

* See discussion by the writer in Transactions American Society of Civil Engineers, Vol. XLII, p. 142.

the voids in the latter. The absurdity of this is apparent, because the two stones are so near a size that the smaller cannot fit into the voids of the latter, and the bulk of the mixture is inappreciably less than the sum of the separate volumes, that is, the mixture still has nearly 50% voids. The principle is just as true, although the total effect is less, if we consider it with reference to the finer particles of the gravel or the crusher-run broken stone and the sand or fine screenings which are to be introduced to fill the voids. The sizes of many of the particles of the latter are so nearly equal to the sizes of the smallest particles of the coarse material that they increase the total bulk instead of reducing the voids. They also get between the surfaces of the stone particles and prevent the stones touching each other.

We might conclude from the above that the best concrete can be made with a coarse stone of uniform size and a sand whose particles are all small enough to fit into its voids; in fact, this is the conclusion reached by the advocates of broken stone of uniform size in preference to crusher-run stone.

Our experiments indicate that while this may be true in theory, in practice in making concrete the graded materials give about the same density and work rather smoother in handling and placing.

The point, however, which is to be emphasized is the inaccuracy of determining the exact volume of sand or mortar by simply measuring the water contents of the voids in the coarse aggregate.

The selection of the proportion of cement by determination of the water contents of the voids in sand is even more inaccurate than the proportioning of sand to stone by void measurement. The varying effect of moisture on the sand so influences the volume of the voids that their determination is chiefly important as an aid to the judgment; and as a matter of fact, although in practice the quantity of cement is supposed to depend upon the volume of voids in the sand, it is customary to select a definite relation of cement to sand varying according to the character of the construction from 1 : 1 to 1 : 3, recognizing, however, that fine sand—and fine sands in an ordinary state of moisture will almost always have the distinguishing characteristic of a lighter weight per cubic foot than coarse sands and a consequently larger percentage of voids—requires more cement for equivalent strength.

As already stated, if the work is too small to warrant a thorough study of the materials by mechanical analysis or volumetric synthesis, or some other scientific method, it is evident from the above discussion that it is nearly as accurate to determine the proportions by arbitrary selection (see p. 178) as by a study of voids.

RAFTER'S METHOD OF PROPORTIONING

Mr. George W. Rafter* has called attention to the method of proportioning the mortar as a percentage of the volume of the stone slightly shaken, the relation of cement to sand having been determined by the required strength of concrete.

Quoting from specifications for the Genesee Dam, the concrete is proportioned as follows:

> In forming concrete such a proportion of mortar of the specified composition will be used as may be found necessary by trial to a little more than fill the voids in the aggregate. Tests of the voids will be made from time to time under the direction of the engineer, and instructions given as to the per cent of mortar of the specified composition to be used. For the information of the contractor, in the way of computing the cost of concrete of the quality herein required, it may be stated that ordinarily the per cent of mortar will be about 33 per cent of the measured volume of the aggregate. In case of the use of a certain proportion of gravel in the aggregate, the proportion of mortar may be reduced to somewhat less than 30 per cent.

This method of proportioning is more accurate than the usual procedure, because there is less apt to be an excess of mortar. It does not, however, take account of the fact that with a coarse aggregate of varying sized particles some of the grains of sand are too large to fit into the voids of the stone, and that therefore the coarse and fine aggregates must be studied together.

An examination of the analysis of the sand used by Mr. Rafter indicates that to its fineness was due the small proportion of mortar to stone which he was able to use. Ninety-two per cent of the sand passed a No. 30 sieve, so that the grains were small enough to enter the voids of the stone without appreciably increasing the bulk of the concrete.

FRENCH METHOD OF PROPORTIONING

In France, proportions are ordinarily stated in terms of the volume of mortar to the volume of stone, and the mortar is described by the number of kilograms of Portland cement to 1 cubic meter or liter of sand.

The following table gives the nominal proportions in English measure based on a volume of 3.8 cubic feet corresponding to similar French proportions based on kilograms of cement to a cubic meter of sand.

*"On the Theory of Concrete" Transactions American Society Civil Engineers, Vol. XLII, p. 104.

American Equivalents of French Proportions. (*See p.* 184.)

French measure, kilograms cement per cubic meter of sand.	American measure, cement to sand by volume.*	Pounds of cement per cubic foot of sand.	French measure, kilograms cement per cubic foot of sand.	American measure, cement to sand by volume.*	Pounds of cement per cubic foot of sand.
200	1 : 8.0	12.5	700	1 : 2.3	43.7
300	1 : 5.3	18.7	800	1 : 2.0	50.0
400	1 : 4.0	25.0	1000	1 : 1.6	62.5
500	1 : 3.2	31.3	1200	1 : 1.3	75.0
600	1 : 2.7	37.5	1600	1 : 1.0	100.0

*Proportions based on standard weight of cement, i. e., 100 pounds per cubic foot

Concrete in France is frequently designated with respect to the ratio of mortar to stone; for example, one volume of mortar to two volumes of stone, the mortar then being designated as indicated in the above table. To express the parts more definitely, the basis is sometimes a cubic meter of sand; for example, 650 kilograms cement to one cubic meter sand to 1.8 cubic meter stone, this corresponding substantially to proportions 1 : 2½ : 4½ by volume, as ordinarily used in America.

MECHANICAL ANALYSIS

Mechanical analysis consists in separating the particles or grains of a sample of any material, — such as broken stone, gravel, sand or cement, — into the various sizes of which it is composed, so that the material may be represented by a curve (see Fig. 55, p. 188) each of whose ordinates is the percentage of the weight of the total sample which passes a sieve having holes of a diameter represented by the distance of this ordinate from the origin in the diagram.

The objects of mechanical analysis curves as applied to concrete aggregates are (1) to show graphically the sizes and relative sizes of the particles; (2) to indicate what sized particles are needed to make the aggregate more nearly perfect and so enable the engineer to improve it by the addition or substitution of another material; and (3) to afford means for determining best proportions of different aggregates.

To determine the relative sizes of the particles or grains of which a given sample of stone or sand is composed, the different sizes are separated from each other by screening the material through successive sieves of increasing fineness. After sieving, the residue on each sieve is carefully weighed, and beginning with that which has passed the finest sieve, the weights are suc-

cessively added, so that each sum will represent the total weight of the particles which have passed through a certain sieve. The sums thus obtained are expressed as percentages of the total weight of the sample and plotted upon a diagram with diameters of the particles as abscissas and percentages as ordinates. The method of plotting and the uses of the curves thus obtained are fully described in the pages which follow. In appendix I there is given a more detailed and mathematical treatment of the method of combining mechanical analysis curves than is given in this chapter.

Sieves and Other Apparatus. The necessary apparatus for a mechanical analysis consists of a set of sieves and scales for weighing. To this may be added a mechanical shaker, see Fig. 54, p. 186, of which various forms are available, but satisfactory results are obtained from hand work. A standard size of sieve is 8 inches in diameter and 2¼ inches high. Sieves with openings of ¼ inch or larger are preferably made of spun hard brass with circular openings drilled to the exact dimensions required. Sieves with openings of 0.10 inch and less are preferably of woven brass wire set into a hard brass frame. Woven brass sieves are made for many purposes, and are sold by numbers which are approximately the number of meshes to the linear inch.

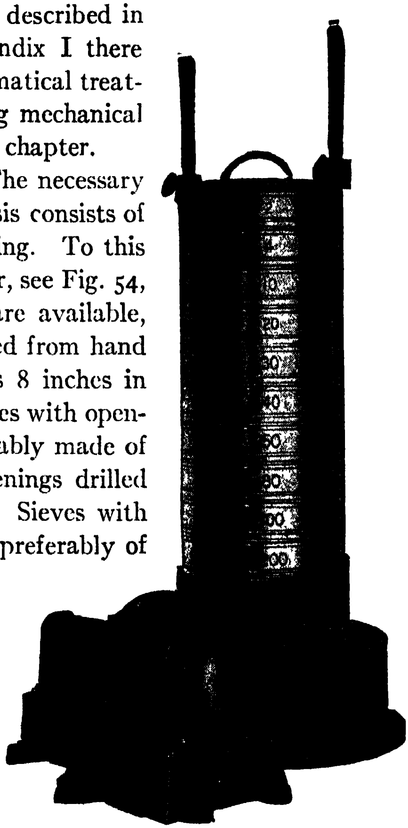


FIG. 54.—Mechanical Analysis Sieves and Shaker. (*See p.* 186.)

For separating particles smaller than those passing through a No. 200 sieve, the winnowing device described on page 88 may be used.

The number and sizes of sieves to be used depends upon the importance of the testing to be done. The manufacturers' catalogues give complete lists where the openings of successive sizes varies very little. For ordinary concrete work a large number is not warranted. The following list of sieves has been found to give satisfactory results in laboratory practice:

Stone Sieves Diameter of Hole Inches.	Sand Sieves		
	Commercial No.	Diameter in inches.	
		Hole.	Wire.
3.00	¼ inch round		
2.50	No. 7	0 111	0.032
2.00	No. 12	0.056	0.027
1.50	No. 20	0 0335	0.0165
1.00	No. 30	0 0198	0.0135
0.75	No. 50	0 0120	0.0080
0.50	No. 90	0.0059	0 0052
0.25	No. 200	0 0029	0.0021

If uniformly varying spacing is desired for sieve mesh, the widths of openings may be made proportional to their logarithms. For example, if 10 sieves are desired, the logarithms of the widths of openings of the largest and the smallest are found and by direct proportion the logarithms of the intermediate sieves are fixed. The width of opening corresponding to each logarithm is then found and the nearest convenient sieve selected.

After the sieves are obtained it is necessary that they should be very carefully calibrated to ascertain the average diameter of the mesh. This should be done by averaging the diameters of the openings measured in two positions at right angles to each other, as the meshes of commercial sieving are not exactly square. Sieves having meshes exceeding 0.10 inch are most conveniently calibrated by ordinary outside calipers; those having meshes of less diameter, by a micrometer microscope.

When many analyses are to be made, it is convenient to have a printed cross section form, with appropriate spaces for filling in the number of the analysis, description of the material, location of the work, and other facts relating to the material.

Plotting Analysis Curves. For those who are unfamiliar with mechanical analysis a detailed explanation of the method of locating the curve is here given. The method can best be understood by referring to the diagrams of typical materials which are also of practical interest as illustrating the curves which may be expected in special cases.

Fig. 55, p. 188, represents a typical mechanical analysis of crusher-run micaceous quartz stone which has been run through a ¼-inch revolving screen so as to separate particles finer than ¼ inch, that is the dust, for use with sand.

For a sample of stone, which may be taken by the method of quartering

described on page 344, 1 000 gr. is a convenient quantity for 8-inch diameter sieves 2¼ inches in depth, and also permits of easy reduction from weights to percentages. To obtain the analysis shown in Fig. 55, the sample of stone is placed in the upper (coarsest) sieve of the nest of stone sieves given in list below, and after 1 000 shakes the nest is taken apart, and the quantity caught on each sieve is weighed begin-

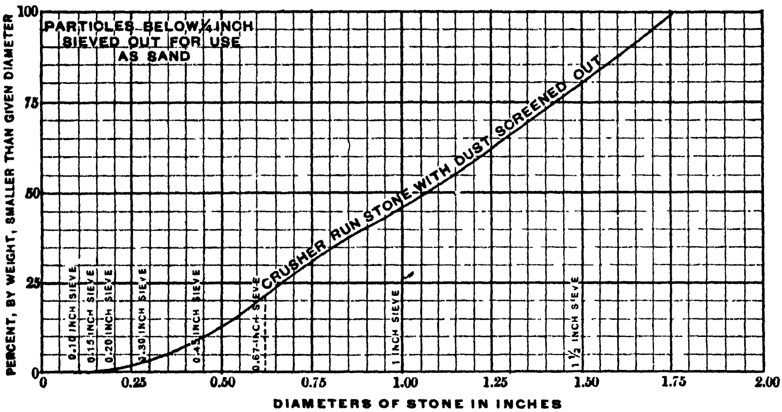


FIG. 55.—Typical Mechanical Analysis of Crusher-Run Micaceous Quartz Stone. (See p. 187.)

ning with the finest and placing each successive residue on the scale pan with that already weighed. The results obtained in the particular case under consideration are illustrated in the following table which shows the method of finding the percentages.

Results of Screening Samples of Stone of Fig. 55.

Size sieve inches.	Amount finer than each sieve grams.	Percentage finer than each sieve %.
1.50	801	80
1.00	457	46
0.67	222	22
0.45	99	10
0.30	27	3
0.20	19	2
0.15	8	1
0.10	0	

The various percentages are plotted on the diagram and the curve drawn through the points. The vertical distance from the bottom of the diagram

to the curve, that is, the ordinate at any point, represents the percentage of the material which passed through a single sieve having holes of the diameter represented by this particular ordinate. Since the percentage of material passing any sieve is always the complement of the percentage of grains coarser than that sieve, the vertical distances from the top of the diagram down to the curve represents the percentages which would be retained upon each sieve if employed alone. For example, taking 1.25, 62%, the distance from the bottom of the diagram, represents the percentage of material finer than 1¼ inch diameter, and 38%, the distance down from the top of diagram, represents the percentage coarser than 1¼ inch.

Fig. 56 represents a typical analysis of crushed trap rock which has been

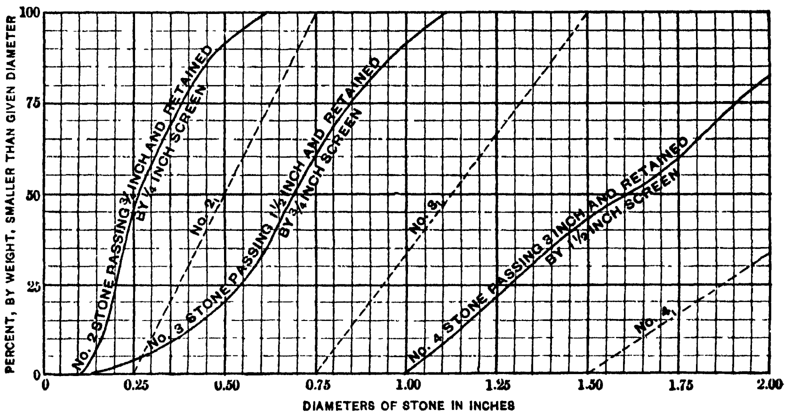


FIG. 56. — Typical Mechanical Analysis of Crushed Trap Rock Separated into Three Sizes by Revolving Screens having 3, 1½, ¾ and ¼ inch perforations. (*See p.* 189.)

separated into stone of three sizes and dust, by a revolving screen 2 feet 6 inches in diameter and 12 feet long set on a slope of 1 foot 9 inches. This was made up of four sections having respectively 3, 1½, ¾ and ¼ inch perforations. The curves not only show the sizes of trap rock which ordinarily pass through crusher screens of given diameter of hole, but also illustrate how inefficient the screening process may be. For example, if the sizes of the particles had corresponded exactly to the diameters of the holes and the screening had been more perfectly done, we should have had curves whose general direction and location is shown by the dotted lines No. 2_1, No. 3_1, and No. 4_1, that is, for example, No. 3, since it represents stone which passes a 1½ inch screen and which is retained on a ¾ inch screen, should occupy a position between the ordinates representing 1.50 and

0.75 diameters. If the stone had rumbled longer in the screen because of flatter slope or screen sections of greater length, the curves would have approached more nearly to these dotted lines.

Typical curves of a fine, a medium well graded, and a coarse sand are shown in Fig. 57. For convenience in plotting, the horizontal scale is ten

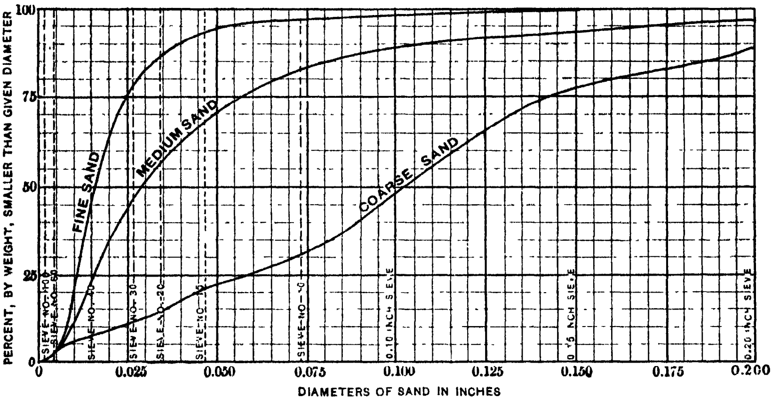


FIG. 57.—Typical Mechanical Analyses of Fine, Medium, Well Graded and Coarse Sands. (*See p.* 190.)

times greater than that of Figs. 55 and 56, the diagram showing diameters ranging from 0 to 0.200 inches diameter. The "granulometric composition" of these sands may be determined if desired by reference to page 162.

The mechanical analysis of crusher dust is apt to vary between the curves of fine sand and medium sand which are shown in Fig. 57.

STUDIES OF THE DENSITY OF CONCRETE

In the year 1901 the writer, through the permission and assistance of Mr. E. LeB. Gardiner, Vice-President, and Mr. J. Waldo Smith, Chief Engineer, of the East Jersey Water Company, was enabled to make an extended series of experiments on the comparative strengths of different proportions of concrete aggregate. Many mixtures of different proportions were made up into beams, their curves of mechanical analyses drawn as explained above, and the strength of the beams determined by breaking tests.*

These tests indicated that the strength of concrete varies with the percentage of cement contained in a unit volume of the set concrete, also with

* The results of these tests are presented in the table on pages 334 and 335.

the density of the specimen. With the same percentage of cement, the densest mixture, irrespective of the relative proportions of the sand and stone, was in general the strongest. These tests further indicated that for the materials used there was a certain mixture of sizes of grains of the aggregate which, with a given percentage by weight of cement to the total aggregate, gave the highest breaking strength. In practice also it was found that the concrete made with this mixture worked most smoothly in placing.

These tests led to a still more extended series by the writer and Mr. Sanford E. Thompson at Jerome Park Reservoir, New York, in 1903 and 1904, under the authorization of the Aqueduct Commission of the city of New York, Mr. J. Waldo Smith, Chief Engineer.

The method of procedure and the results of the tests are given in full in a paper on "The Laws of Proportioning Concrete," by William B. Fuller and Sanford E. Thompson, Transactions American Society Civil Engineers, Vol. LIX, p. 67, 1907. The experiments were begun with a series of tests on the density of different mixtures of aggregate and cement to determine the laws of proportioning for maximum density for different materials, and these density experiments were followed by the manufacture of concrete specimens in the attempt to determine the relation between the laws of strength and the laws of density.

The mechanical analysis diagram furnished a ready means of studying the effect of various sized particles on the density of concrete. For this purpose crusher-run stone and bank gravel were screened into twenty-one sizes ranging from 3 inches down to that passing a No. 100 sieve, having meshes 0.0027 inch in diameter. These sized materials were then re-combined in a predetermined mechanical analysis curve by weighing out the necessary quantities of each size.

This material was next thoroughly mixed with a given weight of cement and the whole amount wet and mixed and tamped into a strong cylinder in which its volume could be measured. This batch was then thrown away and another batch made up according to another mechanical analysis curve and its volume recorded. In this way over 400 different mechanical analysis curves were tested as to volume for the purpose of determining the ideal curve corresponding to the densest concrete mixture.

Both broken stone and gravel were used in the tests, and to reduce the number of variables, most of the experiments were made upon the same proportions, using 10 per cent by weight of cement to the total dry materials, corresponding to proportions 1 : 9 by weight.

In all of the tests instead of following the more usual plan of testing the

aggregate separately, every experiment was performed with a mixture of the aggregate and cement gaged with the water necessary to produce the proper consistency. The water was found necessary both in theory and practice. The cement and water actually occupy space in the mass, since many of the voids are too small for the grains of cement to fit into them without expanding the volume and the water also occupies actual bulk in the concrete. Besides this, a concrete mixed up with water is easier and smoother to handle than a mixture of dry materials alone which tend to separate when being placed.

Curve of Maximum Density. The Little Falls tests made by the writer indicated that the curve at greatest density was substantially a parabola. The Jerome Park tests based on a larger number of experiments define the curve still more accurately as a combination of an ellipse and a straight line.*

One of the most interesting developments was that a curve of substantially the same form would fit different materials whatever the maximum size of the stone. The ½-inch stone, for example, required but very slight change in curve equation from the 2¼-inch stone.

The maximum density curve then was found to consist of a combination of an ellipse† and a straight line, the ellipse being first constructed with its

* Mr. Fuller's method of proportioning the materials so that their mixture will form a smooth, clearly defined curve appears, on its face, to conflict with Mr. Feret's conclusion (see p. 160) that the best mixture of sand and cement for mortar is made up of coarse and fine grains only, with no intermediate grains. For sand mortars, Mr. Feret's methods are undoubtedly more exact than Mr. Fuller's, but for a concrete mixture the conditions are different, and, as we have stated on page 133, more than two sizes of materials are theoretically necessary for obtaining the densest mixture. In practice, too, all classes of materials are more or less varied, and experiments show that the particles will best fit into each other if the sizes are graded. The best proof of the practical efficiency of Mr. Fuller's method lies in the fact that he has employed it day after day for determining the proportions of the aggregate for concrete used in constructing thin, water-tight walls. The proportions used by him for such work are about 1 : 3 : 7, whereas for water-tight construction where the materials are not scientifically graded 1 : 2 : 4 mixtures are commonly used.

The method is exact and scientific and not "rule-of-thumb." The nature of the materials and their variation from hour to hour makes great refinement unnecessary, so that an accuracy of, say, 2% or 3% in the percentages are all that is necessary in practice. Although further tests may show that for other materials the form of the curve varies from that indicated by Mr. Fuller, the general method of analyzing materials and combining the curves is undoubtedly applicable whatever the form of the curve, so that Mr. Fuller's general principles and methods still hold.

† In practice ellipses may be most readily plotted graphically by the Trammelpoint method as follows:

Plot the major and minor axes on the diagram. The major or horizontal axis in all cases is on a line 7% above the base. The minor or vertical axis is at a distance, a, to the right of the vertical zero ordinate of the diagram. Lay a strip of paper or a thin straight-edge upon the major or horizontal axis, and mark upon it two points to represent the length of the semi-major axis, calling one of these points—the point on the zero ordinate—O, and the other point A. Mark off on the strip or straight-edge, in the same direction from O, the length of the semi-minor axis, calling this point B. Now, swing the strip of paper or straight-edge little by little so that the outline of the curve may be marked off by the point O, while the points A and B are kept at all times upon the axes b and a respectively. The straight lines to continue the curves are drawn as tangents to them, or may be readily plotted from the data on the following page.

major axis coinciding with 7 per cent line of percentages, and the equation of the ellipse, using the zero coördinates of the diagram, being $(y - 7)^2 = \frac{b^2}{a^2}(2ax - x^2)$. One of the ideal curves is illustrated in Fig. 58, page 197, showing the general form which it takes.

In practice it was necessary to raise the curve somewhat higher, that is, to use more sand than the very careful laboratory tests would indicate as the ideal mix.

The values of a and b for the different materials, including the cement for the Ideal Mix, based on the Jerome Park stone and Cowe Bay sand and gravel, which, as already stated, were fairly representative materials, are as follows:

Data for Plotting Ellipses in Curves of Ideal Mix.

Materials.	Ideal Mix Axes of Ellipse.	
	a	*b*
Crushed stone and sand....	0.04 + 0.16D	28.5 + 1.3D
Gravel and sand.. . . .	0.04 + 0.16D	26.4 + 1.3D
Crushed stone and screenings................ . .	0.035 + 0.14D	29.4 + 2.2D

In this table, D = the maximum diameter of the stone, in inches.

For the Practical Mix the values of b must be greater so as to give a higher curve with more of the finer material. A quick and sufficiently accurate method of drawing the curves for the practical mix is to draw a straight line from the point where the largest diameter stone reaches the 100% line to the point on the vertical ordinate at zero diameter given in Column (1) in the following table.

Data for Plotting Curves of Practical Mix.

Materials.	Intersection of tangent with vertical at zero diameter (1)	Height of tangent point (2)	Axes of Ellipse.	
			a (3)	*b* + 7 (4)
Crushed stone and sand.	28.5	35.7	0.150D	37.4
Gravel and sand.......	26.0	33.4	0.164D	35.6
Crushed stone and screenings.................	29,0	36.1	0.147D	37.8

Then mark the tangent point on this line where it is intersected by the vertical ordinate for one-tenth the maximum diameter stone. This mark should check with the values given in column (2) of above table. Then plot the location of minor axis of the ellipse from the values of a and b+7, given in columns (3) and (4) in the above table. This point, together with the tangent point and the point at + 7 on the vertical ordinate at zero diameter where the curve begins, gives three points on the ellipse, which is usually sufficient for drawing the curve with the aid of an irregular curve. If more points are wanted, they may be plotted graphically by the trammel point method as given in the note on page 192.

RELATION OF DENSITY TO STRENGTH

Having determined the maximum density curve as just explained, it was important to know if the greatest strength coincided with the greatest density, and for this purpose a large number of beams, six inches square and six feet long, were made up and tested for transverse and crushing strength, for permeability and modulus of elasticity. Some beams were made using the proportions determined by the maximum density curve and other beams according to higher and lower curves to note if there were any decrease in these properties as the maximum density curve was departed from. The full results of the tests are given in the paper referred to,* but in general it may be said that a departure from the maximum density curve represented a reduction in all these properties except that when the curve was modified so as to use a uniform size of coarse stone instead of the graded stone it gave practically the same results as the graded. Any curving above the straight line in the coarse material decreased the density, and also the strength, indicating that the coarse aggregate should not have an excess of medium particles.

LAWS OF PROPORTIONING

From these experiments, laws of proportioning and also laws relating to strength and permeability which are outlined in full in the paper by Messrs. Fuller and Thompson* were evolved.

Those relating specifically to strength are given on page 323 and those relating definitely to permeability on page 304 and reference should be made to these for complete conclusions.

The laws relating especially to the grading of the aggregates are:

* See page 191.

1.—Aggregates in which particles have been specially graded in sizes so as to give, when water and cement are added, an artificial mixture of greatest density, produce concrete of higher strength than mixtures of cement and natural material in similar porportions. The average improvement in strength by artificial grading under the conditions of the tests was about 14 per cent. Comparing the tests of strength of concrete having different percentages of cement, it is found that for similar strength the best artificially graded aggregate would require about 12% less cement than like mixtures of natural materials.

2.—The strength and density of concrete is affected but slightly, if at all by decreasing the quantity of the medium size stone of the aggregate and increasing the quantity of the coarsest stone. An excess of stone of medium size, on the other hand, appreciably decreases the density and strength of the concrete.

3.—The strength and density of concrete is affected by the variation in the diameter of the particles of sand more than by variation in the diameters of the stone particles.

4.—An excess of fine or of medium sand decreases the density and also the strength of the concrete, as will also a deficiency of fine grains of sand in a lean concrete.

5.—The substitution of cement for fine sand does not affect the density of the mixture, but increases the strength, although in a slightly smaller ratio than the increase in the ratio of cement.

6.—It follows from the foregoing conclusions that the correct proportioning of concrete for strength consists in finding, with any percentage of cement, a concrete mixture of maximum density, and increasing or decreasing the cement by substituting it for the fine particles in the sand or vice versa.*

7.—In ordinary proportioning with a given sand and stone and a given percentage of cement, the densest and strongest mixture is attained when the volume of the mixture of sand, cement and water is so small as just to fill the voids in the stone. In other words, in practical construction, use as small a porportion of sand and as large a proportion of stone as is possible without producing visible voids in the concrete.

8.—The best mixture of cement and aggregate has a mechanical analysis curve† resembling a parabola, which is a combination of a curve approaching an ellipse for the sand and a tangent straight line for the

* This very important law requires further tests for confirmation, outside of the limits of the present tests.

† For definition of mechanical analysis, see page 185.

stone. The ellipse runs to a diameter one-tenth the diameter of the maximum size of stone, and the stone from this point is uniformly graded.

9.—The ideal mechanical analysis curve, *i.e.*, the best curve, is slightly different for different materials. Cowe Bay sand and gravel, for example, pack closer than Jerome Park stone and screenings, and therefore require less of the size of grain which the authors designate as sand.

10.—The form of the best analysis curve for any given material is nearly the same for all sizes of stone, that is, the curve for $\frac{1}{2}$-inch, 1-inch, and $2\frac{1}{4}$-inch maximum stone may be described by an equation with the maximum diameter as the only variable. In other words, suppose a diagram in which the left ordinate is zero, and the extreme right ordinate corresponds to $2\frac{1}{4}$-inch stone, with the best curve for this stone drawn upon it. If, now, on this diagram the vertical scale remains the same, but the horizontal scale is increased two and a quarter times, so that the diameter of 1-inch stone corresponds to the extreme right-hand ordinate the best curve for the 1-inch stone will be very nearly the one already drawn for the $2\frac{1}{4}$-inch stone. The chief difference is that the larger size stone requires a slightly higher curve in the fine sand portion.

11.—It follows from this last conclusion that from a scientific standpoint the term *sand* is a relative one. With $2\frac{1}{4}$-inch stone, the best sand would range in size from 0 to 0.22 inch diameter, while the best sand for $\frac{1}{2}$-inch stone would range in size from 0 to 0.05 inch diameter.

APPLICATION OF MECHANICAL ANALYSIS DIAGRAMS TO PROPORTIONING

The mechanical analysis diagram offers a very exact method of determining the proper proportions of any materials for concrete by sieving each of the materials, plotting their analyses and combining these curves so that the result is as near as possible similar to the maximum density curve.

Plot on the diagram the maximum density curve for the given materials to be used; if the equation for this material is not known use the practical equation previously given. Make a mechanical analysis of all of the materials which it is desired to mix together in the right proportions and plot the result of each analysis on the diagram on which the maximum density curve has been plotted.

The aim is to find a new curve representing the mixture of the materials. but which will conform as nearly as possible to the curve of maximum density. The proportions of different materials required to produce this curve will show the relative quantity of each which must be used in proportioning. The theory of the combination and complete discussion of the

methods to be employed with different forms of curves are treated in Appendix I.

A less exact method, but one which is convenient in practice, is by inspection and trial of different percentages. To illustrate this trial plan, the method of forming a curve of a mixture of several materials in stated proportions such as 1 : 2 : 4 will be given, then the curve for the mixture of the same materials which corresponds nearest to the curve of maximum density, and finally the application will be made to material like run of the bank gravel which may be separated into two or three parts.

In reading this discussion it must be borne in mind that the same principles will apply to mixtures of several aggregates, although for simplicity the principal part of the discussion refers to two aggregates. The same

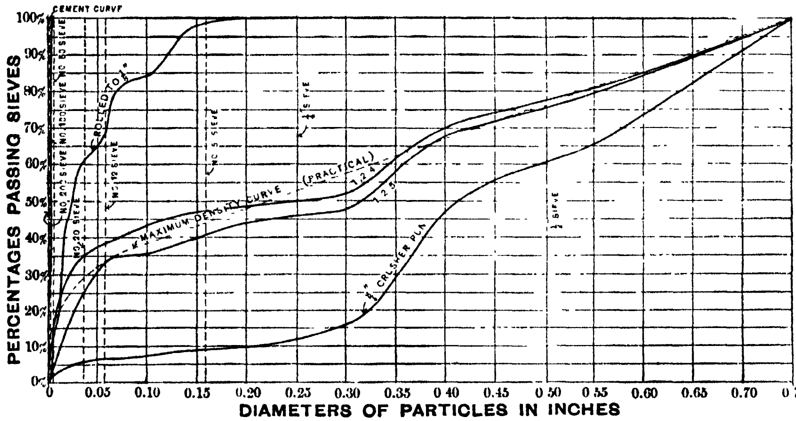


Fig. 58.—Curves of Fine and Coarse Crushed Stone and Mixtures. (p. 197)

approximate plan may be used for the larger number of aggregates or the more exact method in the Appendix may be adopted.

Plotting Curve of Mix in Studying Proportions. In Fig. 58 we have $\frac{3}{4}$-inch Shawangunk grit as one aggregate and the same material rolled to $\frac{1}{8}$-inch maximum size as the other, giving the mechanical analysis curves shown in the diagram.*

In this diagram a curve of cement is also plotted so that the 1 : 2 : 4 curve represents the combination of the three materials. The curve marked 1 : 2 : 4 then represents the analysis of the mixture of cement, screenings

* This diagram and the ones which follow are made up from materials used in subsequent studies by the New York Board of Water Supply, and referred to in the Discussion by Mr. James L. Davis, Transactions American Society Civil Engineers, Vol. LIX, p. 144.

and stone in these proportions. This curve is made up by plotting various points and connecting these by a smooth curve. To find the point, for example, where the curve cuts the ordinate corresponding to the No. 20 sieve, the sums of the percentages of the individual materials at this same ordinate are taken in the proportion which they bear to the concrete mixture. All of the cement is finer than the No. 20 sieve, and since the cement is one part of the seven parts in the mixture, one-seventh of 100 per cent represents the percentage of cement in the mixture at the given ordinate. Similarly, since there are two parts of sand in the seven parts, the sand percentage at the No. 20 ordinate, 61 per cent, is multiplied by two-sevenths, and the stone percentage, 6 per cent, by four-sevenths, thus giving as the point on the No. 20 sieve ordinate in the combined curve:

$\frac{1}{7} \times 100$ per cent $= 14.3$ per cent for cement
$\frac{2}{7} \times 61$ per cent $= 17.4$ per cent for sand
$\frac{4}{7} \times 6$ per cent $= 3.4$ per cent for stone

Total......... 35.1 per cent for the point in the curve.

The other points in the curves are found in a similar manner.

Curve of Mix to Best Fit the Maximum Density Curve. Take the same two aggregates plotted in Fig. 58, but in this case disregard the cement or rather consider it a part of the sand. (Frequently the cement must be considered in the trial mixtures in order to study the part of the curve representing the fine material to see that the percentages of the finest particles are satisfactory). The slide rule is convenient for this proportioning.

Averaging the $\frac{3}{4}$-inch stone by a straight line, we see that it crosses the 0.15 line at about 9%; we note also that the $\frac{1}{8}$-inch sand crosses the same line at 98% and the maximum density curve crosses the line at 43%, that is, along this line it is 34% from the $\frac{3}{4}$-inch stone to the maximum density curve and 55% to the $\frac{1}{8}$-inch sand. The percentages to be used to obtain a 43% mixture would be an inverse ratio of these two numbers to their total, that is, $\frac{34}{89} = 38\%$ of fine material and $\frac{55}{89} = 62\%$ of the coarse material. With the slide rule take these percentages of each curve, add together and plot a new curve, and see if it conforms reasonably with the maximum density curve. If it does not, make another trial of percentages, the plot of the curve indicating by inspection the new percentages.

It must be remembered that the fine portion of the curve includes also the cement, so having decided on the amount of cement to use, say the equivalent of a 1 : 7 mix, which has $12\frac{1}{2}\%$ of cement, the actual proportions would be $12\frac{1}{2}$ parts cement to $38 - 12\frac{1}{2} = 25\frac{1}{2}$ parts fine aggregate to 62 parts coarse aggregate, or translated into the usual nomenclature, 1 : 2.04 : 4.95, or practically 1 : 2 : 5, showing that the ordinary mixture with this particu-

lar material is the best. Supposing, however, the equivalent of a richer mixture, say 1 : 2 : 4, is wanted. This would contain 1 : 6 = $14\frac{1}{2}$% cement and the proportions would be

$$14\tfrac{1}{2} : 23\tfrac{1}{2} : 62,$$

or

$$1 : 1.62 : 4.27,$$

or practically

$$1 : 1\tfrac{2}{3} \quad : 4\tfrac{1}{4},$$

showing that for richer mixtures less fine materials is desirable.

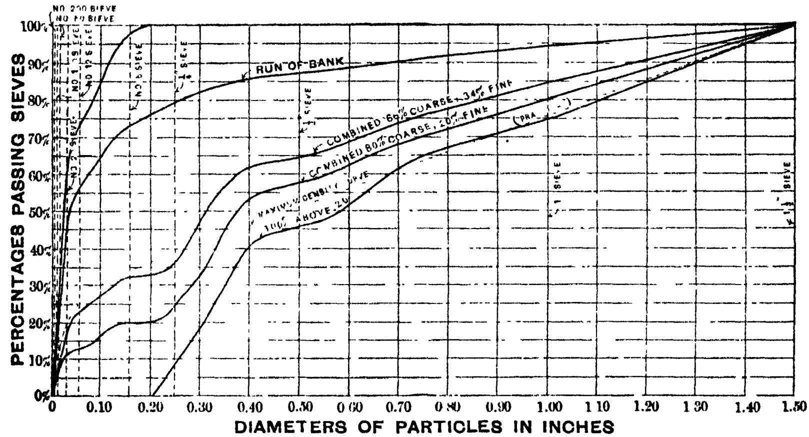


Fig. 59 —Cortland Gravel Screened to Two Sizes. (*see p.* 200)

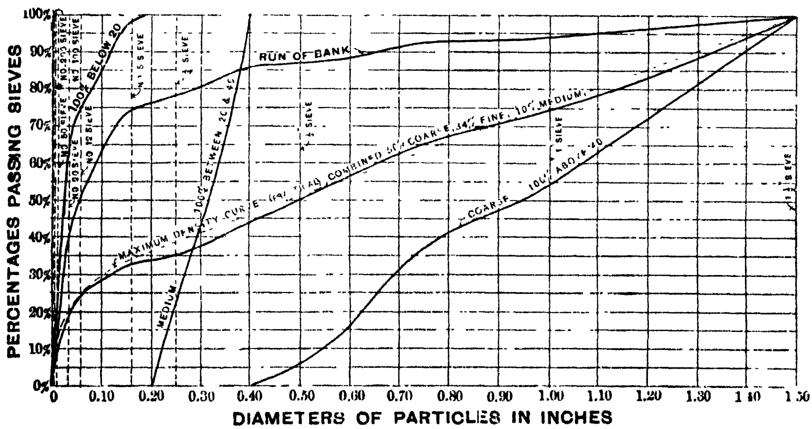


Fig. 60.—Cortland Gravel Screened to Three Sizes. (*see p.* 200.)

Run of Bank Gravel. Gravel as it is found in the natural bank almost always contains too much fine material. In many cases screening this into two sizes produces a good curve which fits very closely to the curve of maximum density.*

Other gravels, especially where the sand is greatly in excess, require two screenings for the best result. Fig. 59 represents a common run of such gravel, showing that screening into two sizes will not permit a mixture fitting very near to the maximum density curve. The figure also shows how far away the original analysis of the run of the bank is from the ideal curve. In Fig. 60 the same sand is shown screened into three sizes, and illustrates the improvement that can be obtained in this case by the extra screening, the effect of which is to leave out some of the medium size particles which are too large to fill the voids of the coarse stones, and therefore decrease the density and the strength of the mixture.

PROPORTIONING BY TRIAL MIXTURES

The density tests at Jerome Park and the relation there found of the strength to the density indicate a method of proportioning by trial mixtures which may be called volumetric synthesis. This may be used to compare the density of the same materials mixed in different proportions or different materials mixed in similar proportions.

Having determined the particular sand and stone which are to be used on any piece of work, a simple and accurate way of determining proportions is by actual trial batches of fresh material. For this it is only necessary to have good scales and a strong and rigid cylinder, say, a piece of 10-inch wrought-iron pipe capped at one end. Carefully weigh out and mix together on a piece of sheet steel or other non-absorbent material all the ingredients, having the consistency the same as is intended to be used in the work. Place these in the pipe, carefully tamping all the time, and note the height to which the pipe is filled. Weigh the pipe before filling and after being filled, thus checking weight of material mixed. Throw this material away before it has time to set, and clean the pipe. Make up another batch, using the same weights of cement and water and the same total weight of sand and stone, but have the ratio of weights of the sand and stone slightly different from the first. Note whether, after placing, the height in the cylinder is less or more than was the height of the first batch, and this will be a guide to further similar mixes, until a proportion is found which gives the least height in the cylinder, and at the same time works

*An illustration of this is given by Mr. James L. Davis, in Transactions American Society of Civil Engineers, Vol. LIX, p. 145.

well while mixing and looks well in the cylinder, all the stones being covered with mortar. This method, if carefully followed, will give very accurate results, but of course does not indicate, as does mechanical analysis, what other changes can be made in the physical sizes of the sand and stones so as to get the best available composition.

Mr. A. E. Schutté, in studying the proportions of materials for bituminous macadam pavement for the Warren Brothers Company, has very effectively developed the method of volumetric synthesis with dry materials. His experiments included various classes and sizes of stone, sand, and screenings ranging from 3 inches diameter down to that which passes a No. 200 sieve. He found that the best method for compacting dry materials, such as sand, gravel or broken stone, is to place them in a vessel the shape of a truncated cone, with the largest diameter at the bottom. The cone is filled with the coarsest material and taken by a laborer, who compacts it by repeatedly striking the cone against the ground, keeping the measure full by adding new material of the same kind. When it ceases to settle, the contents is emptied and mixed with a portion of a finer material, replaced in the measure and compacted as before. By repeated trials the exact size and maximum volume of successive finer materials, which may be added without appreciably increasing the bulk of the coarsest after thoroughly compacting, are determined. Mr. Schutté has found that for different shapes of particles the proportions of each size must be varied, but having determined the required percentages for a certain stone, that is, for a stone from a certain quarry, the proportions of the sizes from day to day need be varied but little.

Practical Proportioning During Progress of the Work. The above methods of mechanical analysis and volumetric synthesis are methods to be used in the office or laboratory in determining the relative values of all the aggregates available for the work. When the work is begun, however, and the same general character of aggregate is used day by day, it is only necessary to see that the material does not change or, if it does, simply to readjust the relation between the fine and coarse aggregate. To do this by the mechanical analysis method, it is only necessary to have a nest of about six 8-inch sieves: say, stone sieves with 1 inch, ½-inch and ¼-inch diameter holes and sand sieves No. 7, 20, 50 and 90, together with a cover and pan. The shaking can be done by hand, and the sievings beginning with the finest emptied into a long glass tube. If a standard sample has been previously put in the tube in the same way and the points of division between the different sievings marked on a paper pasted on the outside of the tube, the difference between the standard and the sample under test can be quickly seen and modifications made in the mix accordingly.

The test by volumetric synthesis is one easily made in a modified way in the field and with care gives good results. Procure a galvanized tin pail and a spring balance graduated to half pounds; take a representative sample of concrete, being careful that it contains no more stones or mortar than the regular concrete; tamp it into the pail until level full and weigh. Any variation from the standard weight will show a change in the character of material, and this change can usually be detected and corrected by observing the materials and mixing. If not, then mechanical analysis methods will have to be used.

PROPORTIONS OF CONCRETE IN PRACTICE

The proportion of cement to aggregate depends upon the strength (see p. 311) and water-tightness (see p. 298) required, as well as upon the character of the inert materials, and, in general, relatively rich mixtures are necessary for loaded columns and beams in building construction, for thin walls subjected to water pressure, and for foundations laid under water. The following table has been compiled to show a few examples in practice.

Proportions in Actual Structures

Class.	Structure.		Proportions	Reference.
Buildings	Massachusetts Institute of Technology Cambridge, Mass.	Special columns	1 : 1 : 2	Sanford E. Thompson
		Superstructure ...	1 : 1½ : 3	
		Foundations	1 : 2 : 4	
	Youth's Companion, Boston, Mass.	Special columns	1 : 1 : 2	Sanford E. Thompson
		Superstructure .	1 : 2 : 4	
		Foundations	1 : 2½ : 5	
	McElwain Factory, Manchester, N. H.	Special columns .	1 : 1½ : 3	Specifications
		Superstructure. . .	1 : 2½ : 4½	
		Foundations	1 : 2½ : 5½	
Arch Bridges	Tunkhannock, Penn.	Viaduct	1 : 3 : 5	Eng. Cont., Apr. 1, 1914, p. 382.
	Fort Worth, Tex.	Viaduct...	1 : 2 : 4	Trans. A. S. C. E., Vol. LXXVIII, p. 1206.

Proportions in Actual Structures.

Class.	Structure.	Proportions.	Reference.
Girder Bridges	Kansas City, Mo. 12th St. Viaduct.	1 : 2 : 4	Eng. News. Jan. 7, 1915, p. 10.
	Portland Me. Harbor Viaduct.	1 : 2 : 4	J. R. Worcester.
Retaining Wall and Piers	Fort Worth, Tex. Viaduct.	1 : 2½ : 5	Trans. A. S. C. E., Vol. LXXVIII, p. 1206.
	Tunkhannock, Penn. Viaduct Piers.	1 : 3 : 5*	Eng. Cont., Apr. 1, 1914, p. 382.
Dams	Kensico, N. Y.	1 : 3 : 6 †	Trans. A. S. C. E., Vol. LXXIX, p. 248–251.
	Medina River, Tex.	1 : 3½ : 6½	Eng. News, Sept. 11, 1913, p. 508.
	Arrowrock, Idaho.	1 : 2½ : 5½ : 2¾‡	Eng. News., Feb. 25, 1915, p. 370.
Subways Tunnels	Boylston St., Boston, Mass.	1 : 2 : 3½	Specifications.
	Stockton St., San Francisco, Cal	§	Eng. Rec. June 27, 1914, p. 728.
Tanks	Middleborough Mass. — Tank .	1 : 1 : 2 1 : 1½ : 3 1 : 2 : 4	Eng. News, Aug. 26, 1915, p. 392.
	Middleborough Mass. — Tower. . .	1 : 2 : 4	
	Middleborough Mass. — Foundations	1 : 2½ : 5	
	Fulton, N. Y. Standpipe	1 : 2 : 4	Eng. News, Jan. 10, 1914, p. 43.
Pavements	National Conference Concrete Road Building — One course	1 : 2 : 3	Eng. Rec., Feb. 26, 1916, p. 275.
	National Conference Concrete Road Building — Two course top coat	1 : 2	
	National Conference Concrete Road Building — Two course base	1 : 2½ : 5	
	Fort Worth, Tex. Viaduct Pavement — Base.	1 : 3 : 6	Trans. A. S. C. E., Vol. LXXVIII, p. 1206.

* Derrick stone imbedded in concrete

† 27 per cent. large rubble.

‡ Cement composed of 55 per cent. Portland cement and 45 per cent. pulverized granite. 2¾ parts of cobbles used.

§ 1:2½ mortar to fill voids in broken stone.

CHAPTER XI

TABLES OF QUANTITIES OF MATERIALS FOR CONCRETE AND MORTAR

This chapter presents the tables and formulas (pages 208 to 217), by which the volumes of materials required for a known volume of concrete may be estimated, and emphasizes the importance of distinctly stating the proportions (p. 205).

The volume of concrete, even when made from materials in the same proportions, varies largely with the character of the materials and the methods of placing it. A mixed aggregate like gravel contains fewer voids and with the same proportions by volume of the same cement and sand produces a larger quantity of concrete than a screened broken stone. The fineness of the sand also largely affects the volume of the concrete and mortar, a fine sand requiring more water, and therefore producing a larger volume of mortar than coarse sand in the same proportions by volume. If the sand is dry, a slightly larger bulk of mortar is produced than with the same sand when containing a larger percentage of moisture, because the latter is less compact (see p. 137). Some cements require more water in gaging than others, and produce a larger amount of paste, which increases the volume of the concrete or mortar. The method of mixing and placing the concrete also affects the resulting volume, since an imperfectly mixed or poorly compacted mass contains voids which increase the volume. An excess of water in mixing affects the resulting volume of the set concrete or mortar to a slight extent, although most of the surplus water is expelled during setting.

It is possible to provide for all these variations, except those relating to improper mixing and placing, in rational formulas from which the resulting volumes may be accurately estimated if the characteristics of all the materials are known. For most practical purposes, however, average values, such as are presented in the tables and curves, are sufficiently accurate for estimating quantities. These average values are based upon a large number of tests in the United States, France, and Germany.

The theory of a concrete mixture is discussed, and formulas for volumes and quantities are given on pages 207 to 213 preceding the tables.

EXPRESSING THE PROPORTIONS

In framing concrete specifications, the proportions of the constituents should be stated so distinctly that there can be no misunderstanding between the engineer and the contractor as to the quantities which will be required for the work. The quantity of cement should invariably be regulated by its weight; if the proportions are stated by volume a definite weight or number of packages of cement must be assumed to the unit volume. For reasons discussed in Chapter X, it is also more accurate and scientific to measure the aggregates by weight than by volume, and since with a properly constructed plant using materials of several sizes, the cost need be no more than volume measure, the authors believe this will become common practice on important construction.

With our present system of weights and measures, it is advisable either to specify the number of cubic feet (or pounds) of sand and gravel, stone, or mixed material to a definite weight of cement, or else to stipulate a definite weight of cement to a cubic yard of concrete tamped in place, with an aggregate of clearly described material proportioned as the engineer may direct.

In stating the proportions for both mortar and concrete, it is now customary in the United States to separate the materials by colons, the first figure always representing the cement, followed by the aggregates in the order of the size of their grains. For example, 1: 3: 6 means 1 part cement (the unit of measurement should be stated), 3 parts sand, and 6 parts coarse material; or 1: 8 means 1 part cement (of defined weight) to 8 parts of graded aggregate. Mortar in proportion 1: 2 signifies one part cement to two parts sand by either weight or volume as specified.

In France, proportions are stated as one or more volumes of mortar to a definite number of volumes of stone, — "un volume de mortier pour deux volumes de cailloux."

Unit for Proportioning. If the proportions must be stated in parts, a bag of cement is assumed as one cubic foot. This unit has been adopted by the Joint Committee on Concrete and Reinforced Concrete and by other authorities. Proportions 1 : 3 : 6 thus represent one bag or 94 lb. cement to 3 cu. ft. of sand to 6 cu. ft. of gravel or stone; or, 1 bbl. cement (4 bags) to 12 cu. ft. sand to 24 cu. ft. gravel or stone.

When stating the proportions by volume, too much stress cannot be laid upon the necessity for the adoption of a standard unit, such as a bag of 94 lb., assumed to measure one cubic foot, or the equivalent assumption that a barrel of cement measures 4 cu. ft., and upon distinctly specifying this standard, as otherwise an unscrupulous contractor may

adopt for his unit the volume of cement very loosely measured, and thus produce too lean a concrete.

It is even inaccurate to state that proportions shall be based on packed or on loose measurement of cement, for either of these terms is very elastic. The authors have personally known engineers to place the volume of a barrel of packed cement all the way from 3.1 to 3.8 cu. ft., corresponding to a variation in weight of from 123 to 100 lb. per cubic foot, while loose

Tests of Capacity of Portland Cement Barrels and Weight of Contents.

(Tabulated by the authors from measurements of Boston Transit Commission, 1896, Howard A. Carson, Chief Engineer.) (*See p.* 206.)

No. of barrels tested results averaged	Brand	Height between heads	Average diameter of barrel	Average horizontal area	Capacity of barrel between heads	Depression of cement below head	Volume of depression	Volume of cement per barrel			Net weight of cement per barrel		Weight per cubic foot				Weight of barrel
								Packed	Loose	Shaken*	Before dumping	After dumping	Packed	Loose	Shaken	Sifted	
		ft.	ft.	sq ft.	cu.ft	ft.	cu.ft.	cu. ft.	cu. ft.	cu. ft.	lb.	lb.	lb.	lb.	lb.	lb.	lb.
5	A	2.12	1.437	1.622	3.446	0.17	0.235	3.21	3.75	3.432	377.4	376.9	117.5	100.5	109.4	90.6	21.1
6	B	2.19	1.430	1.605	3.495	0.12	0.171	3.35	4.17		381.0		113.8	91.4			29.0
3	C	2.07	1.412	1.571	3.249	0.07	0.096	3.15	4.05		387.0		112.8	94.2			22.7
5	D	2.01	1.407	1.554	3.123	0.07	0.093	3.03	3.99	3.522	373.2	371.4	123.2	93.2	105.5		25.6
6	E	2.08	1.403	1.546	3.219	0.04	0.059	3.16	4.19		374.2		118.4	89.2			24.3
1	F	2.13	1.38	1.496	3.186	0.03	0.039	3.15	4.27	3.695	378.0	378.0	120.1	88.5	102.3		22.0
5	G	2.01	1.46	1.662	3.327	0.10	0.148	3.21	4.06	3.598	370.7	370.2	115.7	91.4	102.9	80.3	23.3
Final Averages		2.09	1.42	1.579	3.292	0.09	0.120	3.18	4.07	3.562†	377.4	374.1†	118.8	92.6	105.1†	85.4†	24.0

NOTE.—A and B are American Cements; C, D, E and F are German Cements; G is a Danish Cement; Paper weighs about 1 lb.

*Box rocked over bar.

†Partial averages, to be compared only with like brands.

measurement, on the other hand, is variously fixed at from 3.8 to 4.5 cu. ft. to the barrel, or 100 to 84½ lb. per cubic foot. The extreme actual variation is therefore from 3.1 to 4.5 cu. ft. per barrel, or 123 to 84½ lb. per cubic foot. Proportions 1 : 3 : 6 in the first case would require 1 bag cement to 2.3 cu. ft. of sand and 4.6 cu. ft. of gravel; in the last case, proportions 1 : 3 : 6 would stand for 1 bag cement to 3.4 cu. ft. of sand and 6.8 cu. ft. of gravel. In other words, concrete mixed 1 : 3 : 6 by one man may be called 1 : 4¼ : 8½ by another.

Weight of Cement. Experiments by Mr. Howard A. Carson, for Boston Transit Commission, upon 31 barrels of Portland cement of

American and foreign brands, furnish an interesting illustration of the difference in weight of the same cement in different stages of compactness. The results,* a summary of which is presented in the table on page 206, show a variation from 86 to 118 lb. in the average weights of the same cement, according as it was weighed sifted, or packed in a barrel, while the actual weight of one brand, the average of 5 barrels, was as high as 123 lb. per cubic foot as it came from Germany packed in a barrel.

From the experiments just described, the ratios of volume and weight of the same cements in different degrees of compactness are calculated by the authors as follows:

Ratio of volume of packed cement to capacity of barrel between heads	0.97
Ratio of volume packed to volume loose	0.78
Ratio of volume packed to volume shaken	0.88
Ratio of volume loose to volume shaken	1.13
Ratio of weight packed to weight loose	1.28
Ratio of weight packed to weight shaken	1.13
Ratio of weight packed to weight sifted	1.37

From the table it is evident that the selection of the volume of a barrel is arbitrary. The adopted volume of 3.8 cu. ft. is convenient for calculation because it assumes a cubic foot of cement to weigh approximately 100 lb.

THEORY OF A CONCRETE MIXTURE

The discussion and the formulas which follow relate to plastic mortars and plastic or medium concrete. While a small amount of water in mixing may result, with heavy ramming, in a concrete or mortar of less than average volume, in practice the volume is more apt to be increased by lack of water because of the less perfect mixture and the visible voids. The volume of set concrete or mortar produced by a very wet mixture is approximately the same as that of a plastic mixture, because nearly all of the surplus water is thrown to the surface and expelled by the settling of the solid materials. This the authors have repeatedly proved by experiment.

The frequently repeated assertion that a very wet mixture contains visible air voids because of the drying out of the water is incorrect. This may be proved by carefully pouring neat cement grout into a rectangular mold, one of whose sides is formed by a piece of glass. The surplus water is expelled, and the specimen after setting is dense and glassy with no visible voids. The large visible voids which sometimes occur in very wet

*Tabulated by Sanford E. Thompson in *Engineering News*, Oct. 4, 1900, p. 229.

concrete, similar in appearance to visible voids in dry concrete, are due to the grout running away from the stones, or to too violent agitation in placing.

The volume of fresh concrete or mortar produced by any mixture of cement and aggregate or aggregates is equal to the sum of the volumes of the *separate particles* of the cement, the sand, and the other dry materials, the water contained in the aggregate and added in mixing, and the small volume of air entrained between the particles. The volume of set mortar or concrete is not appreciably different from its compacted volume when fresh or green, except in very wet mixtures, which expel a portion of the water. The volumes of the *particles* of dry materials are termed *absolute volumes*, and it is important to note the distinction between the absolute volumes and the apparent volumes determined by measuring the materials. Absolute volumes are discussed on pages 148 to 152.

The fact that water actually occupies space in a mass of fresh concrete or mortar has been entirely ignored by many writers on the subject of concrete mixtures. As stated on page 204, the fineness of the sand and the moisture contained in it affect the volume of the resulting concrete or mortar. Mr. Feret has proved by experiments (cited on page 140) that fine sands require more water for gaging than coarse. This extra volume of water produces a mortar of less density and consequently less strength; even stones such as are found in gravel or coarse broken stone require a very small percentage of water.

FORMULAS FOR QUANTITIES OF MATERIALS AND VOLUMES

A concrete is therefore made up of solid grains of cement plus water required for the cement, plus solid grains of sand plus water required for the sand, plus solid stone particles plus water required for the stone, plus air voids. The last term, the *air voids*, represents the voids entrained by the sand, which may be considered as a function or percentage of the sand, and the voids due to imperfect mixing of the concrete materials, which may be considered a function or percentage of the stone. Accordingly the volume of a concrete mixture may be expressed as a rational formula, which is applicable to all concrete and mortar mixtures in which the voids of the coarse stone are filled with mortar. The formula (1) which follows is presented to illustrate the theory, but because of the variation in the coefficient with different sands and different proportions, formula (2), page 209, and formulas (3) to (8), which are based on average conditions, are suggested for practical use as sufficiently accurate for most purposes.

Let

c = absolute volume* of cement.

s = absolute volume* of sand.

g = absolute volume* of stone.

m = ratio of the absolute volume of the water plus air voids of the cement, to the absolute volume of cement.

n = ratio of the absolute volume of the water coating the grains of sand plus the air entrained in gaging it, to the absolute volume of sand.

p = ratio of the absolute volume of the water coating the stone particles plus the air voids due to imperfect mixing, to the absolute volume of stone.

W = volume of concrete produced.

In other words, these ratios, m, n, and p, represent the sum of the volumes occupied by the water required for the material in mixing plus the air, in terms of the respective volumes of cement, sand, and stone.

Then

$$W = c + mc + s + ns + g + pg$$

or

$$W = (1 + m)\,c + (1 + n)\,s + (1 + p)\,g \qquad (1)$$

The coefficient n is really composed of two variables, one depending upon the coarseness of the sand, and the other upon the ratio of cement to sand, since a lean mortar contains more air voids. It is possible to express this coefficient as a more complex term with this ratio as a factor, but by what appears to be a peculiar coincidence, experiments show that for ordinary bank sand the variation in voids caused by different proportions may be provided for by taking the cement and sand together; in other words, for different proportions of the same cement and sand, the sum of the water and the air voids in the mortar is approximately a constant. Where there is no sand, or where the stone and sand are mixed, formula (1) must be employed.

The more practical formula may be expressed as follows, employing similar notation to that given above, and letting

r = ratio of the absolute volume of the water plus the air entrained in gaging, to the absolute volume of cement plus sand,

then

$$W_1 = c + s + r\,(c + s) + g + pg$$

or

$$W_1 = (1 + r)\,(c + s) + (1 + p)g \qquad (2)$$

*Absolute volumes are defined on p. 148.

FOR CONCRETE WITH GOOD COARSE SAND

Substituting in formula (2) average values for r and p, which the authors have selected by analyzing the results of a number of exact records in the United States and Europe of the volumes of concrete and mortar made with good coarse sand, the formula becomes

$$W_1 = 1.34\ (c + s\ ) + 1.08\ g \tag{3}$$

This formula may be readily reduced to a practical working form if may be expressed in pounds by substituting for the absolute volume, c, the number of pounds of cement divided by its specific gravity (which may be taken as 3.1) times the weight of a cubic foot of water (62.3 lb.). It may also be expressed in barrels by substituting for the absolute volume, c, the number of barrels, B, multiplied by the net weight per barrel, 376 pounds, and divided, as above, by the specific gravity times the weight of a cubic foot of water [see formula (4)]. The terms relating to sand and stone may be expressed in pounds in a way similar to that just shown for cement, or they may be expressed in measured volume by substituting for the absolute volume, s or g, the measured volume, S or C, multiplied by the proportion of solid material contained in it. Expressing this algebraically, if

Q = quantity of concrete made with B barrels cement,
Q_1 = quantity of concrete made with one barrel cement,
B = number barrels cement,
B_1 = number barrels cement per cubic yard of concrete,
S = volume of loose sand in cubic feet,
S_1 = volume of loose sand in cubic yards per cubic yard of concrete,
G = volume of broken stone or gravel or cinders in cubic feet,
v = absolute voids in sand determined by weight method (p. 127),
v' = absolute voids in stone determined by weight method (p. 128),

then from formula (3), since $c = B\dfrac{376}{3.1 \times 62.3}$

$$Q = \frac{1.34 \times 376}{62.3 \times 3.1} B + 1.34\ (1 - v)\ S + 1.08\ (1 - v')G$$

$$Q = 2.61\ B + 1.34\ (1 - v)\ S + 1.08\ (1 - v')\ G \tag{4}$$

The volume of concrete in cubic feet made by one barrel of cement, assuming that a cubic foot of average loose, moist sand contains 89 pounds of dry sand, and that its specific gravity dry is 2.65, is,

$$Q_1 = 2.61 + 0.723\ S + 1.08\ (1 - v')\ G \tag{5}$$

This formula is applicable to average concrete made with Portland cement of good quality, coarse bank sand measured loose and containing ordinary moisture, and any broken stone or gravel of known voids. Formula (5) has been used in compiling tables on pages 215 and 217, except in the first twelve proportions, page 215, which contain no sand.

If the volume of concrete made from a barrel of cement plus the sand and other aggregate which accompanies it is known, the number of barrels of cement per cubic yard is readily calculated. In formula (5), Q_1 represents the number of cubic feet of concrete made with one barrel cement, hence the number of barrels cement per cubic yard of concrete is 27 divided by Q_1

$$B_1 = \frac{27}{Q_1} \tag{6}$$

Assuming a cubic foot of average sand to contain 89 pounds of dry sand produces the formula employed in calculating tables on pages 230 to 232, and substituting in formula (6) the value of Q_1 from formula (5),

$$B_1 = \frac{27}{2.61 + 0.723\,S + 1.08\,(1 - v')\,G} \tag{7}$$

The formulas may be expressed in parts by volume (such as 1: 2: 4) by multiplying the coefficient of S and G by the assumed volume of a barrel, say by 4.0.

Knowing the number of barrels of cement, B_1, per cubic yard of concrete, the number of cubic yards of sand per cubic yard of concrete, S_1, is evidently

$$S_1 = \frac{B_1 \times \text{quantity sand in cubic feet per barrel of cement}}{27} \tag{8}$$

The quantity of stone is similarly obtained.

If two or more coarse materials, such as broken stone and gravel, are used, they must be mixed in the selected proportions, before weighing, to determine their voids.

FOR CONCRETE WITH VERY FINE SAND

In mortars of extremely fine sands the density $(c + s)$ is apt to be about 0.60 (see Feret's table, sand C, p. 146) and the coefficient of first term of formula (3) becomes $\frac{1.00}{0.60} = 1.67$ instead of 1.34. In plastic mortars of standard Ottawa sand the density $(c + s)$, by tests of the authors, averages about 0.71, hence the coefficient becomes $\frac{1.00}{0.71} = 1.41$ instead of

1.34. Substituting these values, or any others which may be obtained by experiment, in formula (2), the working formulas which follow it may be readily deduced. It is evident from the variation in the coefficient with different sands, that the variation in volume of mortar and concrete obtained by different experimenters is due chiefly to the difference in the materials employed.

The coefficient of $(c + s)$ is also affected, though to a less degree, by the character of the cement, some cements requiring more water than others and therefore producing a greater bulk of paste for a given weight of cement.

FOR CONCRETE OF CEMENT AND COARSE AGGREGATE

In concrete mixtures of cement and coarse stone, with no sand or screenings, formulas (2) to (8) are inapplicable because apparently the air voids do not increase with the leanness of the mixture until the point is reached at which the paste fails to fill the voids in the stone. It is therefore necessary to go back to formula (1), page 209. Since s is zero, the formula becomes

$$W_2 = (1 + m)\,c + (1 + p)\,g \tag{9}$$

An average value of $(1 + m)$ for a first-class American Portland cement has been found by experiment to be 1.65. It varies with the quantity of water required to gage the cement to such a consistency that the voids will be filled, but no free water will exist upon the surface. The selected value, assuming 1% voids in the paste, corresponds to 20% of water by weight. The value of $(1 - p)$ is usually 1.04 to 1.08. An average formula for a concrete of cement and coarse stone may thus be taken as

$$W_2 = 1.65c + 1.08g \tag{10}$$

which is readily reduced to practical forms by the method adopted in evolving formulas (4) to (8) from formula (3).

If the stone is a mixture of sand and gravel, or broken stone and screenings, the coefficient of g must be increased and a figure selected whose value depends upon the relative proportion of fine and coarse material.

TABLES OF QUANTITIES OF MATERIALS AND VOLUMES

Tables on pages 213 to 217 are calculated from formulas (5), (6), (8), and (9). The quantities for rubble concrete are reduced in proportion to the percentage of rubble stone used. These formulas are

used not merely because of their theoretical worth, but because, as stated on pages 204 and 218, the results from them agree with actual experiment.

The values are average values of sufficient exactness for practical use, although, as already suggested, variations in the quality of the materials largely affect the resulting volumes, especially of the mortar.

VOLUMES OF MORTARS AND QUANTITIES OF MATERIALS

Volume of Plastic Mortar and Quantity of Materials per Cubic Yard (*see p.* 212.)

Based on Tests and Experience of the Authors

Proportion by parts		Ordinary Coarse Bank Sand.				Very Fine Sand.			
		Volume Compacted Plastic Mortar.		Materials for 1 Cu. Yd. of Plastic Mortar.		Volume Compacted Plastic Mortar		Materials for 1 Cu. Yd. Plastic Mortar.	
		From one bag (94 lb.) of Cement	From one bbl (376 lb.) of Cement	One bag cement assumed as 1 cu. ft.		From one bag (94 lb.) of Cement.	From one bbl (376 lb.) of Cement	One bag cement assumed as 1 cu. ft.	
Cement.	Sand.			Packed Cement.	Loose Sand.			Packed Cement.	Loose Sand.
		cu. ft.	*cu. ft.*	*bbl.*	*cu. yd.*	*cu. ft.*	*cu. ft.*	*bbl.*	*cu. yd.*
1	0	0.80	3.2	8.31					
1	½	1.02	4.1	6.61	0.49	1.15	4.6	5.91	0.44
1	1	1.38	5.5	4.88	0.72	1.51	6.0	4.48	0.66
1	1½	1.74	7.0	3.87	0.86	1.88	7.5	3.61	0.80
1	2	2.11	8.4	3.21	0.95	2.24	8.9	3.02	0.90
1	2½	2.47	9.9	2.74	1.01	2.51	10.4	2.60	0.96
1	3	2.83	11.3	2.39	1.06	2.97	11.8	2.28	1.01
1	3½	3.19	12.8	2.12	1.10	3.33	13.3	2.03	1.05
1	4	3.55	14.2	1.90	1.13	3.70	14.8	1.83	1.08
1	4½	3.91	15.6	1.72	1.15	4.06	16.2	1.67	1.11
1	5	4.28	17.1	1.58	1.17	4.43	17.7	1.53	1.13
1	5½	4.64	18.5	1.46	1.19	4.79	19.1	1.41	1.15
1	6	5.00	20.0	1.35	1.20	5.15	20.6	1.31	1.17

Note:—Variations in the fineness of the sand and the cement, and in the consistency of the mortar, may affect the values by 10 per cent. in either direction.

All except the first item in the table on page 213 and the first 12 items in tables on pages 214 and 215 are calculated from formulas (5), (6), and (8), pages 210 to 211, with the assumption there outlined. The broken stone in the first twelve items in the concrete tables, pages 214 and 215, except where the voids are 40% or over, is assumed to contain fine material, and the coefficient selected for g, formula (9), varies from 1.08 for 50%, 45%, and 40% voids to 1.14 for 20% voids.

One Bag of Cement (94 lb.) is Assumed as One Cubic Foot. (See p. 212)

Proportions by Parts. Cement.	Proportions by Parts. Sand.	Proportions by Parts. Stone.	Proportions by Volume. Packed Cement. bbl.	Proportions by Volume. Loose Sand. cu ft.	Proportions by Volume. Loose Stone cu. ft.	Volume of Mortar in Terms of Percentage of Volume of Stone. %	Broken Stone. Screened to One Size. 50 % Voids. Cement. bbl.	Sand. cu yd.	Stone. cu yd.	Broken Stone. CRUSHER RUN. NO DUST.* 45 % VOIDS. CEMENT. bbl.	SAND. cu yd.	STONE. cu. yd.	Gravel or Mixed Stone and Gravel. 40 % Voids Cement. bbl.	Sand cu. yd.	Stone. cu. yd.	Scientifically Graded Mixtures. 30 % Voids. Cement. bbl.	Sand. cu. yd.	Stone. cu. yd.	Scientifically Graded Mixtures. 20 % Voids. Cement. bbl.	Sand. cu. yd.	Stone cu. yd.
1		1	1		4	89	4.90		0.74	4.80		0.71	4.62		0.69	4.23		0.63	3.91		0.58
1		2	1		8	49	3.57		1.06	3.37		1.00	3.20		0.95	2.84		0.84	2.56		0.76
1		3	1		12	35				2.60		1.16	2.45		1.09	2.13		0.95	1.90		0.84
1		4	1		16	28										1.71		1.01	1.51		0.80
1		5	1		20	24										1.43		1.06	1.26		0.93
1		6	1		24	22										1.22		1.08	1.07		0.95
1		7	1		28	20													0.94		0.98
1		8	1		32	18													0.83		0.98
1		9	1		36	17													0.75		1.00
1		10	1		40	16													0.68		1.01
1		11	1		44	15													0.62		1.01
1		12	1		48	15													0.57		1.01
1	1	1½	1	4	6	96	3.08	0.46	0.68	2.07	0.44	0.66	2.87	0.42	0.64	2.69	0.40	0.60	2.53	0.38	0.56
1	1	2	1	4	8	73	2.74	0.41	0.81	2.63	0.39	0.78	2.52	0.37	0.75	2.33	0.34	0.69	2.17	0.32	0.64
1	1	2½	1	4	10	59	2.47	0.37	0.91	2.35	0.35	0.87	2.25	0.33	0.83	2.06	0.31	0.76	1.90	0.28	0.71
1	1	3	1	4	12	50	2.25	0.33	1.00	2.13	0.32	0.95	2.03	0.30	0.90	1.85	0.27	0.82	1.70	0.25	0.76
1	1½	2	1	6	8	92	2.39	0.53	0.71	2.30	0.51	0.68	2.22	0.49	0.66	2.07	0.46	0.61	1.94	0.43	0.58
1	1½	2½	1	6	10	74	2.18	0.48	0.81	2.09	0.46	0.77	2.01	0.45	0.74	1.86	0.41	0.69	1.73	0.38	0.64
1	1½	3	1	6	12	62	2.01	0.45	0.89	1.91	0.42	0.85	1.83	0.41	0.81	1.68	0.37	0.75	1.56	0.35	0.69
1	1½	3½	1	6	14	54	1.86	0.41	0.96	1.77	0.39	0.92	1.68	0.37	0.87	1.54	0.34	0.80	1.42	0.32	0.74
1	1½	4	1	6	16	48	1.73	0.38	1.03	1.64	0.36	0.97	1.56	0.35	0.92	1.42	0.32	0.84	1.30	0.29	0.77
1	1½	4½	1	6	18	43	1.62	0.36	1.08	1.53	0.34	1.02	1.45	0.32	0.97	1.31	0.29	0.87	1.20	0.27	0.80
1	1½	5	1	6	20	39	1.52	0.34	1.13	1.43	0.32	1.06	1.35	0.30	1.00	1.22	0.27	0.90	1.11	0.25	0.82
1	2	3	1	8	12	74	1.81	0.54	0.80	1.74	0.52	0.77	1.67	0.50	0.74	1.54	0.46	0.68	1.44	0.43	0.64
1	2	3½	1	8	14	64	1.69	0.50	0.88	1.61	0.48	0.83	1.54	0.46	0.80	1.42	0.42	0.74	1.31	0.39	0.68
1	2	4	1	8	16	56	1.58	0.47	0.94	1.51	0.45	0.89	1.44	0.43	0.85	1.32	0.39	0.78	1.21	0.36	0.72
1	2	4½	1	8	18	51	1.49	0.44	0.99	1.41	0.42	0.94	1.34	0.40	0.89	1.23	0.36	0.82	1.13	0.34	0.75
1	2	5	1	8	20	46	1.40	0.42	1.04	1.33	0.39	0.98	1.26	0.37	0.93	1.15	0.34	0.85	1.05	0.31	0.78
1	2	5½	1	8	22	42	1.33	0.39	1.08	1.26	0.37	1.03	1.19	0.35	0.97	1.08	0.32	0.88	0.98	0.29	0.8[illegible]
1	2	6	1	8	24	39	1.26	0.37	1.12	1.19	0.35	1.06	1.13	0.34	1.00	1.02	0.30	0.91	0.93	0.28	0.83
1	2½	3	1	10	12	86	1.65	0.61	0.73	1.59	0.59	0.71	1.53	0.57	0.68	1.42	0.52	0.63	1.33	0.49	0.59
1	2½	3½	1	10	14	75	1.55	0.57	0.80	1.48	0.55	0.77	1.42	0.52	0.74	1.32	0.49	0.68	1.23	0.46	0.64
1	2½	4	1	10	16	66	1.46	0.54	0.87	1.39	0.51	0.82	1.33	0.49	0.79	1.23	0.46	0.73	1.14	0.42	0.68
1	2½	4½	1	10	18	59	1.38	0.51	0.92	1.31	0.48	0.87	1.25	0.46	0.83	1.15	0.43	0.77	1.06	0.39	0.71
1	2½	5	1	10	20	54	1.31	0.48	0.97	1.24	0.46	0.92	1.18	0.44	0.87	1.08	0.40	0.80	0.99	0.37	0.73
1	2½	5½	1	10	22	49	1.24	0.46	1.01	1.18	0.44	0.96	1.12	0.41	0.91	1.02	0.38	0.83	0.93	0.34	0.76
1	2½	6	1	10	24	45	1.18	0.44	1.05	1.12	0.41	1.00	1.06	0.39	0.94	0.96	0.36	0.85	0.88	0.33	0.78
1	2½	6½	1	10	26	42	1.13	0.42	1.09	1.07	0.40	1.03	1.01	0.37	0.97	0.92	0.34	0.89	0.84	0.31	0.81
1	2½	7	1	10	28	39	1.08	0.40	1.12	1.02	0.38	1.06	0.96	0.36	1.00	0.87	0.32	0.90	0.79	0.29	0.82
1	3	4	1	12	16	75	1.35	0.60	0.80	1.30	0.58	0.77	1.25	0.56	0.74	1.15	0.51	0.68	1.08	0.48	0.64
1	3	4½	1	12	18	67	1.28	0.57	0.85	1.23	0.55	0.82	1.18	0.52	0.79	1.08	0.48	0.72	1.01	0.45	0.67
1	3	5	1	12	20	60	1.22	0.54	0.90	1.16	0.52	0.86	1.11	0.49	0.82	1.02	0.45	0.76	0.94	0.42	0.70
1	3	5½	1	12	22	55	1.16	0.52	0.95	1.11	0.49	0.90	1.06	0.47	0.86	0.97	0.43	0.79	0.89	0.40	0.72
1	3	6	1	12	24	50	1.11	0.49	0.99	1.06	0.47	0.94	1.01	0.45	0.90	0.92	0.41	0.82	0.84	0.37	0.75
1	3	6½	1	12	26	48	1.06	0.47	1.02	1.01	0.45	0.97	0.96	0.43	0.92	0.87	0.39	0.84	0.80	0.36	0.77
1	3	7	1	12	28	44	1.02	0.45	1.06	0.97	0.43	1.01	0.92	0.41	0.95	0.83	0.37	0.86	0.76	0.34	0.79
1	3	7½	1	12	30	42	0.98	0.44	1.09	0.93	0.41	1.03	0.88	0.39	0.98	0.79	0.35	0.88	0.73	0.32	0.81
1	3	8	1	12	32	39	0.94	0.42	1.11	0.89	0.40	1.05	0.84	0.37	1.00	0.76	0.34	0.90	0.69	0.31	0.82
1	4	5	1	16	20	75	1.08	0.64	0.80	1.03	0.61	0.76	0.99	0.59	0.73	0.92	0.55	0.68	0.86	0.51	0.64
1	4	6	1	16	24	63	0.99	0.59	0.88	0.95	0.56	0.84	0.91	0.54	0.81	0.83	0.49	0.74	0.77	0.46	0.68
1	4	7	1	16	28	55	0.92	0.54	0.95	0.88	0.52	0.91	0.83	0.49	0.86	0.76	0.45	0.79	0.70	0.42	0.73
1	4	8	1	16	32	48	0.86	0.51	1.02	0.81	0.48	0.96	0.77	0.46	0.91	0.70	0.42	0.83	0.64	0.38	0.76
1	4	9	1	16	36	43	0.80	0.47	1.07	0.76	0.45	1.01	0.72	0.43	0.96	0.65	0.39	0.87	0.60	0.36	0.80
1	4	10	1	16	40	40	0.75	0.44	1.11	0.71	0.42	1.05	0.67	0.40	0.99	0.61	0.36	0.90	0.55	0.33	0.81
1	5	10	1	20	40	47	0.70	0.52	1.04	0.66	0.49	0.98	0.63	0.47	0.93	0.57	0.42	0.84	0.52	0.38	0.77
1	6	12	1	24	48	46	0.59	0.52	1.05	0.56	0.50	1.00	0.53	0.47	0.94	0.48	0.43	0.85	0.44	0.39	0.78

* Use ordinarily.

Variations in fineness of sand and compacting of concrete may affect volumes 10% in either direction.

With a barrel of 3.8 cu. ft. (which means 100 lb. per cu. ft.) use 5% more cement and ½% less sand and stone.

With a barrel of 3.5 cu. ft. use 12% more cement and 1½% less sand and stone.

One Bag of Cement (94 lb.) is Assumed as One Cubic Foot. (See p. 212)

Proportions by Parts. Cement.	Proportions by Parts. Sand	Proportions by Parts. Stone.	Proportions by Volume. Cement. bbl.	Proportions by Volume. Sand. cu. ft.	Proportions by Volume. Stone. cu. ft.	Volume of Mortar in Terms of Percentage of Volume of Stone. %	Broken Stone: Screened to One Size. 50% Voids. cu. ft	Broken Stone: CRUSHER RUN. NO DUST.* 45% VOIDS. cu. ft.	Gravel or Mixed Stone and Gravel. 40% Voids. cu. ft.	Scientifically Graded Mixtures: 30% Voids. cu. ft.	Scientifically Graded Mixtures: 20% Voids. cu. ft.
1		1	1		4	89	5.4	5.6	5.8	6.4	6.9
1		2	1		8	49	7.6	8.0	8.4	9.5	10.5
1		3	1		12	35		10.4	11.0	12.7	14.2
1		4	1		16	28				15.8	17.8
1		5	1		20	24				18.9	21.3
1		6	1		24	22				22.1	25.1
1		7	1		28	20					28.8
1		8	1		32	18					32.4
1		9	1		36	17					36.1
1		10	1		40	16					39.7
1		11	1		44	15					43.4
1		12	1		48	15					47.0
1	1	1½	1	4	6	96	8.8	9.1	9.4	10.0	10.7
1	1	2	1	4	8	73	9.8	10.3	10.7	11.6	12.4
1	1	2½	1	4	10	59	10.9	11.5	12.0	13.1	14.2
1	1	3	1	4	12	50	12.0	12.7	13.3	14.6	15.9
1	1½	2	1	6	8	92	11.3	11.7	12.2	13.0	13.9
1	1½	2½	1	6	10	74	12.4	12.9	13.5	14.5	15.6
1	1½	3	1	6	12	62	13.5	14.1	14.8	16.0	17.3
1	1½	3½	1	6	14	54	14.5	15.3	16.0	17.6	19.1
1	1½	4	1	6	16	48	15.0	16.5	17.3	19.1	20.8
1	1½	4½	1	6	18	43	16.7	17.7	18.6	20.6	22.5
1	1½	5	1	6	20	39	17.8	18.0	19.9	22.1	24.3
1	2	3	1	8	12	74	14.0	15.6	16.2	17.5	18.8
1	2	3½	1	8	14	64	16.0	16.7	17.5	19.0	20.5
1	2	4	1	8	16	56	17.1	17.0	18.8	20.5	22.3
1	2	4½	1	8	18	51	18.1	19.1	20.1	22.0	23.9
1	2	5	1	8	20	46	19.2	20.3	21.4	23.5	25.7
1	2	5½	1	8	22	42	20.3	21.5	22.7	25.1	27.4
1	2	6	1	8	24	39	21.4	22.7	24.0	26.6	29.2
1	2½	3	1	10	12	86	16.3	17.0	17.6	18.9	20.2
1	2½	3½	1	10	14	75	17.4	18.2	18.9	20.5	22.0
1	2½	4	1	10	16	66	18.5	19.4	20.2	21.9	23.7
1	2½	4½	1	10	18	59	19.6	20.6	21.5	23.5	25.4
1	2½	5	1	10	20	54	20.7	21.8	22.8	25.0	27.2
1	2½	5½	1	10	22	49	21.8	22.9	24.1	26.5	28.9
1	2½	6	1	10	24	45	22.8	24.1	25.4	28.0	30.6
1	2½	6½	1	10	26	42	23.9	25.3	26.7	29.5	32.3
1	2½	7	1	10	28	39	25.0	26.5	28.0	31.0	34.0
1	3	4	1	12	16	75	20.0	20.8	21.7	23.4	25.1
1	3	4½	1	12	18	67	21.0	22.0	23.0	24.9	26.8
1	3	5	1	12	20	60	22.1	23.2	24.3	26.4	28.6
1	3	5½	1	12	22	55	23.2	24.4	25.6	28.0	30.3
1	3	6	1	12	24	50	24.3	25.6	26.9	29.5	32.1
1	3	6½	1	12	26	48	25.4	26.8	28.2	31.0	33.8
1	3	7	1	12	28	44	26.4	27.9	29.4	32.5	35.5
1	3	7½	1	12	30	42	27.5	29.1	30.8	34.0	37.2
1	3	8	1	12	32	39	28.6	30.3	32.0	35.5	39.0
1	4	5	1	16	20	75	25.0	26.1	27.2	29.3	31.5
1	4	6	1	16	24	63	27.2	28.5	29.8	32.4	35.0
1	4	7	1	16	28	55	29.3	30.8	32.4	35.4	38.4
1	4	8	1	16	32	48	31.5	33.2	34.9	38.4	41.9
1	4	9	1	16	36	45	33.6	35.6	37.5	41.4	45.3
1	4	10	1	16	40	40	35.8	38.0	40.1	44.4	48.8
1	5	10	1	20	40	47	38.7	40.0	43.0	47.3	51.7
1	6	12	1	24	48	46	45.9	48.5	51.1	56.3	61.4

* Use ordinarily.

Variations in fineness of sand and compacting of concrete may affect volumes 10% in either direction.

With a barrel of 3.8 cu. ft. (which means 100 lb. per cu. ft.) quantities are 4% smaller.

With a barrel of 3.5 cu. ft., quantities are 11% smaller.

One Bag of Cement (94 lb.) is Assumed as One Cubic Foot. (See p. 212)

Percentage of Rubble in Total Volume of Concrete.	Proportions of Plain Concrete by Parts: Cement.	Sand.	Stone.	Proportions of Plain Concrete by Volume: Packed Cement	Loose Sand	Loose Stone.	Broken Stone. Screened to One Size 50% Voids: Cement.	Sand.	Stone.	Broken Stone. CRUSHER RUN. NO DUST.* 45% VOIDS: CEMENT.	SAND.	STONE.	Gravel or Mixed Stone and Gravel 40% Voids: Cement	Sand	Stone.	Scientifically Graded Mixture 30% Voids: Cement	Sand.	Stone.
				bbl	cu ft.	cu ft	bbl	cu yd	cu yd	bbl	cu yd	cu yd	bbl	cu. yd.	cu. yd	bbl	cu. yd.	cu. yd.
20%	1	2	3	1	8	12	1.45	0.44	0.65	1.40	0.42	0.62	1.34	0.40	0.60	1.24	0.37	0.55
	1	2	4	1	8	16	1.27	0.38	0.76	1.21	0.36	0.72	1.16	0.35	0.68	1.06	0.32	0.63
	1	2	5	1	8	20	1.12	0.34	0.84	1.07	0.32	0.79	1.01	0.30	0.75	0.92	0.28	0.68
	1	2½	4	1	10	16	1.17	0.44	0.70	1.12	0.41	0.66	1.07	0.40	0.64	0.99	0.37	0.59
	1	2½	5	1	10	20	1.05	0.39	0.78	0.99	0.37	0.74	0.95	0.36	0.70	0.87	0.32	0.64
	1	2½	6	1	10	24	0.95	0.35	0.85	0.90	0.33	0.80	0.85	0.31	0.75	0.77	0.29	0.68
	1	3	5	1	12	20	0.98	0.44	0.73	0.95	0.42	0.69	0.89	0.40	0.66	0.82	0.36	0.61
	1	3	6	1	12	24	0.89	0.40	0.79	0.85	0.38	0.75	0.81	0.36	0.72	0.74	0.33	0.66
	1	3	7	1	12	28	0.82	0.37	0.85	0.79	0.35	0.81	0.74	0.33	0.76	0.67	0.30	0.69
30%	1	2	3	1	8	12	1.27	0.38	0.56	1.22	0.37	0.54	1.17	0.35	0.52	1.08	0.33	0.48
	1	2	4	1	8	16	1.10	0.33	0.66	1.06	0.31	0.62	1.01	0.30	0.59	0.92	0.27	0.55
	1	2	5	1	8	20	0.97	0.29	0.73	0.93	0.27	0.68	0.88	0.26	0.65	0.80	0.24	0.60
	1	2½	4	1	10	16	1.02	0.38	0.61	0.97	0.36	0.57	0.93	0.34	0.55	0.86	0.32	0.51
	1	2½	5	1	10	20	0.92	0.34	0.68	0.87	0.32	0.64	0.82	0.31	0.61	0.75	0.28	0.56
	1	2½	6	1	10	24	0.82	0.31	0.73	0.78	0.29	0.70	0.73	0.27	0.66	0.67	0.25	0.60
	1	3	5	1	12	20	0.85	0.38	0.63	0.81	0.36	0.60	0.78	0.34	0.57	0.71	0.31	0.53
	1	3	6	1	12	24	0.78	0.34	0.60	0.74	0.33	0.66	0.71	0.31	0.63	0.64	0.29	0.57
	1	3	7	1	12	28	0.71	0.31	0.74	0.68	0.30	0.71	0.64	0.28	0.66	0.58	0.26	0.60
40%	1	2	3	1	8	12	1.09	0.33	0.48	1.15	0.31	0.46	1.00	0.30	0.45	0.93	0.28	0.41
	1	2	4	1	8	16	0.97	0.28	0.56	0.90	0.27	0.53	0.86	0.26	0.51	0.79	0.23	0.46
	1	2	5	1	8	20	0.84	0.25	0.62	0.80	0.23	0.59	0.75	0.22	0.55	0.69	0.20	0.51
	1	2½	4	1	10	16	0.87	0.32	0.52	0.83	0.30	0.49	0.80	0.30	0.47	0.74	0.27	0.44
	1	2½	5	1	10	20	0.78	0.29	0.58	0.71	0.27	0.57	0.71	0.26	0.52	0.65	0.24	0.48
	1	2½	6	1	10	24	0.71	0.26	0.63	0.67	0.24	0.60	0.63	0.23	0.56	0.58	0.21	0.51
	1	3	5	1	12	20	0.73	0.32	0.54	0.70	0.31	0.51	0.66	0.30	0.49	0.61	0.27	0.45
	1	3	6	1	12	24	0.66	0.29	0.59	0.63	0.28	0.55	0.61	0.27	0.54	0.55	0.24	0.49
	1	3	7	1	12	28	0.61	0.27	0.63	0.58	0.26	0.61	0.57	0.24	0.57	0.50	0.22	0.51
50%	1	2	3	1	8	12	0.90	0.27	0.40	0.87	0.26	0.38	0.83	0.25	0.37	0.77	0.23	0.34
	1	2	4	1	8	16	0.79	0.25	0.47	0.75	0.22	0.44	0.72	0.21	0.42	0.66	0.20	0.39
	1	2	5	1	8	20	0.70	0.21	0.52	0.66	0.20	0.49	0.63	0.19	0.47	0.57	0.17	0.42
	1	2½	4	1	10	16	0.73	0.27	0.43	0.69	0.25	0.41	0.67	0.25	0.40	0.61	0.23	0.36
	1	2½	5	1	10	20	0.65	0.24	0.48	0.62	0.23	0.46	0.59	0.22	0.43	0.54	0.20	0.40
	1	2½	6	1	10	24	0.59	0.22	0.53	0.56	0.20	0.50	0.53	0.20	0.47	0.48	0.18	0.43
	1	3	5	1	12	20	0.61	0.27	0.45	0.58	0.26	0.43	0.55	0.25	0.41	0.51	0.23	0.38
	1	3	6	1	12	24	0.55	0.25	0.50	0.53	0.23	0.47	0.50	0.23	0.45	0.46	0.20	0.40
	1	3	7	1	12	28	0.51	0.23	0.53	0.49	0.22	0.50	0.46	0.20	0.48	0.42	0.18	0.43
60%	1	2	3	1	8	12	0.73	0.22	0.32	0.70	0.21	0.31	0.67	0.20	0.30	0.62	0.19	0.28
	1	2	4	1	8	16	0.63	0.19	0.38	0.60	0.18	0.35	0.57	0.17	0.34	0.53	0.16	0.31
	1	2	5	1	8	20	0.56	0.17	0.41	0.53	0.16	0.40	0.50	0.16	0.37	0.46	0.13	0.34
	1	2½	4	1	10	16	0.58	0.21	0.35	0.55	0.20	0.33	0.53	0.19	0.31	0.49	0.18	0.29
	1	2½	5	1	10	20	0.52	0.19	0.39	0.50	0.18	0.37	0.47	0.17	0.35	0.43	0.16	0.32
	1	2½	6	1	10	24	0.47	0.17	0.42	0.45	0.16	0.40	0.42	0.15	0.37	0.38	0.14	0.34
	1	3	5	1	12	20	0.49	0.21	0.36	0.46	0.21	0.34	0.44	0.19	0.33	0.41	0.18	0.30
	1	3	6	1	12	24	0.44	0.19	0.40	0.42	0.19	0.38	0.40	0.18	0.36	0.37	0.16	0.33
	1	3	7	1	12	28	0.41	0.18	0.42	0.39	0.17	0.40	0.37	0.16	0.38	0.33	0.15	0.34

* Use ordinarily.

Variations in fineness of sand and compacting of concrete may affect volumes 10% in either direction.

With a barrel of 3.8 cu. ft. (which means 100 lb. per cu. ft.) use 5% more cement and ½% less sand and stone.

With a barrel of 3.5 cu. ft. use 12% more cement and 1½% less sand and stone.

Volume of Rubble Concrete from One Barrel (4 bags) Cement

One Bag of Cement (94 lb.) is Assumed as One Cubic Foot. (See p. 212)

Percentage of Rubble in Total Volume of Concrete.	Proportions of Plain Concrete by Parts.			Proportions of Plain Concrete by Volume.			Volume of Mortar in Terms of Percentage of Volume of Stone.	Average Volume of Rubble Concrete Made From One Barrel of Cement.			
								Broken Stone.		Gravel or Mixed Stone and Gravel 40% Voids.	Scientifically Graded Mixtures. 30% Voids.
								Screened to One Size 50% Voids.	CRUSHER RUN. NO DUST.* 45% VOIDS		
	Cement.	Sand.	Stone.	Cement	Sand.	Stone.					
				bbl	cu. ft	cu ft	%	cu ft.	cu ft	cu ft	cu. ft.
20%	1	2	3	1	8	12	74	18.7	19.5	20.2	21.8
	1	2	4	1	8	16	56	21.4	22.4	23.5	25.6
	1	2	5	1	8	20	46	24.0	25.4	26.8	29.4
	1	2½	4	1	10	16	66	23.1	24.2	25.2	27.4
	1	2½	5	1	10	20	54	25.0	27.3	28.5	31.3
	1	2½	6	1	10	24	45	28.4	30.0	31.6	35.0
	1	3	5	1	12	20	60	27.6	29.0	30.1	33.0
	1	3	6	1	12	24	50	30.2	32.0	33.6	36.8
	1	3	7	1	12	28	44	33.0	34.8	36.7	40.5
30%	1	2	3	1	8	12	74	21.2	22.2	23.0	24.0
	1	2	4	1	8	16	56	24.4	25.5	26.8	29.2
	1	2	5	1	8	20	46	27.3	29.0	30.5	33.0
	1	2½	4	1	10	16	66	26.4	27.7	28.8	31.2
	1	2½	5	1	10	20	54	29.6	31.0	32.5	35.7
	1	2½	6	[illegible]	10	24	45	32.5	34.5	36.2	40.0
	1	3	5	1	12	20	60	31.6	33.0	34.6	37.6
	1	3	6	1	12	24	50	34.6	36.5	38.4	42.0
	1	3	7	1	12	28	44	37.6	39.8	42.0	46.3
40%	1	2	3	1	8	12	74	24.8	26.0	27.0	29.2
	1	2	4	1	8	16	56	28.5	29.8	31.3	34.2
	1	2	5	1	8	20	46	32.0	33.8	35.6	39.0
	1	2½	4	1	10	16	66	30.8	32.4	34.0	36.6
	1	2½	5	1	10	20	54	34.6	36.[illegible]	38.0	41.7
	1	2½	6	1	10	24	45	38.0	40.[illegible]	42.3	46.7
	1	3	5	1	12	20	60	36.8	38.7	40.5	44.0
	1	3	6	1	12	24	50	40.5	42.7	44.8	49.3
	1	3	7	1	12	28	44	44.0	46.5	49.0	54.2
50%	1	2	3	1	8	12	74	29.8	31.2	32.4	35.0
	1	2	4	1	8	16	56	34.2	35.8	37.6	41.0
	1	2	5	1	8	20	46	38.4	40.6	42.8	47.0
	1	2½	4	1	10	16	66	37.0	38.8	40.4	43.8
	1	2½	5	1	10	20	54	41.4	43.6	45.6	50.0
	1	2½	6	1	10	24	45	45.6	48.2	50.8	56.0
	1	3	5	1	12	20	60	44.2	46.4	48.6	52.8
	1	3	6	1	12	24	50	48.6	51.2	53.8	59.0
	1	3	7	1	12	28	44	52.8	55.8	58.8	65.0
60%	1	2	3	1	8	12	74	37.3	39.0	40.5	43.8
	1	2	4	1	8	16	56	42.8	44.8	47.0	51.2
	1	2	5	1	8	20	46	48.0	50.8	53.5	58.8
	1	2½	4	1	10	16	66	46.3	48.5	50.5	54.8
	1	2½	5	1	10	20	54	51.8	54.5	62.5	62.5
	1	2½	6	1	10	24	45	57.0	60.3	63.5	70.0
	1	3	5	1	12	20	60	55.3	58.0	60.8	66.0
	1	3	6	1	12	24	50	60.8	64.0	67.3	73.8
	1	3	7	1	12	28	44	66.0	69.8	73.5	81.3

* Use ordinarily.

Variations in fineness of sand and compacting of concrete may affect volumes 10% in either direction.

With a barrel of 3.8 cu. ft. (which means 100 lb. per cu. ft.) the quantities range from 3% to 2% smaller, according to percentage of rubble.

With a barrel of 3.5 cu. ft. the quantities are 9% to 5% smaller.

Before adopting these formulas and compiling the tables from them, comparisons were made in great detail with actual volumes of concrete recorded by a large number of prominent engineers, and, with due allowance for different materials, the average results agreed very closely, with an extreme variation of seldom more than 5 per cent.

Tables of Rubble Concrete. The tables on pages 216 and 217 give the quantities of materials and the volumes of concrete mixed in different proportions and with different percentages of rubble.

The percentages of rubble are based on the ratio of the volume of the concrete after it is laid to the actual volume of the large stone contained in it. In other words, it is the percentage of the finished concrete occupied by the large stone.

CHAPTER XII

PREPARATION OF MATERIALS FOR CONCRETE

The various operations relating directly to the laying of concrete are discussed in detail in this and several succeeding chapters. While the selection of the special methods and machinery, which are described at length in the succeeding chapters, are determined by local conditions, certain general principles apply to all classes of work. The preparation of the materials relates to the storing of cement, the screening of sand and gravel, and the crushing of stone.

STORING CEMENT

Portland cement is not injured by storing in a dry place for a considerable length of time; in fact, contrary to former belief, instead of deteriorating, the quality is often improved by storage. Cement manufacturers when rushed with orders sometimes ship material which, not being sufficiently air-slaked, contains free lime, that exposure to air may change to a hydrate and thus render harmless.

Recognition of the fact that exposure to dry atmosphere does not injure cement has led to packing it in bags instead of in barrels, thus saving both cost of barrel and extra freight upon it. Paper bags avoid loss and return charges on cloth bags, but result in more breakage.

The economy of storing the cement as near as possible to the mixing platform or mixing machine is obvious, but since, on the other hand, it is more easily handled and is always less in volume than sand and stone, these should be given the preference in the matter of location.

SCREENING SAND AND GRAVEL

The three most common methods of screening are (1) by hand, that is, by throwing shovelfuls of the material on to an inclined screen, (2) by dumping or hoisting the material on to a fixed inclined screen, (3) by a revolving screen.

Cost of Hand Screening.* The cost of hand screening depends upon the total amount of material handled rather than upon the quantity of sand or gravel produced. A material most of whose particles run through the screen can be most cheaply screened, because the screen can be moved,

* See "Concrete Costs" by Taylor & Thompson for futher information on costs of preparing materials.

or arranged over a hole, while if a large proportion of the particles are caught they must be shoveled from the foot of the screen.

An average laborer, properly superintended, will throw about 24 cu. yd. of material against a screen in a ten-hour day, but in estimating the cost, allowance must be made for shoveling the material out of the way, moving screen, and superintendence.

The following are approximate costs of screening sand and gravel by hand under ordinary conditions. The prices are from actual records on a number of jobs and are based on labor at $2.00 for ten hours, with a suitable allowance for superintendence and contractor's profit. The minimum prices apply to first-class men.

	Average cost per cu yd.	Minimum cost per cu yd.
Screening sand, coarse stuff wasted	$0.17	$0.12
Screening gravel to remove large stones	0.23	0.16
Screening gravel to remove sand, sand wasted	0.41	0.29
Screening gravel coarse, and fine stuff, both measured	0.21	0.14

If laborers are working alone with no foreman in sight, as is often the case on concrete work, 50% should be added to the average costs.

Inclined Screen fed by Carts, Derrick Buckets, or Endless Chain. The slope of an elevated screen may vary from 35° to 45° from the horizontal, according to the character of the material. Coarser screens are required to pass material of a certain size than for hand screening.

At the new Cambridge Bridge, Boston, the contractors employed a screen about 15 feet long, hinged at the top so that the slope could be varied to suit the material. A hopper located above the screen fed on to a 3-inch bar screen, consisting of parallel iron bars about 3 inches apart, supported by iron cross pieces about 5 inches apart. The stones too large for the concrete ran down this coarse screen, and rolled off one side, while the remainder of the material fell through it on to a screen with 1-inch by ¾ inch mesh, which separated the medium gravel from the sand.

On another large job in Everett, Mass., where an inclined screen was fed by a bucket elevator supplied by carts, 300 to 350 cu. yd. of sand and gravel were screened in ten hours, and an even larger quantity could have been handled had it been supplied with absolute regularity.

The cost of screening by this method depends both upon local conditions and the quantity screened. The average cost may be assumed to be from 4 to 8 cents per cubic yard when large quantities of sand or gravel are handled at once.

Rotating Screens. Rotating screens, cylindrical or hexagonal in shape, although most frequently employed for separating crushed stone

(see p. 224), are also adapted, if power is available, for separating sand from gravel, or for separating gravel into several sizes to remix in the theoretical proportions required for a dense, impervious concrete.

While the first cost of a rotating screen is more than that of an inclined screen, less elevation is required and it may be fed with a bucket conveyor.

A plant for ordinary concrete made from two aggregates, sand and gravel, requires a screen with only two sizes of mesh, the smaller about ⅜-inch and the larger 2, 2½ or 3-inch mesh, as desired. Often no screening is required except to remove the sand, as a few large stones do no harm. The screen may be about 3 feet in diameter by 12 feet in length.

The present tendency, for concrete which is to be subjected to severe stress or to water pressure, is to require more scientific proportioning by separating the aggregate into several sizes and remixing them so as to produce the greatest density. This separation may be accomplished in practice by adding more sections, and thus lengthening the screen, or by employing a double cylinder, which occupies about half the space of a single cylinder.

The inner cylinder of a double-cylinder screen is composed of two or more sections of different sized mesh, and the outer cylinder is composed of two or more corresponding sections which are entirely separate from each other so that each may discharge into a separate bin. Each outer section has a finer mesh than the corresponding section of the inner cylinder. The material, after passing through a section of the inner cylinder, falls upon the outer wire and is again separated, the part which is caught rolling out through an annular opening into one bin and the remainder passing through the mesh into another bin.

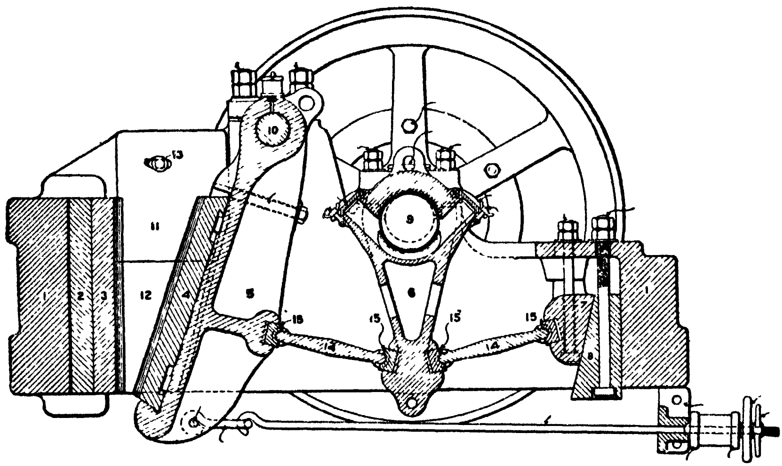
STONE CRUSHING

The crushing of stone for concrete must be approached from a different standpoint than the preparation of material for macadam paving, although the costs will not vary materially from those of a well-arranged portable crushing plant used on road construction.

For city or town macadam paving, where a suitable ledge is available, it is possible to establish a fixed plant with stationary engine, large stone bins, and economical machinery for handling cars, so that the stone can be hauled over a system of movable tracks directly from the ledge to the crusher, while for country road building the plant is arranged with a view to its portability, sometimes even resting on wheels.

For concrete work a plant intermediate in style between these is usually required. Its design is governed by the local conditions and by the quan-

tity of concrete to be made. In some cases where the concrete is laid in excavation it is possible to locate the crusher on the bank, and allow the stone to pass by gravity on to and through an inclined screen, or, if "crusher run" is used, to fall directly into a pile below. Generally the stone from the crusher must be taken by bucket or belt conveyors to bins, located, if possible, above the concrete mixer, or where the stone can be conveniently conveyed to the mixer without shoveling.



NAME AND NUMBER OF PARTS

1 Main Frame	**6 Pitman**	**11 Upper Half Cheek Plate**
2 Round Back	**7 Toggle Block**	**12 Lower Half Cheek Plate**
3 Fixed Jaw Plate	**8 Wedge**	**13 Bolt for Cheek Plate**
4 Swing Jaw Plate	**9 Eccentric Shaft**	**14 Toggle**
5 Swing Jaw	**10 Swing Jaw Shaft**	**15 Toggle Bearing**

Fig. 61.—Jaw Crusher. (*See p.* 222.)

Stone Crushers. Stone crushers are of two general types, jaw crushers and gyratory crushers.

The size of a jaw crusher is designated by the opening into which the stone is introduced. A 16 by 10 inch crusher has jaws 16 inches in width, and the space between the two jaws at the top is 10 inches. A "duplex" crusher has two pairs of jaws operated by the same shaft, but working alternately by means of different eccentrics. Single jaw crushers range in size from 3 by $1\frac{1}{2}$ inches to 36 by 24 inches or even larger.

The operation of a typical jaw crusher is shown in Fig. 61. One of the jaws is fixed, and the other is hinged at the top, and swung back and forth through a very small arc. The motion is imparted by the eccentric shaft, which, in revolving, raises and lowers the "pitman," whose lower end is connected by toggles with the lower end of the movable jaw. The size of

the stone passing through the jaws, that is, the size of the largest particles, is regulated by the opening at the bottom of the swing jaw, which is changed by using longer or shorter toggles.

A section of a gyratory crusher, which is adapted for more stationary plants, is shown in Fig. 62, page 223. It consists essentially of a cone with a gyratory motion within an inverted conical chamber or shell. The

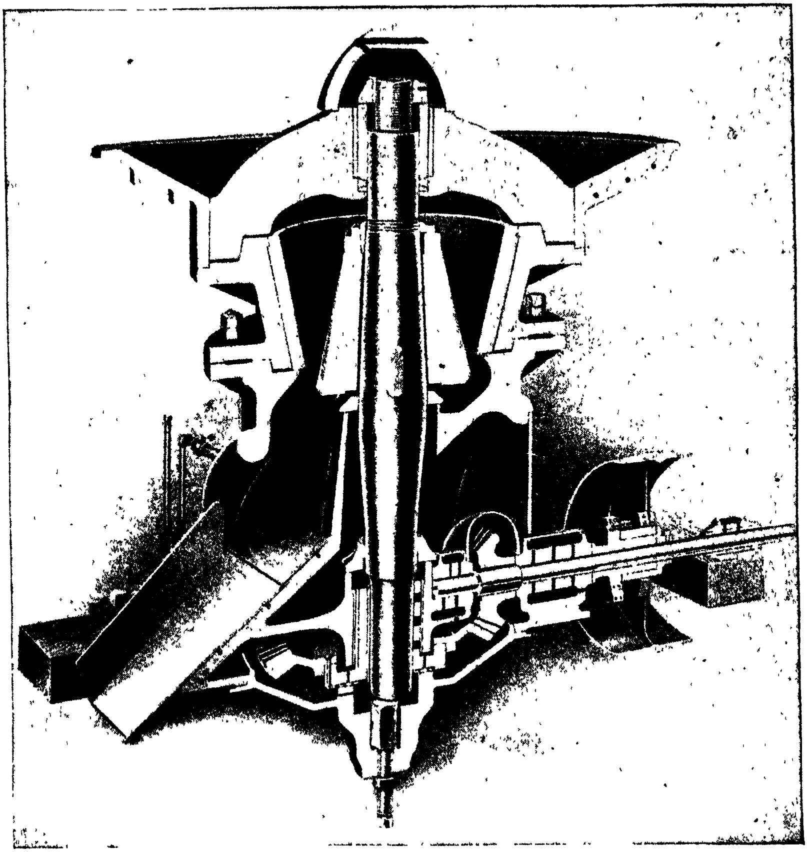


FIG. 62.—Gyratory Crusher. (*See p.* 223.)

size of the crusher is determined by the width of the opening between the top of the cone and the shell, and the circumference. The gyratory motion of the cone shaft is produced by an eccentric keyed to its lower end. As the shaft revolves, the cone is given a kind of a rocking motion which continually directs it toward, and then away from, different portions

of the shell. The size of the broken stone is regulated by raising or lowering the cone on the shaft.

The horse-power required to drive a crusher and its attendant machinery varies largely with the material handled. It is advisable to make ample allowance above the figures given in manufacturers' catalogues. It is, also, economical to use a wider and heavier belt

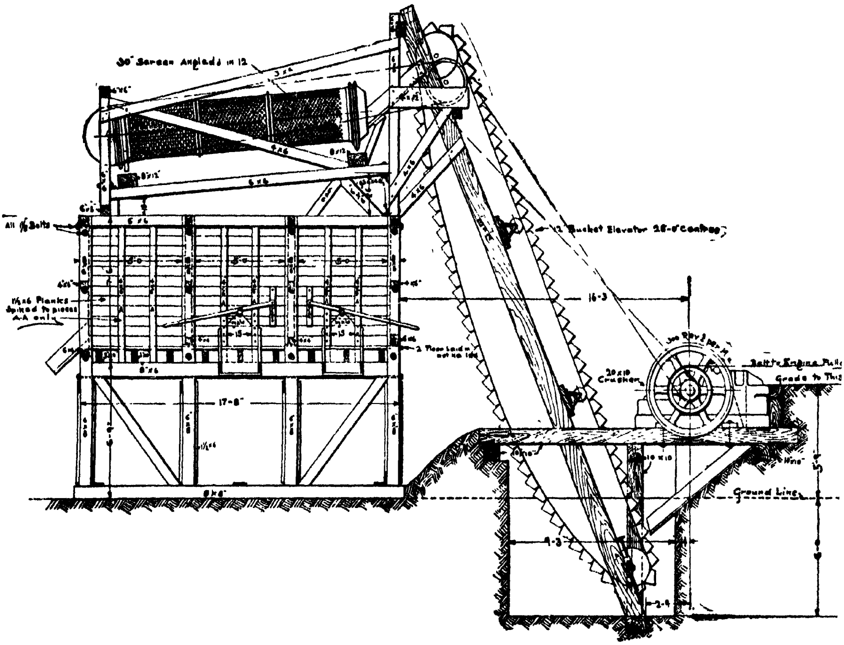


FIG. 63.—Small Crushing Plant with Elevator, Screen, and Portable Bin. (*See p.* 224)

than is generally specified, in order to avoid delays and shutdowns. When ordering almost any kind of machinery the authors make it a practice to require a wider and heavier pulley than the standard width. It is wise to make a pulley at least 2 inches wider than the belt which is to be run upon it.

Crusher Screens and Bins. A typical design, by Mr. Earle C. Bacon, for bins suitable for a plant where the concrete mixer or mixing platform is located at a distance from the crusher is shown in Fig. 63. With slight changes they may be arranged to discharge into hoppers over a concrete mixer. The dimensions of timber employed in the construction may be used as a basis for bins of other sizes.

A safe slope for the bottom of stone bins is 45°, although if lined with sheet iron this may be decreased to 35° or 40°.

Rotating screens for broken stone or gravel are made usually in sections varying in length from 3 to 5 feet, so that they can be bolted together and give as many divisions of sizes as are required. The diameters vary from 24 to 48 inches. The mesh of a rotating screen should be about 20% smaller in diameter than the required maximum size for the stone as there is more or less wear on the screen, which enlarges the holes, and this allowance will also assist in excluding the oblong pieces whose longest dimension is above the limit. For concrete, unless two or more sizes of stone are mixed, no more than two sizes of mesh are required, one, $\frac{1}{4}$-inch to remove the dust, and the other, 2, $2\frac{1}{2}$, or 3-inch to remove the coarse stuff. Often it is necessary only to separate the dust which may then be used as sand.

Stone Bin Gates. On large jobs where the carts, cars, or auto trucks can pull in underneath the bins, a horizontal sliding gate is much used On smaller work where the carts pull up alongside the bins the simplest device is the ordinary trough or chute built in two sections, the lower movable and hinged to the upper so that it can be raised to block it or dropped down to permit a continuous passage from the bin to the cart. For side loading from large bins this type of chute is too heavy to operate successfully and some sort of gate must be used. A satisfactory design of such a gate is shown on page 247 of the second edition of this book.

Cost of Quarrying and Crushing Stone. The cost of excavating and crushing stone for concrete varies with the kind of rock and the equipment used. The largest part of the variation comes in excavating; in crushing, there are fewer variables and each is less affected by different conditions. In excavating, hardness of rock, seaminess, character of quarry, equipment and method of conducting operations, and efficiency of labor, all affect the cost. In crushing, the hardness and structure of the rock and equipment are less important.

Task work has been applied to quarrying with marked success. The General Crushed Stone Company of Easton, Pennsylvania, used the system in quarries of limestone and quartzite and secured high outputs and low costs. Similar results were attained in mining in British Columbia. Both cases are described on pages 179 and 180 of Concrete Costs.

Gang. The arrangement of gang varies considerably, but for ordinary work, permanent, but not large enough for steam shovel equipment and the like, the typical gang per drill is made up of

One drill man	$3.00
One drill helper	2.00
½ fireman @ $2.50	1.25
¼ blacksmith @ $3.00	.75
¼ blacksmith helper @ $2.00	.50
¼ foreman @ $4.00	1.00
Total wages per day per drill	$8.50
Add 15 per cent. for superintendence, overhead charges and contingencies	1.28
	$9.78

The typical crusher gang for a 15 by 9 inch crusher is made up of

One foreman	$4.00
One engineer	2.50
2 men feeding crusher @ $2.00	4.00
One man at crusher on odd work	2.00
2 single carts with one teamster hauling stone to crusher	6 00
3 men loading stone into carts @ $2.00	6.00
Total wages per crusher	24.50
Add 15 per cent. for superintendence, overhead charges and contingencies	3.68
Total cost of crusher per day	$28.18

Cost Table. The table on page 227, giving average outputs and cost of quarrying and crushing stone, is a summary of three tables in "Concrete Costs,"* where the totals given here are fully itemized so that corrections to suit local conditions may be made. The outputs and costs were obtained on actual work, and are therefore reliable in comparing different kinds of rock, and in preparing estimates. For close figuring, the detailed tables in "Concrete Costs" should be used.

The value of rock in ledge is assumed to be, for hard rock, 5 cents per cubic yard in place, for soft rock, 3 cents, and, for very soft rock, 2 cents. The cost of stripping is taken as 3 cents per cubic yard of ledge, depreciation on machinery is 25 per cent. per year, and interest on first cost 6 per cent. per year. Explosives run from 3 cents to 8 cents per ton of rock according to the hardness.

For ordinary city work, add 50 per cent. to the costs given.

Data on Broken Stone. Broken stone should be bought and sold by the ton unless the method and place of measuring is fully specified, for the volume and unit weight depend to a large extent upon the method of handling before weighing. Thus stone shipped 75 miles or more over a railroad settles 8 to 10 per cent. and is correspondingly heavier than when in the crusher bins. Similarly, stone hauled one half mile or more in wagons actually settles 9 to 12 per cent., about half of this settle-

* Taylor and Thompson's "Concrete Costs," p. 208-213.

*Average Outputs and Cost of Quarrying and Crushing Stone** (*see p. 226*)

Costs include stripping, drilling, and blasting† at quarry; sledging rock for crusher, hauling 200 feet to crusher, and crushing.

Costs are based on average conditions. Labor 20 cents per hour. Superintendence, overhead charges, etc., 15 per cent. Profit not included.

Quarrying Rock for Crushed Stone

Classification of Rock.	Specific Gravity.	Weight of Crushed Stone‡ per Cubic Yard.	Spacing of Drill Holes.		Quantity Crushed Stone per Foot of Hole.	Hourly Rate of Drilling.	Hourly Output per Drill Crushed Stone‡	Cost of Excavation per Cubic Yard Crushed Stone.
			From face.	Apart				
		Tons	Ft.	Ft	Cu Yd	Ft	Cu. Yd	$
Very Hard Trap.........	3 0	1 39	4 5	4 5	1 36	2.0	2.7	0.736
Hard Rock: Trap, Granite Conglomerate	2.7	1.25	5.2	5.2	1.82	4.0	7.3	0 333
Medium Rock: Limestone.	2.6	1.20	5.8	5.8	2.27	6.0	13.6	0.208
Soft Rock: Shale	2.6	1.20	6.4	6.4	2.73	8 0	21.8	0.157
Very Soft Rock: Sandstone	2.4	1.11	8.2	8.2	4.55	10.0	45 5	0 114

Crushing Hard Rock

Size of Stone.	Jaw Crusher.		Gyratory.			
	9 x 15	10 x 20	No 3.	No 4.	No 5	No 6.
	Hourly Output of Crushers—Crushed Stone					
	cu. yd.	cu. yd.	cu yd.	cu yd.	cu. yd	cu. yd.
$2\frac{1}{2}''$	7.0	9 4	7.7	13.5	19 3	23.2
$1\frac{1}{2}''$	4.6	6.2	5 0	9.3	13 1	15 4

Cost of Crushing Hard Rock

	$	$	$	$	$	$
$2\frac{1}{2}''$	0.601	0 506	0.579	0.449	0.394	0.374
$1\frac{1}{2}''$	0.765	0.648	0.751	0.549	0.484	0 466

* Summarized from "Concrete Costs", pp. 208 to 213.
† Use 60 per cent dynamite in very hard rock; 50 per cent in most solid, unseamed ledges; 40 per cent in closely seamed medium rock, and 25 per cent in very seamy disintegrated rock.
‡ Assuming 45 per cent voids.

ment occurs in the first hundred feet of haul with very little increase between one half and one mile.*

The weight of broken limestone varies from 2 300 to 2 600 pounds per cubic yard at the crusher. A cubic yard of broken trap varies from 2 400 to 2 700 pounds. The authors have found by repeated measurements that 100 pounds per cubic foot is a fair average weight for screened trap rock after it has been shaken down by hauling, although when measured loose in a small measure an average weight is about 90 pounds. Crusher run stone is about 10 per cent. heavier than this because it contains less voids. Stones having lower specific gravities than trap are correspondingly lighter in weight.†

On macadamized or paved roads, if no steep hills are to be encountered, two horses will haul from 6 000 to 7 000 pounds of broken stone to a load. Very high side boards are of course necessary to carry this quantity.

In case stone is bought by number, it should be remembered that No. 1 is the finest size in some localities and the coarsest in others.

WASHING SAND AND STONE

Sand and gravel frequently require washing to remove silt, clay, and loam, and broken stone must sometimes be washed free of dust. Washing a pile or a cart or barrow-load with a hose is practically useless. Two methods are in general use: (1) Washing the material down a trough or sluice, the water and waste passing off through stationary screens in the bottom of the trough or inclined in the opposite direction to the trough, and (2) Washing through one or more revolving screens, which gives better results, principally because of the additional rubbing action imparted. Although revolving screens are best adapted to large stationary plants, portable plants for construction work have been put on the market.

Trough Method. In trough washing the material is shoveled or dumped into the trough and water is led in from an overhead hopper or from pipes. If material of one size only is washed at a time, in the bottom of the trough is placed a screen fine enough to allow the aggregate to pass over it into the storage bin, while the dirt and water drop through the screen into a second trough and are carried away. The Aberthaw Construction Company has used ¼-inch screens for washing stone and gravel, and No. 20 mesh screens, supported by frequent cleats, for washing sand. If sand is to be screened from gravel in the washing

* Prof. Ira O. Baker, Bulletin 23, University of Illinois, 1908.

† See table p. 123.

process, use a screen at the end of the trough slightly inclined in the opposite direction to the trough. Water, dirt, and fine material, of size determined by the screen selected, will pass through into a settling tank and coarse aggregate will roll over the end of the screen into a storage

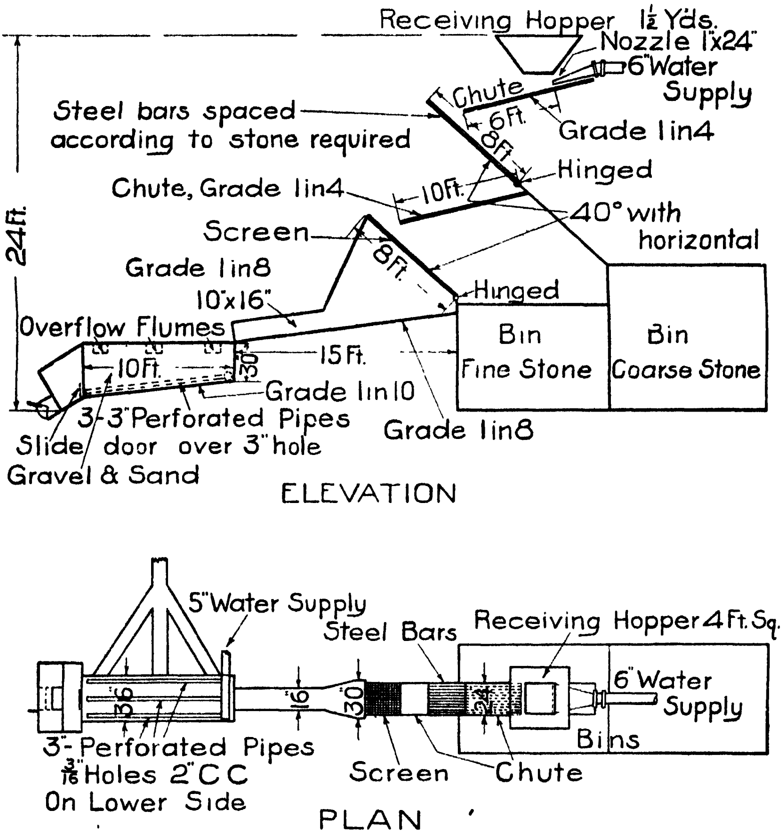


FIG. 64.—Washing and Screening Plant. (*See p.* 229.)

bin. In the settling tank the overflow of water will carry off the dirt.*

Gravity Screens. For separating into several sizes, where accurate grading is unnecessary and the material is not too dirty, inclined screens give good results. A plant of this kind is shown in Fig. 64, page 229. The slope of the screens varies from 35° to 45° and with a series, are arranged to reverse the direction of the material as shown, since, at

* A design for such a tank is given by W. H. Wilms in *Engineering News*, November 19, 1914, p. 1008.

best, the velocity becomes so great that only short lengths are effective. To break up a cemented gravel, a heavy bar screen on a 35° slope should be used. For reasonably clean sand and gravel, about 1½ gallons of water per minute per cubic yard is enough, one-half entering the hopper with the material, while the rest may be divided up among the screens. Gravel may be screened direct to bins, but sand, water, and dirt all pass into a settling tank where the water and dirt overflow together.

Revolving Screens. Because the revolving action makes possible a flatter slope, gravel moves slower over a revolving screen than over a gravity screen and this, together with the additional rubbing action, makes more accurate sizing and better cleaning possible.

For separation into several sizes, independent screens are generally installed. The pitch of the working surfaces runs about 1½ inches per foot, and the pitch of the intermediate chutes between about 1½ to 3 or 4 inches per foot, according to the size of the material and amount of water. It is important to check or reverse the travel of material just before it starts through a given screen. One gallon of water per minute per cubic yard is enough for revolving screens where 1½ gallons are needed on gravity screens.

One type of portable plant consists of a screen, fitted on to a frame of 1-inch bars spaced one inch apart, and suspended and partly immersed in a pan of water.* As the screen rotates the bars break up and grind the material in the water so that the fine dirt is washed through the screen, leaving the balance to work along the screen into a hopper at the end.

Another type consists of a cylindrical steel shell fitted on the inside and also on the channel iron shaft, with longitudinal vanes or shelves.† A steady stream of water passes through the cylinder, keeping the lower part always full. While the stone or gravel works along in the opposite direction to the water. As the cylinder revolves, the shelves pick up gravel from the bottom, carry it to the top and drop it on to the central shelves, from which it drops off into the water. This grinding and rinsing action removes all fine particles and the water carries them out at the opposite end from the clean gravel. The dirtiest gravel receives the first washing with the dirtiest water and the cleanest gravel its final washing with the cleanest water.

* *Engineering and Contracting*, July 15, 1914, p. 70.

† *Engineering and Contracting*, June 14, 1914, p. 709.

CHAPTER XIII

MIXING CONCRETE

The method employed for mixing concrete is immaterial, provided the result is a homogeneous mass of the required uniform consistency, containing the various aggregates and cement in proper proportions. If the color of the mass is not absolutely uniform, that is, if uncoated particles of sand or stone are visible, if masses of stones are separate from the mortar, or if some portions of the mortar are dryer than others, the mixing has not been thorough.

Hand vs. Machine Mixing. First-class concrete may be produced with careful superintendence, by either hand or machine-mixing, but machine-mixed concrete may be about 25% stronger than hand-mixed (see p. 320).

The relative cost of the two methods depends entirely upon circumstances, and must be estimated for each individual case. If the job is a small one, so that the cost of erecting the plant plus the interest and depreciation, divided by the number of cubic yards to be made, is a large item, or if frequent moving is required, concrete may be and often is mixed cheaper by hand than by machinery. The information which follows concerning both methods will serve as a guide for comparison in special cases.

MIXING CONCRETE BY HAND

The methods employed by different engineers and contractors for handling the materials and arranging the men are nearly as varied with hand-mixed as with machine-mixed concrete. Concrete mixing is seemingly so simple an operation that it is often neglected by the inspector, and poor workmanship escapes detection.

The inspector should lay the greatest stress upon (*a*) exact measurement of the gravel or broken stone, (*b*) thorough mixture of the cement and sand, (*c*) thorough mixture of the mass, and (*d*) care in dumping the concrete into place. The quantity of water used in the mixing and the proper ramming or puddling of the concrete in place are equally important but are less likely to be overlooked.

In proportioning the ingredients, it is poor economy to make allowance for insufficient mixing or improper handling of the materials. The addi-

tional cement will be much more expensive than the extra time expended by laborers in securing homogeneous mixture.

In the first place the mixing platform should be located as near the work as possible, and so situated that the coarse materials can be conveniently dumped on one side of it and the sand on the other. It should be not less than 15 to 20 feet square if all the work is to be done upon it, and except for a very small job should be of 2-inch plank, planed one side, spiked to, say, 2 by 4-inch stringers about 5 feet apart, so that it can be moved from place to place as required. A 2 by 3-inch strip around the edge will prevent loss of material. If the sand and cement are made into a mortar before mixing with the stone, the platform may be narrower and a mortar box employed in addition.

Methods of Measuring Material. Cement should invariably be measured by weight. In practice this is accomplished not by weighing on scales but by counting packages, since bags or barrels of cement have standard weights.*

The volumes of sand and stone or other aggregate should be distinctly stated in the porportions in terms of the number of cubic feet of each material to a barrel of cement, or else by parts, coupled with the explanation that one part, or bag, represents a definite volume, such as one cubic foot. In specifications where the proportions are given by parts with no unit of measurement, the contractor undoubtedly has the legal right to base the volumes of aggregate on the loose measurement of cement hence the necessity of exact statement of units, as prescribed on page 205.

For measuring sand and stone for hand mixing, the bottomless box shown in Fig. 5, page 8, is most satisfactory and reliable. The dimensions are determined by the required volume.

For machine mixing, especially where the materials must be wheeled from stock piles or dropped to the mixer from bins, work with this type of bottomless box is altogether too slow. Where wheel barrows are used, it is customary to measure into the barrow the proper amount of sand or stone, note the appearance of the barrow, and see that successive barrows are filled to about the same point. A better plan is to have a special barrow of the exact size required or else put a small bottomless box into the barrow and fill the box, lifting it out before the wheeler starts for the hopper. The better quality of work and the saving in materials offsets the slightly increased cost of this method.

Automatic measuring devices for use in connection with bins is taken up on page 239.

* See page 2.

Hand Mixing. The concrete ingredients may be mixed in various ways, any one of which is satisfactory so far as strength is concerned, provided the mass is turned a sufficient number of times. For satisfactory quality with lowest cost, the sand and cement should be mixed dry—turning three times—the stone shoveled on top of this mixture, water added, and the whole turned three times. Time studies show that it costs 3 per cent. more to mix concrete by shoveling the sand and cement on to the stone than by this method, and $11\frac{1}{2}$ per cent. more if sand and cement is made into mortar and shoveled on to the stone. The details of the methods are described in full on page 21. The systematic arrangement of men and insistance upon shoveling from the bottom of the pile and then turning the shovels completely over are essentials for thoroughly mixed concrete. Water should be poured from buckets in order to measure accurately. The quantity of water should be governed largely by the appearance of the concrete and its use, always remembering that an excess of water beyond a plastic consistency causes a decrease in strength. For properly imbedding steel, a concrete which flows sluggishly is necessary, while for heavy construction a dryer, plastic consistency should be used. A little more water is needed at the beginning of the day's work, for as successive layers are placed the water rises to the top from the layers beneath.

Distribution of Mixing Gang. Whatever the methods of mixing, the chief requisites for economy are such an arrangement of the gang that each man will have definite duties, and that the number of men on one set of operations will perform their work in the same length of time required by another set of men to perform a different operation or set of operations. A gang should be as large as practicable in order to lessen the cost of superintendence and the general expense.

It is generally economical to have two batches of concrete in preparation at once, although one set of men usually can measure and mix the sand and cement for two mixing gangs. While one batch of concrete is being shoveled to place or wheeled in barrows, the other batch, either in a different location on the same platform or on a separate platform, may be spread and mixed.

The method of handling a small gang is described on page 21. The arrangement of gangs on two well managed actual jobs is illustrated in the following outline:

(1) Gang on a core wall for a dike where the sand and cement were mixed dry and spread on to the stone, then wet as the mass was turned.

The large mixing platform was located 30 to 50 feet distant from the excavation, and the concrete was handled in wheelbarrows.

One foreman.

One man wheeling sand to measuring box.

Two men, working alternately at the two ends of the mixing platform, opening cement, and mixing sand and cement dry.

Three or four men, working alternately at each end of platform, shoveling gravel into bottomless boxes.

Six men working alternately at each end of platform, mixing concrete (turning it three times).

Two men handling water.

Four men wheeling concrete, each filling his own barrow.

Four men leveling and ramming.

The average quantity of concrete in proportions 1 : 2 : 5 laid by this gang per day of ten hours was about 65 batches or 47 cubic yards, with a maximum of about 90 batches or 65 cubic yards.

(2) Gang for a 6-inch foundation for a street pavement, where the sand and cement were made into a mortar and spread on to the stone, and where two mixing platforms were used, one on each side of the street, with a mortar box between them.

One foreman.

Two men mixing mortar in one mortar box.

Four men shoveling stone alternately into two measuring boxes.

Four men working alternately on the two mixing platforms, spreading mortar on stone, mixing concrete, and shoveling to place.

Three men leveling and ramming and also assisting to shovel to place.

One man carrying water and doing other odd work.

The total quantity of concrete in proportions 1 : 2 : 5 laid per day of ten hours averaged from 40 to 46 batches or 29 to 33 cubic yards per day for the gang. The gang was not quite up to the average, for under given conditions they ought to have turned out regularly 34 cubic yards per day of ten hours.

Approximate costs of concrete mixing are discussed on page 24.

MIXING BY MACHINERY*

On all large contracts, machinery for mixing concrete is universally replacing hand labor. The economy of this usually is due as much to the appliances introduced for handling the raw materials and the concrete

* See "Concrete Costs" by Taylor and Thompson, pp. 321–445, for a complete treatment of the subject, including description of plants of various types with costs of construction and operation.

as to the saving in the actual labor of mixing. Any arrangement which requires the measuring and spreading of materials by shovelers before entering the mixer results simply in saving the process of hand turning of the concrete and the labor of shoveling it into the vehicle, and this saving is partly balanced by the cost of maintaining and operating the mixer. On a small job this last item almost invariably exceeds the saving in hand labor and renders the expense with the machine greater than without it.

The design of the appliances or plant for handling the materials, and to some extent the selection of the type of mixer, depends upon local conditions, the quantity to be mixed per day, and the total volume of concrete. For a large mass of concrete masonry it is evident that it pays to invest a considerable sum in machinery to reduce the number of men and horses, but if for any reason only a small quantity, we will say not over 50 cubic yards, can be deposited in a day, the cost of expensive machinery cuts a very large figure and hand labor is generally cheaper. In estimating the interest on the cost of the plant which must be charged against a cubic yard of concrete, instead of dividing the interest per day by the usual daily output, the interest for the year must be divided by the total amount of concrete to be laid in the year. In other words, allowance must be made for the days when inclement weather prevents work. To find the depreciation, the value of the entire plant when new, minus its value after the job is completed, is divided by the total number of yards of concrete. Some of the other running expenses, such as the wages of the engineman, may continue from day to day whether or not any concrete is being laid.

Concrete Mixers. An effective concrete mixer not only stirs the mass, which may tend to separate the light and heavy particles, but cuts it again and again, and repeatedly transfers the materials from one part of the machine to another, so that in whatever order they are introduced, the product will be homogeneous. Continuous turning alone does not accomplish the result so quickly or thoroughly as the more complicated motions. The appearance of the concrete as it falls from the mixer will often distinguish the better of two machines.

The larger the machine, the more economical it will be, provided the arrangements for supplying it with material and conveying the concrete to the work permit running at full capacity.

Concrete mixers are of two general classes: (1) continuous mixers into which the materials are fed constantly, usually by shovelfuls, and from which the concrete is discharged in a steady stream, and (2) batch mixers, designed to receive at one charge, say, a barrel or a bag of cement with its proportionate volume of sand and stone, and after mixing to discharge it

in one mass. It is impossible to separate these two classes very distinctly because many of the machines are adapted to either continuous or batch mixing.

The authors are opposed, as a rule, to the use of continuous mixers, unless the materials are measured and fed mechanically, because of the difficulty of uniform feeding. When the ingredients are measured out by hand, spread in layers one above another, and then, starting at one edge, are shoveled into the mixer, the proportions of the materials in the resulting concrete are regulated by the thickness of the layers of the different ingredients rather than by the dimensions of the measuring barrels or boxes. If in one portion of the pile the layer of cement is thicker than in another, the resulting concrete will be proportionally richer. With batch mixers all the materials enter the machine at once; the homogeneity of the product depends upon the character and length of time of mixing rather than upon the care exercised by the laborers in feeding, and less inspection is necessary.

The regulation of the water supply in machine-mixing as in hand-mixing must be based on judgment. Having determined the quantity to produce the required consistency, the amount for each batch should be accurately measured and the quantity changed only when a variation in the materials require it. When the concrete is laid to a considerable depth the water works up through from one layer to the next, so that less water may be necessary near the top than lower down to maintain the proper consistency.

The selection of the type of mixer is often governed by local conditions. If, for example, there is to be a large quantity of concrete, and the machinery can be located at one place, a stationary machine, mounted perhaps on timber framework, with derricks, elevators, or belts, to raise the materials, may be economical. On running work, like a conduit or retaining wall, more portable machines are required, while for thin layers, like pavement foundations, if any machine is used it must be very light or easily moved. If stone for the aggregate is to be broken on the spot, a stationary plant may be built, or the stone may be hauled from the crusher bin to the mixer. In some cases the conformation of the ground will permit of dropping the materials into or through the machine by gravity. Frequently the volume of concrete to be laid is limited by the construction of forms, and a machine of small size is sufficient.

Mixers may be classified in four general types:

Rotating mixers.

Paddle mixers.
Gravity mixers.
Pneumatic mixers.

Rotating or rotary mixers, as they are usually termed, mix the materials by tumbling them in a drum or cubical box, which is usually provided with deflectors, blades, or plows.

The rotating mixer, drum type, Fig. 65, (see p. 238,) contains deflectors or blades. It may be mounted on an elevated platform to facilitate the removal of concrete, and may be provided with a sliding pivot hopper in order that the unmixed materials may be raised without hand labor. There are a multitude of different arrangements to suit different conditions, all described in mixer catalogues, and many times a special wooden frame work must be erected (see Fig. 67, p. 242) in which case the mixer may be purchased without auxiliary apparatus.*

It has been recommended that the periphery of the drum move at an average speed of about 200 feet per minute. An automatic measuring device for the water, with an automatic locking device on the discharging mechanism to prevent premature dumping, are also recommended.

Paddle mixers consisting of a trough with a single or duplex shaft with paddles were formerly used to a large extent either as continuous mixers in which material was shoveled and impelled by the paddles or blades, falling out at the end or as batch mixers (for which the duplex form is adapted) in which the material is mixed and then dropped out through a gate in the bottom. Unless fed very uniformly by continually measured feed, less uniform concrete results than with a batch mixer. The duplex mixer is adapted to use with bituminous material and also for mortar mixed fairly stiff.

Gravity machines, properly so-called, require no power, the materials being mixed by striking obstructions which throw them together in their descent through the machine or else by flowing through successive hoppers. It must be borne in mind that the only appreciable saving in cost in gravity mixing is the power of turning the mixer. If the raw materials have to be raised higher than with a power mixer this saving is overbalanced. The gravity machine must be designed to insure thorough mixing.

A gravity concrete mixer is illustrated in Gillmore's Treatise on Limes, Hydraulic Cements and Mortars,"† first published in 1863. In this

* Special types of mixers are illustrated in our advertising pages.
† Page 229.

machine the concrete fell into successive hoppers opened and closed by hand-levers. This scheme is used by a common modern type of mixer.

Pneumatic mixers have not been found in practice altogether satisfactory, delivering poorly mixed concrete to the forms. It was necessary on the New York Rapid Transit Subways* to supersede the pneumatic mixer with a rotary mixer, using the compressed air simply to force the concrete through the conveyor pipe to place. The use of compressed air in depositing is discussed in Chapter XIV.

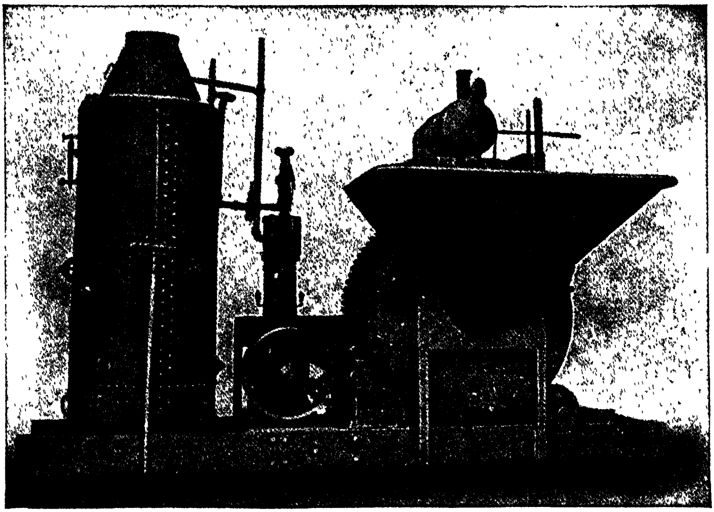


FIG. 65.—Rotary Mixer; Drum Type. (*See p.* 237.)

Time of Mixing. In batch mixing there is a tendency to use too wet consistency and dump before the materials are thoroughly mixed. This produces a concrete of low strength and is one of the most dangerous elements in concrete construction. Furthermore it is actually uneconomical because if the low strength is permissible it may be obtained more cheaply by thorough mixing of a leaner concrete. A minimum time for mixing in a rotating mixer measured from the time the mixer is filled to the time ready to dump should be preferably not less than one minute and longer mixing, which with less water produces a smoother consistency, gives a still better concrete. In no case should the mixing time, with the ordinary revolving mixer, be less than 40 seconds.

* *Engineering News*, March 16, 1916, p. 406.

Automatic Measurers for Concrete Materials. The accurate measuring of concrete materials by mechanical means has not been extensively developed. One difficulty, if methods of volumes are employed, lies in the inaccuracy of measuring cement by volume.

One patented device consists of several drums, one for each material, placed directly under the bins containing the cement, sand and stone, and rotating upon the same horizontal shaft. The quantity of each material is regulated by the position of the gates in the bins and by the speed of rotation of the drum.

Another machine delivers the different materials through separate troughs containing Archimedean screws.

Another consist of one or more bottomless storage cylinders, from under which the material flows out on to revolving discs or tables, and is peeled off by stationary adjustable knives which rest upon the disc and project into each material a distance determined by the quantity of each required.

A partially automatic measuring arrangement was employed on one section of the Boston Subway, in 1896. Each material fell into a closed chute arranged with gates at such distances apart as to enclose the required volume, whence it dropped into a hopper above the mixer.

Partially automatic measuring arrangements were used on one of the Ford factories* and on the Elephant Butte dam†. Overhead bins on the Ford job dumped through chutes into a hopper above the mixer which was divided into sections. When each section was full, the chute leading to it was closed by a gate operated by compressed air. On the Elephant Butte dam overhead bins emptied directly at each mixer to vertical cylindrical boxes provided with conical ends so as to fill completely full and dump readily. The cylindrical sides telescoped so that the volume of the charge could be regulated. The bottom of the bins and the bottom of the measuring boxes were closed by cylindrical gates, operated by hydraulics.

Measuring Water. The water for each batch of concrete should be measured. The quantity of water used in different batches must be varied occasionally because of the conditions of the materials, but even in such cases the amount can be regulated best by measurement. A tank with a float connected with an indicator on the outside is easily constructed.

On the Elephant Butte dam tanks were placed above the mixers and the water siphoned down in definite quantities, regulated by setting the siphon.

* *Engineering Record*, February 14, 1914, p. 182.

† *Engineering Record*, October 4, 1913, p. 368.

Small overhead tanks with adjustable siphons are standard attachments that can be purchased for all mixers. One such is illustrated in Fig. 65, page 238. A barrel arranged with a float indicator is sometimes used. Any such scheme is better than leaving the matter to the judgment of a laborer.

Proportioning by Weight. Attention has been called on page 205 to the fact that not only cement, but also sand, stone, and gravel, can be more accurately proportioned by weighing than by volume measurement. When a large amount of concrete is to be mixed, it is possible to arrange apparatus for weighing each material in such a way that less labor will be required than for proportioning by volume. The first cost of the scales may often be more than counterbalanced by the accuracy in proportioning, which permits of leaner mixtures, while at the same time greater uniformity is assured.

In view of these facts, the authors predict that engineers will gradually recognize the advantage of proportioning by weight. In most cases excessive cost may prohibit the use of standard scales, but if the materials are accurately screened and subdivided, the relative weights of each on the same job will be so nearly constant that the weighing can be performed by a simple system of counterweights and levers. With properly constructed gates to the bins it might be possible to arrange for their automatic closing after the required weight of each material had been received in the hopper.

Measurements by weight are employed to excellent advantage by Warren Brothers Company at their various plants where the materials, which consist of stone, sand, and binding material, are prepared for their bituminous macadam pavement. Eight bins containing aggregates of different coarseness drop their materials through gates into a hopper which forms the platform of the scales and is located directly above the mixer. The scale-beam is compound, with as many arms as there are ingredients to be weighed, and each of the arms has a sliding weight and a stop so arranged that the sliding weight can be moved only to the point on the beam which will balance the required weight of one of the materials. When the sliding weights are all at zero and the hopper is empty, the scale balances. The weight on one of the arms is moved out by the laborer who operates the apparatus until it comes to the stop fixed at the point corresponding to the weight of the material to be used from a certain bin. The gate of this bin is opened, and the material allowed to run into the hopper until the scale balances. The weight on the next lever is then slid out, and the second material deposited in like manner upon the first. When all the materials are thus weighed, the entire mass is dropped into the mixer below.

CONCRETE PLANTS*

The design of the plant for handling the raw materials and the concrete usually has more to do with an economical production than the type of the mixing machine. The plant should be drawn or sketched on paper and accurate estimates made of its cost and the expense of operation, so as to determine whether the volume of concrete is sufficiently large to warrant its installation. The authors have occasionally seen expensive machinery which could not be readily transported to another job, installed on a section of work where, because of the small total volume of concrete and on account of its distribution, hand-mixing was really more economical. On some sections of the New York Subway the yardage of concrete was so small that hand-mixing proved more economical than machine-mixing.

It is evident that the arrangement of any plant must be determined by local conditions, such as the contour of the ground, the distance from which the raw materials are transported, and the class of construction. A description of several plants, successful and economical in operation, may afford suggestions for other work. The illustrations are intended to show the arrangement of the gang and conveying machinery rather than the type of mixer.

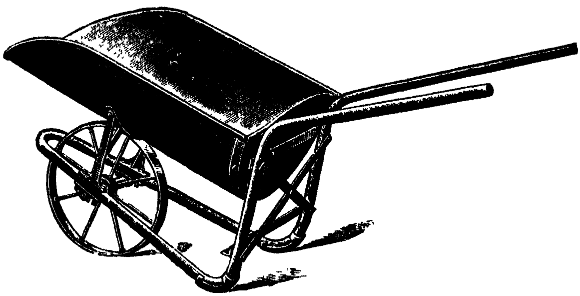


FIG. 66.—Wheelbarrow for Charging Mixer. (*See p.* 242.)

Loading by Barrows. One of the common types of plants consists of a mixer mounted just high enough to discharge into barrows, carts, or cars, and loaded from storage piles on the ground by wheel-barrows, which also measure the aggregates. These are usually wheeled up an incline to dump directly into the mixer. Mixers are also designed with a hopper arranged to be hoisted by the mixer engine and dumped into

* For further illustrations and costs and treatment in detail, see "Concrete Costs" by Taylor and Thompson, pp. 321 to 380.

the drum. This type of plant is economical where the yardage is not large enough to justify an elevated platform with storage bins and expensive machinery for hoisting materials from the ground to the bins. It is adapted to any method of placing, either with derrick and

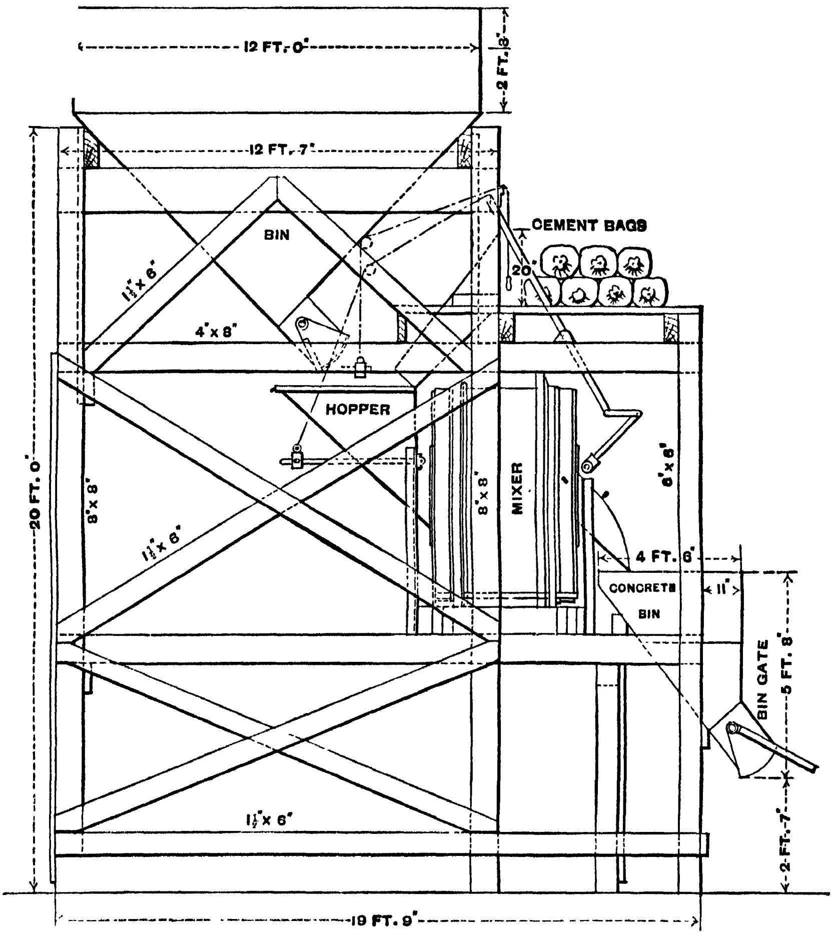


FIG. 67.—Stationary Mixing Plant with a One-Yard Rotary Batch Mixer (*See p.* 243)

bucket, or with towers and chutes, as well as by direct haul. A useful type of barrow adapted to accurate measurement and quick dumping is shown in Fig. 66, p. 241.

Gravity Plants. Where the conformation of the ground permits, the mixer is sometimes placed at the foot of a bank with a platform built over it, through which the materials may be dumped.

Elevated Hopper. A satisfactory arrangement for a stationary batch mixer is illustrated in Fig. 67, page 242. The bin above the hopper is divided into two compartments for the sand and stone, and these are measured by feeding them to definite heights in the hopper, while the cement is dumped into the chute in front.

This general type with many important variations in detail is in common use. On some jobs the ground slopes so that material can be delivered to bins direct from railroad cars or teams without any trestle or runway. Where the ground is not favorable a careful estimate is necessary to make the hopper feed economical, for in addition to the elevated bins, belt or bucket conveyors, trestles, or similar plant, may be required. The Turner Construction Company, for example, in installing two separate plants on a large building job was able to use an adaptation of the hopper type at one end of the lot where the ground sloped enough to make an expensive trestle unnecessary, while at the opposite end the ground was level and a careful estimate indicated that a mixer served by a gang of wheelbarrow men would give better results. With a one-yard mixer in each, the record output of the gravity plant was 349 batches in 8 hours with 19 men, while the record output of the second plant was only 286 batches in 8 hours with 30 men.*

Building Construction. The common plant for building construction consists of a mixer located on the ground or in a pit, dumping into a single bucket operating in a light timber frame tower, or on steel guides as shown in Fig. 68, page 246. The mixer is fed by barrows or cars, and the concrete transported by barrows, cars, or chutes.

In the construction of the buildings of the Massachusetts Institute of Technology, Cambridge, by the Stone and Webster Engineering Corporation the aggregates were dumped from railroad cars running on low trestles to a platform below, shoveled to barrows, wheeled, and dumped through a hatch into a large car in a pit. This car after receiving the cement from bags was then hauled up an incline by a hoisting engine and tipped into the mixer. The concrete was hoisted in towers to a hopper which fed a distributing chute, as shown in Fig. 73, page 256. Instead of chuting direct to place, the chutes were maintained at an angle of 27° and led to hoppers, from which the concrete was distributed by two-wheeled barrows. This system made it possible to

* *Engineering Record*, September 20, 1913, p. 319.

maintain a smooth uniform mix, not at all sloppy. The cost was substantially the same as by chuting direct to place and a much better concrete was produced.

Belt Conveyor. On the Highland Park factory, Detroit,* belt and bucket conveyors were used to carry the material from the railroad car to the overhead bins. Laborers shoveled from the car to an open trough under which ran a belt conveyor, carrying the material to a bucket conveyor elevating it to chutes running to the bins. The width of such a belt should be not less than 18 inches and the slope no greater than about 22°, which corresponds to $2\frac{1}{4}$ feet horizontal to one foot vertical. Idlers for giving the proper V-shape to the belt were placed at proper intervals.

Conveyors were used similarly on the Elephant Butte Dam.† Cars carrying buckets ran under the concrete hoppers and thence to cableways which picked up the bucket and delivered the concrete on the dam.

The plan in Fig. 69, page 247, shows the design of Mr. William B. Fuller of a plant used at the Parsippany Dike of the Jersey City Water Supply Company, N. J. The sand was brought to the bins and the stone to the crusher in wagons. A belt conveyor delivered the crushed stone to the bins. At the outlet of each bin a measuring hopper (shown in a detail section in the drawing) containing about 8 cubic feet, received the sand or stone from the bin, and at the ring of a bell the proper quantity of each material for one batch of concrete was dropped upon the conveying belt. The cement was emptied from bags on top of the sand and stone as they were carried past the cement shed. The bin over the mixer had two hoppers. As soon as a batch was delivered to hopper No. 1, the bell rung again and another batch started into hopper No. 2, and while this was filling, No. 1 batch was dumped into the mixer.

Movable Plants. Where the work extends over a long strip of territory as in tunnels, retaining walls, and sometimes buildings, a movable plant may be economical. On small jobs the mixer is sometimes mounted on wheels and the loading gang follows it about with wheelbarrows. The ordinary paving mixer is a well known example of this method.

The Rock Island R. R. uses a 3-car concreting train‡ in building retaining walls on track elevation work. The train runs on the lower level. Two cars with bins carry the material and a belt conveyor conveys it to the third car, on which are the mixer, hoist, tower, and chutes,

* *Engineering Record*, February 14, 1914, p. 182.

† Louis C. Hill in *Engineering Record*, October 4, 1913, p. 368.

‡ *Engineering News*, April 8, 1915, p. 674. See also for description of a similar train used on a Baltimore viaduct, *Engineering News*, Nov. 16, 1913, p. 927.

or spouts. The cement and aggregates are dumped directly to the bin cars from trains on the upper level. This train is typical of those used by railroads on such work.

An interesting plant, used in concreting a large number of small separate buildings,* consisted essentially of a platform with mixer and stiff-leg derrick having a horizontal boom 45 feet above ground, which hoisted the concrete in a bucket and ran it out to place. The whole plant, mounted on wide wheels, was moved by a cable anchored by a dead man.

For the retaining walls of a part of the Chicago Drainage Canal,† mixing plants were built on flat cars running on standard gage double tracks 12 feet on centers, along one side of the wall for its entire length. The center line of the nearest track was 10 feet from the face of the wall. Two standard flat cars were solidly united by heavy timber platforms, so as to form practically one very wide car. A timber framework built upon this platform supported 3 working floors. An engine, boiler, and one mixer were located on the first floor and 2 mixers on the second floor, the machinery being all driven by line shafting and friction gearing.

A small derrick and hoisting engine on the third floor hoisted materials from a surface track alongside and dumped into three 15-cubic yard hoppers, two holding stone and the third divided in the middle for cement and screenings. This double hopper fed through a measuring hopper into a mixer on the first floor, where the cement and screening were mixed dry‡ and then raised by two bucket elevators to measuring hoppers over two mixers on the second floor. The broken stone was drawn from the bins through mixing hoppers direct to these two mixers. These two mixers delivered the concrete to inclined bucket elevators which dumped it into a hopper on the middle line of the wall. From this hopper the concrete ran to place in 10-inch pipe chutes. The cost of both of these plants complete was about $21 000.‖ The cost of the two plants divided by the total quantity of concrete laid gives a unit plant cost of $0.21 per cubic yard. This, however, does not include the repairs, or fuel, or the cars and locomotive handling the materials.

In building a dam at Chaudiere Falls, P. Q.,§ tracks were laid just above and below the site of the dam and parallel to it and a traveling

* R. C. Hardman in Engineering and Contracting, May 12, 1915, p. 427.

† Plant is shown in *Engineering Record*, February 17, 1906, p. 190.

‡ This preliminary mixing of the cement and sand is not generally considered necessary in machine mixed concrete. The same scheme of plant design could be used effectively without this feature.

‖ Personal correspondence with Mr. L. K Sherman, Assistant Chief Engineer.

§ *Engineering News*, May 7, 1903, p. 403.

platform containing the mixer was constructed so as to straddle the dam. The mixer discharged the concrete into the upper end of a tube fitted with a lower telescoping section, so that it could be deposited directly on any part of the dam.

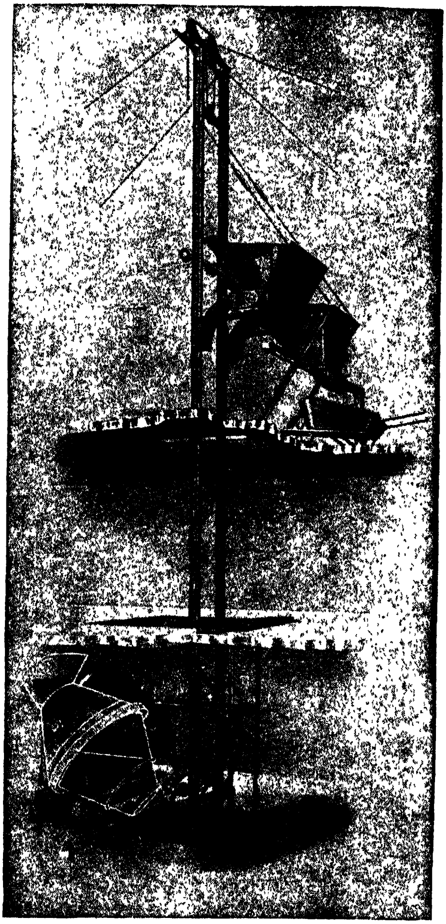


FIG. 68.—Automatic Dumping Concrete Elevator. (*See p.* 243.)

Pneumatic Mixing Plants. Pneumatic mixing plants have been used on some jobs of considerable size. On an arch viaduct at Saskatoon, Canada,* ½-yard batches were delivered over distances of 375 feet at

* *Engineering News*, March 4, 1915, p. 434.

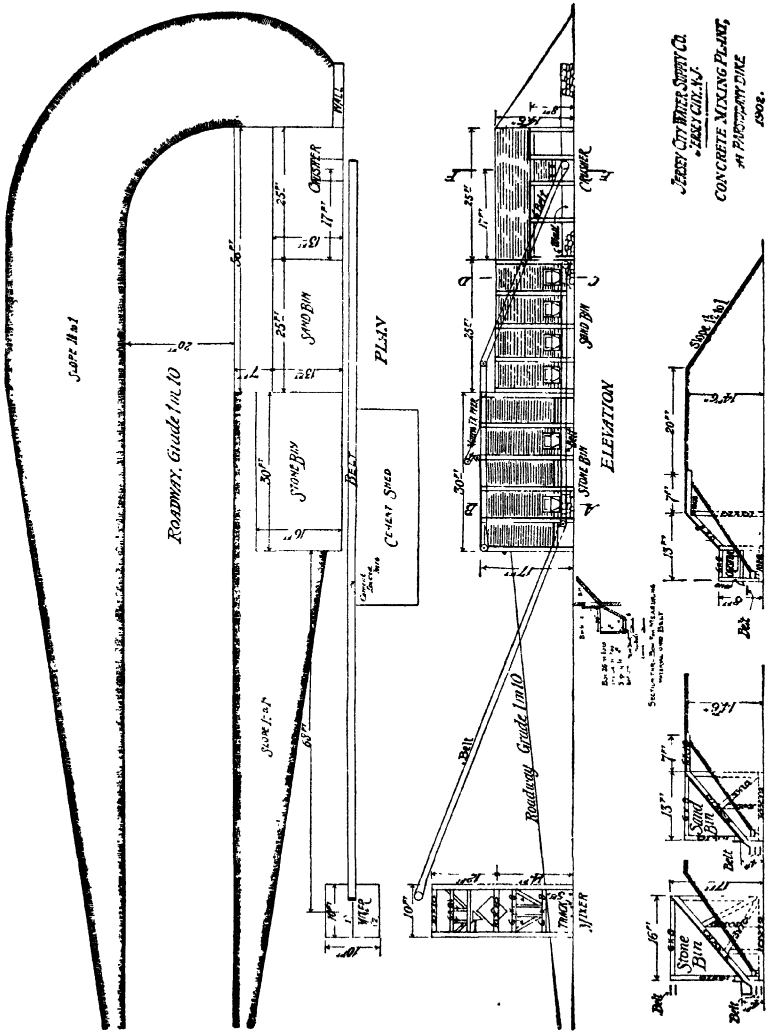


FIG. 69.—Mixing Plant Employing Belt Conveyor. (See p. 244.)

the rate of 50 batches per hour and to 1 000 feet with 35 batches per hour. The concrete was delivered into a box at the end of the pipe line and from it distributed to the forms. The materials may be charged into the mixer from bins, or by wheelbarrows, as in any other type of mixer.

River and Harbor Work. In river and harbor work, it is sometimes economical to put the plant on a scow that can be moved alongside the bridge, wharf, or retaining wall as needed. Aggregates are frequently brought in by scows and transferred to overhead storage bins.

In building the substructure of the Cambridge Bridge, Boston, Mass.,* the concrete plant was located on a pier resting on piles. The gravel for the concrete was dredged from the harbor and dumped from scows into the water close to the pier. An "orange peel" bucket, operated from a dredging machine on a scow, lifted the gravel, and dropped it into a hopper whence it ran by gravity upon the combination inclined screen described on page 220, which separated the sand, pebbles, and the coarse waste material. Bucket elevators raised the sand and pebbles to bins above the mixer, and from the bins, which were V-shaped, the materials feed by gravity into the measuring hoppers, which also received the cement. These hoppers were arranged in two sets, an essential requirement for maximum output, so that one batch could be measured while another was being dropped into the mixer.

A Central Plant. The establishment of a central plant from which the mixed concrete may be hauled to various points as required may be economical in some cities or large towns. This plan has been adopted in St. Louis, Mo.,† for concrete, and is employed in many places for tar and asphalt paving. The plant may be located at a gravel bank or stone crusher, or near a railroad siding, permanent machinery provided which will mix the concrete at a much lower cost than could be done by hand-mixing, and the concrete hauled in carts to the work at but slightly higher cost than the hauling of the dry materials. Most Portland cement concrete will not be injured (see p. 173) if laid within an hour or two after mixing. It is absolutely necessary, however, to use a cement of a slow enough set so that the initial set will not be reached before the concrete is in place.

* For full description, see article by Sanford E. Thompson in *Engineering News*, October 17, 1901. p. 282.

† D. G. Fisher in *Engineering News*, March 10, 1904, p. 231.

CHAPTER XIV.

DEPOSITING CONCRETE

The methods to be selected for handling and depositing concrete must be governed by the size and nature of the construction and by the local conditions. As in mixing, the choice of machinery is controlled largely by economical considerations.

Whatever the methods adopted, the following essentials must be observed:

(1) The stones must not be allowed to separate from the mortar.

(2) The consistency must be controlled to see that (a) for dry and for medium consistency there is enough water to produce a concrete without visible voids or stone pockets, and (b) for wet consistency there is only enough water to produce a sluggishly flowing mass.

(3) The tamping or ramming of concrete (a) for dry or for medium consistency always must be sufficient to flush the mortar to the surface, while (b) for wet consistency, it must be limited to that required to place the concrete in all parts of the forms and to surround the reinforcement.

When Portland cement first came into general use, in the period from about 1880 to 1895, a very dry consistency, substantially that of dry earth, was used, probably because an excess of water is specially injurious to the natural cement concrete previously used. With the advent of reinforced concrete and the development of structures above ground, where the surface appearance is of so much importance, the necessity for more water was apparent. This led to the use in many cases of an excess of water, to a thin sloppy mix, which tests and experience show will produce a very weak concrete because of poor crystallization. (See pp. 251 and 320).

Volume and Weight of Loose Concrete. The volume and weight of loose concrete is of importance in designing the implements or vehicles for transporting it and in estimating the quantities which can be handled under different conditions. The weight of well-proportioned concrete after setting, as stated on page 9, generally ranges from 143 to 155 lb. per cubic foot. When green, it will weigh, after ramming, slightly more than this, say from 150 to 160 lb. The weight per cubic foot loose, that is, in the vehicle which transports it from the mixer to place, depends largely upon

the consistency. If mixed very wet, it will settle down to very nearly the volume it has after it is placed, perhaps within 5% of it; but if of dry consistency, the volume of the rammed mass is apt to be as much as 25% less than the loose. A fair average weight of loose concrete may be estimated, then, at about 140 lb. per cubic foot, or 1.9 tons per cubic yard, when mixed wet, and 120 lb. per cubic foot, or 1.6 tons per cubic yard, when mixed dry. The weights and volumes vary, of course, with the proportions used in the mixture and the specific gravity of the stone in the aggregate, but for rough estimates these figures are sufficiently accurate. The volumes of loose mixed concrete required for a cubic yard of rammed concrete, based on the above percentages, are 28 cu. ft. of a very wet mixture and 36 cu. ft. of a dry mixture.

The volume of concrete contained in an iron wheelbarrow load of average size is 1.9 cu. ft. place measurement. A large load is about 2.2 cu. ft. place measurement. Special concrete barrows are also made with a capacity up to 6 cu. ft. (see Fig. 70, p. 253). Further data is given in Chapter I.

A single cart on ordinary construction roads will carry about half a batch of concrete of average proportions, which may be assumed as 1 barrel cement to 2½ barrels sand to 5 barrels stone, while with a properly constructed cart which will not overflow or leak, 50% more than this, or about three-quarters of a batch, can be drawn over macadam and paved streets.

CONSISTENCY OF CONCRETE

The maintenance of a proper consistency or plasticity in freshly mixed concrete is of the utmost importance. The consistency is determined not only by the amount of water used, but by the manner and time of mixing, since a long, thorough mixing produces a smoother concrete, and also by the nature of the fine aggregate, a certain amount of fine particles being necessary to prevent a "harsh" mixture.

In this treatise the term *dry mixture* is applied to concrete of the consistency of damp earth, from which the water rises to the surface only after prolonged ramming, and then simply in a glistening film. A *medium* or *quaking mixture* means a tenacious, jelly-like consistency which shakes on ramming. A *wet or mushy mixture* is one which will not hold its shape in a pile and will flow sluggishly in a trough or in the forms.

As a result of a series of tests and of practical experience, the authors advocate varying the consistency according to the class of work, and present the following general conclusions:

Dry concrete may be employed in dry locations for mass foundations which must withstand severe compressive strain within one month after placing, provided it is carefully spread in layers not over 6 inches thick and is thoroughly rammed.

Medium or quaking concrete is adapted for ordinary mass concrete, such as foundations, heavy walls, large arches, piers, and abutments.

Wet or mushy concrete is suitable for rubble concrete and for reinforced concrete, such as thin building walls, columns, floors, conduits and tanks. Very wet, sloppy concrete should never be used.

The experiments of the authors show that while dry concrete, very carefully mixed and rammed, is stronger on short time test, medium mixtures will attain nearly equal strength after six months' time. One of the arguments against very dry mixtures is the difficulty of obtaining a uniform consistency. Occasional batches invariably will be too dry, and it is impossible with ordinary care in placing and ramming to avoid visible voids or pockets of stone which form weak places.

A wet mixture is more suitable for rubble concrete or concrete rubble because the large stones more readily settle into place and bed themselves. In thin walls wet concrete can be more easily "joggled" into position so as to conform to the molds and give a smooth surface. The use of a mixture sufficiently wet to flow under and around reinforcement is one of the essentials for the preservation of metal. (See p. 292). Ordinarily the thick, sluggishly flowing consistency is best.

Stone pockets are liable to occur with very wet concrete because of the mortar running away from the stones. This may appear an imaginary danger to many users of concrete who have never employed a very wet consistency, but the authors have seen concrete mixed with too much water, which after setting and the removal of the forms had the appearance of being mixed too dry because of the stone pockets.

Laitance. "Laitance" is a French word, quite generally adopted in the United States and England for the light-colored powdery substance which is held in suspension by the water when cement or concrete is deposited below the surface. On land the same substance forms on the surface of concrete which has been mixed very wet.

The analysis of a sample of laitance* is as follows:

Silica (SiO_2)	16.00%
Alumina and Iron (Al_2O_3, Fe_2O_3)	8.66 "
Lime (CaO)	47.40 "
Magnesia Oxide (MgO)	2.40 "
Ignition loss	23.60 "

* Analyzed for the authors by Mr. Clifford Richardson.

If calculated to a water and carbonic acid free basis the analysis becomes:

Silica (SiO_2)	20.94%
Alumina and Iron (Al_2O_3, Fe_2O_3)	11.30 "
Lime (CaO)	62.04 "
Magnesia Oxide (MgO)	3.14 "

Mr. Richardson notes that this composition corresponds with that of a normal Portland cement except that it is unusually high in alumina and iron, a fact which may be explained by the large amount of magma detected in the thin section examined. He further states:

I have had a thin section ground, but find that it shows no structure which is characteristic. The section consists largely of amorphous material of an isotropic nature, that is to say, it does not affect polarized light. It reveals a considerable amount of a yellow substance which seems to be the undecomposed magma contained in the original cement. I have formed a material very similar to the "laitance" by shaking Portland cement with water, decanting the finer portion and allowing it to settle out and harden. This material, like your "laitance," is rather soft, and this is due to the fact that the Portland cement is much more thoroughly decomposed under these conditions than under ordinary ones, and this accounts for its character.

It is evident from these facts that the milky laitance which appears on concrete laid under water represents an actual loss of cement, which should be prevented by confining the mass until it reaches its position.

HANDLING AND TRANSPORTING CONCRETE

In handling and transporting, as noted above, it is essential to prevent separation of the stones from the mortar. This is liable to occur, on the one hand, if the concrete is of dry consistency so as to be incoherent, and, on the other hand, if it is so wet that the mortar flows away from the stone. With the modern slow-setting cement, in the use of which some time may elapse without injury between mixing and placing (see p. 174), there is less difficulty in handling concrete than formerly, and it can be transported readily to a considerable distance. Moreover, a soft, sluggishly flowing mass is easier to handle than either a dry or a very wet mixture.

For transporting concrete in small quantities, either hand-mixed or machine-mixed, wheelbarrows of the ordinary contractor's type, or else for larger work the two-wheeled barrow illustrated in Fig. 70, page 253, are used. On larger jobs, derricks are suitable if the mass is concentrated near the mixer, otherwise cars running on a track, (see Fig. 71, p. 254,) or in some cases wagons, afford means of conveyance. A combi-

nation of car and derrick work is readily effected by using flat cars with derrick buckets or trays upon them. Cableways may be satisfactory on a large job, but the output, which is limited by the speed on the cableway, must be carefully figured. For a high thin wall, galvanized iron-buckets are sometimes useful. A bucket elevator is a poor contrivance for elevating concrete because the mortar sticks to the buckets and the ingredients separate as the concrete is thrown from them. A controllable bottom dump bucket is illustrated in Fig. 72, p. 255.

DEPOSITING CONCRETE ON LAND

Formerly it was specified that concrete should be of dry, damp-earth consistency, and placed in 6-inch layers, which must be allowed to attain a hard set before the next layer was placed. With the use of wetter mixtures and slow setting Portland Cement, it is possible to deposit concrete to any depth desired so long as it is properly compacted.

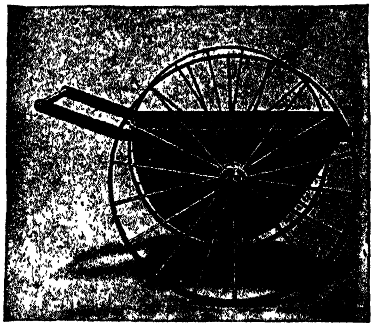


FIG. 70—Two-Wheeled Concrete Barrow. *(See p. 252.)*

The methods to be selected for depositing concrete depend largely upon the structure but are also affected by the consistency. If of medium or of mushy consistency, it can be dropped vertically to any depth. A wetter consistency can be passed through an inclined trough or chute. On the other hand, the stones in a dry or "damp earth" consistency will separate from the mortar on the slightest provocation.

In a thin wall or a structure requiring especial care, such as a tank, it may be advisable to shovel the concrete from the wheelbarrows or from a platform. Stones which tend to separate can be thus mixed in with the mortar and a thin layer formed in the molds, so as to avoid possibility of separation.

Chuting Concrete. Chutes or troughs for handling concrete have come into use with the wetter mixtures. In construction such as dams, where the conformation of the ground permits, the concrete may flow from the mixer to place by gravity. In buildings, bridges, and other elevated structures, the concrete often is hoisted in a tower and deposited through an inclined chute either directly to place or to a hopper, from which it is conveyed to the forms in barrows or other vehicles.

There is danger in the use of chutes discharging directly to place unless steep enough slope is maintained to permit of a concrete not excessively wet or sloppy. With a flat slope and the very wet, sloppy mix necessary to flow in it, concrete of low strength is produced, because of injury to the cement and there is a tendency to form a scum or laitance on the surface of each layer while the cement itself is injured by the excess water. (See p. 251.) In one case, for example, Mr. Thompson found 4 inch thicknesses of laitance on the top of basement columns of a completed 6-story building. This had to be cut out and replaced by good concrete. In another instance pockets of laitance some 8 inches thick were found in the center of a

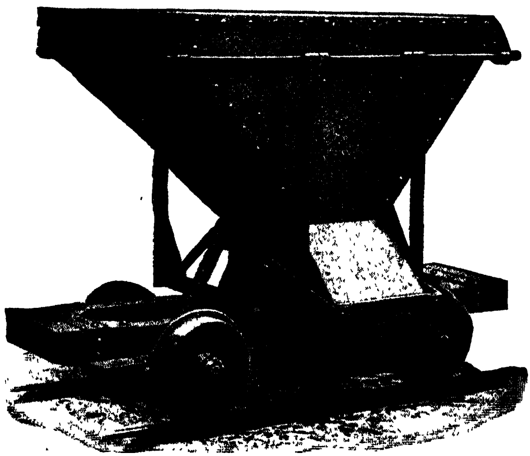


FIG. 71.—Radial-Gate Hopper Car. (*See p.* 252)

dam built by chuting very wet concrete. Even when layers of laitance are not formed, the excess water and churning affect the chemical action of the cement and a light colored concrete of low strength is thus produced.

To prevent the ingredients separating when flowing down an incline, the lower end of the pipe or trough should run into a hopper with a gate so that the concrete may be drawn out as required.

As a result of tests on the new buildings of the Massachusetts Institute of Technology, in 1916, the Construction Superintendent, Mr. Thomas A. Carr, found the best slope for both gravel and broken stone concrete to be 27° with the horizontal, that is, 2 horizontal to 1 vertical. This permits a smooth sluggish flowing concrete.

The minimum slope to permit depends in a measure upon the smoothness of the mix, which in turn, is governed by the nature and grading of the aggregates, but other experimenters have confirmed the conclusion that 27° is the proper slope for such work as building construction. In mass work where a wetter or dryer consistency may bė permissible the slope should be in no case less than 18° (3 to 1) nor greater than 37° ($1\frac{1}{3}$ to 1). The chutes at Technology led into hoppers from which the concrete was wheeled to place in two-wheeled barrows. The cost of this plan was estimated to be no greater than chuting direct to the forms, while much better concrete resulted. Special tests indicated that the strength (which for 1 : 2 : 4 concrete at 28 days

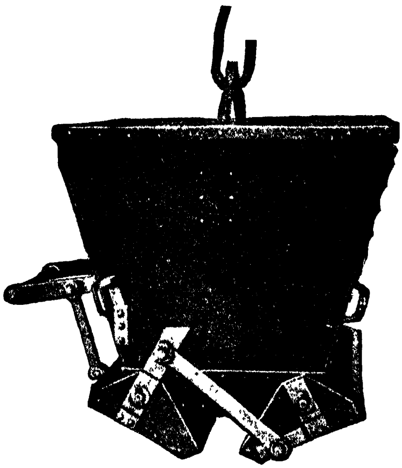


FIG. 72.—Controllable Bottom Dump Bucket. (*See p.* 253.)

averaged 2 110 lb. per sq. in. and for 1 : $1\frac{1}{2}$: 3 concrete 2 520 lb.) was actually increased by flowing through the chute in this manner. Fig. 73, page 256, shows the arrangement of tower, chute, and hopper.

In Fig. 74 is shown at the left a sluggish mass flowing on a slope of 27° and on the right an excessively wet sloppy mix flowing into a hopper. Note the foot of the sluggish mass just above the cleat in the left hand picture.

Pneumatic Placing of Concrete. The transmission of concrete by air pressure through long pipes has been adopted in certain cases in tunnels, subways, and similar construction, when conditions require a plant occupying little space and a small amount of hand labor. To resist the excessive wear on the pipe under the abrasive action of the

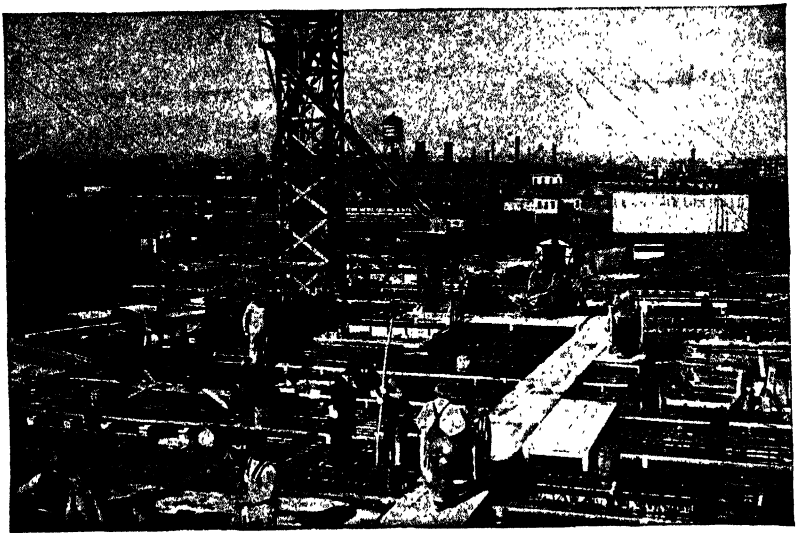


FIG. 73.—Chuting Concrete—Massachusetts Institute of Technology Buildings (*See p.* 255.)

Sluggish Concrete Flowing Down Chute

Excessively Wet Concrete Flowing into Hopper

FIG. 74.—Consistency of Concrete. (*See p.* 255.)

concrete travelling at a high rate of speed, 8-inch steel pipe with flanged joints was used on recent tunnel work in San Francisco.* This delivered 6 000 cubic yards of concrete before wearing out.

Manganese steel elbows of large radii were found economical on this work in spite of high first cost. Deflections and bends must all be braced against the powerful blow of the charge and the forms also must be rigid. In raising concrete from one level to another the use of a vertical pipe is always advisable because on a slope the air tends to pass over the concrete and escape. The same difficulty is met on long horizontal lines and the charge must be collected by inserting a vertical U similar to the expansion joints used in steam pipe lines. On the New York subways a T connection was used at the end of the pipe line by means of which the concrete would be delivered in any direction in a vertical plane.†

Monolithic Concrete. For water-tight work or for maximum strength the concrete should be placed, if conditions permit, so as to form a monolith. To do this on a large structure two or three shifts are employed in twenty-four hours, so that no portion of the mass commences to set until fresh concrete has been laid on top of it. In a large reservoir wall at Little Falls, N. J., built *en masse* to sustain a 40-foot head of water, the only point where the moisture appeared on the surface was at a layer where the work was stopped for one hour at noon. In most structures it is possible to divide the work into sections, each of which is a monolith. Wherever joints are unavoidable they must be made with a neat cement bond as described on page 259.

RAMMING OR PUDDLING

The method of compacting the concrete or forcing out the air after placing, and the kind of tools to employ for this, depend upon the consistency of the material.

In concrete mixed with a small amount of water the thickness of layers is usually specified at 6 to 10 inches, the former being the most common, but with a very wet or mushy concrete 12 to 15 inches may be placed at once, the chief object being to expel bubbles of air by puddling or joggling. In using very wet concrete there is danger of too much ramming, which results in wedging the stones together and forcing the finer material, the sand and cement, to the surface.

The style of rammers ordinarily used for dry mixed or medium con-

* *Engineering and Contracting*, March 17, 1915, p. 235.

† A. E. Comstock in *Engineering News*, March 16, 1916, p. 496.

crete are similar to the forms shown in Fig. 76. The style on the left of the figure is the ordinary type, and on the right is a style convenient for use close to the forms.

FIG. 75.—Rammer for Mushy Concrete. (*See p.* 258.)

A "post-hole" tamping bar with iron shoe, shown in Fig. 75, has been successfully used by the authors for mushy concrete. A piece of 2 by 3-inch studding cut to the required length and smoothed off so as to be readily grasped by the hands is also a serviceable tool.

For face work an ordinary flat spade or an ice chisel may be used to slice down along the forms. Care must be taken not to pry with the spade, as the form will spring unless it is excessively strong.

In narrow forms where a man cannot stand in the concrete, a piece of 2-inch by 3-in scantling,—with the upper portion rounded and the tamping end wedge-shaped,—of a length determined by the depth of the form, is convenient and cheap.

Labor of Ramming. Where the work is not cramped, a laborer of average ability should level and tamp 11 cubic yards of plastic concrete and 18 cubic yards of wet or mushy concrete per day of 10 hours. With labor at $2.00 per day, allowing for superintendence and contractor's profit, the cost per cubic yard is approximately 24 cents for the plastic concrete and 15 cents for the wet or mushy concrete. First-class men should do one-third more work than this and, if working at the same wages, at three-quarters of these costs. In small columns and thin walls, the costs are high and may be two or three times these figures.

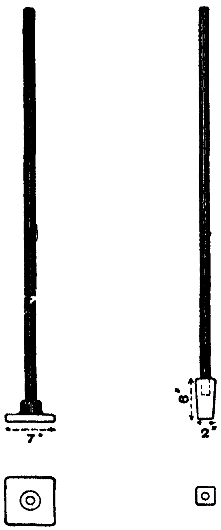


FIG. 76.—Rammers for Dry Concrete. (*See p.* 258.)

BONDING OLD AND NEW CONCRETE

To bond old and new concrete, thoroughly clean the old surface of all dirt and laitance; roughen

smooth surfaces; soak with water; coat with neat cement paste; and immediately place the new concrete.

A 1 : 1 or 1 : 2 mortar (always richer than the mortar of the concrete) is sometimes used, but for a surer bond and always for watertight work a neat cement paste is necessary. The consistency of this should be about that of brick mortar, and it should be brushed on or spread to a depth of about $\frac{1}{16}$ inch. Dry cement thrown on to a wet surface does not produce a satisfactory bond. The body is insufficient and the cement does not have its full value unless worked up with water before placing.

In mass concrete subject to compressive stresses only, no precautions other than cleaning are necessary.

Dowelling with steel bars is frequently useful in transferring stress, and in mass concrete fairly large stones or plums may be placed at the end of a day's work. Methods of treating expansion joints are discussed on page 260.

Dilute hydrochloric acid is sometimes used for cleaning and roughening the surface of the set concrete. The acid must be thoroughly washed off before placing the new concrete or mortar.

In reinforced concrete, joints should be made so as to least affect the strength. In columns the joint is at the lower surface of girder or at bottom of haunch or capitol, if any. In a floor system or in reinforced walls resisting pressure, it is best to make the joints perpendicular to the surfaces and locate them at or near the center of the span. If a beam intersect a girder near its center the joint should be offset a distance equal to twice the width of the beam. In beams carrying shear where a joint is to be located, the joint may be inclined as much as 30° to the perpendicular.

CONTRACTION JOINTS

Concrete, like any other material, expands with heat and contracts with cold. Temperature cracks occur in brick and stone walls, but are less noticeable because they usually follow the joints instead of cutting across smooth surfaces, as in concrete. Besides being affected by temperature, concrete contracts for a period while setting and hardening in air, and is also affected to some extent by moisture.

The effect of expansion must be provided for only in exceptional cases where angles occur in long surfaces which cannot move freely, such as a depression in a large floor surface or a change in grade of pavement or where dangerous compressive stresses are liable to be produced. Ordinarily, it is the contraction which gives trouble by forming cracks,

expansion being taken care of by the strength and elasticity of the concrete.

The effect of contraction may be taken care of by expansion joints or by reinforcement. Reinforcement suitably placed (see Chapter XXII) will reduce both the size and number of cracks, and if the structure rests on columns so as to be free to move, it may prevent them entirely. It is frequently better, even in structures up to say 1 000 feet long, which are free to move above the foundations, to reinforce throughly and avoid expansion joints. In such cases, however, it must be determined in advance that if cracks do occur they will produce no structural damage. This matter is discussed more fully in connection with the reinforcement in Chapter XXII.

In unreinforced structures contraction joints are necessary. Retaining walls, dams, and subways, may be built in alternate or successive sections, with time enough for one section to harden before fresh concrete is poured against it. Longitudinal steel will aid in preventing cracking between joints. If desirable, some non-adhesive material, such as tar paper, may be used to separate adjacent sections. In thin walls joints are needed about every 30 feet and in thicker walls every 50 or 60 feet. In horizontal surfaces, unreinforced, cracks are apt to occur about 20 feet apart.

In work that must be reasonably water-tight, such as dams and subways, sheet lead or copper flashing sometimes is used at joints. In dams and walls where loads must be transferred from one section to another, V-shaped grooves may be built into the face of the first section and filled with concrete when the next section is placed. Sometimes wells are left and filled afterward with concrete or puddled clay.

In reservoir work special precautions are needed, for if one wall slides by another built into it at an angle, considerable leakage may result. In such cases, water-tight joints may be made by leaving slits about ½ inch wide and filling them with a plastic material, one of the best for this purpose being pure asphalt of medium hardness. Lime dust is sometimes mixed with the asphalt. Another way of forming a joint is to insert two or more thicknesses of roofing paper. In building a reservoir floor in sections, the joints may be filled with asphalt or asphalt and limestone dust, but the joint must be backed up with a narrow concrete beam to prevent the head of water from forcing out the filling.

Temperature Expansion and Contraction. The coefficient of expan-

sion of concrete as determined by tests* and measurements of structures may be taken as 0.0000055 per degree Fahrenheit of temperature change, the test results varying from about 0.0000050 to 0.0000065. This coefficient corresponds so closely to the coefficient of expansion in steel that they may be assumed to, and actually do, work together in a reinforced concrete structure.

To figure the actual movement in a structure, the coefficient is simply multiplied by the length times the number of degrees temperature change. Thus, for a structure 500 feet in length, subjected to 50° temperature variation, the total change in length may be estimated at $0.0000055 \times 500 \times 50 = 0.1375$ feet, or 1.65 inches.

Measurements of structures in general agree fairly well with the theoretical, although other conditions, such as, in mass concrete, the heat of setting, (see p. 93) affect the results. Moisture also influences the volume since wet concrete expands and dry concrete contracts. Concrete under continuous load has been shown to react to the loads and actually flow. The interior of large masses of concrete, such as dams, are only slightly affected by atmospheric temperature changes. In the Arrowrock Dam† no effect was detected 20 feet or more from the face. The effect of setting and temperature changes in dams is discussed in Chapter XXVII.

Shrinkage and Expansion due to Atmospheric Changes. Tests indicate that concrete in actual structures shrinks about 0.05 per cent. as soon as it is allowed to dry out. The corresponding expansion if kept wet is much smaller—possibly about 0.01 per cent.

Prof. A. H. White‡ has shown that concrete, even when twenty years old, expands if wetted and shrinks if dried, and that with rich mortars these variations cause changes much greater than those due to temperature. Successive long immersions with intermediate dry periods cause progressive expansion. A small bar cut from a sidewalk after 20 years service elongated 0.175 per cent. by successive immersions at room temperature.

Tests§ by F. R. McMillan give a shrinkage of 0.080 per cent. after one year in 1 : 2 : 4 concrete beams, 4 by 5 inches by 4 feet long mixed

* "The Coefficient of Expansion of Concrete" Journal Western Society of Engineers, Vol. VI, p. 549, republished in *Engineering News*, November 21, 1901, p. 380. Also, see tests by Professor Hallock in Burr's "Materials of Engineering," 1903, p 378.

† "Temperature Changes in Mass Concrete" by Paul and Mayhew, Trans. American Society of Civil Engineers, Vol. LXXIX, 1915, p. 122.

‡ Proceedings American Society for Testing Materials, Vol. XIV, 1914, p. 203, and Transactions International Engineering Congress. Paper 103.

§ Bulletin University of Minnesota, March 1915.

with ¾-inch limestone, and reinforced with about 0.3 per cent. steel. The total shrinkage does not appear to be affected by any temporary method of storage; concrete stored in air begins to shrink at once, but shrinkage of concrete stored for a few months in water or under moist cloths proceeded to shrink rapidly enough upon exposure to dry air to make up for the lost time. Beams in wet storage showed a tendency to expand while wet and beams in dry air expanded rapidly if transferred to water storage.

German tests* of concrete specimens mixed with ordinary coarse aggregate setting in air give a shrinkage of 0.032 per cent. or not much more than one-third of Mr. McMillan's results. The rate of shrinkage was about the same in both cases; two-thirds of the total being attained in forty days with a more gradual increase to the maximum which was reached in from 200 days to one year.

FACING CONCRETE WALLS†

A pleasing appearance of concrete surfaces requires, first, straight, sharp lines or edges, and, second, uniformity or regularity in texture. A rough surface is generally preferable because it conceals slight defects.

Plastering is unsatisfactory on walls exposed to the weather of variable climates and is almost sure to discolor and crack off. The concrete itself should be relied upon with as little treatment as possible. At the same time with attention to details various effects can be produced as required by the character of the structure.

With carefully built form work, the use of a mushy mix, neither too wet nor too dry, and spading against the form so as to flush the face and keep back the coarse aggregate, a smooth even surface can be produced. At the same time the surface will show the imprints of the joints, knots, and other markings of the form, besides having a dead color that is not pleasing. By removing the surface skin the monotony can be relieved. To show better the coarse aggregate it may be pressed against the form with a spade, when laying the concrete.

Scrubbing. The least expensive method of removing the skin is to scrub it with brushes and water before it becomes hard, so as to expose and bring into partial relief the particles of the aggregate. In Fig. 77, page 263 is shown in full size a finish thus obtained where a granolithic mixture with aggregate of ¼ inch dark shale was placed against the form

* Deutscher Ausschuss für Eisenbeton, Heft 23: 1913

† The authors are indebted to Mr. Henry H. Quimby for cuts and much of the text in this section.

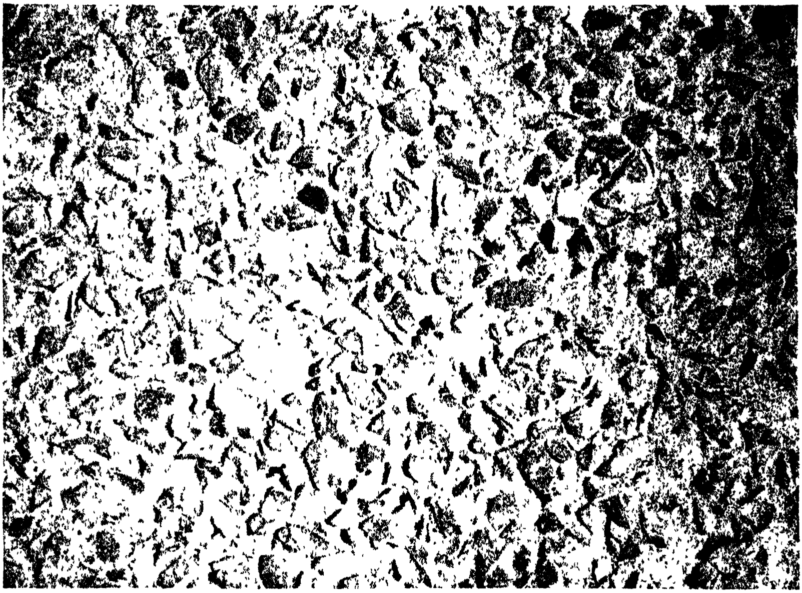


FIG. 77.—Surface of Washed Concrete. (See p. 262.)

Surface left by forms is shown on left and picked surface on right.
FIG. 78.—Surface of "Picked" Concrete. (See p. 266.)

as the body concrete was poured. A bridge parapet made with concrete having a $\frac{3}{4}$-inch trap-rock aggregate and scrubbed the day after pouring is shown in Fig. 79, page 264. The panels are formed with colored tile. In Fig. 80, page 265, is shown a surface faced with small white pebble granolithic and scrubbed.

Surfaces must be scrubbed within eight to twenty-four hours after pouring, depending upon the setting quality of the cement, the temperature of the atmosphere and the character of the fine aggregate. Of course care must be exercised to be sure that the concrete has set sufficiently to be safe for the removal of the face forms. The scrubbing

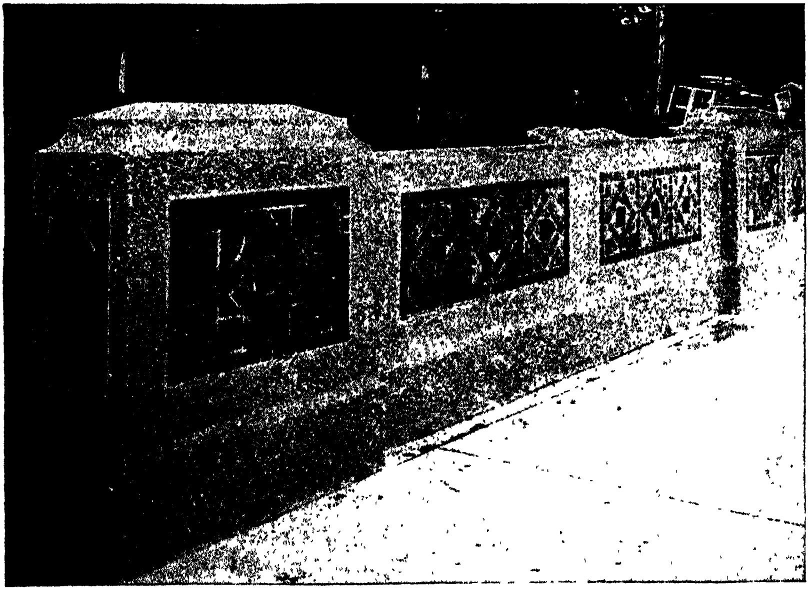


FIG. 79.—Bridge Parapet of Scrubbed Concrete. (*See p.* 264.)

must be done immediately after removal of the forms, as the surface hardens quickly when exposed to the air. With the proper degree of hardness, which is while the surface is still "green" or friable, an ordinary fibre scrubbing brush is the only implement needed, water being supplied freely and the surface being rinsed clean. If a hose carrying a constant stream of water is not available a tin can, with a nail hole punched through the side near the bottom can be used with a convenient bucket of water as a supply. Care is necessary in removal of forms to avoid spalling corners of the tender concrete. Any repairing needed

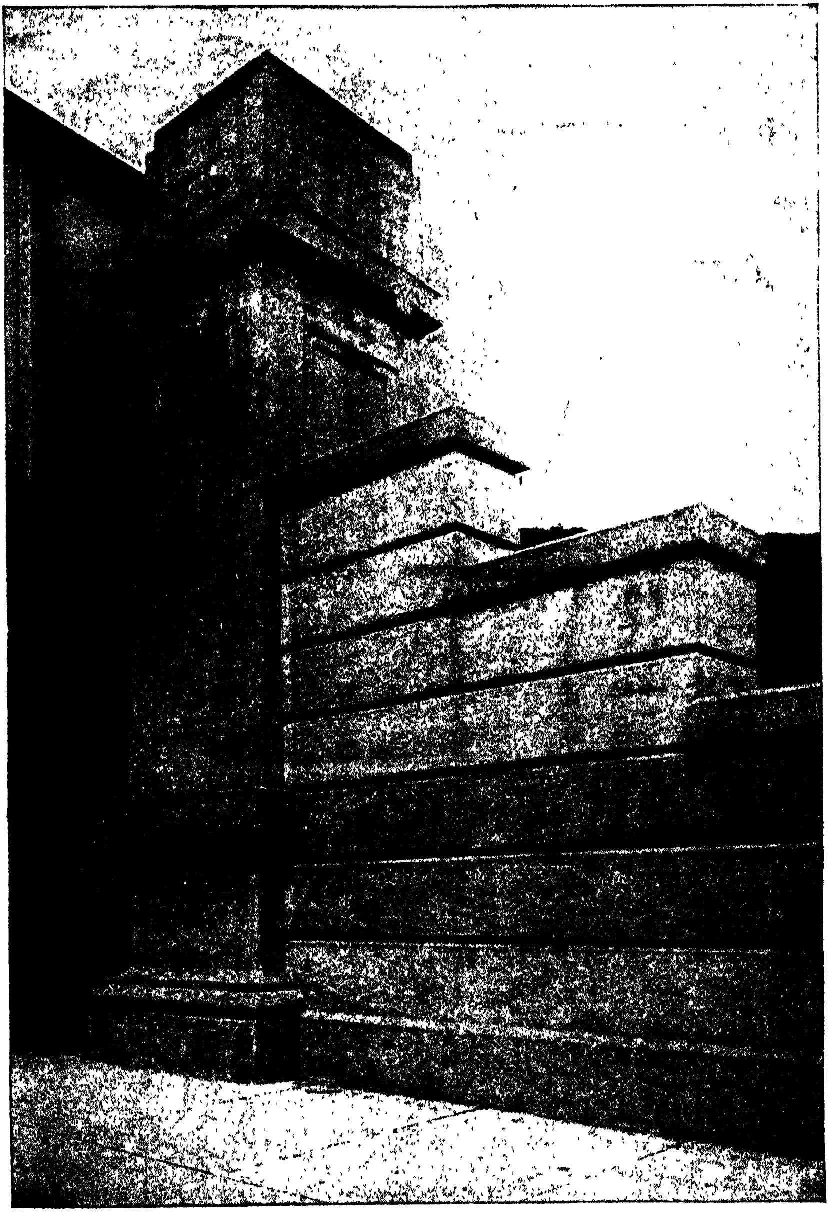


FIG. 80.—Bridge Abutment with Paneled Surface, Scrubbed. (See p. 264.)

should be done immediately after completion of scrubbing and the patches scrubbed as soon as they are sufficiently set.

The cost of scrubbing is very low if done at the right time. A laborer can easily scrub and rinse as much as one hundred square feet in an hour.

In case the concrete has hardened, as it may in hot weather if left over night, a wire brush is advantageous. If the surface is too hard for this, a brick or block of wood with sand and water will rub off the skin, but in such cases the resulting finish will be comparatively smooth and of a different texture. Carborundum and water give best results on hard concrete.

In cold weather scrubbing is somewhat uncertain in results and economy because of irregularity of the setting of the concrete and the difficulty in determining just when to remove the face forms, also if the concrete is under load, as in columns, care is necessary to insure sufficient strength to prevent fall or flow of the soft concrete.

When scrubbing is to be done on a large face where a day's pouring is only a portion of the whole height, the face planking is erected in courses, supported by cleats and nailed to the uprights or studs, which are set at several inches from the face, so that the courses of planks can be removed individually and re-set higher up for the next day's pouring. A much smaller amount of face planking is thus required, which offsets the labor in constructing.

Tooling. A very satisfactory but somewhat more expensive process of finishing a concrete surface is tooling it—either axing, bush-hammering or pointing—when the surface is too hard for scrubbing. When done by hand the cost will run from $1\frac{3}{4}$ to 5 cents per square foot, according to the efficiency of the workman and the wages paid him. The most that one man can be expected to do in one hour is ten square feet. A pneumatic tool should do two or three times as much. The photograph, Fig. 78, page 263, shows at the left a concreted surface and at the right the surface after picking.

Acid. The hardened skin also can be removed by the use of muriatic acid diluted with six parts of water. A stronger solution is not more effective and is liable to stain the aggregate. The process is expensive and troublesome, particularly on vertical surfaces, for repeated applications are required to obtain satisfactory results. The acid should be thoroughly washed off.

Color. Color effects, as well as different textures, are obtained by selecting the aggregates and, at the same time, combinations of colors

are produced as, for example, yellow pebbles in panels and black shale stone in borders.

Plaster Forms. Perfectly smooth forms that will not leave imprints on the concrete face have been made by plastering metal mesh with plaster-of-paris to which the concrete will not adhere. Such forms have been made interchangeable and repeatedly used, the corners and other joints at each setting being filled and smoothed with the same plaster.

White Cement. White Portland Cement, of high strength and mixed with sand is used to good effect in surface treatment.

Panels. In connection with any of these treatments, sunken or intaglio panels, or, on the other hand, raised panels, i.e., built in relief by special form construction, may be used to relieve the plain even surface of the concrete. Tile and terrazzo panels are also used for ornamentation, as, for example, in subway stations, and ornamental bridges.

Mortar Facing. If a mortar surface is required, it is best obtained by depositing the mortar and concrete together, the mortar close to the form. To place it, a movable form, preferably of steel, is held one or two inches behind the face, to govern the thickness of the mortar, and gradually withdrawn as the mortar and concrete are deposited.

DEPOSITING CONCRETE UNDER WATER

Concrete is usually placed under water by pouring through a tube or tremie in a continuous flow or by molding large blocks on land to be placed by machinery or floats after hardening. Derrick buckets are sometimes used for depositing but the results are less satisfactory. Cofferdams, not necessarily watertight, are usually required to prevent the concrete from spreading and the cement from washing away.

The consistency should be quite wet, wetter than is good practice for work above water. Dry concrete, dry materials mixed without water, should never be deposited under water.

Depositing through Tremies. In using tremies it is absolutely essential that the concrete shall not be allowed to wash; the pipe must be kept full of concrete at all times, and the bottom of the pipe moved slowly about to allow the concrete to run gradually out with a minimum disturbance of the water. In case the charge is lost and the pipe fills with water, extra cement should be used in the concrete until the pipe is once more full and in proper working order. Some sort of a traveler, scow, or derrick, is necessary to move the tremie about.

The size of tremie depends a good deal upon the size of the plant. For small work with the concrete deposited from wheelbarrows a diam-

eter of about one foot at the top is enough. On large jobs with large mixers larger pipes are required. The diameter at the lower end should be from $\frac{1}{4}$ to $\frac{1}{3}$ as large again as the top to avoid plugging and permit telescoping of the section.

The use of tremies is referred to as long ago as 1863 by Gilmore in his "Treatise on Limes, Hydraulic Cement and Mortars."

Depositing from Buckets. The best type of bucket for the depositing of concrete is a box open at the top with a bottom that can be dropped down for emptying. The ordinary construction bucket that dumps by tipping and allows the water to stir up the concrete and wash out the cement cannot be used. Usually the cost is greater than with tremies and the results much less satisfactory.

Depositing in Bags. In the past bags varying in size from small paper or muslin bags to jute sacks containing 100 tons* sometimes have been used for holding concrete together as it passed through the water. In some cases the concrete has been placed in the bags dry.† This is bad practice because concrete, unless thoroughly mixed, does not attain satisfactory strength or density.

Molded Blocks. The molded block method is especially practical in tidal water where concrete deposited in place would be liable to serious wash unless expensive, water-tight cofferdams were used. Large blocks weighing many tons may be cast and then lowered to place. References to work constructed in this manner are given in Chapter XXXIII.

CONCRETE IN SEA WATER

For concrete laid in cofferdams in sea water the essential requirements of construction are:

(1) Select materials adapted to sea water use (See Chapter XV by R. Feret).

(2) Proportion for maximum density using a mix as rich as 1:2:4.

(3) Employ a medium consistency scarcely soft enough to flow.

(4) Make cofferdams tight to prevent flow of water through green concrete.

(5) Be sure that concrete is hard before it is subjected to sea water.

(6) At joints between set or partly set and fresh concrete, clean surface and make a neat cement bond. (**See p. 259.**)

The disintegration of concrete by sea water occurs chiefly between low

* Proceedings Institute of Civil Engineers, Vol. XXXIX, p. 126, and Vol. LXXXVII, pp. 101 and 126.

† Lt. Col. J. A. Smith, *Engineering Record*, March 23, 1895.

and high tide, and is produced by a combination of frost and of chemical action. If laid with the best of workmanship by methods outlined above it should resist the elements.

Special precautions are necessary to prevent the tide from rising and falling on or through the fresh concrete. A number of failures of sea water construction have been due to this cause alone. The water washes the cement out of the green concrete and leaves a porous mass readily acted on by the sea water so as to be completely disintegrated. It is interesting to note in this connection that in reinforced concrete the steel may not be affected. In one instance in Boston Harbor, where the repairs were made under the direction of one of the authors, the concrete in certain sections which had been injured during construction by the tide was soft enough in places to pick out with the fingers, but the steel, which had had no opportunity to dry out, was intact after two years' exposure.

In southern waters and below low tide concrete is affected but slightly. The effect of frost is illustrated in the sea wall of a power house in Boston Harbor, where the wall washed by the cold salt water is badly disintegrated, while the portion reached by the warm water discharge is unaffected. Concrete blocks or piles thoroughly hardened before exposure to sea water resist sea water excellently.

Materials. The characteristics of aggregates required are discussed in the following chapter. Density is of the utmost importance to prevent water flowing into or through the mass and thus affecting the cement. Tests by the authors using concentrated sea water, in connection with the construction of the South Boston power house of the Boston Elevated Railway Company indicate that the choice of the cement should be governed largely by a low percentage of aluminum. A maximum of $6\frac{1}{2}\%$ alumina is specified by the Boston Transit Commission for cement in sea-water as a result of tests of pats of neat cement ranging in composition from $4\frac{1}{2}\%$ to 8% alumina. The cements low in alumina in general resisted decomposition for a much longer time than those high in alumina.*

DRILLING CONCRETE

In factory construction concrete floors must be drilled for bolts to anchor machinery. On a larger scale drilling is required for the re-

*Report of Boston Transit Commission, June 30, 1914, p. 53; June 30, 1915, p. 53.

moval of old concrete. The equipment and the selection of hand or machine tools depends upon the amount of work to be done.

An ordinary star brick hand drill will cut holes for machinery by striking light quick blows to avoid chipping or breaking through. Similar methods have been used on the Boston Subway by the line and grade parties in drilling holes for lead plugs. A light pneumatic drill is also used for such work.

In drilling a large number of holes in $1 : 2\frac{1}{2} : 5$ rubble concrete* ten months old for $1\frac{1}{2}$ inch anchor bolts, hand drillers averaged 1.8 linear feet of hole per hour, or about 14 linear feet per 8-hour day, while the pneumatic drill, cut 4.7 linear feet per hour, or about 38 linear feet per 8-hour day, with a maximum of 54 linear feet. In concrete three months old the average progress was about 85 feet per day. Wet or damp concrete drilled badly. One sharpening of a drill was required per 8 feet of hole.

A section of retaining wall at Newton Highlands was torn out to provide room for an extension of the station platform. A large steam drill on a tripod with 2″ drill was used. A time study showed the work to consist of four operations with average times as follows:

(1) Getting ready to drill and starting hole, a constant per hole, 7.5 minutes; (2) Drilling, per linear foot of depth of hole, 2.1 minutes; (3) Moving drill from hole to hole, varying with spacing, per linear foot of distance, 0.5 minutes; (4) Lost time, in terms of (1), (2) and (3), 23%.

Holes were about 25 inches deep and about 3 linear feet of drilling were required per cubic yard of concrete. The gang consisted of 10 men,—foreman, blacksmith and helper, fireman, drill runner and 5 laborers. By using the times given, an approximate estimate can be made for various conditions.

* J. R. Taft in *Engineering Record*, September 3, 1910, p. 269.

CHAPTER XV

EFFECT OF SEA WATER UPON CONCRETE AND MORTAR*

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The principal conclusions which have been reached by the author of this chapter, as discussed in the following pages, are as follows:

(1) No cement or other hydraulic product has yet been found which presents absolute security against the decomposing action of sea water (See p. 271.)

(2) The most injurious compound of sea water is the acid of the dissolved sulphates, sulphuric acid being the principal agent in the decomposition of cement. (See p. 272.)

(3) Portland cement for sea water should be low in aluminum (see p. 274), and as low as possible in lime. (See p. 273.)

(4) Puzzolanic material is a valuable addition to cement for sea water construction. (See p. 279.)

(5) As little gypsum as possible should be added, for regulating the time of setting, to cements to be used in sea water. (See p. 272.)

(6) Sand containing a large proportion of fine grains must never be used in concrete or mortar for sea-water construction. (See p. 278.)

(7) The proportions of the cement and aggregate for sea water construction must be such as will produce a dense and impervious concrete. (See p. 278.)

EXTERNAL PHENOMENA

At present there is no hydraulic product which is known to be capable of resisting absolutely the decomposing influence of sea water. It is true that some concrete masonry has remained intact for a very long time in salt water, but with our present knowledge it is impossible to say why these structures have resisted so well, and there is little doubt that the cements from which they were made might have decomposed rapidly if they had been used under different conditions. In some cases, on the other hand, similar large structures subject to the action of sea water were

*The authors are indebted to Mr. Feret for this chapter, which has been especially prepared by him for this Treatise.

ruined in a few years and were torn down and completely rebuilt. Notable instances of this kind are the failures which occurred in the ports of Aberdeen,* Dunkerque, and Ymuiden.

Such occurrences have aroused great interest in the subject of the action of sea water upon mortars, and but few questions have received more careful study. In spite of this, however, it cannot be said that any sure means of preventing these failures have been found.

The decomposition manifests itself in various ways: sometimes the mortar softens, and little by little becomes disintegrated; sometimes the mortar becomes covered with a crust which finally cracks off; more often fine white veins develop on the surface of the mortar, these gradually grow large and open, the mortar swells, cracks, and falls off in small pieces or collapses in a pulp-like mass. Almost always the interior of the decomposed mortar is found to contain a soft white material which may be easily separated from it. The chemical composition of this substance is not, however, constant.† Generally, the more advanced the state of decomposition, the more readily the white material can be extracted from the mortar and the richer it is in magnesia. The proportion of sulphuric acid in it also increases with the degree of decomposition, though less uniformly.

ACTION OF SULPHATE WATERS

For several years the injurious action of sea water upon hydraulic compounds was attributed chiefly to the magnesia in the water. It is noteworthy, however, that chloride of magnesia is almost without action, while sulphate of magnesia acts very energetically upon cement, and it has now been ascertained that magnesia plays only a secondary part, while in fact it is the sulphuric acid combined as a soluble sulphate which is the real cause of the decomposition.

This has been confirmed in practise by the destruction of masonry washed by water which has traversed earth containing gypsum, or built from mortar made with sand which has been extracted from strata containing sulphate of lime.‡ A consideration of this fact makes it apparent how dangerous it is to use, in concrete or masonry subject to the action of sea water, cements to which the gypsum has been added for the purpose of regulating the rate of their setting or of increasing their initial strength.§

There are numerous instances in which brick masonry has rapidly de-

*Smith, Proceedings Institution Civil Engineers, Vol. CVII, 1891-92.
†Feret, Annales des Ponts et Chaussées, 1892, II, p. 93.
‡Bied, Annales des Ponts et Chaussées, 1902, III, p. 95.
§Feret, Annales des Ponts et Chaussées, 1890, I. p. 375.

composed because the bricks, burned with coal, contained alkaline sulphates which when drawn out by water attacked the mortar of the joints.*

These practical observations combined with certain laboratory experiments intelligently conducted have demonstrated that sulphuric acid is the principal agent in causing decomposition. Indeed, we must attribute to dissolved sulphates most of the damage which many American writers have improperly explained as due to the action of "*alkalies*" on the cement.

CHEMICAL PROCESSES OF DECOMPOSITION

Messrs. Candlot,† Michaelis,‡ and Deval§ have discovered successively by different methods that aluminate of lime $Al_2 O_3 3 CaO$, which exists in cements in company with other calcareous salts, such as silicates, possesses the property of combining with sulphate of lime so as to give a double salt $Al_2 O_3 3 CaO, 3 (SO_3 CaO)$ combined with a large quantity of water with great increase in volume. This substance, moreover, has no firm coherence. It is soluble in pure water, but insoluble in lime water, a fact that explains its existence in a solid state in mortars.

On the other hand, even if the cements do not contain free lime when they are anhydrous, their setting under the action of water frees a part of the lime which was combined with the acid elements, principally with silica. If a soluble sulphate other than sulphate of lime is placed in contact with a hydraulic binding material during hardening or after having set, it produces, with the freed lime, sulphate of lime, which in turn combines with the aluminate, giving "sulpho-aluminate," and produces the swelling which causes the disintegration of the mortar. The same reactions would be produced, moreover, without the intervention of free lime as a result of the reaction of the sulphuric acid of the salt dissolved by the water upon a part ot the lime of the binding material.

Although the formation of the sulpho-aluminate of lime seems to be the principal cause of the decomposition of cement by sea water and sulphate waters, it may not be the only one: the setting and the hardening of the cement in contact with water result in the separation of compounds rich in lime, in salts less calcareous, and in free lime. According to the nature of the medium and the conditions affecting its preservation, this reaction may be modified or counteracted in such manner that the hardening cannot

*Zamboni, Industria, October 15, 1899.

†Ciments et Chaux Hydrauliques, Paris, 1891, p. 257.

‡Der Cement-Bacillus, Berlin, 1892.

§Bulletin de la Société d'Encouragement pour l'Industrie Nationale, 1900, I, p. 49.

follow its regular course; likewise, the lime set at liberty may be dissolved little by little in the water which penetrates the mortars, and may disappear by exosmose, giving place to other more or less injurious compounds.

These various phenomena are yet far from being satisfactorily explained, nevertheless, it appears that those cements which are richest in lime are the most quickly decomposed.

SEARCH FOR BINDING MATERIALS CAPABLE OF RESISTING THE ACTION OF SEA WATER

For a long time the efforts of experimenters have been directed toward finding a cement of such composition that it cannot be decomposed by sea water. Thinking at first that the destructive action of the water resulted from the substitution of the magnesia which it contained, for the lime of the cement, the idea was conceived of making cement by burning dolomitic limestone which consequently was composed largely of salts of magnesia. But it was found that the magnesia which this contained, since it was burned necessarily at a very high temperature, was slaked with great difficulty, and by its tardy hydration caused the mortar to swell. Cements were also made experimentally of baryta, a laboratory product whose high price does not permit its introduction into regular practice.*

After the discovery of the sulpho-aluminate of lime, the question changed its aspect, and alumina was considered a dangerous element in cement, the proportion of which ought to be reduced as much as possible. At present the specifications adopted by the Administration of Public Works in France limit to 8% the maximum amount of alumina allowed in cement intended for use in sea water, and this limit would be placed much lower were it not for the fact that in many localities it would be very difficult to obtain products containing less alumina. On the other hand, the percentage of alumina cannot be greatly reduced without at the same time rendering more difficult the burning of the cement, in which operation this element acts as a flux. Accordingly, it was suggested that the alumina be replaced by iron oxide. Cements have been made in the laboratory which were absolutely free from alumina and rich in iron, and these resisted sea water very well.† The various hydraulic cements and limes produced by the works of Teil, whose reputation is world-wide, contain not more than 2% of alumina, and some of them usually last much better

* Le Chatelier, Annales des Mines, May and June, 1887.

† Le Chatelier, Congrès International des Matériaux de Construction, held at Paris in 1900, Vol. II, Part 2, p. 51.

in sea water than most of the Portland cements which contain between 7% and 8% of alumina. These too, however, become decomposed under certain conditions, but with this peculiarity — that their disintegration is not usually accompanied by any increase of volume.

It has been noted that the cements which are the richest in lime decompose the most quickly in sea water. Based upon this observation, the experiment was also tried of making cements for marine use by burning mixtures less rich in carbonate of lime than the ordinary Portland cements. This diminished the strength of the cement, but the falling off in strength was only of secondary importance. The principal difficulty lay in the process of manufacture. In burning cements of this class there was produced in the kilns a considerable quantity of powder possessing only a comparatively feeble hydraulic power, which obstructed the draught. This difficulty was lessened by mixing ferruginous materials (ore, etc.), or even sulphate of lime,* with the raw materials before burning. Also, the use of rotary kilns prevents the choking of the draught. As has just been said, cements low in lime do not attain as great strength as the ordinary Portland Cements, but they generally resist the decomposing action of sea water better.

When the proportion of limestone is small, the burning can be done only at a very low temperature, and the cement obtained sets very quickly. Some of these low lime cements appear to resist chemical decomposition satisfactorily, while others resist no better than most of the Portland cements, a difference which has not yet been explained. In any case, on account of the rapidity of set, this class of cements cannot readily be used on large work, and, in fact, their use is mainly limited to special cases.

Another means of neutralizing the bad effects of the excess of lime liberated by the setting of Portland cement consists in mixing with the latter, before using, materials capable of combining with this lime so as to produce insoluble compounds. Puzzolans have been found to be the most useful material for this purpose. Laboratory tests, verified by experiments on a larger scale,† have shown that mortars made in this way generally resist sea water better than if they had been made from similar cements without puzzolanic material. Sometimes, too, their strength is increased by this mixture.

*Candlot, paper delivered at the meeting of the French and Belgian members of the International Association of the Materials of Construction, on April 25, 1903.

†Feret, Annales des Ponts et Chaussées, 1901, IV, p. 191.

METHOD OF DETERMINING THE ABILITY OF A BINDING MATERIAL TO RESIST THE CHEMICAL ACTION OF SULPHATE WATERS

One method is to gage the cement to be tested with sufficient water to obtain a plastic paste, spread this paste on glass plates so as to form cakes or pats with thin edges, immerse the pats in sea water, and observe them from time to time. But with this method the amount of deformation in the pats depends to a large extent upon the hardness of the paste at the time of immersion, so that a cement which cracks when immersed before setting may stand a long time without showing any trace or alteration if the pat is not placed in contact with the water until twenty-four hours after gaging. Further, the surface of the pat is quickly covered by a crust more or less thick resulting from the partial carbonization of the freed lime, so that the substitution of magnesia for a part of this lime and the presence of this crust may influence the decomposition of the underlying cement.

Another and more exact method consists in molding a block of cement or of mortar of a sufficient thickness; for example, a briquette such as is used for a tensile test. Allow this to harden in the usual way, say for twenty-eight days, then cut out from the center of this block a small solid parallelopiped with sharp edges, and immerse it in sea water or in a sulphate solution (saturated gypsum, sulphate of magnesia, etc.). In order to prevent all new superficial carbonization of the specimen, carbonic acid should not be allowed to come in contact with or be present in this liquid. When decomposition occurs in the cement it is indicated by cracks which appear at the edge of the parallelopiped after a lapse of a variable time.

As a third test, sea water under pressure can be made to filter continuously through mortars made with fine sand. The author of the present chapter uses for this test mortars containing from 250 to 450 kilograms (551 to 991 lb.) of cement per cubic meter (35.3 cu. ft.) of sand (corresponding approximately to proportions 1:6 to 1:3 by weight) which he gages to a plastic consistency and molds into cubes 50 square centimeters (7.74 sq. in.) on a face, with a tube of brass penetrating to the center of the block. After a few days the brass tubes are attached with India rubber tubes to a vessel containing sea water under a head of 2 meters (6.52 ft.). The amount of water which flows through each cube in a given time is accurately measured from time to time, the cube being immersed in sea water in a glass receptacle, where the state of preservation of the mortar can be closely observed.

Finally, the following quite rapid method is used in the laboratory at Boulogne. A mixture is made consisting of 100 parts of cement to be

tested and 300 parts marble ground to a fine powder. To this is added gypsum in the form of a very fine powder, varying progressively from 0% to 20% of the weight of the cement. Plastic mortars are then made from each of these mixtures, which are molded into prisms 2 by 2 by 12.5 centimeters (0.8 by 0.8 by 4.9 in.), allowed to harden for seven days in moist air, and then immersed in fresh water after the length of each has been exactly measured. The water is frequently renewed and at stated periods the lengths of the prisms are again measured, at which time their state of preservation is also examined.

The ability of the cement to resist decomposition by sulphates is indicated by the time taken for the prisms to expand abnormally and to develop cracks, and also by the quantity of gypsum which the binding material is able to bear for a given time without deterioration.

As a result of a long series of experiments, especially of those made by the last two methods, the conclusion has been reached that no binding material has as yet been found which will not be decomposed sooner or later when subjected to these tests, so that at present no cement can be looked upon as absolutely safe from the action of sea water.

MECHANICAL PROCESSES OF DISINTEGRATION

It seems possible to divide the phenomena of disintegration into two classes according as the destruction of the mortar is produced by a sort of progressive dissolution of its elements without appreciable change in volume, or as the products of decomposition, collecting in the pores, enlarge them and produce a scaling off and a weakening of the mortar. This second class of phenomena is much the more frequent and serious.

In both cases decomposition may be produced when the mortar is simply immersed, because of the penetration of the water into its pores and its renewal by the double phenomenon of endosmose and exosmose. But when the masonry is subjected to different degrees of pressure upon its opposite faces, as is usually the case, this tends to establish a current of water through it and the replacement of the dissolving elements goes on more actively. However, disintegration may, under these conditions, proceed more slowly if the current of water is strong enough to carry away the solid products of decomposition as they are formed. The writer has cited in a former paper* experiments which plainly show the difference between these two methods of decomposition: if lean mortars, made with the same cement and sands of different granulometric compositions, are kept in absolutely quiet sea water, those which disintegrate most rapidly are the ones

*Annales des Ponts et Chaussées, 1892, II, pp. 106 to 116.

into whose composition there enters no fine sand, but only medium sand or, and above all, coarse sand. These latter are the mortars that contain the voids of largest size. On the contrary, if a series of similar mortars are subjected to a continuous filtration of sea water, those made from coarse sand remain intact, while decomposition is more and more active for mortars containing more and more fine sand. *In practise this latter is the most frequent case, and, in fact, it has been verified that the destruction of concrete or mortar by sea water has in most cases been due to the use of too fine sands.*

This is a point which cannot be too strongly insisted upon, and experiments show that a rather lean mortar of coarse sand is much preferable to a mortar of fine sand, even when a very large quantity of cement is introduced into the latter. Fine sands ought to be banished relentlessly from sea water construction even when the cost of coarse sand is very high.* When stone is at hand, an excellent sand can be obtained economically by crushing it.

PROPORTIONS FOR MORTARS AND CONCRETES

From the preceding it is evident that the best means of fighting against sea water is to prevent as far as possible its penetration into the mortars and concretes, and accordingly to make these of great density. It has been suggested in a preceding chapter (Chapter IX) with what size of sand and what quantity of cement this result can best be attained in mortars: the author of the present chapter has ascertained that the maximum density is obtained with a mortar composed of material having about two parts of very coarse grains to one of fine grains, including cement. Usually, natural sands, even the coarsest, contain a proportion of relatively fine sand sufficient to make it useless to add more with the cement. If a sand is used from which the fine grains have been screened, and this is mixed with about one-half of its weight of cement, a mortar is obtained at once very dense and of great strength, but whose use would often be too costly. In such cases the cement can be replaced by a mixture of sand and cement prepared in advance, such as the product known as "sand-cement," for the making of which a few factories have been built in Europe and also in America. It must be borne in mind, however, that this solution, excellent for mortars destined to remain in the air or to come in contact only with fresh water, would be poor to use in sea water, for very fine sand intimately mixed with cement separates its grains and increases the surface of attack, and various experiments have shown that this kind of mortar suffers severely in sea water.

* See also, Feret, Baumaterialienkunde, 1896, p. 139, and "Le Ciment," 1896, p. 212.

For use in sea water, on the contrary, if a good puzzolanic material can be procured on favorable terms, it is advantageous to grind this with the cement to take the place of the fine sand, so that in the mortar it may play both a mechanical and a chemical role, assuring to it a great density, and at the same time forming, with the lime freed by the setting, compounds which tend to harden the mortar and render it impermeable.

For concretes it has not yet been possible to express a general law. However, maximum density appears to be attained, in general, when the fine grains, including the cement, constitute, according to conditions, from 25 per cent. to 40 per cent. of the total mixture, the remainder being made up chiefly of large aggregate with little or no medium sized materials. In all cases it is necessary to see that the concrete does not contain voids, and above all that the cement is not diluted by an excess of fine sand, which must always be considered as the greatest enemy of masonry in sea water.

In every case the sea water should be prevented from coming in contact with the work for as long a time as possible, so that the setting of the cement may be already considerably advanced. Yet it must not be forgotten that when the mortar contains a puzzolanic material its hardening can be properly effected only in the presence of moisture.

MIXTURES OF PUZZOLAN AND SLAG WITH CEMENTS

Tests by M. Vetillart and the writer, described in detail in a paper published in Annales des Ponts et Chaussées, 1908, I, page 121, indicate that Puzzolanic material may be of great value when mixed with Portland cement for concrete construction in sea-water, materially increasing the durability of the concrete without increasing its cost.

The conclusions reached in these tests are as follows:

The use of Puzzolan in hydraulic mortars in combination with the cement increases the strength, and in a great many cases appreciably retards disintegration by sea-water. It should be employed then, at least experimentally, in accordance with the following recommendations:

Grind the Puzzolan to the fineness of Portland cement.

Mix it mechanically with the cement so as to obtain an absolutely thorough mixture.

For Portland cement and a good natural Puzzolan, take two parts by weight of cement to one part of Puzzolan.

Select only Puzzolan of known good quality; the use of gaize slightly roasted is especially recommended.

If other kinds of cement or limes are used with Puzzolan, or if the Puzzolan is of doubtful quality,—especially if it is obtained from granulated slag or a similar industrial by-product,—determine the proportions of the mixture by means of preliminary trials based on tests of strength.

Add to the sand the mixture of cement and Puzzolan as pure cement would be added, and in the same proportions; mix and place the mortar in the usual manner.

Always use for comparison with the Puzzolan mortar, specimens of mortar, of the same proportions and made under identical conditions, in which the mixture of cement and Puzzolan is replaced by the same weight of pure cement.

Allow the Puzzolan mortar to harden in the presence of moisture.

It is as yet impossible to suggest detail rules for the acceptance and control of Puzzolan cements. The recommendation is made, however, that their ability to resist the decomposing action of the salts in sea-water be compared to the resistance of pure cements by means of the test with sulphate magnesia already referred to.*

VARIOUS PLASTERS AND COATINGS

Various methods have been tried to prevent sea water from wetting masonry too soon, either by coating the work with materials designed to obstruct the pores, or by covering it with a layer more or less thick and more or less impermeable, consisting usually of a rich mortar, clay, bituminous materials, etc.

This method of protecting the work is generally rather costly and is not applicable to all kinds of construction. Besides, it presents this disadvantage, that if by accident there is any break in the continuity of the covering, the sea water finds a passage towards the heart of the masonry and creeps in from one place to another, so that often the coating offers only an illusory security.

In certain cases, a coating is formed spontaneously by the carbonization of the lime in the parts of the mortar near the free surface, and this action is aided by the development of sea organisms such as sea-weed and shellfish. This cause, together with the differences in the saltness and the temperature of the water, and the course of the ocean currents, is the one which is most often called upon to explain why mortars decompose more quickly in some regions than in others.

* See also Annales des Ponts et Chaussées, 1908, I, p. 107.

CHAPTER XVI

LAYING CONCRETE AND MORTAR IN COLD OR FREEZING WEATHER

The results of practice and experiment with cement and concrete exposed to frost or cold, which are discussed in detail in the following pages, may be summarized as follows:

(1) Concrete work in winter is more difficult and somewhat more expensive than in summer, but policy frequently makes winter work necessary and even economical, and with precautions to prevent freezing first-class work results.

(2) The setting and hardening of Portland cement concrete, or mortar is retarded by the cold even if not frozen and the strength at early periods is low.

(3) Concrete which has been frozen may attain eventually, after thawing and hardening, an ultimate strength nearly as high as ordinary concrete.

(4) A thin scale is apt to crack from the surface of concrete walks or walls which have frozen before thorough hardening. **(See p. 282.)**

(5) Frost expands cement masonry or concrete and settlement results with the thawing. **(See p. 282.)**

(6) Heating materials hastens setting and retards the action of frost. **(See p. 286).**

(7) Salt and calcium chloride lower the freezing point of water and if used in small quantities do not appear to affect the ultimate strength of the concrete or mortar. **(See p. 287.)**

EFFECT OF COLD OR FREEZING

Cold, even if the temperature is not below freezing, retards the hardening of concrete and mortar. When also the cement or the sand possesses slow hardening properties, the concrete may remain soft enough to break with the fingers for several months after laying. This is particularly the case with concrete placed below ground, as in piles. On the other hand, concrete of Portland cement, even if actually frozen, will eventually attain after thawing fair ultimate strength with only slight surface injury, if moisture is present or applied to permit proper hydration of the cement. This freedom from much permanent injury from frost

may be due in part to the internal heat of crystallization, especially in the interior of a large mass.

A thin crust about $\frac{1}{16}$ inch thick is apt to scale from the surface of granolithic or concrete pavements which have frozen, leaving a rough instead of a trowelled surface. A similar result occurs in walls.

The settlement of masonry due to contraction from thawing is another factor that must be allowed for in case of freezing. This settlement occurs both in concrete and in stone or brick masonry. Failures of newly laid brick walls, for example, have occurred through the frozen mortar thawing out on the surface next to the sun, with a resulting settlement which causes the wall to topple over.

If for any reason the concrete actually freezes during construction, care must be taken to be sure that any laitance or scum which had risen to the surface and which, when frozen, resembles good concrete, is chipped off.

Freezing Experiments. An extensive series of tests of frozen mortars was conducted by Mr. Thomas F. Richardson during the construction of the Wachusett Dam in Massachusetts, indicating that Portland cement mortar is not permanently injured by freezing. The results of tests extending up to one year showed that although briquettes mixed one part cement and three parts sand, had less strength at the end of seven days than those which had not been frozen, the frozen specimens after longer periods, especially at the end of one year, gave as high or higher strength than those kept at ordinary temperatures.

Mr. Richardson describes* the tests which were made in the middle of winter as follows:

During the progress of the masonry work on the Wachusett Dam briquettes were made each week and submitted to the same conditions as the masonry, the molds being filled with mortar and placed out of doors in the air, not in the water, immediately after filling. At the same time briquettes were made and kept in the laboratory, both in air and in water, those in the air approximating more closely the conditions which obtained on the masonry construction at the dam. About $\frac{1}{2}$ of the briquettes out doors were exposed to temperatures as low as 9° above zero in the first 24 hours, and some of them to temperatures as low as 12° below zero in the first week. Salt was used in most of the experiments, the quantity ranging from 4 to 16 pounds per barrel of cement, the average being about 6 pounds or about 3% by weight of water. Our experiments indicate that 8 pounds of salt per barrel of cement is sufficient, even in the coldest weather, and the results from 4 pounds are very nearly as good; 16 pounds do not seem to give quite as good results.

* Kindly furnished by Mr. Richardson for this Treatise.

The following table gives the average results of the experiments:

Effect of Frost upon Tensile Strength of 1:3 *Mortar.* (*See p.* 282.)
BY THOMAS F. RICHARDSON.

Briquettes Kept	No. of Briquettes	Tensile Strength, lb. per sq. in.				
		7 d.	28 d.	3 mo.	6 mo.	1 yr.
Water in laboratory..................	20	268	304	359	370	401
Air in laboratory.....................	20	298	352	364	392	517
Out doors, below freezing............	80	139	238	344	435	627

The briquettes were made in sets of 5, consequently 4 experiments are shown for water and air in laboratory, and 16 for out doors.

In France similar results have been reached by Mr. P. Alexandre* as to the effect of temperatures slightly above freezing.

Mr. Charles S. Gowen† also has concluded from his tests that "there is no indication that freezing reduces the ultimate strength of the mortar, although it delays the action of setting."

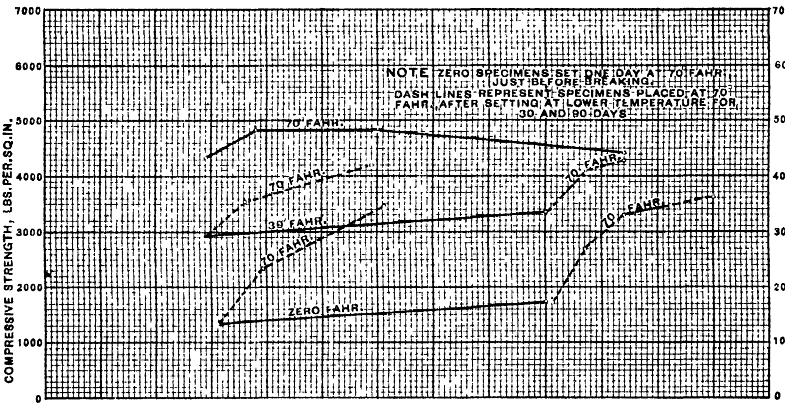


FIG. 81.—Strength of Neat Portland Cement Mortar, 2-inch Cubes, Set in Air at Different Temperatures. (*See p.* 283.)

The effect of different uniform temperatures upon neat cement mortar is illustrated in Fig. 81 by curves made up by the authors from a series of experiments by Mr. J. E. Howard‡ at the Watertown Arsenal. The results with sand mortars are similar to the neat cement. Specimens were stored for thirty days at temperatures of 70° Fahr. (21° Cent.)

* Annales des Ponts et Chaussées, 1890, II, pp. 302 and 422.
† Proceedings American Society for Testing Materials, 1903, p. 393.
‡ Tests of Metals U. S. A. 1901, p. 530.

38° Fahr. (3° Cent.) and 0° Fahr. (−18° Cent.) Part were broken after setting one extra day at 70° Fahr. to thaw and the others were placed in a temperature of 70° Fahr. and broken one week and three weeks later. The balance were kept till 90 days old at the lower temperatures and then part broken and part placed at 70°. The curves show the low strengths even at the age of 90 days with the lower temperatures and the sharp increase as soon as the cubes were placed in warm air.

Cold retards setting. Prof. Tetmajer* found, for example, that 1 : 3 Portland cement mortar which attains its initial set at $2\frac{3}{4}$ hours and its final set at $8\frac{1}{2}$ hours when mixed at 65° Fahr. (18° Cent.), at a temperature of freezing reaches its initial and final set at 21 and 38 hours respectively.

Effect of Temperature on Growth in Strength of Concrete. Tests at the University of Illinois† show that concrete stored at temperatures ranging from below freezing to about 90° Fahr. gains strength with age in proportion to the temperature. The lower the temperature the lower the strength at all ages, and the slower the growth in strength. Specimens subjected to alternate freezing and thawing disintegrated badly, but those maintained well below freezing gained slightly in strength. Fig. 82 indicates the effect of temperature on strength of concrete at different ages. The curves are drawn by the authors from the results of the tests, but are plotted to apply to concrete testing 2 000 pounds‡ per sq. in. at 28 days and 70° Fahr. (the common standard for 1 : 2 : 4 concrete). The diagrams may be used in practice to indicate approximately the growth in strength of concrete in building construction.

CONSTRUCTION IN FREEZING WEATHER

It is frequently necessary to erect concrete structures in winter, notwithstanding the extra cost, because of the economic value of early use. In such cases full precautions should be taken to prevent freezing and, in fact, to guard against low temperatures, for temperatures slightly above freezing retard setting and hardening. Although the tests cited above indicate that concrete, frozen after mixing, attains some strength if allowed to remain long enough before subjected to alternate thawing and freezing, it is a dangerous proceeding to permit on account of the possibility of trouble and it should be avoided except under most extra-

* Johnson's Materials of Construction, 1903, p. 616.

† A. B. McDaniel. University of Illinois, Bulletin No. 81, 1915.

‡ The actual breaking strength in the 28 day tests was about 1365 pounds per square inch, a low strength even for soft limestone.

ordinary circumstances. If necessary to continue work through the winter, precautions must be taken by (1) careful selection of materials, (2) heating of materials and (3) protection from the cold.

Materials to Use in Cold Weather. Natural cement of ordinary composition, as is shown conclusively by tests and practice, never regains its strength after freezing, and therefore never should be used where liable to freeze.

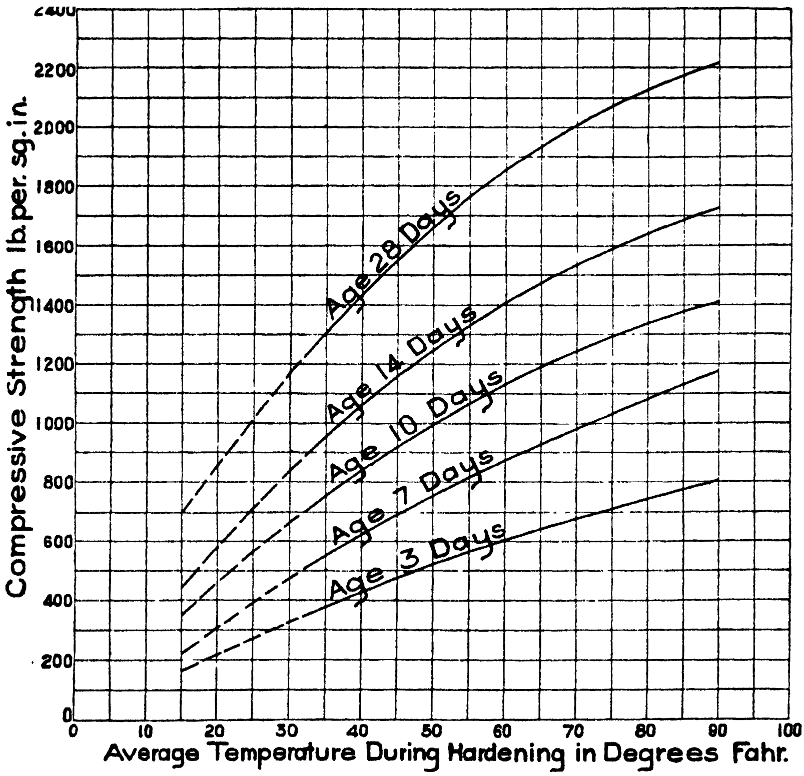


FIG. 82.—Effect of Temperature on Growth in Strength of Concrete. (*See p.* 284.)

A slow hardening Portland cement (the hardening determined not by the set but by the tensile strength at early periods) should be avoided for cold weather use. Not only should the cement pass the Standard Specifications, but it is sometimes advisable to require a higher than usual strength (in the laboratory tests) at the 24 hour neat and 7 days sand mortar periods. It is frequently advisable to select a brand which normally gives high strength at these early periods.

Of equal importance with the selection of the cement is the choosing of the sand, especially should a fine sand or a sand containing even a minute quantity of vegetable loam be avoided, as the reduction in the early strength incident to this may even be a contributary cause for failure in cold weather.

Heating Materials. At moderate temperatures of say 30° to 40° Fahr., heating of the materials is sufficient to prevent trouble, provided cement and sand are satisfactory. If a drop in temperature below freezing is liable, the concrete should be protected, as with canvas supported so as to provide an air space, or with straw which is absolutely free from manure. The water, sand and stone should be heated and in extreme cases the reinforcement and the forms may be steamed just before placing the concrete and a steam jet may even be played into the mixer drum* to warm it up.

To heat the sand and gravel or stone, steam pipes may be run as grids or flat coils through the bins or storage piles. It is possible to arrange the piles so that the material will drop through the grids when required for use so as to have a thorough and even heating. On small work, as for hand mixing, fires under boiler plates can be used but are inadequate on large jobs.† If not too chilled, the steam from the sand and stone coils may be run to the water tank to be used there as a coil or a jet, the former being preferable. Water should be heated to 100° to 120° Fahr. Excessive heating should be avoided as it may accelerate too much the setting of the cement.

Exhaust steam in any form, or live steam in a jet is of little use in heating concrete materials.

Protection from Frost. Covering with straw or canvas is inadequate for low temperatures, say below 28° Fahr. unless artificial heat is used.

In building construction in addition to heating the materials, which should be done in any case, it is customary in order to prevent freezing and maintain a temperature for proper hardening, to enclose in canvas as soon as the column forms are erected on a floor. Stoves or salamanders are placed on the finished floor to keep the cold from the surface of the new slab, while on the newly laid work straw is spread, or better still, canvas supported so as to provide an air space which may be warmed by steam coils or by allowing warm air to come up from the floor below through holes left for the purpose. The number of stoves

* See *Engineering Record* March 16, 1912, p. 295.

† A combined water, sand, and stone heater designed for easy transportation is illustrated in the second edition of this Treatise, p. 324.

or salamanders required depends upon the temperature. For low temperatures, say below zero, one stove for three hundred square feet* of area may be necessary, while ordinarily a stove for every six hundred square feet is sufficient.

Care should be taken to prevent overheating, especially of granolithic surfaces, thus avoiding checking.

Addition of Salt. If the concrete is kept from freezing or from reaching too low a temperature, no further precautions are necessary. Salt, because it lowers the freezing point of water, permits the laying of concrete at comparatively low temperatures. Glycerine and alcohol, have been experimented upon, but tend to lower the strength of the mortar. Salt should not be used where there is danger of electrolysis, nor in reinforced concrete where there is liable to be an excessive amount of moisture present Rules have been formulated for varying the percentage of salt with the temperature of the atmosphere. Prof. Tetmajer's† rule, for example, reduced to Fahrenheit units, requires 1% by weight of salt to the weight of the water for each degree Fahrenheit below freezing.

A rule frequently cited in print, which practical tests by the authors have proved to be entirely inadequate, is to require one pound of salt to 18 gallons of water for a temperature of 32° Fahr. and an increase of one ounce for each degree of lower temperature. Since the temperature of the air usually cannot be determined in advance, an arbitrary quantity is as suitable as a variable one. In the New York Subway work in 1903, 0% of salt to the weight of the water was adopted. On the Wachusett Dam, during the winter of 1902, 4 pounds of salt were used to each barrel of cement. For 1 : 3 mortar this corresponded to about 2% of the weight of the water.

Experiments show that ordinary "quaking" concrete in proportions 1 : 2½ : 5 requires about 130 pounds of water per barrel of Portland cement, hence 10% of salt in average concrete is equivalent to 13 pounds per barrel of Portland cement. Ordinary 1 : 2½ mortar requires about 120 pounds of water per barrel of Portland cement, hence 10% of salt in average mortar is equivalent to about 12 pounds salt per barrel of Portland cement. Salt is sometimes added in sufficient quantity to "float a potato" or an egg. About 15% of salt to the weight of the water is required to float a potato, and about 11% to float an egg.

* Turner Construction Company, *Engineering News*, September 24, 1914, p. 636

† Johnson's Materials of Construction, 1903, p. 615.

Recent experiments, by Mr. Gowen[*] and Mr. Richardson,[†] extending up to a period of one year, tend to show that salt in a quantity corresponding to at least 10% of the weight of the water does not lower the ultimate strength of ordinary mortar. The time of setting, however, is considerably lengthened and the strength at short periods is lowered. The effect, at laboratory temperature, of 10% salt with 1:3 Portland cement mortar is illustrated in the following table:

Tensile Strength of 1:3 Mortars made with Fresh and Salted Water.
BY CHARLES S. GOWEN.

	1 week.	1 mo.	3 mos.	0 mos.	9 mos.	12 mos
Fresh water used.........	112	183	268	335	351	458
Salted water used.........	68	131	215	266	301	413

In Mr. Richardson's experiments[‡] smaller percentages of salt proved beneficial. Portland cement mortar in proportions 1:3, mixed with 4 and 8 pounds of salt per barrel cement (corresponding respectively to about 2% and 4% of the weight of the water), gave slightly higher tensile strength than the unsalted mortar at all periods from 7 days to one year.

Experiments by Mr. E. S. Wheeler[§] indicate that the use of 10% of salt tends to prevent the swelling of briquettes in the molds, even if the specimens freeze.

Practical Proportion of Salt. Since in practice it is impossible to tell how low the temperature will fall before the concrete sets, Mr Thompson has adopted the arbitrary rule of 2 pounds of salt to each bag of cement to be used when the temperature is expected to fall several degrees below freezing, and if experience shows this to be insufficient to prevent the frost catching the surfaces, 3 pounds of salt are to be used.

The salt can be added most conveniently by putting it into the mixing water. To determine the amount of salt per barrel or per tankful of water, the quantity of water used per bag of cement must be noted and from this the amount can be readily figured.

Calcium Chloride. Experiments indicate that calcium chloride added in quantities not exceeding 2% of the weight of the cement is an effective agent for lowering the freezing point of the concrete. It should be used with caution, however, since a larger quantity than this is likely to so hasten the set as to make the concrete difficult to handle.

*Proceedings American Society for Testing Materials, 1903, p. 393.

†Report Metropolitan Water and Sewerage Board, 1903, p. 112.

‡See page 283.

§Report Chief of Engineers, U. S. A., 1895, pp. 2963 to 2971.

CHAPTER XVII

DESTRUCTIVE AGENCIES

FIRE PROTECTION

Tests and experience have proved reinforced concrete to be the best all-round building material in ability to resist fire. The surface of the concrete is apt to be injured, but the main body and the imbedded steel is amply protected against all but the worst fires. Hence, a proper thickness of concrete outside the steel, over and above the section required to carry stress, acts as fireproofing during the fire and can be replaced or repaired if necessary, without endangering the structure. Just what constitutes a proper thickness depends upon conditions that vary from a small fire in an out-of-the-way passage to a conflagration.

In members of little importance, liable to low temperatures, only ¾ inch of concrete is sufficient protection for the reinforcement; for main beams and columns, liable to ordinary fires, 1½ to 2 inches is needed; for slabs ¾ to 1 inch is customary. Where there is a possibility of such an intensely hot fire as that in the Edison plant at West Orange, N. J., in 1914, it is not economical to design with sufficient structural protection to insure against structural damage and an overhead sprinkler system and similar protective devices should be used. The question as to whether or not the protective covering in columns should be taken as a part of the section depends upon the conditions to which they are to be subjected. If the building contents are liable to be inflammable, at least 1½ inches should be considered as protective covering and not included in the effective section.

As for the concrete itself, the denser and richer it is, the more effective is its resistance. The character of the aggregate has a good deal of influence; cinder concrete is a specially good non-conductor of heat, but too weak for important work. Of the stronger aggregates, trap is particularly good; any stone containing much quartz tends to split and crumble, while limestone is more liable to disintegration. In the design certain precautions are feasible; columns can be built circular and sharp, projecting corners avoided in all cases as far as possible, so that the flames cannot attack from two sides at once. Hooping will keep the vertical column steel from springing in case the concrete spalls off.

Repairs in Case of Fire. In ordinary cases the repairs to buildings injured by fire consist in removing all injured concrete and replacing with new mortar. To repair the unusually extensive damage done the Edison buildings* it was necessary to remove injured concrete, wind the reduced cross-sectional area with closely spaced spiral hooping—supported by vertical spacing bars—place circular column forms, and fill with mortar. In case the old vertical reinforcement had buckled with the heat, new bars were placed. Where complete collapse had occurred, the floor above was jacked up and a new column built.

THEORY OF FIRE PROTECTION

Mr. Spencer B. Newberry, in an address delivered before the Associated Expanded Metal Companies, Feb. 20, 1902,† gives the following explanation of the fire-proof qualities of Portland cement concrete:

The two principal sources from which cement concrete derives its capacity to resist fire and prevent its transference to steel are its *combined water and porosity*. Portland cement takes up in hardening a variable amount of water, depending on surrounding conditions. In a dense briquette of neat cement the combined water may reach 12%. A mixture of cement with three parts sand will take up water to the amount of about 18% of the cement contained. This water is chemically combined, and not given off at the boiling point. On heating, a part of the water goes off at about 500° Fahr., but the dehydration is not complete until 900° Fahr. is reached. This vaporization of water absorbs heat, and keeps the mass for a long time at comparatively low temperature. A steel beam or column embedded in concrete is thus cooled by the volatilization of water in the surrounding cement. The principle is the same as in the use of crystallized alum in the casings of fireproof safes; natural hydraulic cement is largely used in safes for the same purpose.

The porosity of concrete also offers great resistance to the passage of heat. Air is a poor conductor, and it is well known that an air space is a most efficient protection against conduction. Porous substances, such as asbestos, mineral wool, etc., are always used as heat-insulating material. For the same reason cinder concrete, being highly porous, is a much better non-conductor than a dense concrete made of sand and gravel or stone, and has the added advantage of lightness. In a fire the outside of the concrete may reach a high temperature, but the heat only slowly and imperfectly penetrates the mass, and reaches the steel so gradually that it is carried off by the metal as fast as it is supplied.

* Report on Fire of Edison Phonograph Works, by the National Fire Protection Association and National Board of Fire Underwriters, January 30, 1915, p. 34.

† *Cement*, May, 1902, p. 95.

TESTS OF FIRE RESISTANCE

Prof. Ira H. Woolson of Columbia University has made several series of tests* to determine the effect of heat upon the strength and elastic properties of the concrete and upon the thermal conductivity of the concrete and the imbedded steel.

Effect Upon Strength. Tests to determine the effect of heat treatment upon the strength and elastic properties of different mixtures showed that the trap concrete was least affected. Concrete two months old, in proportions 1:2:4, the crushing strength of which before heating was about 2500 pounds per square inch tested in 7-inch cubes, after being subjected to a heat of 1500° Fahr. for two hours gave a strength of about 1000 pounds per square inch. However, since this reduction in strength was due at least in part to the reduction in the effective area because of the surface deterioration (if the surface was injured to a depth of 1¼ inches the effective area would be reduced from 49 sq. in. to 20 sq. in.), it is probable that the interior of the blocks was affected very little. The concrete made with gravel, which in these tests was nearly pure quartz having a high coefficient of expansion, was affected to a much greater extent. Cinder concrete, which showed a normal crushing strength of about one-half that of the trap, after heat treatment gave a corresponding weakening.

The modulus of elasticity of the concrete was always greatly reduced by heat treatment.

CONDUCTIVITY OF CONCRETE AND IMBEDDED STEEL

As a result of the conductivity tests, which were made upon specimens of trap, gravel and cinder concrete having thermo-couples for measuring heat transmission imbedded so as to indicate the temperature at points varying from ½ inch to 6 inches from the heated face, Prof. Woolson drew the following conclusions:†

All concretes have a very low thermal conductivity, and herein lies their ability to resist fire.

When the surface of a mass of concrete is exposed for hours to a high heat, the temperature of the concrete one inch or less beneath the surface will be several hundred degrees below the outside.

A point 2 inches beneath the surface would stand an outside temperature of 1500° Fahrenheit for two hours, with a rise of only 500° to 700°, and points with three or more inches of protection would scarcely be heated above the boiling point of water.

* Proceedings of American Society for Testing Materials, Vol. V, 1905, p. 335; VI, 1906, p. 433; VII, 1907, p. 404.

† Proceedings American Society for Testing Materials, Vol. VII, 1907. p. 408.

The fact that cinder concrete showed a higher thermal conductivity than the stone concrete would indicate that its well-known fire-resistive qualities are due, in part at least, to the incombustible quality of the cinder itself.

The thermal conductivity of the gravel concrete* was fully as low as that of the trap, but the specimens of gravel concrete cracked and crumbled in many cases when the trap and cinder specimens under similar treatment remained firm and compact.

In the tests on the conductivity of imbedded steel with the end projecting from concrete, Prof. Woolson found practically the same results with concrete from all three aggregates. With the temperature of the end surface of the concrete and the projecting end of the bar 1700° Fahrenheit, a point in the bar only 2 inches from the heated face of the concrete developed a temperature of only 1000° Fahrenheit, while at a point 5 inches in the concrete the temperature was only 400° to 500°, and at 8 inches the temperature reached only the heat of boiling water.

From these results Prof. Woolson concludes that "where reinforcing metal is exposed in the progress of a fire, only so much of the metal as is actually bare to the fire is seriously affected by it."

Tests by the National Fire Protection Association† in 1905 upon beams 8 inches by 11¼ inches by 6 feet long, of different kinds of concrete, showed that the strength of rods imbedded 1 inch from the lower surface was reduced about 25 per cent after heating to a temperature of 2000° Fahrenheit for one hour. With rods imbedded 2 inches a similar reduction in strength occurred after 2 hours and 20 minutes heating, and the strength of the concrete was appreciably reduced to a depth of 4 inches from the sides and bottom.

The hardest and densest mixtures were usually the poorest conductors of heat; the cinder concrete gave, however, a slower rise of temperature than the others.

PROTECTION OF STEEL FROM RUSTING

Concrete, if mixed wet, protects steel reinforcement from rusting. The wet concrete flows around the steel and forms a thin film of cement that prevents attack by impurities in the atmosphere or in the aggregates. If mixed dry, stone or cinder pockets will form along the steel in which rusting is liable to begin. Cracks due to load or to tempera-

*As stated in connection with the tests on preceding page, this gravel was nearly pure quartz In other tests, concrete with gravel containing a larger percent of slate or other similar material has given much better results.

† *Cement*, January, 1906, p. 273.

ture stresses are scarcely ever large enough at point of contract with the steel to afford the slightest danger.

In razing buildings reinforcing and structural steel imbedded for many years in first-class dense concrete has been found in perfect condition.

The structural steel in the Boston subway,* imbedded for twelve years in concrete or protected by the cement mortar joints of brick arches, was found upon examination during changes in the structure to be free from rust. The only exception to this was under the rather large base plates (21 by 24 inches) of columns, where a thin layer of rust frequently was found, having tubercles sometimes $\frac{1}{4}$ inch thick. This was evidently due to the settling of the finer parts of the concrete under the plates. The small base-plates were practically free from rust.

It has been seriously questioned whether the minute cracks which open in a concrete beam and slab even under loads which are absolutely safe do not permit corrosion of the steel reinforcement. Tests by A. Probst† in Germany, in 1907, indicate very conclusively that steel in reinforced beams, laid in ordinary wet concrete used in practical construction, is in no danger of rusting through the cracks formed in the concrete under tension, until near the breaking point of the steel. The specimens, 34 beams, which contained both plain and deformed bars and rusted and unrusted steel, were subjected in loading to the action of a mixture of oxygen, carbon dioxide, and steam, for a period of from 3 to 12 days. Unprotected steel subjected to this mixture was badly rusted in two hours. After breaking up the specimens of concrete no rust was found even on steel stressed to its elastic limit, although some was discovered on steel stressed nearly to its breaking point, which could be attributed to large cracks extending to the metal and uncovering it.

EFFECT OF ACIDS

Dilute acids will attack green concrete and prevent its hardening. Therefore, if concrete is to be laid under water the purity of the water both from acid and strong alkalis must be determined. The discharge of a pulp mill into a river may prevent the hardening of concrete bridge piers built in the river below the mill, but concrete that is well cured will resist successfully such acids as those in sewage even if quite concentrated. If acids from factory wastes are to be discharged direct to the concrete sewers, special investigations are necessary.

* Personal correspondence with Mr Howard A Carson, Chief Engineer.

† Report of the Royal Department of Testing Materials in Gross Lichtenfelde, West Prussia

Manure has no effect on seasoned concrete, although it is liable to injure green concrete.

Strong, concentrated acids will attack nearly every material and concrete is no exception, the acid acting on the carbonate of lime in the cement. Nevertheless, mortar is used either alone or in combination with tile as a lining for digestors in pulp mills, where sulphurous acid is present under high heat pressure, and also for lining acid tanks. An unlimited supply of ground water containing sulphuric acid will in time cause complete disintegration.*

A dense rich concrete or mortar is essential in resisting acid action. The failure of sewage tanks in one or two cases has been traced to poor concrete.

EFFECT OF ALKALIES

There have been numerous failures in the West due to alkali in soil and ground water, but there can be no doubt that nearly all are due to failure to recognize the conditions and provide for them. A permeable concrete on the other hand allows ground water to filter through, depositing salts that expand when crystallized and disintegrating the concrete in a manner similar to the action of sea water described by Mr. Feret on page 272.

EFFECT OF OILS

Oils and fats—mineral, animal, and vegetable—can with few exceptions be safely handled in buildings of first-class concrete that was properly set. Floors in soap factories and machine shops have shown no harmful injury in many years.

In certain manufacturing processes, on the other hand, where animal fats are heated to high temperatures concrete has been badly disintegrated. Concrete tanks are liable to attack in this manner and floor slabs above the tanks, if subjected to the steam or vapor from the tank. Cocoanut oil and olive oil have proved destructive in tests. Mineral oils usually have no effect. Cold lard oil has no effect.

Mr. Toch† states that the action of fat or vegetable oil is due to expansion caused by the formation of crystals of sterate and oleate of lime.

References to detailed accounts of the action of acids, alkalis, and oils, are given in Chapter XXXIII.

* New York Board of Water Supply, 8th Annual Report, 1913, p. 56.

† *Engineering News*, April 20, 1905, p. 419.

ELECTROLYTIC ACTION

Injury to reinforced concrete from electrolysis is rare in practical construction, and much of the damage attributed to it has been due probably to other causes. Plain concrete, as shown by tests and experience, is never injured. The danger to structural steel, even if encased in concrete, is greater than to reinforced concrete. In locations where electrolysis is liable to be present, certain precautions should be taken, such as insulating electrical transmission and, since conduction is greatly accelerated by moisture, in making the concrete water-tight.

Exhaustive tests by the Bureau of Standards* indicate two forms of injury from electrolysis; (1) When the reinforcement is positive, which occurs when the electricity flows from the steel to the concrete, rusting the steel with consequent splitting of the concrete; (2) When the reinforcement is negative, the current flowing from the concrete to the steel, softening the concrete around the steel and destroying the bond. This softening of the concrete appears to be caused by the concentration along the steel, under the action of the current, of the sodium and potassium in the cement in sufficient quantities to attack the cement.

High voltages (gradients of 60 volts or more per foot) are necessary to produce enough rusting to split the concrete; on the other hand with a negative current even low voltages may destroy the bond to the steel and this action is not easily detected until actual failure begins. In structures liable to electrolysis no salt or calcium chloride should be used in mixing the concrete, since the rate of corrosion is increased many hundreds of times by the presence of these materials and low voltages therefore may be dangerous.

* Paper by E. B Rosa, Burton McCollum and O. S. Peters; Journal-American Concrete Institute November, 1914.

CHAPTER XVIII

WATER-TIGHTNESS

Concrete with first-class workmanship may be made practically impermeable on ordinary construction work by proper proportioning, mixing, and placing, and no other means, such as surface coatings, foreign ingredients, or membranes, need be used. Structures that are to be water-tight require special skill in design and construction; cracks must be prevented or at least controlled; proportions should be worked out for proper density; and the mixing and placing must be handled with care and skill.

CONCRETE FOR WATERTIGHT WORK

To secure water-tight work it is important to:

(1) **Adopt a fairly rich mix.** (See p. 298.)

(2) **Proportion aggregates to secure a dense mixture.** (See p.298.)

(3) **Mix concrete to quaking or wet consistency.** (See p. 298.)

(4) **Place concrete carefully to avoid stone pockets.** (See p. 299.)

(5) **Lay entire structure if possible in one operation, without joints.** (See p. 297.)

(6) **If joints are unavoidable, clean and roughen old surface, wet it thoroughly, and coat with a layer of neat cement paste.** (See p. 297.)

(7) **Provide for contraction in long structures by special joints or by steel reinforcement.** (See p. 297.)

(8) **For continuous structures or where poor workmanship is feared, introduce membrane waterproofing.** (See p. 302.)

If leakage occurs through concrete walls it is almost invariably through horizontal or vertical joints, through cracks caused by temperature contraction, or through porous stone pockets due to poor construction. Where these difficulties cannot be overcome or when the damage or inconvenience in case of leakage is liable to be considerable, it is economical to use some supplementary method. Of the three methods in general use mentioned above, textile or felt membranes coated with asphalt or tar pitch, although expensive, are most reliable, and are used to advantage on such structures as subways and bridges

where traffic must be maintained. (See p. 302.) The use of certain foreign ingredients is relatively cheap and in certain cases should receive consideration. Nevertheless, the question should always be considered whether the ingredient should not be extra Portland cement. Surface coatings, with a few exceptions, are of doubtful value. A plaster coat mixed with some ingredient is likely to split off and is expensive to apply. Both integral waterproofing and plaster coatings fail in the event of cracks.

Design and construction are of equal importance for structures that are to be water-tight. The special considerations are thickness of wall to prevent seepage, reinforcement to reduce effect of temperature contraction, and water-tight joints, proper proportioning, consistency, mixing, placing, and curing.

Thickness of Wall. A wall properly designed to resist the stresses is generally thick enough to resist percolation of water. A minimum thickness under any condition may be considered as 6 inches, so as to give room for placing of reinforcement and proper placing of the concrete around it. Examples are cited in Chapter XXXIII of a 15-inch wall sustaining a head of 40 feet of water, and a $5\frac{1}{2}$ foot wall, a head of 100 feet.

Reinforcement. To avoid cracks in water-tight construction due to unequal settlement, shrinkage in setting, and temperature contraction, reinforcement always should be used except in mass work where special contraction joints are provided. The cross-sectional area of the steel should be at least $\frac{1}{3}$ of 1% of the gross cross-section of the concrete. Where openings occur which reduce the cross-section, a little more steel not simply in per cent. but in actual cross-section, should be used than in the solid slab portion of the wall, because there is less concrete to assist in taking tension. At the point of reduction in height of a wall, additional reinforcement must be introduced because the stress at this point is governed by the cross-section of the higher wall.

Expansion Joints. If cracks cannot be avoided entirely in reinforced concrete or if the construction is of mass concrete, expansion joints must be provided. To make the joints water-tight, copper or sheet lead flashing has been used in dams, subways, and walls. In reservoirs the joints have been filled with asphalt and the joint backed up by a reinforced beam or slab to prevent the water pressure forcing the asphalt out.

Bonded Joints. To avoid percolation through horizontal joints between two days' work, the surface of the old concrete must be absolutely

cleaned of all dirt, scum, and laitance, down into the true concrete. This surface must be thoroughly soaked and immediately before laying the fresh concrete a layer of neat cement paste (not dry cement) must be spread, using a thickness of at least $\frac{1}{8}$ inch to $\frac{1}{4}$ inch, and the new concrete placed before this has begun to stiffen. For vertical joints between two days' work, similar procedure is necessary in addition to the reinforcement, which should extend through the joint a sufficient distance for a complete bond. (See Chapter XXII.)

Proportions. On important work it is advisable to make special laboratory tests for the determination of the best available materials and the proper proportioning and grading of the aggregates.

Proportions for water-tight concrete range usually from 1:1:2 to 1:$2\frac{1}{2}$:$4\frac{1}{2}$; the most common mixtures being 1:$1\frac{1}{2}$:3 and 1:2:4. With a small coarse aggregate up to, say, $\frac{1}{2}$ inch, the concrete is not much better than a mortar, and rich proportions, such as 1:1:2 or 1:$1\frac{1}{2}$:3 are required; while with a coarser stone, up to say $1\frac{1}{2}$ inch, a 1:2:4 mix will be satisfactory. A stone larger than this, while theoretically better, requires more care in placing to avoid stone pockets.

With accurate grading by scientific methods, such as are described in Chapter X, water-tight work has been obtained with proportions as lean as 1:3:7. (See p. 175.) In mass work, such as dam construction, the authors have recommended, where fine crushed screenings are available, proportions as lean as 1:4:7, using for the fine aggregate specially prepared screenings with a large proportion of dust. Lean proportions have the advantage over a richer mix of less shrinkage on setting and therefore less tendency to crack. A finer sand is permissible for water-tight construction than for maximum strength because in the former the size of the voids rather than the percentage of voids is one of the chief factors.

Proportioning by mechanical analysis, as described in Chapter X, is the best way to produce a water-tight concrete with the leanest possible mixture.

Consistency. The maintaining of a proper consistency is one of the most important requirements for water-tight work. The sluggishly flowing consistency, such as is recommended for reinforced concrete work (see p. 251), is also best for water-tight construction. If mixed too dry, the mass is porous and will permit penetration of water and also the formation of stone pockets. If mixed too wet, the mortar will run away from the stones, leaving stone pockets, the cement will be chemically affected (see p. 251), and there is a tendency to form laitance

on the surface of layers or even through the mass, which permits the penetration of water.

Tests are given on page 319 showing the effect of different percentages of water on permeability, strength, and density. For maximum water-tightness a slightly softer consistency than medium quaking appears desirable.

Mixing the Concrete. Care must be exercised to maintain the proportions accurately for every batch. Thorough mixing must be insisted upon. Much concrete is rushed through the mixer so rapidly that it is not thoroughly mixed, and an unnecessarily large part of the cement remains unhydrated and inert.

Placing the Concrete. Special care must be exercised in transporting to avoid separation of the ingredients. Trowelling, tamping, or spading, which brings the cement to the surface (provided the mix is not wet enough to produce laitance) increases the surface tightness. The formation of water-tight joints has already been referred to (see p. 297).

Curing Concrete. To avoid shrinkage and the formation of cracks, the surface should be protected for a period of at least a week or ten days.

SPECIAL METHODS FOR WATERPROOFING

If the concrete is made and placed in accordance with the recommendations on pages 296 to 299, no additional treatment or waterproofing is required except, as suggested, for very long structures where temperature cracks are unavoidable or in places where the importance of the structure or the danger of poor workmanship makes the extra cost of special methods permissible; also where an old structure made with poor concrete must be waterproof, special methods must be employed.

SURFACE TREATMENT

Surface Washes. Surface washes in general have been found ineffective.* Long time tests showed in certain cases fair results, but indicated the necessity of additional applications from time to time. Alum and lye wash has been used by the U. S. Army Engineers. C. B. Hegarbt† employed a mixture of one pound concentrated lye, 2 to 5 pounds alum, and 2 pounds water.

* See Report of Committee on Waterproofing Materials, Proceedings American Society for Testing Materials, Vol. XIII, 1913, p. 459.

† Report of Chief of Engineers, U. S. A., 1902, p. 2482.

Grout. For a surface which is to come in contact with water and be kept wet a coating of cement grout spread on with a brush serves to fill surface voids and tends to assist in preventing penetration of water.* This is worthless if exposed to the air.

Trowelled Surface. The water-tightness of horizontal or inclined layers can be greatly increased by trowelling the concrete. This brings the cement to the top and produces a hard dense surface. With proper precautions for bonding, a hard trowelled mortar may be applied also to set concrete, and if the concrete is not too porous may resist pressure even when placed on the back of the wall. Hydrolithic finish is a special treatment of this type.

In experimenting upon the permeability of concretes, the authors have noticed that even the light joggling necessary to compact a wet concrete and the spading along the forms increases the impermeability of the surface. Even after chipping off the top of the specimen for a depth of $\frac{1}{8}$ to $\frac{1}{4}$ inch, the flow may be several times less than when the pressure is directed on the under surface of the concrete.

Plastering. Ordinary plastering of the surface is usually ineffective unless it is placed on the back of a wall and an additional wall of brick or concrete built up against it. In any case it does not prevent the formation of contraction cracks. In constructing a subway station on the Hudson and Manhattan Railway under compressed air, a 2-inch mortar coat of clay and cement, one part finely divided clay and one part Portland cement, was placed against the lagging and timbering of the tunnel and inside of this the concrete lining was built. The tunnel was water-tight under the high head of salt water.

Granolithic Finish. On horizontal or inclined surfaces a granolithic finish of mortar may be laid and trowelled as in sidewalk construction, placing it immediately after the concrete is laid.

Paraffin. Concrete has been effectively waterproofed by a coating of hot paraffin. On the Strawberry Valley Project† of the U. S. Reclamation Service, concrete subject to a fluctuating head of water at a temperature of 50° Fahr. below zero, scaled badly and it was necessary to close the pores of the surface. The horizontal surfaces were cleaned and paraffin, boiled to drive off any water, was applied with a paint brush, and driven into the pores with a gasoline torch.

Alum and Soap. Vertical surfaces in this Reclamation work were

* See J. W. Schwab, Transactions American Society of Civil Engineers, Vol. LI, 1903, p. 123; *Engineering News*, December 5, 1912, p. 1061; and Zentralblatt der Bauverwaltung, October 2, 1912.

† *Engineering News*, April 15, 1915, p. 707.

water-proofed with successive applications of alum and soap solutions. The alum solution consisted of 2 ounces of alum per gallon of hot water, and the soap solution of ¾ pound of castile soap per gallon of hot water. The alum solution was applied first and worked in with a stiff brush and immediately followed by the hot soap solution. The temperature of both washes was maintained at 100° Fahr.

INTEGRAL WATERPROOFING

While the addition of cement, that is, the use of rich proportions, is usually the cheapest kind of integral waterproofing, under certain conditions a mixture of foreign materials is beneficial from the standpoint of increased water-tightness or economy. One result of the introduction of foreign materials is the necessity of more thorough mixing than usual and part of the benefit may be attributed to this. The U. S. Bureau of Standards, after an extensive series of tests,* reports: "The addition of so-called 'integral' waterproofing compounds will not compensate for lean mixtures, nor for poor materials, nor for poor workmanship in the fabrication of the concrete. Since in practice the inert integral compounds (acting simply as void filling material) are added in such small quantities, they have very little or no effect on the permeability of the concrete. If the same care be taken in making the concrete impermeable without the addition of waterproofing materials, as is ordinarily taken when waterproofing materials are added, an impermeable concrete can be obtained."

Hydrated Lime. The addition of hydrated lime tends to reduce the size of the voids and increase the water-tightness of a comparatively lean mix. It also "greases" the mortar so as to make the concrete flow into place more readily. Whether it is economical to use, instead of additional cement, must be determined in each individual case. The percentage of hydrated lime to use varies with the proportions of the concrete and the character of the materials. Permissible quantities in practice range from 5% to 15% of the weight of the cement.† Hydrated lime paste occupies about 2½ times the bulk of paste from the same weight of Portland cement. It is therefore efficient in void filling. Unslaked lime must never be used under any circumstances. (See p. 172.)

Pulverized Clay. Clay when finely powdered and free from any trace

* Tests of Waterproofing Materials, by R. J. Wig and P. H. Bates, Technologic Paper No. 3, 1912.

† "Permeability Tests of Concrete with the Addition of Hydrated Lime." by Sanford E. Thompson. Proceedings American Society for Testing Materials, Vol. VIII, 1908, p. 500.

of vegetable matter acts as a void filler. The proportion depends upon conditions, but about 5% of the weight of the sand is generally effective. Clay, acting as a colloid, in combination with an electrolyte such as alum sulphate, has been suggested* for increasing water-tightness.

Pulverized Rock. For mortars 1 : 3 and leaner, the addition of finely pulverized rock increases water-tightness as well as strength.†

Alum and Soap Solution. An integral mixture of alum and soap similar to the mixture described for surface treatment (p. 300) is termed the Sylvester Process, and has been used with satisfaction in certain cases.‡

Mr. Albert Grittner of Budapest reports§ successful results in waterproofing concrete with an 8% solution of potash soap substituted for the mixing water.

MEMBRANE COATINGS AND ASPHALT

Membrane waterproofing, consisting of one or more layers of waterproof paper or textile material coated with tar or asphalt, is the most expensive treatment for waterproofing, but at the same time the most reliable where absolute security is required and especially where temperature cracks are apt to occur as in subways and other long structures. It has the advantage of being to some extent elastic and hence permits a certain amount of expansion and contraction without cracking. This is not true of surface and integral methods. On the other hand, leaks are liable to occur through poor workmanship or through failure under the action of impurities in the ground water and when this occurs the water may work along behind the lining and finally penetrate it, frequently at considerable distances from the original leak. Another disadvantage is that in subways and tunnels membrane waterproofing prevents the radiation of heat. This objection proved so great in New York subways‖ that it was decided to omit all waterproofing on the roof and the sides down to a point 2 feet above ground water line, taking special precautions to secure a good water-tight mix. The results have proved satisfactory. Brick laid in mastic are more durable than paper or fabrics in the presence of gas drip, organic matter, and other injurious materials.

* Richard H. Gaines in Transactions American Society of Civil Engineers, Vol. LIX, 1907, p. 159.
† See Chimie Appliquée by R. Feret, 1897 pp. 477 and 493.
‡ See Report Chief of Engineers, U. S. A., 1901, p. 918.
§ International Association for Testing Materials, 1912, XV2.
‖ F. Lavis in *Engineering News*, November 5, 1914, pp. 950 and 1127.

Much bridge work is waterproofed, to protect the steel and the traffic under the bridge, by one or more coats of asphalt alone or with fabric, or by asphaltic concrete. The vibration to which many bridges are subject, in addition to ordinary expansion and contraction, make the membrane method—relatively elastic—much to be preferred either to the surface or integral method. Expansion joints, where any material movement is to take place, should be filled with asphalt and covered with a copper plate; the asphalt and fabric will take care of small movements.

The methods of placing and the properties of first-class materials are fully described in "General Specifications for Waterproofing Solid Floor Railroad Bridges," by Samuel T. Wagner in Transactions American Society of Civil Engineers, Vol. LXXIX, 1915, p. 311. Tests of asphalts, felts, and fabrics, are appended to the specifications.

Asphalt is generally to be preferred for structures subject to vibration or where ductility and adhesion are required. Coal tar pitch becomes brittle at 40° Fahr. and fluid in summer. In a general way pitch gives better results under water than asphalt, and asphalt is preferable to pitch when exposed to air.

Method of Laying Paper or Felt. The waterproof layer of a floor may be laid directly upon the ground if the soil is fairly dry and firm, but is usually spread upon a layer of concrete from 4 to 8 inches thick. In the former case* the first layer consists of strips with a 2 to 6-inch lap cemented with asphalt, and the remaining layers are mopped on. Upon a concrete base it is customary to first spread a layer of the asphalt or tar upon the concrete, although, if the concrete is damp, the bottom layer of paper or felt may be placed dry, as described above.

The "ply" in waterproofing,—that is, the number of layers which cover all parts of the surface,—varies from 2-ply to 10-ply. It is considered better practice to "shingle" the strips than to place each ply or layer independently. If the surface to be waterproofed is rough it may be leveled with cement mortar. It must be dry before applying the tar or asphalt. The asphalt is heated and brought, generally in buckets, to the work. Several rolls of paper are started consecutively. Ahead of each roll, as it is unrolled, the liquid asphalt is swabbed upon the concrete with a mop, so that the paper or felt is spread directly upon the fresh hot stuff. As soon as the first roll is started the second is placed to overlap the first, a width depending upon the number of ply to be laid. For example, if the felt is 32 inches wide and is laid

* This method was followed in portions of the floor in the approaches to the East Boston Tunnel.

3-ply, the second roll is lapped upon the first about 22 inches. As this is unrolled (in the same general direction as the first roll) the surface ahead of it is mopped with asphalt, as described above. A third roll is immediately started, lapping both of the two others, and so on for the entire width of the surface to be covered.

A waterproof course of this character always forms a distinct joint in the mass, thus destroying its cohesion upon that plane, and the strength of the concrete in bending on the two sides of the layer must be considered independently.

LAWS OF PERMEABILITY

The following conclusions have been reached with reference to the permeability of concrete and mortar. Many of these are based on experiments of Messrs. William B. Fuller and Sanford E. Thompson as presented in the paper on "Laws of Proportioning Concrete"* and in the paper on "The Consistency of Concrete"† by Mr. Thompson.

(1) The permeability or flow of water through concrete is less as the percentage of cement is increased, and in very much larger inverse ratio.‡

(2) The permeability is less as the maximum size of the stone is greater. Concrete with maximum size stone of $2\frac{1}{4}$-inch diameter is, in general, less permeable than that with 1-inch maximum diameter stone, and this is less permeable than that with $\frac{1}{2}$-inch stone.‡

(3) Concrete of cement, sand and gravel, is less permeable than concrete of cement, screenings and broken stone; that is, for equal permeability, a slightly smaller quantity of cement is required with rounded aggregates like gravel than with sharp aggregates like broken stone.‡

(4) Concrete of mixed broken stone, sand and cement, is more permeable than concrete of gravel, sand and cement, and less permeable than similar concrete of broken stone, screenings and cement; that is, for watertightness, less cement is required with rounded sand and gravel than with broken stone and screenings.‡

(5) Permeability decreases materially with age.‡

(6) Permeability increases nearly uniformly with the increase in pressure.‡

* Transactions American Society of Civil Engineers, Vol. LIX, 1907, p. 67.

† Proceedings American Society for Testing Materials, Vol. VI, 1906, p. 358.

‡ "Laws of Proportioning Concrete," by Fuller and Thompson, Transactions American Society Civil Engineers, Vol. LIX, 1907, p. 72.

(7) **Permeability increases as the thickness of the concrete decreases, but in a much larger inverse ratio.***

(8) **Of mortars containing the same percentage of cement but of variable granulometric composition, the most impermeable are those containing equal parts of coarse grains, G, and fine grains, F (see p. 156), the latter including the cement.†**

(9) **Decomposition by the passage of sea-water through mortars mixed in equal proportions by weight increases as the sand contains more fine grains.†**

(10) **Medium and fairly wet consistencies produce concrete much more water-tight than dry consistencies, and slightly more water-tight than very wet consistencies.‡**

(11) **The surface of concrete as molded is much more water-tight than the bottom of a specimen, because of the fine material which rises to the top.‡**

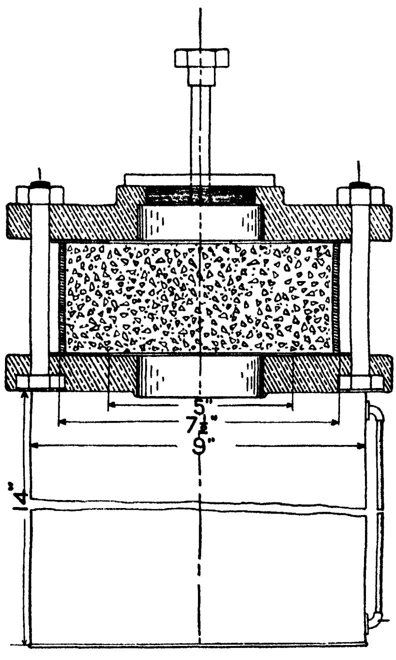


FIG. 83.—Permeability Specimen used by U. S. Bureau of Standards. (*See p.* 306.)

* "Laws of Proportioning Concrete," by Fuller and Thompson, Transactions American Society Civil Engineers, Vol. LIX, 1907, p. 72.

† R. Feret in Annales de Ponts et Chaussées, 1892, II, p. 109.

‡ "The Consistency of Concrete," by Sanford E. Thompson, Proceedings American Society for Testing Materials, Vol. VI, 1906, p. 358.

TESTS OF PERMEABILITY

Permeability tests are made by forcing water under the desired head against one surface of a concrete block, the block being so confined that the only outlet for the water is through the block and out of the opposite side. A successful design used by the U. S. Bureau of Standards is shown in Fig. 83, p. 305. The water is confined to the center of the face by rubber gaskets and the specimen itself remains in its cast iron form or mold during the test.

Another less expensive apparatus designed by one of the authors is shown in Fig. 84. The pipe is enlarged to 4 inches diameter to give a good surface of concrete and to permit thoroughly chipping it, while at the same time the external pipe connections are small, so that tight joints can be made readily. The walls of the mold may be coated with neat cement as well as the bottom, if desired, the concrete being placed in any case before the neat cement has begun to stiffen.*

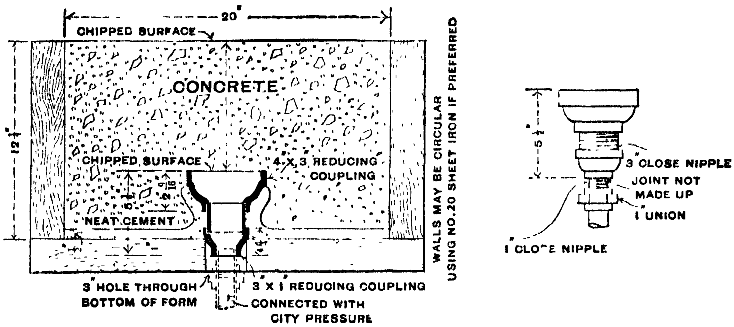


FIG. 84.—Detail of Specimen for Testing Permeability. (*See p.* 306.)

In testing, allow no water to pass along the face of the specimen, between the concrete and the mold. Cut down the surfaces, through which the water enters and leaves the specimen, so that the effect of the more impermeable skin coating will not confuse the results. Make the mix uniform and to insure this govern the size of the specimen by the maximum size of aggregate and use enough water to give a medium wet mix that works easily. Use a slight excess of fine sand to prevent large voids; small voids are less likely to be continuous through the concrete. If a coating of neat cement is used on the sides of the block to prevent

* For example of the method adopted in earlier experiments, see "Consistency of Concrete," Sanford E. Thompson, Proceedings American Society for Testing Materials, Vol. VI., 1906, p. 374.

the escape of the water, the coat must be molded with the concrete or else the hardened concrete must be chipped rough and soaked with water before applying the paste. Soak the specimen 24 hours before testing.

RESULTS OF TESTS OF PERMEABILITY

The following tests bear out the foregoing discussion on the behaviour of concrete under water pressure.

The first table given is a summary of tests carried out by the Bureau of Standards and shows the decreased permeability due to increase in thickness, rich proportions, and age.

Effect of Age, Thickness, and Amount of Cement on Permeability of Concrete

Summarized from tests by U. S. Bureau of Standards* (*See p.* 307.)

Leakage given in grams per minute after 7 hours.

Pressure about 20 pounds per square inch. Concrete mixed with river sand and broken limestone. Area of specimens about 20 sq. in.

Proportions.	Specimens 2 inches thick.			Specimens 3 inches thick.		
	Age in Weeks.			Age in Weeks.		
	4	8	26	4	8	26
	grams	*grams*	*grams*	*grams*	*grams*	*grams*
1:2·4	0.628	0	0	moist	0	0
1:3 6	1.140	0	......	0.255	0	0
1:4 8	2.160	2.02	......	10.280	1.650	12.040

* Technologic Paper No. 3, by Rudolph J. Wig and P. H. Bates, 1912, p. 90.

The foregoing results as regards richness check those obtained by Messrs. Fuller and Thompson at Jerome Park in 1906. These tests shown in the following table indicate that (1) Gravel and sand make a more water-tight mix than broken stone and sand, which in turn is better than broken stone and screenings; (2) Richness of mix increases very materially the water-tightness, especially in the case of broken stone and screenings; (3) Flow increases with the increase in pressure and nearly in proportion to it.

The flow in these tests remained nearly constant for four hours, but the water was pure and the surface of the concrete clean. In practice these conditions do not hold, and the seepage through concrete may be expected to decrease regularly as the pores silt up.

Effect of Thickness of Concrete Upon Permeability. Other experi-

ments, not here recorded, indicate that the rate of flow increases as the thickness of the concrete decreases, but in a much larger inverse ratio. Specimens 17 inches in length in proportion 1 :6.5 by weight were practically water-tight, whereas specimens of half that length were not.

Effect on Permeability of Percentage of Cement, Character of Aggregate and Pressure,

By Fuller and Thompson* (See p. 307.)

Thickness of Specimens 18 inches. Area of contact 36 square inches.
Maximum diameter of stone 2½ inches.

Proportions by Weight	Percentage of Cement to Total Dry Materials	Kind of Material: Stone	Kind of Material: Sand	Time in which Water Appears after Starting Pressure	Rate of Flow of Water in Grams per Minute, at the Following Pressures, per Square Inch: 20 lb.	40 lb.	60 lb.	80 lb.
	%			min.				
1 : 11.5	8.0	Crushed stone	Screenings	7	25	161	237	273
1 : 9	10.0	"	"	3	11	24	37	49
1 : 7	12.5	"	"	3	15	22	30	38
1 . 5.8	15.0	"	"	5.5	5	8	10	12
1 : 8.8	10.2	Crushed stone	Sand	9	4	11	17	22
1 : 6.9	12.7	"	"	10	2	2	3	3
1 : 5.5	15.6	"	"		0	0	0.7	1.4
1 : 10.8	8.5	Gravel	Sand	3	15	25	38	[illegible]
1 : 8.4	10.6	"	"	17	1	3	5	6
1 : 6.5	13.0	"	"	100	0	0	0	0.5
1 : 5.3	15.9	"	"	98	0	0	0	1.4

Effect of Size of Stone on Permeability

By Fuller and Thompson† (*See p.* 309.)

Thickness of Specimens 18 inches. Area of contact 36 square inches.
Aggregates, crushed stone and natural sand.

Proportions by Weight	Percentage of Cement to Total Dry Material	Maximum Size of Stone	Time in which Water Appears	Rate of Flow of Water in Grams per Minute at the Following Pressures per Sq. In.: 20 lb.	40 lb.	60 lb.	80 lb.
	%	in.	min.				
1 : 2.9 : 5.7	10.2	2½	7	1	4	8	12
1 : 2.9 : 5.7	10.2	1	26	0	5	10	15
1 : 2.9 : 5.7	10.2	½	29	0	10	17	20

* Transactions American Society Civil Engineers, Vol. LIX, 1907, p. 132.
† Transactions American Society of Civil Engineers, Vol. LIX, 1907, p. 136.

Effect of Size of Stone upon Permeability. The foregoing table gives the comparative permeability of concrete in the same proportions mixed with stone of different maximum size. The difference in this case is evidently due to the greater density of the concrete composed of the large stone.

Effect of Coarseness of Sand upon Permeability. The effect of size of sand is shown in the following table and shows, as do tests by Mr. Feret, that more fine sand is required for maximum water-tightness than for maximum strength.

Tests to determine Relative Permeability of Concrete with Coarse and Fine Bank Sand

By Sanford E. Thompson (*See p.* 309)

Proportions 1 : 3 : 6 *by Volume or* 1 : 2.8 : 5.7 *by Weight. Age* 32 *days*

CHARACTER OF SAND	DENSITY $c+s+g$	WATER PASSING IN GRAMS PER MINUTE
(1) All coarse	0.853	145.1
(2) $\frac{2}{3}$ coarse, $\frac{1}{6}$ fine	0.846	10.4
(3) $\frac{3}{4}$ coarse, $\frac{1}{4}$ fine	0.843	43.0
(4) All fine	0.813	30.2

Analyses of Natural Bank Sand and Screened Gravel used in Tests

SIEVE	TOTAL PER CENT PASSING SIEVES		
	Coarse Sand	Fine Sand	Screened Gravel
	%	%	%
1 inch			100
$\frac{1}{2}$ inch			50
$\frac{1}{4}$ inch	100		0
No. 5	88		
No. 12	77	100	
No. 40	32	96	
No. 200	3	27	

References to permeability tests on concrete and patented compounds are given in Chapter XXXIII.

CHAPTER XIX

STRENGTH OF PLAIN CONCRETE

The strength of plain concrete, that is, of concrete without steel reinforcement, is governed primarily by

(1) The quality of the cement.

(2) The texture of the aggregate.*

(3) The quantity of cement in a unit volume of concrete.

(4) The density † of the concrete.

(5) The thoroughness of mixing.

(6) The consistency.

The effect of the percentage of cement and the density of the concrete, which are of special importance to the user in determining the proportions of materials may be expressed more explicitly as follows:

(1) With the same aggregate the strongest concrete is that containing the largest percentage of cement in a given volume of concrete, the strength varying nearly in proportion to this percentage.

(2) With the same percentage of cement but different arrangements of the aggregates, the strongest concrete usually is that in which the aggregate is proportioned so as to give a concrete of the greatest density, that is with the smallest percentage of voids. In many cases relative densities nearly correspond to relative weights.

The amount of water is a most important factor. A very wet mix will give a concrete two-thirds or less as strong as a concrete of medium consistency made of the same materials. (See p. 315.)

These various characteristics and others are discussed in this chapter, which takes up the compressive strength of plain concrete, the tensile, shearing, and transverse or bending strength of concrete, and the testing of concrete specimens.

COMPRESSIVE STRENGTH OF CONCRETE

A compressive strength of 2 000 pounds per square inch may be expected of first-class concrete of medium consistency, in proportions one part cement, 2 parts sand, 4 parts broken stone or gravel, at the age of 28 days; at 14 days about 1 600 pounds may be expected, and

* The word aggregate is defined on page 9.

† The meaning of density is illustrated on pages 133 and 134.

at 7 days about 1 300 pounds. At one year the strength increases to about 3 500 pounds and at two years to about 4 000 pounds, provided the conditions are such that moisture has access to the concrete. In dry atmosphere the increase after 28 days is comparatively small.

The compressive strength of concrete is affected by the characteristics of the cement. Certain cements harden faster than others, giving higher strengths at early periods; while other cements harden slowly but eventually obtain strengths as high or higher than those reached by the quicker hardening cements.

The strength of two concretes of different proportions made with the same cement is approximately proportional to the percentage of cement in the mixture and a rough idea of comparative strengths can be obtained from this rule. More exact methods of determination and the various conditions affecting the strength are discussed in the pages that follow.

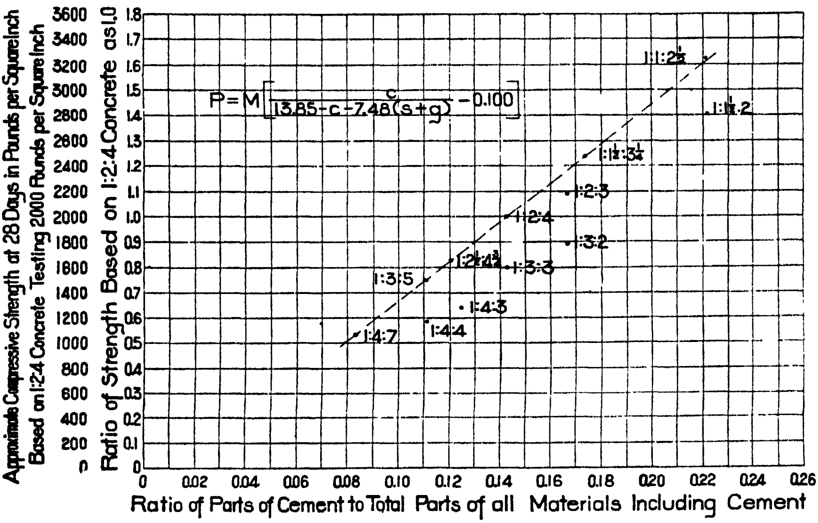
In considering the effect of the amount of cement upon the strength it must not be forgotten that the character of the aggregate and the relative sizes of particles affect the strength to a marked degree. Frequently, by proper selection and proportioning of aggregates the required strength can be obtained at much less cost than by increasing the amount of cement.

Safe Strength of Concrete. Working unit stresses are discussed fully in the chapter on Design of Reinforced Concrete. The percentage of the ultimate strength that may be used varies with different kinds of stress and the character of the structure. For different proportions and conditions the discussion in the following pages may be used as an aid to the judgment. The importance of the structure governs to a certain extent the stresses to be used and relatively high values frequently may be used with conservatism. Many times, however, concrete that will ultimately carry very light loads must be strong enough to do so a short time after placing and a much richer mix and larger sections must be used than would otherwise be necessary.

Strength of Proportions in Practice. In selecting proportions to use in any structure the strength which can be attained at the required age with the available materials must be considered. In some cases a high strength required at early ages, because of immediate loading or early removal of forms, may necessitate a richer concrete than would be selected for a similar structure which carries but little dead load and does not receive its load for a considerable period after placing. If a wet consistency is to be used because of certain conditions of economy

in construction due allowance must be made in determining proportions for the relative weakness of a wet mixture.

The refinement to which proportioning of aggregates should be carried, in accordance with Chapter X, must be governed by practical considerations. It is wise in any case to give due consideration to the possibility and to figure the relative costs of different methods of treatment.



PROPORTIONS ADOPTED ON THE DASH LINE ARE THOSE GIVING, FOR AVERAGE CONDITIONS, MIXTURES WITH VOIDS WELL FILLED. STRENGTHS ARE BASED ON AUTHOR'S FORMULA, CONFIRMED BY TESTS.

FIG. 85.—Approximate Crushing Strength of Concrete of the Same Materials in Different Proportions. (*See p.* 313.)

Diagram for Compressive Strength. The approximate relative strength of concrete made with the same aggregates in different proportions is shown by the diagram, Fig. 85 page 312. The diagram may be read in terms of ultimate strength with 1 : 2 : 4 concrete considered as 2 000 pounds per square inch; also in ratios which may be used conveniently when the strength of the 1 : 2 : 4 concrete is more or less than this. Vertical lines indicate the proportion of cement to total material including the cement. Thus the proportion or ratio of cement in the 1 : 2 : 4 mixture is $\frac{1}{7}$ or 0.143.

The light line in the diagram indicates the strength for standard

proportions, using approximately twice as much coarse as fine aggregate. For rich mixes the proportion of sand is slightly decreased and for lean mixes the proportion is increased, because the cement acts with the sand in filling the voids in the coarse aggregate.

To find the compressive strength of concrete in any proportions or the ratio of strength to that of 1 : 2 : 4 concrete, select the proper point in the diagram, or else interpolate between values given, and follow horizontally to the left.

Formula for Strength of Concrete. The diagram described is made up from a formula which in turn is based on a large number of tests. The formula is useful in estimating the strength of other proportions than those covered in the diagram, and also for comparing the strength of special mixtures and materials with different percentages of voids. The formula is similar in a general way in form to Feret's formula for strength of mortar. (See p. 155.)

Let

P = compressive strength of concrete in lb. per sq. in.
c = barrels of cement per cubic yard of concrete.
s = cubic yards of sand per cubic yard of concrete.
g = cubic yards of gravel, or stone, per cubic yard of concrete.
0.1 is an empirical constant.
M = a coefficient varying with the strength of the cement, the texture of the coarse aggregate, and the age of the specimen, but constant for all proportions of the same materials mixed and stored under similar conditions. A table of values of M is given on page 314.
v_s = voids in sand.
v_g = voids in gravel.

$$1.95 = \frac{376}{62.3 \times 3.1} = \frac{\text{weight of a barrel of cement}}{\text{weight of a cubic foot of water} \times \text{specific gravity of cement}}$$

Then

$$P = M\left[\frac{1.95c}{27 - 1.95c - 27\left[(1 - v_s)s + (1 - v_g)g\right]} - 0.1\right] \qquad (1)$$

Assuming the specific gravity of cement to be 3.1, the specific gravity of sand and stone to be 2.65, and the voids in the sand and stone to be each 46%, the formula becomes

$$P = M\left[\frac{c}{13.85 - c - 7.48(s + g)} - 0.1\right] \qquad (2)$$

The values of c, s, and g can be obtained from the tables of quantities of materials, page 214, or may be computed from the formulas on pages 210 to 212.

The term in the large brackets, that is, without the M, may be used as a ratio. The values of the coefficient, M, which may be adopted for different conditions, are given in the table below, and when substituted in the formula give P in terms of pounds per square inch. It must be understood that the formula is only correct when the voids of the coarse aggregate are filled with fine aggregate and cement; thus the formula would not be correct for such proportions as 1 : 1 : 6, in which the voids of the coarse aggregate evidently are not filled.

Approximate Values of Coefficient, M, for Use in Formulas (1) and (2) *(See p. 313)*

Note that these are not strength values

Age.	Values of Coefficient M							
	Granite or Trap.		Gravel or Hard Limestone.		Soft Limestone or Sandstone.		Cinders.	
	Consistency.		Consistency.		Consistency		Consistency.	
	Medium.	Very Wet	Medium.	Very Wet.	Medium.	Very Wet	Medium.	Very Wet
7 d.	2 480	1 360	2 260	1 220	1 700	920	680	370
14 d.	3 020	1 780	2 740	1 610	2 060	1 210	830	490
1 mo.	3 780	2 330	3 440	2 120	2 580	1 590	1 040	640
3 "	5 130	2 790	4 660	2 540	3 500	1 900	1 410	760
6 "	5 870	3 780	5 330	3 440	4 000	2 580	1 610	1 040
1 yr.	6 700	5 030	6 080	4 570	4 550	3 420	1 840	1 380

For small sized stone, say $\frac{1}{2}$ inch maximum, the values should be reduced about 20%.

Example: What approximate strength at the age of six months may be expected of a granite concrete of medium consistency in proportions 1 : 2 : 5 made with special aggregates, the sand having 46% voids and the stone 40% voids? The specific gravity of the cement is 3.1 and a barrel of 4 cubic feet weighs 376 pounds.

Solution: From Quantity Tables on page 214 the proportions require 1.26 barrels cement; 0.37 cubic yards of sand; and 0.93 cubic yards of stone. From the table on page 314 we find the value of the co-efficient, M, to be 5870. Substituting these values and also the voids in formula (1) gives

$$P = 5870\left[\frac{2.46}{27 - 2.46 - 27\,[(1 - 0.46)\,0.37 + (1 - 0.40)\,0.93]} - 0.1\right]$$

or $P = 2950$.

COMPRESSIVE STRENGTH OF CONCRETE IN PRACTICE

From study of a large number of actual tests of concrete specimens, confirmed by numerous tests of concrete cut out of completed structures, it is possible to present approximate values for the strength that may be expected with different proportions of mixtures. These strengths are based on a fair quality of aggregate and commercial Portland cement. Strengths are given for two different consistencies: (1) medium, which may be assumed to include not only a plastic mix but a mix of a consistency of very thick pea soup in which the coarse aggregate will not separate from the mortar in handling or in flowing down a slope, in fact, a mix just wet enough to flow very sluggishly into the forms and around the steel in reinforced concrete construction; and (2) very wet, or sloppy consistency representing a very wet mixture in which the mortar readily separates from the stones. These of course are simply relative terms, the strength gradually decreasing with the addition of water as soon as the sluggishly flowing consistency is passed. With accurate grading of the aggregates, as stated on page 310, the strength may be increased without additional cement. Relation of strength as affected by different coarse aggregates is indicated in the table on page 316. A dry mixed concrete is somewhat stronger at the age of one month, but approximately the same at the age of six months, as the medium consistency. Conditions of storage, as indicated on page 320, may appreciably affect the growth in strength.

The following table presents the approximate strength that may be expected of first-class concrete of different proportions, ages, and consistencies, tested in cylinders 8 inches in diameter by 16 inches high. Instead of assuming nominal proportions, where the volume of sand is

Approximate Compressive Strength of Concrete With Coarse Aggregate such as Gravel, Granite, or Hard Limestone. (See p. 315.)

Proportions by Volume.	Medium Consistency.		Very Wet Consistency.	
	Age 1 mo.	Age 6 mo.	Age 1 mo.	Age 6 mo.
	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.	lb. per sq. in.
1 : 1 : $2\frac{1}{2}$	3 240	5 020	1 990	3 260
1 : $1\frac{1}{2}$: $3\frac{1}{4}$	2 470	3 830	1 520	2 480
1 : 2 : 4	2 000	3 100	1 230	2 010
1 : $2\frac{1}{2}$: $4\frac{3}{4}$	1 650	2 560	1 020	1 660
1 : 3 : 5	1 500	2 320	920	1 500
1 : 4 : 7	1 060	1 640	650	1 060

one-half that of the stone, more practical relations are chosen which allow for the effect of the cement and sand in filling the voids. The values agree with the formula given on page 313 and with Fig. 85, page 312.

Maximum Strengths Recommended by the Joint Committee. The Joint Committee on Concrete and Reinforced Concrete, recommend the following maximum values for ultimate strength to be used in design. To use for working stresses and to allow a sufficient factor of safety, these of course, must be multiplied by the required percentages. (See Chapter XXII.)

Limiting Strengths of Different Mixtures of Concrete Suggested by Joint Committee

(In Pounds per Square Inch)

Aggregate.	1:1:2	1:1½:3	1:2:4	1:2½:5	1:3:6
Granite, trap rock	3 300	2 800	2 200	1 800	1 400
Gravel, hard limestone, and hard sandstone	3 000	2 500	2 000	1 600	1 300
Soft limestone and sandstone	2 200	1 800	1 500	1 200	1 000
Cinders	800	700	600	500	400

Relation of Percentage of Cement to the Strength of the Concrete. As already stated, the strength of a concrete varies approximately with

Comparative Density and Strength of Similar Concrete with Different Percentages of Cement and 2½-inch Stone Graded as an Ellipse and Straight Line

By Fuller and Thompson. (*See p.* 316.)

Materials.		Density with Different Percentages of Cement*				Modulus of Rupture at 60 Days, Different Percentages of Cement.*				Compressive Strength at 140 Days, Different Percentages of Cement.			
Stone.	Sand.	8%	10%	12½%	15%	8%	10%	12½	15%	8%	10%	12½%	15%
Crushed	Screenings	0.829				188				980			
"	"		0.846				250				1 129		
"	"			0.832				245				1 [illegible]8	
"	"				0.839				326				1 634
Gravel	Sand	0.871				163				990			
"	"		0.855				245				1 715		
"	"			0.865				307				1 890	
"	"				0.867				339				2 040
Averages		0.850	0.850	0.848	0.853	176	248	276	332	985	1 428	1 654	1 837
Strength computed as proportional to the percentage of cement, based on strength with 8% cement						176	220	275	330	985	12 30	1 540	1 850

* In gravel and sand mixtures the percentage by weight of cement was increased in each case to balance the difference in specific gravity between this and the crushed material.

the percentage of the cement in the mass so that this relation may be used as a rough guide for practical purposes. The preceding table gives the results of the Jerome Park tests* by Messrs. Fuller and Thompson, where the density of the concrete was maintained nearly constant. The actual compressive strength and also the modulus of rupture is low because a very wet mixture was used in making up the specimens. In these tests, the strength of the concrete with screenings was less than with sand.

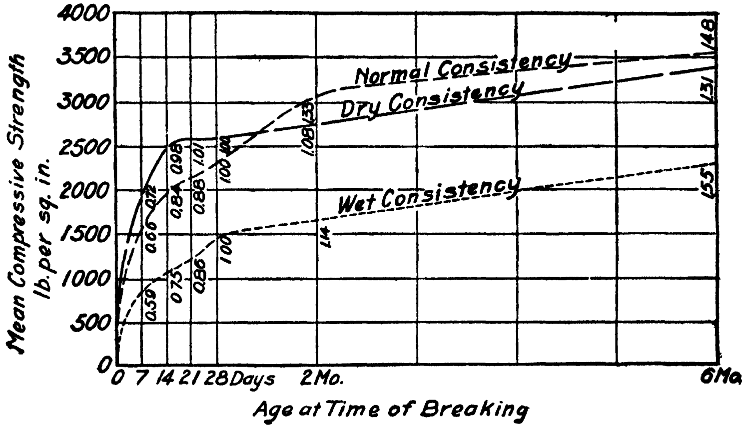


FIG. 86.—Growth in Strength of Concrete of Wet, Normal, and Dry Consistencies. (*See p.* 318.)

Effect of Consistency. The reduction in strength of concrete by the use of an excess of water is very marked. **Very wet concrete, such as is sometimes used in practice in order to flow down a chute of flat slope, never attains the strength secured by a more plastic mixture and should never be used where normal strength is required by the design. Tests and experience show that even for reinforced concrete building construction it is possible to use a very slow flowing, sluggish consistency that will fill the forms and imbed the steel without being so sloppy as to affect appreciably the ultimate strength.** (See p. 250.)

Concrete made with a dry mixture attains higher early strength than medium consistency up to the age of, say 6 weeks, when it is apt to be overtaken and passed by the wetter mix. In scarcely any case, therefore, is it wise to use in practice a mixture in which the concrete requires appreciable tamping to bring the mortar to the surface.

* Transactions American Society of Civil Engineers, Vol. LIX, p. 67, 1907.

The relative strength of very wet, medium, and dry mixed concrete at different ages is shown in Fig. 86, page 317. These tests are based on experiments made by various laboratories under the direction of the Aggregate Committee of the American Concrete Institute.*

The curves in Fig. 87 are plotted from experiments by the authors† upon the strength, density‡, and permeability of the concrete mixed with different percentages of water. In the three curves the points of maximum density, strength and water-tightness all lie not far from the medium quaking consistency, although for maximum water-tightness a still softer consistency appears to be slightly more efficient.

These tests further indicate that (1) the consistency which will produce the densest concrete will result in the greatest ultimate strength provided an excess of water is not employed; (2) dry mixtures attain highest strength at short periods, but mixtures of quaking consistency approach the dryer specimens after longer setting; (3) very wet mixtures, especially of lean proportions, may be chemically injured, by excess of water.

Effect of "Laitance." Whenever concrete is laid under water, the water is likely to be clouded by what appear to be particles of cement floating up from the mass which is being laid. This whitish substance is generally termed "laitance." A similar formation occurs on the surface of concrete laid with too much water.

The authors have found serious defects in structures in which the concrete was laid by chuting with a large excess of water. At the top of the basement columns in one completed six-story structure was found a thickness of laitance varying from ½-inch to 4 inches, which had to be cut out and replaced.

Chemical and microscopical analyses, which Mr. Clifford Richardson has very kindly made for us, show that this laitance has nearly the same chemical composition,§ except for a large loss on ignition, as normal Portland cements, but consists largely of amorphous material of an isotropic nature,—that is to say, it does not affect polarized light, and has almost no setting properties.

It is evident, therefore, that when concrete or mortar is laid under water, or with a large excess of water, a portion of the cement is rendered incapable of setting, and the strength of the mass is consequently reduced in proportion to this loss. The conclusion is naturally reached that for concrete

* Report of Committee on Specifications and Methods of Tests of Concrete Materials, Sanford E. Thompson, Chairman, Journal of American Concrete Institute, October–November, 1914, p. 422.

† Proceedings of American Society for Testing Materials, Vol. VI, 1906, p. 358.

‡ See p. 10 for definition and p. 149 for method of determining density.

§ See p. 251.

laid under water, or in locations where a large excess of water is required in mixing, a higher percentage of cement than usual, about one-sixth more, should be employed.

A lean mixture has been found to be more seriously injured by an excess of water than a rich one, probably because the water has a greater opportunity to penetrate the mass, and therefore to dissolve the cement.

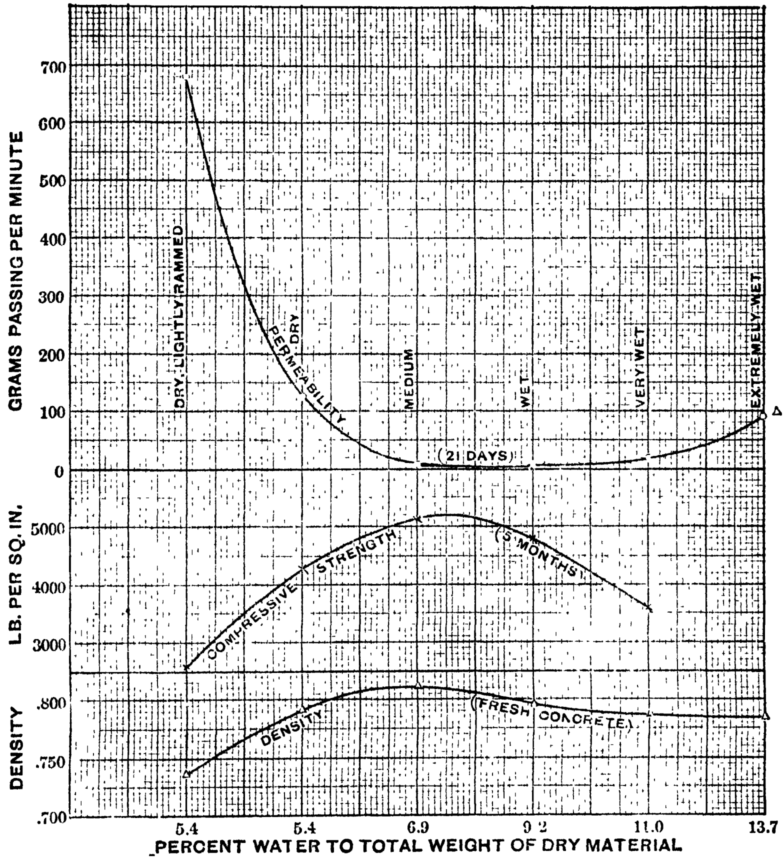


FIG. 87.—Comparative Permeability, Strength and Density of 1 : 2½ : 4½ Concrete, mixed with Different Percentages of Water, By Taylor and Thompson. (*See p.* 318.)

Machine Versus Hand Mixed Concrete. Machine mixed concrete on actual work and, when properly handled, in the laboratory, may be

counted on for greater strength and uniformity than hand-mixed concrete. In mixing laboratory specimens, however, it is difficult when a few specimens are made at a time to prevent the cement and mortar sticking to drum of mixer and thus influencing the proportions. Tests of a large number of 6-inch cubes of 1:2:4 concrete by the University of Illinois* gave an average of 2 200 pounds per square inch for hand-mixed specimens and 2 800 pounds per square inch for machine mixed specimens.

Tests by the authors of laboratory-made specimens in comparison with specimens taken from the mixer on the job show that with first-class workmanship the laboratory specimens are representative of the job concrete made with the same material. Furthermore, specimens of concrete cut from actual structures usually show higher strength than specimens taken either in the field or mixed in the laboratory.

Comparative Strength of Concrete at Different Ages and Consistencies Stored Under Different Conditions.†

Each Value is an Average of four 6 by 6-inch Cylinders. Proportions 1:2:4 by weight.

Normal Consistency

Tests at University of Illinois.

Consistency.	Water in Mixing, %	Storage.	Compressive Strength at Different Ages. lb. per sq. in.							
			7 Days.	14 Days.	21 Days.	28 Days.	2 mo.	6 mo.	1 year.	2 years.
Dry	8.4	Damp sand	1751	2140	2658	2615	3056	3941	3700	4890
Normal	9.3	Damp sand	1390	1775	1816	1820	3063	3431	3768	4042
Wet	10.2	Damp sand	1103	1354	1623	1657	2410	3281	3760	3914
Normal	9.3	Air	1481	2061	2126	2116	2232	2049	2350	2189
Normal	9.3	Coated with paraffine, Damp sand	....	....	....	2314	2521	3339	3675	4235
Normal	9.3	Damp sand‡	....	....	....	....	2734	3433	3945	....
Normal	9.3	Air‡	....	....	....	....	2208	1888	2000	....

Effect of Curing on Strength. Tests indicate a marked effect on the strength of concrete by the manner of curing. If specimens are kept in the dry air of the laboratory, comparatively little gain in strength is evidenced after the age of 28 days. Results of tests at the Uni-

* University of Illinois, Bulletin No. 71, p. 176.

† Journal American Concrete Institute, October-November, 1914, p. 435.

‡ Made from dry stone; for all other test pieces the stone had been thoroughly wet before mixing.

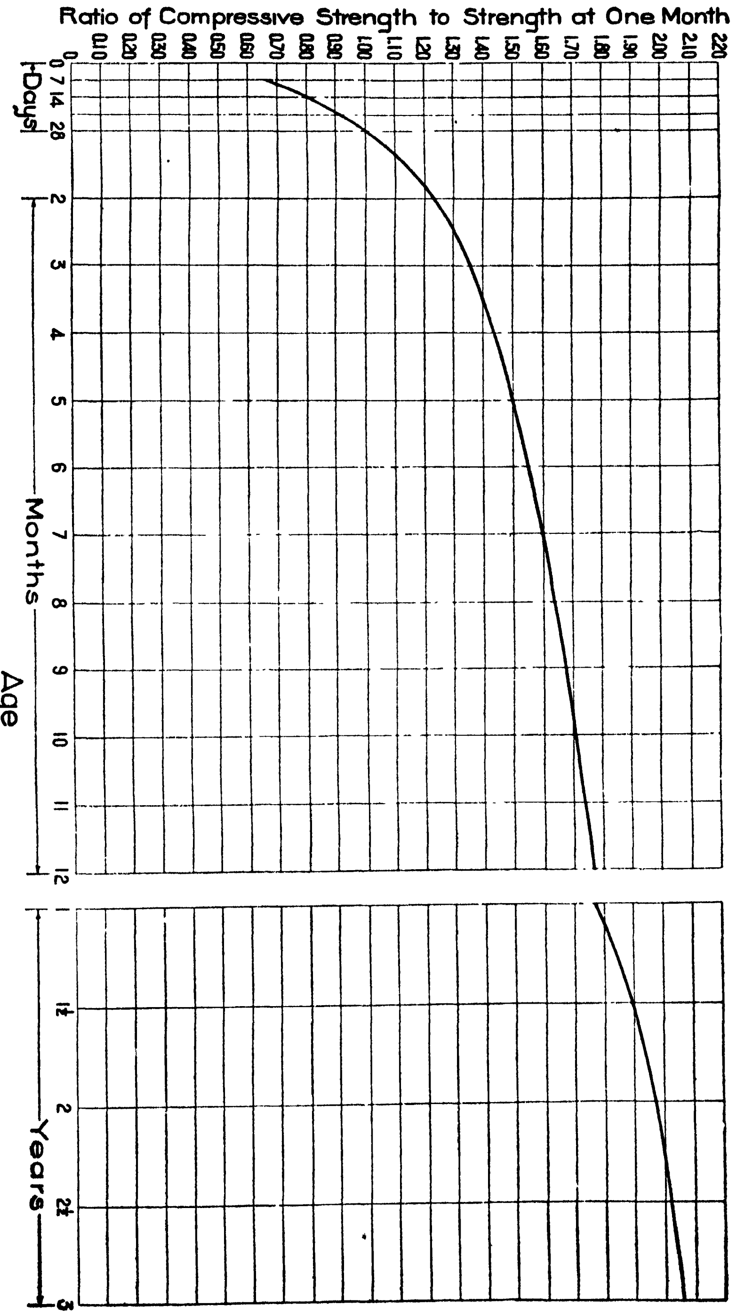


FIG. 88.—Growth in Compressive Strength of Portland Cement Concrete. (*See p.* 322).

versity of Illinois with specimens cured under different conditions are indicated in the following table. Although such tests have not yet been carried far enough to determine the effect on actual structures, the results show that a dry atmosphere should be taken into account in the construction of any structure which is to be closed from the weather at an early period. Certain temporary properties are noticed in concrete subjected alternately to wet and dry conditions. For example, a loss in strength is noted in air-cured specimens when first placed in water, but the strength is gradually regained after soaking.*

GROWTH IN STRENGTH OF CONCRETE

Long-time compressive tests of concrete indicate a fairly uniform growth in strength with no such falling off with age as is frequently observed in tensile tests on neat cement and sometimes in mortar briquets. In Fig. 88, page 321, is plotted an average curve showing a growth in strength representing some fifteen series of tests carried out in the United States, France, and Germany. The curve extends to the age of 3½ years, and tests as far as 9 years show a further slight increase.

Important in practical construction is the fact that the strength at 3 years is more than twice the strength at 28 days.

For laboratory tests, the ratio of strength at 7 and 14 days is of interest as forming a basis for short-time tests when such are necessary to obtain advance information on aggregates or on special conditions.

Comparison of various tests indicate no marked variations in growth with different proportions and different aggregates. Variations due to consistencies are referred to on page 317, and the effect of storage on page 320. A weak aggregate (see page 323) may limit the ultimate strength.

EFFECT OF AGGREGATES UPON THE STRENGTH OF CONCRETE

Effect of Size of Coarse Aggregate. The larger the maximum size of coarse aggregate, the higher the strength, with other conditions similar. In Fig. 89, page 323, are shown the results of tests by Messrs. Fuller and Thompson,† which show the increase in strength as the stones increase from ½ to 2¼ inch, maximum size. These tests and other series show that this increase in strength is due primarily to increased

* Prof. J. L. Van Orman in Transactions of American Society of Civil Engineers, Vol. XXVII, 1914, p. 438.

† Transactions American Society of Civil Engineers, Vol. LIX, p. 67, 1907.

density with the larger size stone. **The tests show that with $\frac{1}{2}$-inch stone, one-third more cement is needed than when the maximum size of stone is $2\frac{1}{4}$ inch, and with 1-inch stone, one-sixth more cement is needed than with $2\frac{1}{4}$ inch, assuming in both cases similar grading.**

The selection of maximum size of particles is apt to be made from practical considerations rather than the strength of the concrete. For mass concrete, a maximum of 3-inch is customary, although 6 and even 8-inch stone has been put through a mixer with satisfactory results. For reinforced concrete, a limit of 1 to $1\frac{1}{2}$-inch maximum size is necessary in order to properly flow around the steel. For face walls, washed or picked, a better appearance is secured by limiting the maximum size to 1-inch.

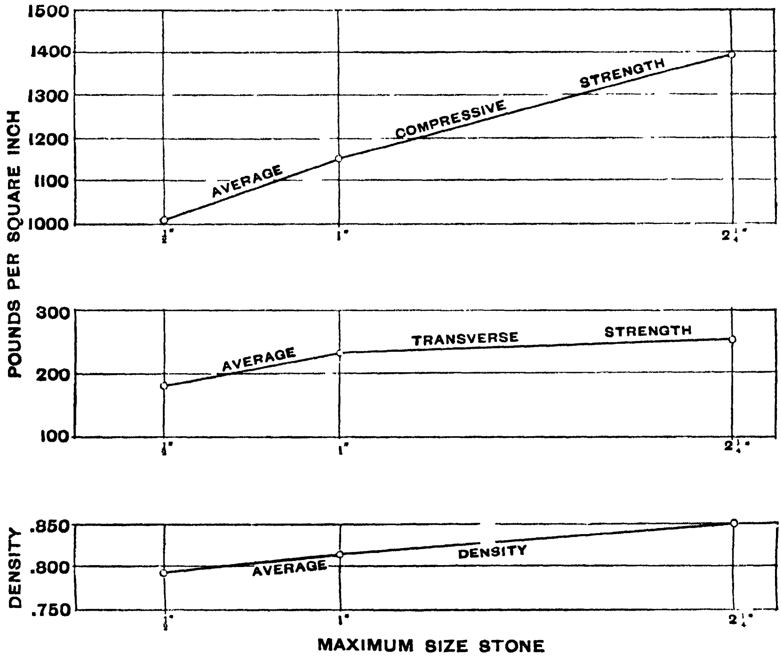


FIG. 89.—Comparative Density and Strength of Concrete made from Broken Stone of different Maximum Sizes. Proportions 1:3:6. Age, 140 Days. (*See p.* 322.)

Effect of Quality of Stone. Weak aggregates eventually limit the ultimate strength of the concrete because a thoroughly hardened concrete will break through the coarse aggregate instead of pulling out the stone.

It is evident, therefore, that the strength of the stone is an important requirement and furnishes an indication of its value. In general, furthermore, the strength of the stone varies, at least to a partial degree, with its specific gravity. A stone of heavy specific gravity, therefore, can be expected to produce in general a stronger concrete. The compressive strength of stone varies from 5 000 to 20 000 lb. per sq. in. according to the texture. The approximate strengths of concrete with different coarse aggregates are given on page 316.

Gravel Versus Broken Stone. Comparative tests of concrete made with broken stone and with gravel in the same proportions by volume show almost always that concrete made from hard broken stone such as trap gives higher compressive strength than concrete made with gravel. This appears to be the rule not only when the materials are mixed by measured volumes regardless of the percentage of voids, but also when the broken stone and gravel are each screened to substantially the same size. The choice, however, between the two aggregates is more often a matter of relative cost and availability than of the actual strength value, because the difference in strength, which usually is not above 8% to 10%, is not likely to be enough to be the governing factor. Furthermore, gravel makes a smoother mix so that the stones slip into place without so much tendency to separate from the mortar. For this reason gravel is usually better for watertight work and in places where it is especially necessary to eliminate surface voids.

These conclusions are further confirmed by tests of the U. S. Geological Survey at St. Louis* and by E. Candlot in France†.

Comparative tests of concrete with different coarse aggregates are shown in Fig. 90, p. 325, representing tests by Messrs. Wm. B. Fuller and Sanford E. Thompson at Jerome Park Reservoir, New York City.‡ Because of the greater density, the proportions by volume being the same, the specimens made with gravel and sand contain in the set concrete a slightly larger percentage of cement, so that the strength of the gravel concrete is slightly greater than if allowance had been made for this. The relatively low strength of the concrete with broken stone and screenings may be due in part to the character of the screenings, which were of gneiss rock and of poorer quality than that produced from a true granite.

* Bulletin 344 U. S. Geological Survey, 1908.

† See Concrete, Plain and Reinforced, 2nd Edition, p. 385.

‡ Transactions American Society of Civil Engineers, Vol. LIX, 1907, p. 67.

These tests show that a concrete with an angular coarse aggregate, such as broken stone, is stronger than one with a rounded coarse aggregate, like gravel, using the same sand and cement. The stronger adhesion of cement to broken stone outweighs the greater density of gravel concrete.

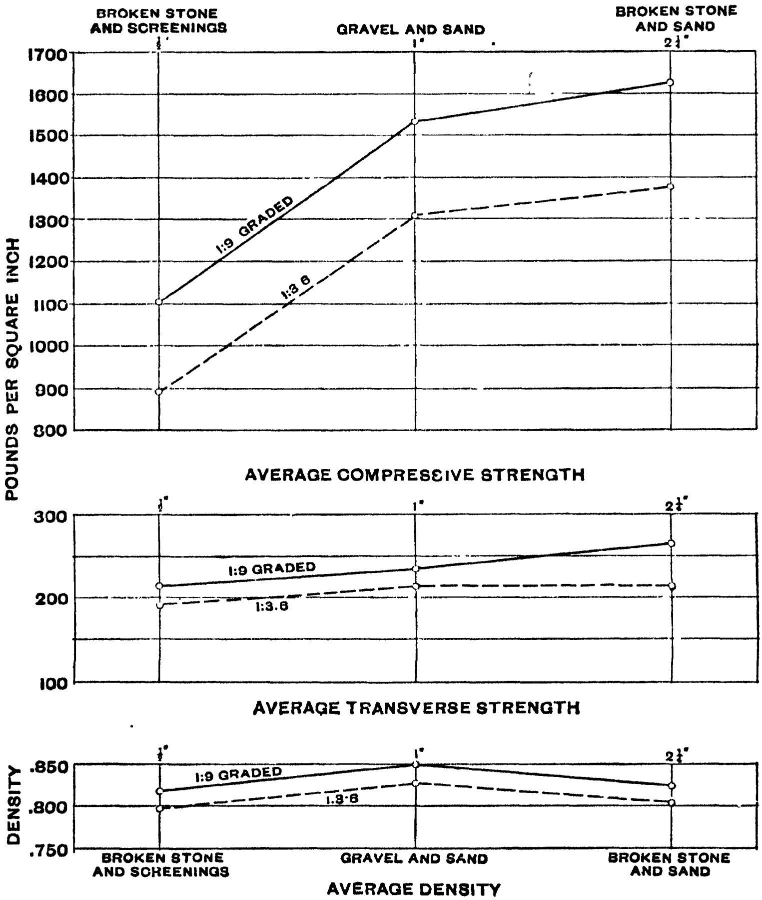


FIG. 90.—Comparative Density and Strength of Concrete made with Different Aggregates. (See p. 324.)

Replacing the sand with screenings of the same size and using broken stone produces a weaker concrete than sand and gravel, probably because of the low density.

The gravel must always be clean. In a bank it is frequently covered with a film of dirt or loam which it is naturally impossible to remove without thorough washing in a special plant. (See p. 228.) A dirty gravel may reduce the strength as much as 25%.

The stone with the smaller percentage of voids if proportioned by volume gives the lower strength. To illustrate, a cubic foot of stone measured loose with 40% voids contains more solid material than stone with 50% voids, and hence makes a greater bulk of concrete with the same proportions by volume. This is further illustrated in the table on page 214. Consequently, there is less cement in a unit volume of the concrete when the stone has 40 per cent voids; and while the density is slightly greater, it is not enough greater to counterbalance the decrease in the percentage of cement. If the proportions had been altered so as to use less sand with the stone having 40 per cent voids, the concrete would have been stronger, with the same amount of cement per cubic yard of concrete, because of the greater density.

From this it must not be inferred that the aggregate with the largest percentage of voids is best to use. As indicated above, it requires more cement to a given volume of concrete, and the concrete is apt to be slightly less dense than with an aggregate having fewer voids, so that the latter is usually the more economical even although it is sometimes slightly inferior in strength. From the standpoint of a contractor, therefore, gravel concrete is cheaper than broken stone concrete when proportioned by volume for the reason that gravel has a smaller percentage of voids and therefore makes a larger volume of concrete with the same measured materials.

It is almost always cheaper to screen bank gravel, recombining the sand and screened gravel in the desired proportions because (1) bank gravel seldom runs uniform enough to depend upon the right proportions of fine to coarse aggregate and (2) the sand is apt to be in excess, thus requiring more cement to the cubic yard. Ordinarily the cost of screening is more than balanced by the saving in cement.

Slag. Slag, a by-product from blast furnaces producing pig iron, has proved a satisfactory coarse aggregate for concrete, but more care must be used in selecting it than is the case with gravel and broken stone. The important requirements are that it shall contain little sulphur, shall be tough, and dense, and shall have been cooled for six to twelve months on large slag heaps, on to which it flows from the furnaces in layers about 6 inches thick. The weight per cubic foot of first-class

* See tests of Howard A. Carson, 7th Annual Report, Boston Transit Commission, 1901, p. 39.

slag (assuming 45% voids) should be not less than 70 pounds per cubic foot. This is an important requirement because it eliminates the soft, porous material. Slag has been used in important concrete structures in the blast furnace regions for many years and its durability is well established for mass concrete provided a first class quality is used. Its use for reinforced work is more questionable because of the difficulty of obtaining uniformly good material.

Tests indicate that a first-class slag may be expected to show strengths at least as high as limestone and gravel.*

Mr. W. A. Aiken† reports a large series of tests on 6-inch cubes of 1:2:4 concrete ranging in age from 28 days to one year. At 28 days, 3 months, and 6 months, 100 specimens each were broken and at 9 months and one year 50 specimens each. There is a substantial growth in strength although the absolute strengths themselves are low, probably, Mr. Aiken states, because of a fine sand.

Compressive Strength of Slag Concrete

Age	28 Days	3 Months	6 Months	9 Months	One Year
Strength	1 561	1 952	2 589	2 841	2 797
Ratio	1 00	1.25	1.66	1.83	1.79

Cinders. Cinders, usually from soft coal, are one of the most variable materials used as a concrete aggregate, and need special care in selection and mixing to secure satisfactory or even safe work. They can never be safely used in design without tests to determine the breaking strength of the concrete. The heavier the cinders, and the less the amount of material passing the quarter-inch sieve, the stronger the concrete. (See p. 328.) Cinders containing fine impalpable ash are unfit for use, and cinders from industrial plants must be investigated to insure freedom from all injurious acids and alkalies. Tests and experience in building construction prove that steel properly imbedded in cinder concrete of wet consistency is not liable to rust. (See p. 293.)

The following tables of tests at the Massachusetts Institute of Technology and at Columbia University show clearly the effect of coarseness and weight on compressive strength. They show also that specially selected cinders will produce concrete of higher strength than the values suggested by the Joint Committee (see p. 316).

* *Engineering News*, August 10, 1911, p 185, and *Engineering Contracting*, April 30, 1913, p. 483.
† Proceedings American Society for Testing Materials, Vol. XIV, 1914, p 280

Compressive Strength of Cinder Concrete (See p. 327.)*
8 by 8 by 16-inch Prisms. Proportions 1 : 2 : 5

Kind of Coal.	Description.	Mech. Analysis % Passing.				Weight of Concrete. lb. per cu. ft.	Compressive Strength. Age 28 Days lb. per sq. in.
		1"	¾"	½"	¼"		
Georges Creek, Cumberland	Dirty, brown, soft, nearly all fine, some unburned coal	100	96	84	71	89	400
Bituminous	Dirty dark brown, soft, traces of vitreous clinker, no unburned coal	100	93	78	65	95	492
Mixture: Anthracite, Coke screenings No. 2 buckwheat, bituminous	Clean black, hard, considerable unburned coal	100	93	76	60	91	645
Variety of bituminous coal	Dirty black, not much gritty or hard, some slag and unburned coal and coke	100	89	63	43	97	812
New River Bituminous coal	Dirty light brown, soft, no unburned material	100	81	70	50	104	828
Dominion coal	Dirty black, very soft, some slag and unburned coal and coke	100	84	57	36	101	868
New River Bituminous coal	Dirty gray, soft, small amount of unburned coal and coke	100	80	49	33	109	883
Nova Scotia coal poor gravel	Black, gray, very heavy, very little unburned, large particles look like slag	100	66	53	26	113	1088
Pocohontas mixed with buckwheat	Brown, hard, well graded, no dust, no unburned coal or coke	100	83	47	26	111	1246
½" broken stone coarse sand						145	1620

* Tests at Massachusetts Institute of Technology under the direction of Prof. C. M. Spofford and Prof. H. W. Hayward. Published by permission.

Compression Tests of Cinder Concrete

*8 by 8 by 16-inch Prisms. Tests at Columbia University**

Kind of Coal.	Proportions.	Description.	Mech. Analysis. % Passing.					Weight of Concrete. lb. per cu. ft.	Compressive Strength. lb. per sq. in.			
			$1\frac{1}{4}''$	$1''$	$\frac{3}{4}''$	$\frac{1}{2}''$	$\frac{1}{4}''$		1 mo.	2 mo.	6 mo.	1 yr.
Anthracite...	1-2-5	.....	98	97	93	86	63	107	407	701	933	913
Anthracite .	1-1-5†	.....	100	98	93	76	34	100	507	662	754	813
Anthracite...	1-2-5	.....	94	...	80	57	24	107	818	1254	1744	1465
Anthracite ..	1-2-5	.....	98	96	88	65	25	109	980	1035	1478	1475
Anthracite..	1-2-5	....	.	...	....	....	..	113	1533	2066	....	2570
Anthracite..	1-2-5	......	100	94	90	81	74	113	1355	....	....	....

* Harold Perrine and George E. Strehan in Transactions Am. Soc. C. E., Vol. LXXIX, 1915, p. 523

† Hand mixed—two turns.

Coke Breeze. Coke Breeze, ordinary coke that drops through the tines of the loading forks and runs below $1\frac{1}{2}$ or 2 inches in size, gives unexpectedly high strength when used as a concrete aggregate, attaining in one series of tests‡ about three-fourths the strength expected from broken stone concrete. This aggregate, weighing only about 30 pounds per cubic foot, may be useful where a concrete of extremely light weight is required; because of its combustible nature it cannot be used for fireproofing.

Variations in Tests of Concrete Aggregates. Tests by the Bureau of Standards§ tend to confirm the opinions indicated by the authors elsewhere that with our present methods of tests the only specification for either fine or coarse aggregate that can be considered final is the requirement for strength of specimens mixed in the proportions to be used. The Bureau reports tests on 18 limestones running from 1 276 to 3 984 pounds per square inch; on 11 gravels running from 888 to 4 126 pounds per square inch; and 3 granites running from 2 376 to 3 054 pounds per square inch. Taking the low value as 100%, the range for limestone is 213%; for gravel, 354%; and for granite, 29%. Comparatively few granites were tested. One sample of cinder concrete tested at 1 647 pounds per square inch. All proportions were 1:2:4 and the age for the above strengths was 4 weeks.

It is further shown, as is evident from the laws of mechanical analysis (see Chapter X) that the relative values of different sands for use in concrete cannot be estimated accurately unless tested in combination with the coarse aggregate, because the grading of the total mixture is the determining factor.

‡ Tests at Massachusetts Institute of Technology under the direction of Prof. C. M. Spofford and Prof. H. W. Hayward. Referred to by permission.

§ Technologic Paper No. 58, U. S. Bureau of Standards, June 20, 1916.

EFFECT OF CONCENTRATED LOADING

In concrete foundations for piers and in concrete footings it is customary to load an area smaller than that of the surface of the concrete. The question at once arises whether the stress shall be based upon the load divided by the total area of the concrete footing or by the area of contact. Experiments made upon concrete and other materials show that neither of these methods is correct, but that an intermediate area should be selected for computation.

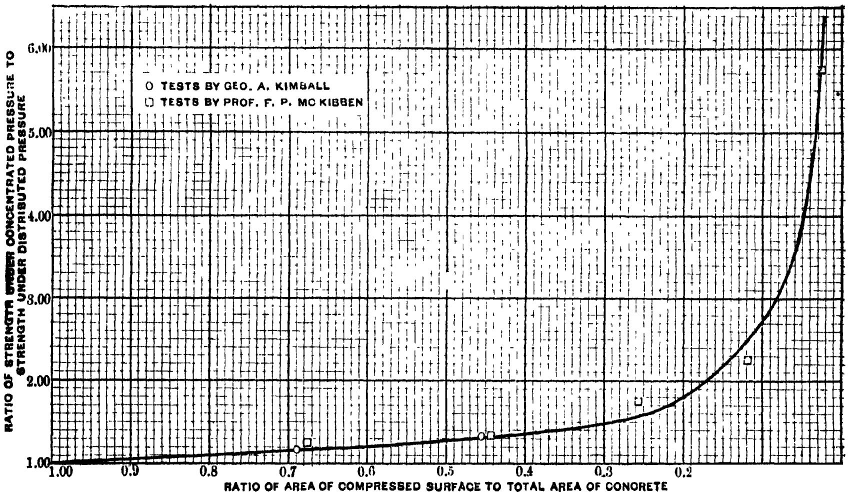


FIG. 91. Concentrated vs. Distributed Loading. (*See p.* 330.)

In connection with the designing of concrete footings for the Boston Elevated Railway, 12-inch cubes were crushed by concentrating the load upon plates 10 by 10 inches and 8 by 8¼ inches.* At Lehigh University in 1908 a set of experiments was made upon the strength of 6 by 6 inch cubes of 1:2:4 proportions where the compressed area varied from the entire area of the specimen down to 1.21 square inches.

In the diagram, Fig. 91, both sets of values† are plotted. The two sets agree where they overlap, and also are similar in general direction, and, in fact, in actual values of the ordinates, to curves drawn by Prof. J. B. Johnson‡ illustrating Bauschinger's tests upon other materials than concrete.

* Tests of Metals, U. S. A., 1899, p. 740.

† From data presented to the authors by Mr. George A. Kimball and by Prof. Frank P. McKibben.

‡ Johnson's Materials of Construction, p. 33.

In considering the smaller areas, as indicated by the smaller ratios of area, the fact must be considered that the compressed surface deforms, that is, actually compresses under the load, and the amount of deformation, which may be approximately estimated from the modulus of elasticity, may sometimes be the limiting consideration. Also, in the small areas the possibility of punching through must be considered.

To use the curve for determining the additional strength gained by the enlarged area under a pedestal or column, find the ratio of the compressed area to the total area, and from the point on the curve corresponding to this ratio find from the values at the left the increased ratio of strength to be expected. Thus, if a compressed area is one-half or 0.5 of the total area, the strength is increased 1.29 times. The use is further illustrated by the following examples.

Example 1.—What dimensions of pedestal would be required to safely support a load of 40 tons concentrated upon a plate 10 inches square, assuming an allowable distributed stress upon the concrete of 650 lb. per square inch?

Solution.—Forty tons or 80 000 pounds on 100 square inches represents 800 lb. per square inch, and the ratio of pressure required under the concentrated load to the allowable pressure is therefore $\frac{800}{650} = 1.23$; hence from the curve, the total area of concrete necessary is $\frac{100 \text{ sq. in.}}{0.55} = 182$ square inches.

Example 2.—The breaking strength of a 12-inch cube of 1 : 2 : 4 concrete having chamfered edges, so that the area of contact of the load is reduced to 9 by 9 inches, or 81 square inches, is 324 000 pounds. What may be considered as the ultimate strength of the concrete when loaded over its full area?

Solution.—The strength per square inch of the cube figured on its chamfered surface is $\frac{324\ 000}{81} = 4\ 000$ lb. per square inch. The ratio of the compressed surface to the total area is $\frac{81}{144} = 0.56$, and from the diagram we find the ratio of strength to be 1.22. Dividing 4 000 pounds, the unit strength on the concentrated surface by this gives as the probable ultimate of the concrete when loaded over its full area, 3 280 lb. per square inch.

TENSILE STRENGTH OF CONCRETE

The tensile strength of concrete is usually of little importance in design because even when the tensile value is taken into account it is a

matter of cross bending or modulus of rupture rather than of pure tension. Furthermore, tensile tests producing accurate results are hard to make on account of the difficulty in making up the specimens and breaking them. The following table gives results for medium consistencies.

*Tensile Strength of Concrete**

Proportions.	Tensile Strength at 28 Days.		Compressive Strength at 28 Days.
	lb. per sq. in.	per cent of compressive strength.	lb. per sq. in.
1 : 1 : 2	210	6.4	3290
1 : 1½ : 3	175	7.0	2500
1 : 2 : 4†	140	8.0	1750
1 : 2½ : 5†	110	9.2	1190

* Tests at Massachusetts Institute of Technology under the direction of Prof. C. M. Spofford and Prof. H. W. Hayward. Published by permission.

† The strengths of these proportions are abnormally low in compression and the tensile strengths may be assumed correspondingly low. The ratios, however, agree substantially with results from other tests.

Comparing the tensile strengths with Mr. Fuller's transverse tests of beams given on page 334, it will be seen that the tensile strength is from one-half to one-quarter the transverse strength. Just how much of this is due to the difficulty in molding and testing tensile specimens cannot be estimated, but since Fuller's specimens were made from a wet mix, the values may be considered as conservative and more representative of practical conditions than the tensile tests.

The true relation between tensile and compressive strength or flexure and compression are probably more accurately indicated by the mortar tests of Mr. R. Feret on pages 334 to 335.

TRANSVERSE STRENGTH OF CONCRETE

The strength of a beam of plain concrete is limited by the tensile strength of the concrete at the place of greatest strain, which, with vertical loading, is its lowest surface. The value of this transverse "fiber" strength or modulus of rupture is of less importance than the crushing strength, because, on account of the brittleness of concrete in tension, that is, its liability to crack from shrinkage or sudden loading, it is seldom safe, and usually is not economical, to construct beams or girders without metal

reinforcement. Most formulas for reinforced design disregard the tensile strength of the concrete. In certain computations, however, the tensile strength must be considered. Since concrete beams can be broken with less powerful and less expensive apparatus than crushing specimens, this form of specimen is often convenient for comparing the relative strength of different mixtures or different materials, and while the ratios thus obtained will not exactly coincide with those for crushing strength, they will be sufficiently close for many purposes.

Fuller's Beam Tests. The table* on page 334 gives the results of a comprehensive series of tests of 6 by 6 by 72-inch beams made by Mr. William B. Fuller at Little Falls, N. J. Although different materials than those used by Mr. Fuller will of course show slightly different strength, the table is sufficiently representative of average conditions to permit its use for comparisons of different proportions, and, with a proper factor of safety, as a working guide to the safe transverse strength of concrete.

The proportions are given by weight but can be transformed to volume measure by referring to the footnote. The various columns present valuable data on weights and volumes and voids.

The curves in Fig. 92 are plotted from the results in the table, and illustrate also the proportions corresponding to maximum strength for a given per cent. of cement.

Tests by other authorities are mentioned under Strength of Beams in References, Chapter XXXIII.

Formula for Transverse or Bending Stress in Plain Concrete. The common formulas for representing the longitudinal forces of compression and tension upon a beam are usually expressed with the following notation:

Let

f = intensity of stress at any point in the beam.
M = bending moment.
I = moment of inertia about its neutral axis of section containing the point under consideration.
y = distance of the point from the neutral axis.
b = breadth of beam.
h = height of beam.

Then $$f = \frac{My}{I} \qquad (3) \qquad \text{also, } M = \frac{fI}{y} \qquad (4)$$

* Especially prepared for this treatise by Mr. Fuller

Data Concerning Composition and Transverse Strength of Concrete Beams Tested at Little Falls, N. J., by Wm. B. Fuller, C. E.

During the year 1901. Beams, 6 x 6 x 72 inches. Spans, 30 and 60 inches. Atlas Portland Cement, River Silica Sand. Crusher Run Trap Rock, ¼ to 3 inches nominal diameter. (*See p.* 333.)

Item.	Proportions by weight.	Weight in Pounds of Material in one cu. ft of Beam as Mixed.								Calculated Volume, in cu ft. of Material in one cu. ft. of Beam as mixed.							Volume of Voids in one cu. ft.			Modulus of Rupture.					
							Totals.															Pounds per sq. in.			
	C.S G	Cement.	Sand	Stone.	Total Dry Mix.	Water.	As Mixed.	Minimum.	Maximum	Cement.	Sand.	Stone.	Total Sand and Stone.	Total Dry.	Water, 62.4.	Total.	Cement.	Aggregate	Total.	Age. Days.	Number of Breaks.	Maximum.	Minimum.	Average.	Per cent. probable error of average.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
(1)	1:0:0	112.9			112 9	24.1	137.0	121.9	138 8	.585				.585	.386	.971	.415	.000	.415	32	6	968	856	906	1.0
(2)	1:0:1	69 1		69.1	138.2	15.4	153.6	143.7	155.2	.358		.370	.370	.728	.247	.975	.254	.018	.272	32	6	872	668	772	2.8
(3)	1:0:2	49 9		99.8	149.7	12.9	162.6	153.7	162.6	.259		.534	.534	.793	.207	1.000	.183	.024	.207	33	6	802	668	731	2.4
(4)	1:0:3	38.0		113.8	151.8	11.2	163.0	154 8	163.9	.197		.609	.609	.806	.180	.986	.140	.054	.194	34	6	724	580	622	2.4
(5)	1:0:4	27.4		109.4	136.8	9.7	146.5	139.0	154 0	.142		.585	.585	.725	.155	.880	.101	.174	.275	34	3	251	236	241	2.3
(6)	1:1:0	64.9	64.9		129.8	15.3	145.1	135.0	146 7	.336	.393		.393	.729	.245	.974	.238	033	.271	34	6	866	628	734	3.8
(7)	1:1:1	47.0	47.0	47.1	141.1	12.3	153.4	144 0	154.8	.244	.285	.252	.537	.781	.107	978	.172	.047	.219	34	6	744	649	708	1.6
(8)	1:1:2	37.2	37.2	74 4	148.8	10.3	159.1	151 8	160.3	.103	.225	.308	.623	.816	.165	081	.136	.048	.184	33	6	798	646	710	3.0
(9)	1:1:3	30.1	30.2	90 4	150.7	10.8	161.5	153.1	161.8	.156	.183	.483	.666	.822	.173	995	.110	.068	.178	34	6	732	573	655	2.3
(10)	1:1:4	25.9	25.9	103.6	155.4	9 7	165.1	157 5	165.1	.134	.157	.554	711	845	.155	1.000	.095	.060	.155	33	6	512	446	486	1.9
(11)	1:1:5	22.6	22.6	113.0	158 2	7.8	166.0	100.0	167 1	.117	.137	.604	.741	.858	.125	.983	.083	059	.142	34	6	542	481	504	1.6
(12)	1:2:0	43.5	86.9		130.4	12.9	143.3	133.9	145.9	.225	.527		.527	.752	.207	.959	.160	.088	.248	33	6	640	592	616	0.9
(13)	1:2:1	34.1	68.3	34.1	136.5	12.9	149.4	139.2	150.7	.177	.414	.182	.596	.773	.207	.980	.125	.102	.227	33	6	572	459	523	2.6
(14)	1:2:2	28.6	57.1	57.1	142.8	11.7	154.5	145.1	155.3	.148	.346	.305	.651	.709	.188	.987	.105	.096	.201	33	6	552	485	528	1.8
(15)	1:2:3	25.3	50.6	76.0	151.9	7.4	159.3	153.9	161 6	.131	.307	.406	.713	.844	.119	.963	.093	.063	.156	33	1			471	∞
(16)	1:2:4	22.3	44.7	89.4	156.4	7.4	163.8	158.2	164.8	.116	.271	.478	.740	.865	.119	.984	.082	.053	.135	33	6	480	399	439	3.0
(17)	1:2:5	19.8	39.5	98.5	157.8	7.4	165.2	159 4	166.0	.103	.239	.527	.766	.869	.119	.988	.073	.058	.131	33	6	413	349	380	2.2
(18)	1:2:6	17.5	35.0	105.1	157.6	8.2	165.8	159.0	166.0	.091	.212	.562	.774	.865	.131	.996	.064	.071	.135	35	5	410	234	319	9.5

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)
(19)	1:3:0	32.0	95.8		127.8	12.2	140.0	130.4	143.6	.166	.581		.581	.747	.196	.943	.117	.136	.253	35	6	432	392	418	1.2
(20)	1:3:1	27.4	82.3	27.4	137.1	10.9	148.0	139.3	150.4	.142	.499	.146	.645	.787	.175	.962	.101	.112	.213	35	5	392	274	360	3.3
(21)	1:3:3	21.0	63.2	63.1	147.3	8.6	155.9	149.0	158.0	.109	.383	.337	.720	.829	.138	.967	.077	.094	.171	33	6	369	338	355	1.2
(22)	1:3:5	16.6	49.9	83.2	149.7	8.0	157.7	151.0	160.1	.086	.302	.445	.747	.833	.128	.961	.061	.106	.167	33	3	308	262	285	3.3
(23)	1:3.6	15.3	45.9	91.9	153.1	8.5	161.6	154.3	162.6	.070	.278	.491	.769	.848	.136	.984	.056	.096	.152	33	5	246	213	226	2.1
(24)	1:3:7	14.3	42.8	100.0	157.1	8.2	165.3	158.2	165.3	.074	.259	.535	.794	.808	.132	1.020	.053	.079	.132	33	2	257	220	239	5.4
(25)	1:3:8	13.1	39.5	105.0	157.6	7.2	164.8	158.6	165.8	.068	.239	.561	.800	.868	.115	.983	.048	.084	.132	33	4	102	159	178	2.9
(26)	1:3:9	12.1	36.3	109.1	157.5	6.5	164.0	158.5	165.0	.063	.220	.583	.803	.866	.104	.970	.044	.090	.134	33	3	176	123	145	8.2
(27)	1:4:0	25.3	101.1		126.4	11.3	137.7	128.4	142.4	.131	.613		.613	.744	.181	.925	.093	.163	.256	33	6	294	262	279	1.8
(28)	1:4:2	18.9	75.7	37.9	132.5	12.8	145.3	134.0	147.5	.098	.450	.203	.662	.760	.205	.965	.060	.171	.240	34	4	235	198	210	3.1
(29)	1:4:4	15.8	63.0	63.1	141.9	10.7	152.6	143.2	154.3	.082	.382	.337	.719	.801	.171	.972	.058	.141	.199	34	3	219	202	209	1.8
(30)	1:4.6	13.6	54.2	81.4	149.2	10.0	159.2	150.3	159.6	.070	.328	.435	.763	.833	.160	.993	.050	.117	.167	34	2	184	114	149	16.7
(31)	1:4:7	12.7	50.6	88.6	151.9	9.5	161.4	152.9	161.4	.066	.307	.474	.781	.847	.153	1.000	.047	.106	.153	34	2	100	170	181	4.3
(32)	1:4.8	11.7	46.8	93.5	152.0	9.7	161.7	152.9	161.6	.061	.284	.500	.784	.845	.155	1.000	.043	.112	.155	34	2	158	156	157	0.4
(33)	1:4:9	11.1	44.3	99.8	155.2	8.7	163.9	156.1	163.9	.058	.268	.534	.802	.860	.140	1.000	.041	.099	.140	34	2	127	120	124	2.0
(34)	1:4:10	10.6	42.6	106.5	159.7	7.3	167.0	160.5	167.0	.055	.258	.570	.828	.883	.117	1.000	.030	.078	.117	34	2	133	130	132	0.8
(35)	1:5:0	20.9	104.7		125.6	13.0	138.6	127.3	141.6	.108	.635		.635	.743	.208	.951	.077	.180	.257	33	4	180	170	173	1.0
(36)	1.5:3	15.5	77.5	46.5	139.5	10.8	150.3	140.7	152.0	.080	.470	.249	.719	.799	.173	.972	.057	.144	.201	34	2	153	149	151	0.9
(37)	1:5.5	13.4	67.1	67.1	147.6	10.2	157.8	148.7	157.9	.069	.407	.350	.766	.835	.163	.908	.049	.116	.165	34	2	163	150	161	0.9
(38)	1:5.7	11.8	58.8	82.3	152.9	8.0	161.8	153.8	161.8	.061	.356	.440	.790	.857	.143	1.000	.043	.100	.143	34	2	134	123	120	3.0
(39)	1:5.9	10.6	53.3	95.9	159.8	6.8	166.6	160.6	166.6	.055	.323	.513	.836	.891	.109	1.000	.039	.070	.109	33	2	113	105	109	2.6
(40)	1:5:11	9.3	46.7	102.6	158.6	7.5	166.1	159.3	166.1	.048	.283	.549	.842	.880	.120	1.000	.034	.086	.120	33	2	120	113	116	2.1
(41)	1:6:0	17.4	104.3		121.7	13.8	135.5	123.1	130.0	.090	.632		.632	.722	.221	.943	.064	.214	.278	33	2	94	92	93	0.8
(42)	1:6:2	14.5	87.1	29.1	130.7	12.4	143.1	131.9	145.7	.076	.528	.156	.684	.760	.190	.959	.053	.187	.240	33	2	102	102	102	∞
(43)	1:6:4	13.1	78.4	52.3	143.8	10.8	154.6	144.8	154.8	.068	.475	.280	.755	.823	.173	.906	.048	.120	.177	33	2	115	111	113	1.2
(44)	1:6.6	11.2	67.5	67.5	146.2	10.0	156.2	147.1	156.9	.058	.400	.361	.770	.828	.160	.988	.041	.131	.172	33	1			78	∞
(45)	1:6.8	10.2	61.5	82.0	153.7	8.5	162.2	154.5	162.2	.053	.373	.438	.811	.864	.136	1.000	.037	.099	.136	33	1			84	∞
(46)	1:6:10	9.3	56.0	93.2	158.5	7.2	165.7	150.2	165.7	.048	.339	.498	.837	.885	.115	1.000	.034	.081	.115	33	2	91	87	89	1.6
(47)	1:7.0	15.6	109.2		124.8	12.8	137.6	120.0	140.8	.081	.662		.662	.743	.205	.948	.057	.200	.257	33	1			95	∞
(48)	1:8.0	14.0	112.4		126.4	11.5	137.9	127.5	141.8	.073	.681		.681	.754	.184	.938	.051	.105	.246	33	1			41	∞

Volumes, cu. ft. per 100 lb., as mixed, — cement 1.00, sand 1.08, stone 1.02. Specific gravity cement paste, 1.81; cement, 3.00; sand, 2.64; stone, 2.99. Weights, pounds per cu. ft. as mixed,—cement, 100; sand, 93; stone, 98, water, 62.4. Tensile strength of cement, lb. per sq. in., neat; 7 days, 834; 28 days, 919. Mechanical Analyses — Per cent by weight, of grains below diameter in inches; sand, 100%, 0.25; 90%, 0.10; 75%, 0.06; 50%, 0.028; 25%, 0.014; 0%, 0.003; stone, 100% 2.1; 0%, 0.17. Col. 9 = Col. 3 x 0.98 + Col. 6. Col. 10 = Col. 6 + Col. 20 x 62.4. Cols. 9 and 10 represent minimum and maximum weights per cubic foot.

For rectangular sections, $I = \frac{bh^3}{12}$ and up to the elastic limit for beams of homogeneous material (but not for reinforced beams), $y = \frac{1}{2} h$.

Hence for rectangular beams of homogeneous material,

$$f = \frac{6M}{bh^2} \qquad (5) \qquad\qquad \text{also, } M = \frac{1}{6} f b h^2 \qquad (6)$$

In considering the strength of a beam, since the stress is greatest at one or the other of the surfaces, y is generally understood to represent the distance of the most strained fiber from the neutral axis, and f the intensity of stress upon this fiber.

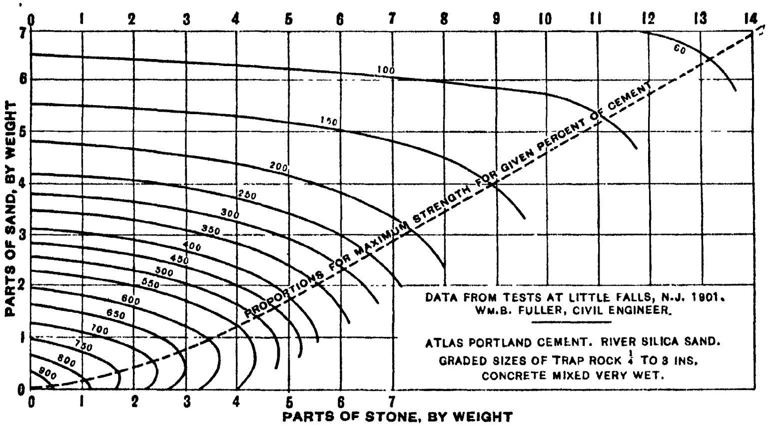


FIG. 92.—Curves showing strength of beams in pounds per square inch for various proportions by weight of sand and stone to one part Portland cement. Age 34 days. (*See p.* 333.)

The foregoing formula is based upon the assumption that the neutral axis passes through the center of gravity of the cross section. For unreinforced mortar and concrete this is true, only in the early stages of the loading. But, although it is not correct after the elastic limit is passed, the comparative results computed on different beams of similar materials are relatively correct.

For convenience in designing, a table is given in Chapter XXII for bending moments caused by uniformly distributed loads and for loads concentrated at different points. Also, in the same chapter, the moments of inertia, I, for various sections are tabulated. These tables are applicable for the most part to both plain and reinforced beams.

Relation of Compressive to Transverse Strength of Concrete. The compressive strength of concrete varies from 4 to 8 times the transverse strength. The ratio varies with different ages, for the growth of compressive strength appears to be faster than the growth of tensile and transverse. This is specially true of concrete mixed with weak aggregates such as cinder. There appears to be very little difference in these relations between different proportions of the same materials.

Tests by the U. S. Geological Survey at 28 days showed limestone and gravel concrete to be six times as strong in compression as in the tensile fibre stress in bending. For granite, the ratio was 7.5. The average ratios for all materials were 6, 7.5, and 8, at 4, 13, and 26 weeks, respectively. The beams used in this series were full size and should therefore be more reliable than the small specimens used in other tests which have given somewhat lower ratios. The proportions were 1 : 2 : 4.

Shearing Strength of Concrete

By Prof. Charles M. Spofford.

Massachusetts Institute of Technology. (*See* p. 337)

Age of Concrete 24 to 32 days

Mixture.	Method of Storing.	Shearing Strength lb. per sq. inch.			Average Compressive Strength in lb. per sq. inch.	Ratio of Shear to Compression
		Maximum.	Minimum.	Average.		
1 : 2 : 4	Air	1630	960	1310	2070	0.63
1 : 2 : 4	Water	2090	1180	1650	2620	0.63
1 : 3 : 5	Air	1590	890	1240	1310	0.95
1 : 3 : 5	Water	1380	840	1120	1360	0.82
1 : 3 : 6	Air	1450	950	1180	950	1.24
1 : 3 : 6	Water	1200	1040	1120	1270	0.88
Average Ratio for 1 : 2 : 4 and 1 : 3 : 5 Concrete						0.76

STRENGTH OF CONCRETE IN SHEAR

Tests indicate that the strength of concrete in direct shear ranges from 60 to 80 per cent. of the compressive strength. These ratios from tests by Prof. Spofford in the table just given agree substantially with experiments made by Prof. Arthur N. Talbot at the University of Illinois.* Prof. Talbot concluded that the resistance to shear is dependent upon the strength of the stone as well as upon the strength of the mortar, and for the richer mixture the strength of the stone probably exerts the greater influence.

* University of Illinois, Bulletin No. 8, 1906

This direct shear must not be confused with shear in a beam involving diagonal tension where the concrete may break when the shearing unit stress is 10% of the crushing strength.

It is difficult to determine satisfactorily the resistance of concrete to direct shear owing to the difficulty of eliminating the effect of bearing action, diagonal tension, and beam stresses in general. At the Institute the test specimens were cylinders 5 inches in diameter by 18 inches long, and in testing, the end thirds of the cylinders were held rigidly by cast iron yokes, the pressure being applied through a cast iron half cylinder bearing, fitting between the two yokes, so as to shear the concrete across two planes. To compare the compressive strength of the concrete with the shearing strength, six extra cylinders of the same dimensions were crushed.

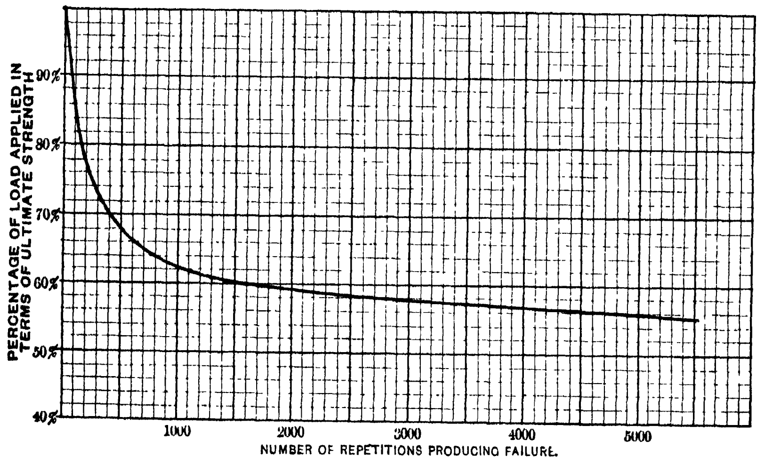


FIG. 93.—Fatigue of Neat Cement under Compression. (*See p.* 338.)

THE FATIGUE OF CEMENT

The action of cement under repeated stresses has been slightly investigated by Prof. J. L. Van Ornum* at Washington University. The experiments were made upon 2-inch neat Portland cement cubes four weeks old. The results of tests on 92 of these blocks are shown in the diagram in Fig. 93.

* Transactions American Society of Civil Engineers, Vol. LI, p. 443.

PLASTICITY OF CONCRETE

Plasticity of concrete is the property of flowing, or yielding very slowly under the continued pressure of heavy loads. Under ordinary working loads, concrete deforms in accordance with Hooke's law, but if the load, after being applied, is left in place the deformation resulting from the plastic property may be from three to five times* as great as that immediately following the application of the load itself. Tests indicate that under a fixed load these progressive deformations continue for a more or less definite time and then cease.† Under a computed 500-pound unit stress in 1: 2: 4 gravel concrete beams, this period was about two weeks, and under a 1 000-pound stress a few days longer than two weeks.

Poisson's Ratio, the ratio of the deformation at right angles to the stress, to the deformation in the direction of the stress, has been found by various experimenters to range from 0.05 to 0.20. All things considered, a fair average appears to be about 0.10, and this value may be taken in computations requiring its use.

DETERMINING PROPORTIONS OF OLD CONCRETE

The approximate ratio of cement plus fine aggregate to coarse aggregate in concrete already in place can be determined by cutting out a piece of the concrete, crushing it to destruction in a testing machine and separating out the stones with a small hammer.

The ratio of cement to fine aggregate can then be determined by dissolving out the cement in a strong solution of muriatic acid provided the aggregates are themselves insoluble. If the sand contains limestone, as is frequently the case where the country rock is limestone, a sample must be subjected to a separate acid test in order to correct for the amount dissolved out with the cement. When the coarse aggregate is limestone and the amount of dust originally in it is unknown, exact determination cannot be made.

Mr. Nathan C. Johnson, has suggested a method‡ of determining the proportions of old concrete by making a microphotograph of a polished section of the surface and planimetering the areas of cement, sand, and stone. The method is said to give good results.

* Tests by F. R. McMillan, Bulletin of the University of Minnesota, March, 1915.

† Earl B. Smith, *Engineering Record*, March 4, 1916, p. 329.

‡ *Engineering Record*, February 27, 1915, p. 263.

MACHINE FOR COMPRESSION TESTS.

A convenient compression testing machine operated by a hydraulic jack is shown in Fig. 94, page 340, as designed by Mr. William O.

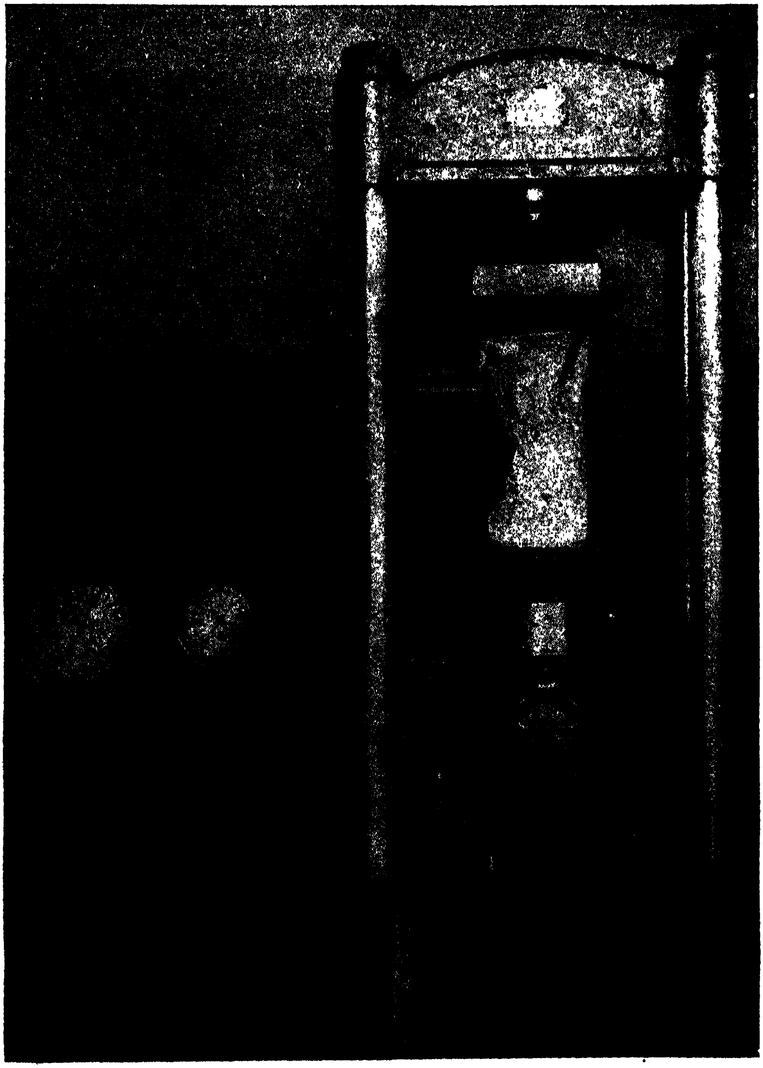


FIG. 94.—Hydraulic Compression Testing Machine. (See p. 340.)

Lichtner* and installed in the laboratory of Sanford E. Thompson. It takes specimens up to 12 inches square and its capacity is 250 000 pounds.

The screw type of testing machine, which may be used for both tensile and compressive tests, is shown in Fig. 95, page 341. The 10 000 000-pound compression testing machine of the U. S. Bureau of Standards, the largest in the world, is a screw machine.

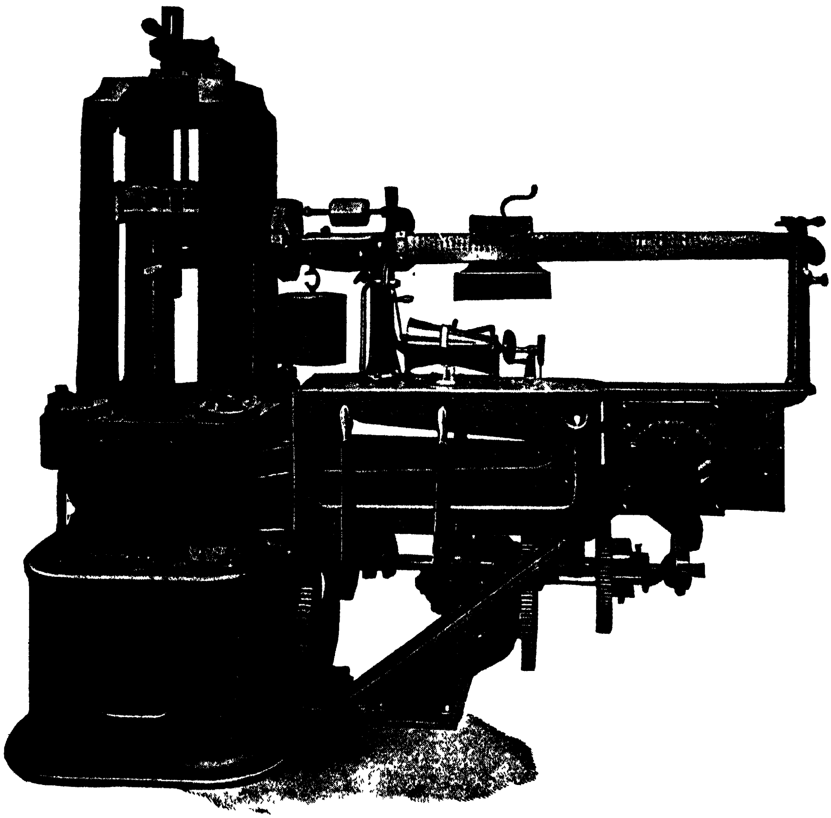


FIG. 95.—Motor Driven Screw Type Compression Testing Machine. (*See p.* 341.)

SAWING TEST SPECIMENS

Concrete blocks cut from structures for compression tests usually must be shaped to the form of prisms, with two parallel bearing faces.

* See description in paper by William O. Lichtner, Proceedings American Society for Testing Materials, Vol. XIV, Part II, 1914, p. 535.

The saw shown in Fig. 96, page 342, is being used successfully in the author's laboratory for sawing the concrete. Two 30-inch discs $\frac{1}{8}$-inch thick, of soft Norway iron, are revolved at a speed of 70 revolutions per minute by a $1\frac{1}{2}$ horse-power motor. No. 20 carborundum mixed with a medium grade of automobile grease and kerosene oil is fed on to the rims of the blades. The block is clamped down to a traveling table that feeds against the saw automatically as the cut deepens. The saws are held in place by nuts and the shaft is threaded the entire length so that the width between blades can be varied from 4 to 12 inches, according to the thickness of block required.

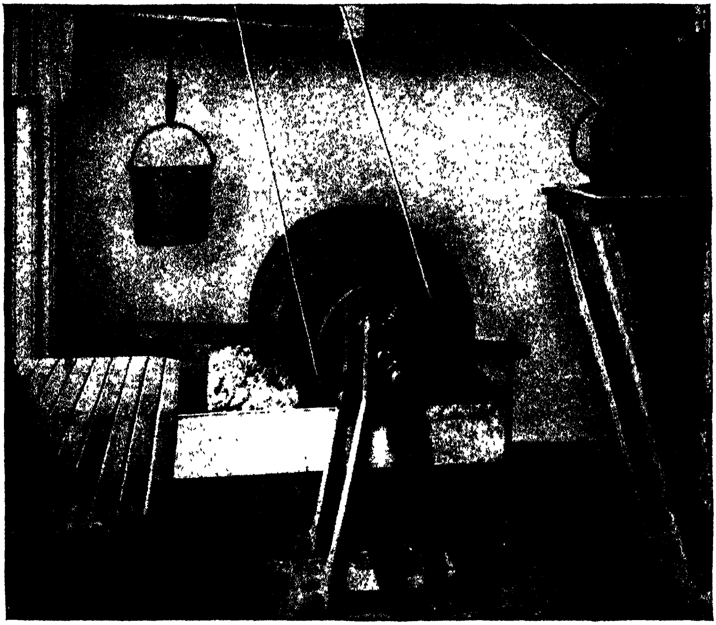


FIG. 96.—Saw for Shaping Concrete Test Specimens. (*See p.* 342.)

MICROPHOTOGRAPHY OF CONCRETE

Mr. Nathan C. Johnson by the use of the microscope* has succeeded in photographing concrete surfaces enlarged to 200 diameters. The voids known to exist in concrete, are made visible to the eye and the action of the sea water and ground water salts that crystallize in these voids, sometimes disintegrating the concrete, has been demonstrated.

* Series of six articles in *Engineering Record*, starting January 23, 1915, p. 98.

METHODS OF TESTING CONCRETE

The methods of testing concrete and the interpretation of results, as investigated by various individuals and by the American Concrete Institute, are discussed in the following pages.

Specimens for Compressive Tests. A compression test specimen in the form of a cylinder with the height equal to twice the diameter is recommended by the Committee on Specifications and Methods of Tests for Concrete Materials of the American Concrete Institute.* The diameter of the cylinder should be at least equal to four times the maximum size of the particles of the coarse aggregate. Where possible, the 8 by 16-inch cylinder should be used, but for small stone 6 by 12-inch is satisfactory and, because of its lighter weight, more convenient for field specimens taken from such work as building construction. Cubes, provided the strength is corrected for length (see p. 344), or prisms, may be used, but cylinders are to be preferred on account of greater ease in securing homogeneous specimens. Cylinders and prisms of the same dimensions give substantially the same results in unit strength.

The theoretical angle of crushing is about 60° with the horizontal and a prism, to allow for two such 60° pyramids, one upright and the other inverted, must be twice as high as it is wide. The specimen shown in the testing machine in Fig. 94, page 340, shows the way in which a cylinder breaks.

Effect of Ratio of Height to Width. Tests by various laboratories under the direction of the American Concrete Institute show the variation in strength as the ratio of the height to width of specimen changes. When the height is more than twice the diameter, that is, when the ratio is greater than 2, there is very little variation in strength, but when the height is less than twice the diameter, the strength increases rapidly with the shorter specimens. The curve in Fig. 97, page 344, shows the variation. By this curve, cubes and prisms of other than the standard dimensions may be used if unavoidable and the results corrected to allow for the standard specimen.

Making Test Specimens. In making concrete specimens, if the nominal proportions are by volume, determine the weight of each material and correct to give the corresponding proportions by weight. Estimate and weigh up the amount of each material for the batch.

Only experienced men should mix concrete for experimental specimens. There is a certain knack which can be acquired only by practice, in

* Journal American Concrete Institute October-November 1914, p. 422.

properly turning the materials so as to mix them thoroughly, and the amount and manner of ramming or puddling is so important that specimens may be rendered worthless by improper manipulation.

Usually several specimens are made of the same proportions to secure a reliable average result. These specimens should be mixed one at a time from individual batches in order to avoid variations in proportions and consistency throughout the batch. This involves more labor but increases the value of the results.

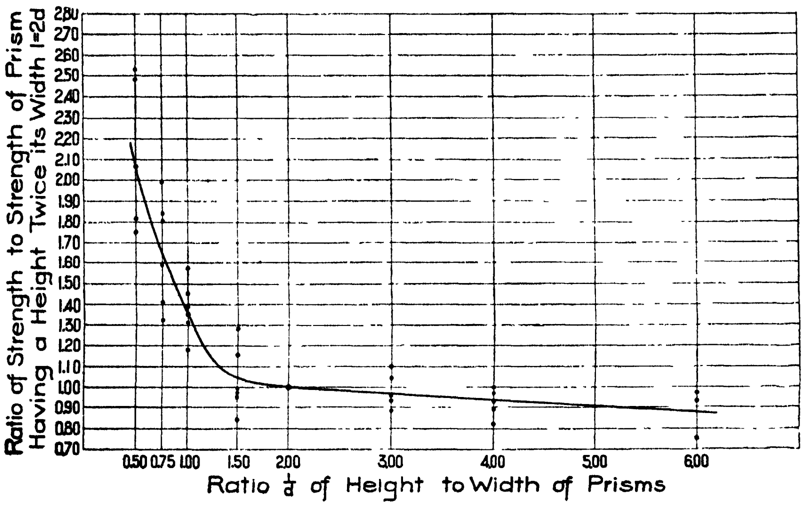


FIG. 97.—Comparative Strength of Concrete Prisms of Different Heights. (See p. 343.)

Method of Quartering. To obtain an average sample from a pile of sand, gravel, or stone, the method of quartering is useful. Shovelfuls of the material are taken from the various parts of the pile, mixed together and spread in a circle. The circle is quartered, as one would quarter a pie, two of the opposite quarters are shoveled away from the rest, and the remainder is thoroughly mixed, spread, and quartered as before. The operation is repeated until the quantity is reduced to that required for the sample.

Weights and Voids. To determine weights per cubic foot, fill a measure having a capacity of ¼ cubic foot or more with the sand or gravel; lift the measure 2 inches above the floor and drop it; repeat, raising and dropping five times; fill measure full; strike off with a straight-edge, and weigh.

From the weights and the specific gravity (see pp. 123 and 124) the voids can be determined.

Mixing. All mixing should be done on a surface of metal or other impervious material. For large specimens and batches, a sheet of zinc or sheet iron is convenient to use. A batch for a single 6 by 12-inch cylinder can be mixed in a galvanized iron pan about 24 inches square and $1\frac{1}{2}$ inches deep, using a 10-inch bricklayer's trowel blunted by cutting off 2 inches of the point. If the mixing is done on a wooden platform or concrete floor, the surface should be thoroughly wet several minutes before mixing is begun.

The procedure for hand mixing is as follows: Mix the cement and sand until a uniform color is obtained; this will require not less than seven turns. Spread out the dry mixture in a layer of uniform thickness and on this spread uniformly the coarse aggregate. (At this point, gravel should be dry; porous crushed stone should be dampened.) Mix these materials at least four times. Form a shallow crater and pour into it about two-thirds the required amount of water. Turn into the crater the dry material from the edges until the water is absorbed, and turn the mass until the batch is of a uniform color and consistency throughout, adding, by sprinkling, from time to time the remainder of the water used. The consistency of the concrete for the specimen in the laboratory, to obtain results comparable with the sluggishly flowing concrete recommended for reinforced concrete in practice, should be a little stiffer than this,—about like a thick oatmeal mush. About seven turns of the wet concrete are necessary.

Making Specimens. Place concrete in the molds in layers 3 to 4 inches thick, using care to prevent pockets of stone against the form. Tamp the concrete to bring the mortar to the surface and imbed all stones. Level off the top of the specimen with a trowel, but avoid working longer than is necessary.

Capping. The bottom of the specimen is sufficiently prepared by casting on a machined metal plate or piece of plate glass. The top of the specimen is apt to be irregular, and if the mold is not quite full after setting, the cylinder may be filled flush with mortar and leveled off with a piece of plate glass left in place until the mortar sets. Both ends are thus made parallel. In testing, pieces of blotting paper may be placed between the end of the cylinder and the heads of the machine. Plaster of Paris is also used to square up the ends of slightly irregular specimens or neat cement paste may be used if much out of true.

Molds. The most satisfactory molds are of cast iron with machined metal base plates.

A less expensive metal mold than cast iron, although not so satisfactory, may be made with a piece of sheet metal or galvanized iron of 18 or 20 gage shaped to the proper diameter with two longitudinal flanges about an inch wide. The mold may be made tight with clamps on the flanges or by holding the flanges together and slipping them into saw cuts in 2 by 4-inch timbers held rigidly in a horizontal position at the proper heights. If machined metal bases are not provided, plate glass coated with oil is satisfactory.

For mortar, cylinders 2 inches in diameter by 4 inches long are recommended tentatively by the American Society for Testing Material. (See p. 81.)

Wood molds, although saturated with oil or coated with paraffine, are apt to absorb water.

Storage of Specimens. Test specimens must be kept moist; if stored in dry air, the gain in strength is much below normal, and a series carried out by the University of Illinois for the American Concrete Institute showed no gain at all up to two years. (See p. 320). Moisture may be supplied by burying in damp sand or by covering with cloths suspended so as to cover and surround the specimens without touching them, and kept wet. Specimens properly stored show little or no loss in weight after removal from forms up to time of testing.

Method of Testing. Use a spherical bearing block on top of the specimen, the diameter of the block at least as great as that of the specimen. Keep the upper section of the adjustable block in motion as the head is brought to a bearing on the specimen, thus insuring a central bearing and preventing the block from being pulled aside, as frequently happens when the block is allowed to adjust itself.

The moving head of the testing machine should travel at a rate of from 0.04 to 0.10 inches per minute.

Note the character of failure and appearance of specimen and its behavior during test.

Specimens for Field Tests. On important work samples should be taken regularly for tests at 14 and 28 days. For beams, columns and girders the concrete may be taken from barrows just before depositing; but wherever possible it should be taken from place just after depositing. The following procedure gives good results: Shovel the concrete into a 14-quart galvanized iron pail, carry to the molding yard and remix to eliminate segregation of materials due to carrying, and

Expt. No.
File *Waltham Reservoir.*
Date *2/9/06.*

Form for Recording Data on Concrete Specimens

(Figures in ( ) refer to Item Numbers.)

ITEM.		
1.	Nominal Proportions	*1 : 1.8 : 4.1*
2.	Car No.	*00*
3.	Kind of Cement	*Atlas*
4.	Kind of Sand	*¾ u c. ¼ G*
5.	Analysis No.	*420 and 421*
6.	Kind of Coarse Aggregate	*W. Gravel*
7.	Analysis No.	*422*
8.	Weight of Cement Used	*3.12*
9.	Weight of Sand Used	*5.72*
10.	Weight of Coarse Aggregate Used	*12.85*
11.	Weight of Water Used	*1.77*
12.	Per Cent Water to Weight of Cement plus Sand	*20%*
13.	Temperature of Water	*60° F.*
14.	Temperature of Laboratory	*70° F.*
15.	Total Weight of Material (8) + (9) + (10) + (11)	*23.46*
16.	Weight of Mold Empty	*3.00*
17.	Weight of Mold Filled	*26.30*
18.	Weight of Concrete Net	*23.30*
19.	Weight of Concrete Left Over	*0.00*
20.	Weight Unaccounted for—Assumed as Solid Material*	*0.16*
21.	Weight Unaccounted for—Assumed as Water	*0.00*
22.	Volume of Fresh Specimen (cu. ft.)	*0.1527*
23.	Weight of Specimen—Mold Removed	*22.7*
24.	Method of Storage	*Air*
25.	Weight of Specimen Before Testing	*22.5*
26.	Measurements of Specimen Before Testing	*7.09″ × 8.02″ × 4.12″*
27.	Date and Hour Specimen Made	*2/9–3 p.m.*
28.	Date Tested	*3/9–10 a.m.*
29.	Specific Gravity Cement *3.15* 30. Sand *2.65* 31. Stone *2.75*	
32.	Weight of Cement in Fresh Concrete $(8) \times \frac{(18)}{(18)+(19)+(20)}$	*3.09*
33.	Weight of Sand in Fresh Concrete $(9) \times \frac{(18)}{(18)+(19)+(20)}$	*5.68*
34.	Weight of Coarse Aggregate in Fresh Concrete $(10) \times \frac{(18)}{(18)+(19)+(20)}$	*12.76*
35.	Weight of Water in Fresh Concrete $(11) \times \frac{(18)}{(18)+(19)+(20)}$	*1.76*
36.	Absolute Volume Cement in Fresh Concrete (assume 1 cu.ft. water, 62.4 lb.) $\frac{(32)}{(22) \times 62.4 \times (29)}$	*0.103*
37.	Absolute Volume Sand in Fresh Concrete $\frac{(33)}{(22) \times 62.4 \times (30)}$	*0.225*
38.	Absolute Volume Coarse Aggregate in Fresh Concrete $\frac{(34)}{(22) \times 62.4 \times (31)}$	*0.487*
39.	Absolute Volume Water in Fresh Concrete $\frac{(35)}{(22) \times (62.4)}$	*0.184*
40.	Total Absolute Volume Materials (36) + (37) + (38) + (39)	*0.999*
41.	Density (36) + (37) + (38)	*0.815*
42.	Remarks	

Computed by *G. B.*
Checked by *S. E. T.*

*** Adhering to Tools and Trays. Divide the Total Loss, (15) — [(18) + (19)], by Estimation into Items (20) and (21).**

pour into iron molds set on an iron plate and imbedded in moist sand. Use 8 by 16-inch cylinders if the aggregate runs up to 2 inches or more, but 6 by 12-inch cylinders will prove more convenient if the maximum size of stone is only $1\frac{1}{2}$ inches. Tamp the concrete with a 6-inch ice chopper, taking about the same precautions as are employed on regular work. The time of dumping into the molds and tamping should never exceed 5 minutes. Trowel the top surface just previous to initial set. After 48 hours take the blocks out of the sand, remove from the molds and rebury in moist sand until the day before testing.

Specimens for Rough Tests. If the quality of sand is questioned and a laboratory is not available, a rough test may be made by mixing up a block of mortar or concrete, using the same aggregates mixed in the same proportion and to the same consistency that is to be employed in the work, and examining the specimens from day to day. If kept at a temperature of 65° to 70° Fahr. under a moist cloth, the mortar or concrete should harden after 24 hours so as to resist the pressure of the thumb, and at the end of a week in the air it should be hard and sound.

Recording Test Data. Tests are so expensive to make that it is always worth a little extra trouble to record enough data to make them of general use to engineers. Through failure to do this, a large proportion of the tests that have been performed are of local value only. The form on page 347 is designed for recording data used while making specimens. To this may be added blanks for recording the properties of the materials used and for recording the results of the test.

The form presented is designed to record not merely the information required for a compression specimen but also to include the data and to give the routine method of computing the density of the concrete, (see p. 148) a matter of the greatest importance in studying the comparative qualities of different materials and mixtures.

CHAPTER XX

THEORY OF REINFORCED CONCRETE

Reinforced concrete is concrete in which steel or other reinforcing metal is imbedded to increase its strength. The reinforcement in general exercises an auxiliary function as it is not self-sustaining but requires the support of the concrete to develop its resistance. Thus most often reinforcement consists of small bars of little stiffness in themselves but which, when imbedded in concrete to secure lateral support and bond, are capable of developing tensile or compressive resistance equal to that of self-sustaining structural steel. An arch of the Melan type (see Chap. XXVI) may be considered a reinforced concrete structure, provided the metal ribs, even if otherwise strong enough to carry all the load, are not connected by lateral bracing and therefore have insufficient stability without assistance of the concrete. A similar arch or a girder structure consisting of metal ribs connected laterally by metal bracing and strong enough to carry the entire load should not be considered as a reinforced concrete structure, since it is in reality a metal bridge encased in concrete, which serves, not for stress bearing purposes, but for the auxiliary purpose of protecting the steel against corrosion and giving the structure the appearance of a masonry structure. In a similar way concrete columns and other members may be reinforced with structural steel. When, however, as in steel frame structures fireproofed with concrete, the steel is self-supporting, designed to take the whole or nearly the whole of the stresses with the concrete merely as an auxiliary for, or chiefly for, protection, the member is not reinforced concrete.

The theory of the design of reinforced concrete is definitely established. The action of combinations of steel and concrete in tension and compression and shear has been analyzed so that a thoroughly rational treatment is possible. In practice, in beam design, the straight line theory, as it is termed (see p. 352), which was selected and adopted by the authors for the first edition of this Treatise, in 1905, has since that time been accepted as the simplest to employ in computation and as giving results which may be used in design with safety and economy.

In this chapter is presented the analysis of this straight line theory of stresses for rectangular beams (p. 352) followed by the same theory

applied to T-beams (p. 355). The analysis of a beam with steel in top and bottom is given on page 358, and the analysis of beams with the concrete assumed to bear tension on page 360. The analysis of shear and diagonal tension is on page 362.

The theory of columns of reinforced concrete reinforced with vertical steel bars is treated on page 375, and that of columns reinforced with vertical steel bars and spirals on page 377. Analyses and formulas are presented for the distribution of stresses in reinforced concrete under combined thrust and bending moment (p. 377) for use in arch design and in the design of columns and beams with eccentric load or thrust. The theory of reinforced concrete chimney design is treated on p. 390.

Formulas to use in practical design with illustrations of methods of treatment will be found in Chapter XXII. Tests of reinforced concrete covering all usual features of design are taken up in Chapter XXI.

GENERAL PRINCIPLES OF REINFORCED CONCRETE BEAMS

Concrete is very strong in compression but is brittle and unreliable in pull or tension. Therefore, it cannot be used economically where tensile stresses have to be resisted. Steel, on the other hand, being a comparatively ductile material, is well adapted for resisting pull, but is more costly than concrete for resisting compression. The economy in the use of reinforced concrete is obtained by placing concrete where compressive stresses are to be resisted, and steel where tensile stresses are to be resisted. The high bond and shearing resistance of concrete holds the steel and concrete together so that they act as one unit.

Requirements for Formulas for Reinforced Concrete Beams. The behavior of reinforced concrete beams under load, as discussed on page 405, is different from that of homogeneous beams. The location of the neutral axis for varying intensities of load is not constant. The compression in the concrete is nearly proportional to the load but the pull in the steel is not proportional to the load because of the variable amount of pull resisted by concrete (see p. 407). Although it is thus impossible to make formulas which represent actual conditions during the whole process of loading, the common formulas for design of beams give safe and economical results. They must satisfy the requirements that:

(1) The compressive stresses in concrete for working loads must not exceed the allowable unit stress.

(2) The beam must have the required factor of safety based on ultimate loads and elastic limit of steel.

The first requirement fixes the unit stresses for concrete. Formulas satisfying these requirements produce a design having a larger factor of safety against compression failure than against tensile failure because concrete is less uniform in its qualities and also it may be called upon during construction to resist stresses before its full strength has been attained.

To satisfy the second requirement, it is necessary in the analysis of beams to eliminate the variable amount of tensile stress carried by the concrete (see p. 405), and assume that all the tensile stresses are carried by steel. Analysis based on this assumption will not represent the actual conditions in a beam under working load because the actual stress in steel will be less than the computation will show, but it will give the required factor of safety and therefore be correct for design. On page 412 is given a comparison between actual stresses and stresses computed by the accepted formula for beams with different percentages of steel. It is seen that for earlier stages of loading, the actual stress is much less than the computed. The difference, which is due to the tensile resistance of concrete, decreases with the increase of the load. At the elastic limit of steel the computed stresses agree fairly well with the actual stresses. The action is shown by the tests, illustrated in Fig. 119, page 413, which give the deformations of concrete and steel at various loads. Stresses, of course, are proportional to the deformations.

The elastic limit of the steel corresponds to the ultimate strength of the beam in tension. Therefore the factor of safety must be based on strength at the elastic limit and formulas must be used which give correct results at this period of the loading.

In analyzing the results of the tests in the early stages of the loading, it is sometimes necessary to consider the tensile stresses in concrete. Formulas for such a case are given on page 360.

ASSUMPTIONS

In the analysis of beams, the following assumptions will be made:

(1) A plane section before bending remains plane after bending. (See p. 403.)

(2) Tension is borne entirely by the steel. (See p. 351.)

(3) Initial stresses are absent in the steel.

(4) Adhesion of concrete to steel is perfect within the elastic limit of the steel.

(5) Modulus of elasticity of concrete is constant. (See p. 400.)

Reasons for selecting these assumptions are as follows:

(*a*) Beams designed by formulas based on them have the required factor of safety.

(*b*) The method of design is the simplest.

(*c*) Adoption by the highest authorities in America and Europe.

ANALYSIS OF RECTANGULAR BEAMS*

Bending Moment and Moment of Resistance. In a beam subjected to bending, the bending moment due to the external forces, or loads, is resisted by the moment of the internal resisting forces, which will be called stresses. Since by simple mechanics, the bending moment for equilibrium must be equal to the resisting moment of the internal forces, or stresses, the unknown stresses in the materials may be found by equating the known external bending moment to the internal resisting moment.

Straight Line Formula. The stresses cause deformation in the material and the consequent deflection of the beam. At any vertical section through this beam, the compressive stresses above the neutral axis cause shortening of the fibers, and the tensile stresses below the neutral axis cause lengthening of the fibers. Assuming that a plane section before bending is plane after bending, that is, that a plane section through a beam simply swings its position without warping when the beam is bent, the deformation, or change in length, in any fiber is proportional to its distance from the neutral axis, as shown in Fig. 98, page 353. With a constant modulus of elasticity (see p. 400), stress is always proportional to deformation; therefore, the variation of the resisting stresses from the neutral axis upward can be represented by a straight line, as seen from Fig. 98, page 353. The compressive stresses then form a triangle having for its base the stress in the extreme fiber, f_c, and for its height the distance from the extreme fiber to the neutral axis, kd. The total compression may be considered as concentrated at the center of gravity of the triangle, which is distant $\frac{1}{3}\ kd$ from the extreme fiber. The tensile stresses may be considered as acting in the center of gravity of the steel, which for one layer of bars is at the center of the bar, and for more layers, at the center of gravity of the set of bars.

For equilibrium, the sum of all forces must equal zero, or the total compression must be equal to the total pull. The total tension and total compression, which are equal and acting in opposite directions, form a couple with a moment arm equal to the distance between the center of steel and the center of gravity of the triangle of compressive stresses.

*The formulas in this chapter were originally prepared by Prof. Frank P. McKibben.

The moment caused by this couple is the resisting moment. For equilibrium this must equal the bending moment due to exterior forces.

FORMULAS FOR RECTANGULAR BEAMS

For this and succeeding analyses, let

h = total depth of beam.
t = thickness of T-beam flange, i.e., thickness of slab.
b = breadth of rectangular beam or breadth of flange of T-beam.
b' = breadth of web of T-beam.
A_s = area of cross-section of steel.
p = ratio of steel in tension to area of beam, bd.
In beams with steel in top and bottom:
p_1 = ratio of tensile steel to area of beam, bd;
p' = ratio of compressive steel to area of beam, bd.
f_c = compressive unit stress in outside fiber of concrete.
f'_c = tensile unit stress in outside fiber of concrete.
f_s = tensile unit stress, or pull, in steel.
f'_s = compressive unit stress in steel.
E_c = modulus of elasticity of concrete.
E_s = modulus of elasticity of steel.
$n = \frac{E_s}{E_c}$
d = depth from outside compressive fiber to center of gravity of steel.
a = ratio of depth of compressive steel to depth, d, of beam.
k = ratio of depth of neutral axis to effective depth of beam, d.

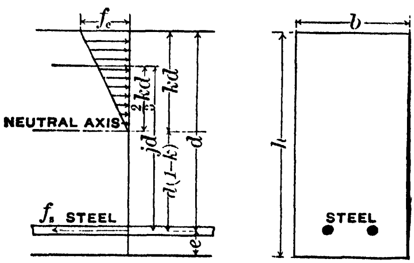


FIG. 98.—Resisting Forces in a Reinforced Concrete Beam. (See p. 352.)

kd = depth of neutral axis below the compressive surface in a beam.
j = ratio of lever arm of resisting couple to depth d.
jd = distance between centers of tension and compression.
e = thickness of concrete below center of gravity of tensile steel.
M = moment of resistance or bending moment in general.
C = constant in Table 15, page 596.

Since it is assumed that a plane section before bending remains a plane section after bending, we have the proportion

$$\frac{\text{stretch in steel}}{\text{deformation in outside compressive concrete fibers}} = \frac{d(1-k)}{kd}$$

and since deformation $= \dfrac{\text{stress per square inch}}{\text{modulus of elasticity}}$, we have

$$\frac{\dfrac{f_s}{E_s}}{\dfrac{f_c}{E_c}} = \frac{d(1-k)}{kd} \quad \text{or} \quad \frac{f_s}{nf_c} = \frac{1-k}{k} \quad (1) \qquad \text{and} \quad k = \frac{1}{1+\dfrac{f_s}{nf_c}} \quad (2)$$

Solving formula (1) for f_c

$$f_c = f_s \frac{k}{n(1-k)} \qquad (3)$$

Now, as stated above, for equilibrium the total tension in the steel must be equal and opposite to the total compression in the concrete. The total tension in the steel is its unit stress, f_s, multiplied by the area of the steel, pbd, and the total compression in the concrete is represented by the area of the pressure triangle, $\frac{1}{2}f_c kd$, times the breadth of the beam, b. Equating these two forces and canceling out the bd which occurs in both,

$$pf_s = \frac{f_c k}{2} \qquad (4)$$

If the value of k in formula (2) be substituted for the k in formula (4), we have

$$p = \frac{1}{2\dfrac{f_s}{f_c}\left(1+\dfrac{f_s}{nf_c}\right)} \qquad (5)$$

For any given percentage of steel the values of f_s and f_c cannot be assumed independently, as they bear a constant ratio to each other.

Substituting the value of f_c in formula (3) for f_c in formula (4) we have

$$p = \frac{k^2}{2n(1-k)} \qquad (6)$$

Solving this quadratic equation and adopting the positive sign before the square root,

$$k = -np + \sqrt{2np + (np)^2} \qquad (7)$$

From formula (7) the location of the neutral axis may be determined for any percentage of steel, p, and any assumed ratio of moduli of elasticity, n. Values for k are given in table on page 596.

The center of gravity of the compressive stresses is distant $\frac{1}{3}\,kd$ from the top of the beam so that $jd = d - \frac{1}{3}\,kd = d(1 - \frac{1}{3}\,k)$.

Since the total compression equals the total tension, the moment of resistance of the beam may be obtained by multiplying either the total tension, $pbdf_s$, or the total compression, $\frac{1}{2}f_c bkd$, by the moment arm, jd.

$$M = pf_s jbd^2 \quad (8) \quad \text{and} \quad f_s = \frac{M}{pjbd^2} \qquad (9)$$

$$M = \frac{f_c kjbd^2}{2} \quad (10) \quad \text{and} \quad f_c = \frac{2M}{kjbd^2} \qquad (11)$$

For a given quality of concrete and steel, the values of f_s, f_c, p, k, and j, are constant so that we may consider the terms $pf_s j = \frac{1}{2}f_c kj$ equal to a constant $\frac{1}{C^2}$. This changes the formulas (8) and (10) to

$$M = \frac{bd^2}{C^2} \quad \text{and} \quad d = C\sqrt{\frac{M}{b}} \qquad (12)$$

To obtain the total depth of beam, h, a value e (see Fig. 98) must be added to the theoretical depth, d. Then, $h = d + e$.

For the use of these formulas in design, see pages 481 to 484.

FORMULAS FOR T-BEAMS

If a reinforced concrete beam is built monolithic with the slab, the beam may be considered as a T-beam in which a portion of the slab acts as a flange.

The formulas for T-beams given below are based on the same assumptions as for rectangular beams. Tension is considered as taken entirely by the steel and the variation of stresses in concrete is according to a straight line. Unless the slab is very thick the neutral axis is located below the flange.

For notation see page 353.

Case I. *Neutral Axis Below Flange, $kd > t$*

Formulas Neglecting Compression Below Flange. Neglecting the slight amount of compression in the stem between the neutral axis and

the bottom of the slab and referring to Fig. 99, page 356, we have similarly as for rectangular beams:

$$k = \frac{1}{1 + \frac{f_s}{nf_c}} \tag{13}$$

The total tension is equal to the unit stress in steel, f_s, multiplied by the area of steel, A_s. The total compression in the concrete is represented by a trapezoid, the sides of which are f_c and $f_c \frac{kd - t}{kd}$; and the depth is equal to t. The total compression, therefore, equals $f_c \frac{2kd - t}{2kd} bt$.

By equating total tension to total compression acting on the section

$$A_s f_s = f_c \frac{2kd - t}{2kd} bt. \tag{14}$$

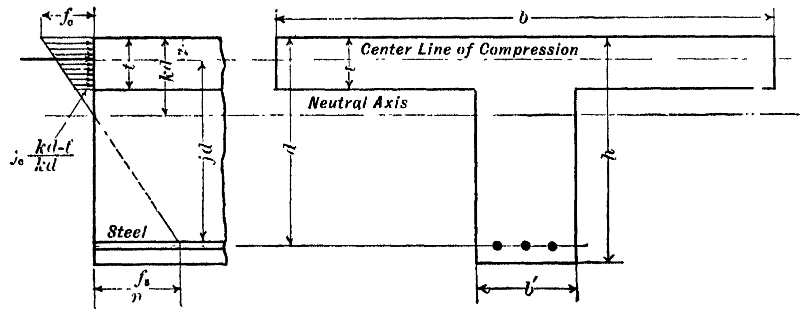


FIG. 99.—Resisting Forces in T-shaped Section of Beam. (See p. 356.)

Solving the two above equations for kd and eliminating f_c and f_s, we get

Position of neutral axis

$$kd = \frac{2nd\,A_s + bt^2}{2n\,A_s + 2bt} \tag{15}$$

The distance of the center of compression from upper surface of beam

$$z = \frac{3kd - 2t}{2kd - t} \cdot \frac{t}{3} \tag{16}$$

Arm of resisting couple

$$jd = d - z$$

Moment of resistance

$$M = A_s jd f_s \quad (17) \quad \text{and} \quad M = \frac{2kd - t}{2kd} btjdf_c \qquad (17a)$$

Fiber stresses

$$f_s = \frac{M}{A_s jd} \qquad (18)$$

$$f_c = \frac{Mkd}{bt\,(kd - \frac{1}{2}t)\,jd} = \frac{f_s}{n}\,\frac{k}{1 - k} \qquad (19)$$

Area of steel

$$A_s = \frac{M}{f_s jd} \qquad (20)$$

Formulas Considering Compression Below Flange. For large beams where the stem forms a large part of the compression area the above formulas do not give results accurate enough for practical purposes. For such cases formulas given below are recommended, which take into account the compressive stresses in the stem as well as in the flange. The following formulas are derived by the same principles used in derivation of formulas in the previous analysis.

Depth to neutral axis

$$kd = \sqrt{\frac{2ndA_s + (b - b')t^2}{b'} + \left(\frac{nA_s + (b - b')\,t}{b'}\right)^2} - \frac{nA_s + (b - b')\,t}{b'} \qquad (21)$$

$$z = \frac{(kdt^2 - \frac{2}{3}t^3)\,b + \left[(kd - t)^2\left(t + \frac{1}{3}(kd - t)\right)\right]b'}{t\,(2kd - t)\,b + (kd - t)^2\,b'} \qquad (22)$$

Arm of resisting couple

$$jd = d - z \qquad (23)$$

Moment of resistance

$$M = A_s jd f_s \quad (24) \quad M = \frac{f_c}{2kd}\left[(2kd - t)\,bt + (kd - t)^2 b'\right] jd \qquad (25)$$

Fiber stresses

$$f_s = \frac{M}{A_s jd} \quad (26) \quad \text{and} \quad f_c = \frac{2M\,kd}{\left[(2kd - t)\,bt + (kd - t)^2 b'\right] jd} \qquad (27)$$

Case II. *Neutral Axis in Flange or at Underside of Flange, $kd \leqq t$*

In this case, which occurs only with slabs that are very thick in proportion to the depth of the beam, use the rectangular beam formula, considering the T-beam as a rectangular beam of the same depth, the breadth of which is the breadth of the flange. The percentage is then based on the total area *bd*.

REINFORCED CONCRETE BEAMS WITH STEEL IN TOP AND BOTTOM

In beams reinforced with steel placed both in the compressive and tensile portions of the beam, the steel in the compressive portion, as in columns (p. 375), may be considered as taking its share of compression according to the ratio of moduli of elasticity of steel to concrete (see p. 353). Neglecting the tension in concrete, as in beams without compressive steel, all the tension may be considered as resisted by the bottom steel. Referring to Fig. 100, page 358, the total compression consists of the compressive stresses in concrete represented by a triangle and the compressive stress in steel. The compressive unit stress in steel equals the unit stress in concrete at the same level multiplied by the ratio of their moduli of elasticity.

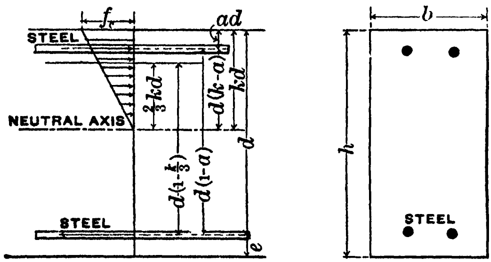


FIG. 100.—Resisting Forces with Steel in Top and Bottom of Beam. (*See p.* 358.)

The sum of all the horizontal stresses acting on a cross section must equal zero; therefore, the total tension in steel must be equal to the compression in concrete plus the compression in the top steel. The resisting moment, that is, the moment of the internal stresses, may be obtained either by multiplying the total tension or compression by the distance between center of tension and center of compression, or by taking moments about the center of tension steel, the center of compression in concrete, or the center of compression in steel. The moments of resistance obtained by either of the four methods must be equal. To

find the stresses for a certain loading, the moment of resistance taken in any one of these ways is equated to the known bending moment.

Formulas. Deformations, as usual, are assumed to vary directly as distance from neutral axis, hence from Fig. 100, using notation on p. 353,

$$\frac{\frac{f_s}{E_s}}{\frac{f_c}{E_c}} = \frac{d(1-k)}{dk} = \frac{1-k}{k} \qquad \text{Whence } k = \frac{1}{1+\frac{f_s}{nf_c}} \tag{28}$$

By comparing the above equation for k with that given for simple beams, page 354, it is evident that for any ratio of $\frac{f_s}{nf_c}$, the position of the neutral axis is the same irrespective of whether the beam is provided with compressive steel or not.

By similarity of triangles in Fig. 100, page 358, the following relations between the unit stresses may be obtained:

$$f'_s = f_s \frac{k-a}{1-k} \quad (29) \quad \text{and} \quad f'_s = nf_c \frac{k-a}{k} \tag{30}$$

$$f_s = nf_c \frac{1-k}{k} \quad (31) \quad \text{and} \quad f_c = \frac{f_s}{n} \frac{k}{1-k} \tag{32}$$

The total tension in steel equals bdp_1f_s and the total compression in steel and concrete is

$$bd\frac{f_ck}{2} + bdp'f'_s = bd\left(\tfrac{1}{2}f_ck + p'f'_s\right)$$

Since the sum of all the stresses must equal zero, the total compression acting on the cross-section of the beam equals the total tension, or

$$bd\left(\frac{f_ck}{2} + p'f'_s\right) = bdp_1f_s$$

$$\text{Whence } p_1 = \frac{1}{f_s}\left(\frac{f_ck}{2} + p'f'_s\right) = \frac{1}{f_s}\left(\frac{f_s}{2n}\frac{k^2}{1-k} + p'f_s\frac{k-a}{1-k}\right)$$

$$\text{Hence} \qquad p_1 = \frac{k^2}{2n(1-k)} + p'\frac{k-a}{1-k} \tag{33}$$

Solving equation (33) for k,

$$k = \sqrt{2n(p_1 + p'a) + n^2(p_1+p')^2} - n(p_1+p') \tag{34}$$

Taking moments about the center of compressive stress in the steel, we have

$$M = bd^2\left[f_s p_1\left(1 - a\right) - \frac{f_c k}{2}\left(\frac{k}{3} - a\right)\right]$$

or by eliminating f_c

$$M = f_s bd^2\left[\frac{p_1 (1-a)\, 2n\, (1-k) - k^2\left(\frac{k}{3} - a\right)}{2n\,(1-k)}\right] \qquad (35)$$

From which

$$f_s = \frac{M}{bd^2}\,\frac{6n\,(1-k)}{6n\,p_1\,(1-k)\,(1-a) - k^2\,(k-3a)} \qquad (36)$$

By substituting this value of f_s in equations for f_c and f'_s respectively, we get

$$f_c = \frac{M}{bd^2}\,\frac{6k}{6n\,p_1\,(1-k)\,(1-a) - k^2\,(k-3a)} \qquad (37)$$

and

$$f'_s = \frac{M}{bd^2}\,\frac{6n\,(k-a)}{6n\,p_1\,(1-k)\,(1-a) - k^2\,(k-3a)} \qquad (38)$$

It may be noted that the denominator in the three above equations is the same. A simplified method of using the formulas in practical design is given in the chapter on design.

STEEL IN BOTTOM OF BEAM, CONCRETE BEARING TENSION.

It is often required to find the actual stresses in reinforced concrete beams during the first stage of loading, for instance, to determine the load at the first crack, or the tensile stress in concrete at the first crack. In the first stage, which, as explained on page 405, lasts till minute cracks open, concrete may be considered as bearing its share of tension. Therefore, the formulas given below, considering tension in concrete and based on straight line distribution of stress, may be used for the above purpose. **These formulas must not be used, however, in designing reinforced concrete beams.**

Formulas. Assume that the ratios of moduli of elasticity, $\frac{E_s}{E_c} = n$, are equal for concrete in tension and compression. Since elongation of steel and concrete at the same point must be equal and the cross-sec-

tional planes are assumed to remain plane during bending, we have from Fig. 101 the following equations using notation on page 353:

$$\frac{\frac{f_s}{E_s}}{\frac{f'_c}{E_c}} = \frac{d - kd}{h - kd} \quad \text{hence} \quad f_s = nf'_c \frac{d - kd}{h - kd} \tag{39}$$

$$f_s = nf_c \frac{1 - k}{k} \tag{40}$$

$$f_c = f'_c \frac{kd}{h - kd} \quad (41) \qquad \text{also } f'_c = f_c \frac{h - kd}{kd} \tag{42}$$

Equating horizontal forces on the section, we have

$$\frac{bf_c kd}{2} = p\ f_s bd + \frac{f'_c b\ (h - kd)}{2} \tag{43}$$

Expressing f_s and f'_c in terms of f_c and simplifying, we have,

$$\frac{kd}{2} = p\ d\ n \frac{1 - k}{k} + \frac{(h - kd)^2}{2kd} \tag{44}$$

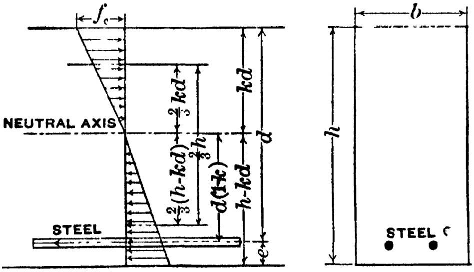


FIG. 101.—Resisting Forces with Concrete Bearing Tension. (See p. 361.)

From which

$$p = \left(\frac{1}{2n}\right)\left(\frac{h}{d}\right)\frac{2k - \frac{h}{d}}{1 - k} \tag{45}$$

This solved for k gives

$$k = \frac{1}{2}\frac{\left(\frac{h}{d}\right)^2 + 2pn}{\frac{h}{d} + pn} \tag{46}$$

Taking moments about the center of the steel and expressing f'_c in terms of f_c, we have, for the moment of resistance:

$$M = \frac{f_c b}{2}\left[kd^2\left(1 - \frac{k}{3}\right) - \frac{(h - kd)^2}{kd}\left(d - \frac{kd}{3} - \frac{2h}{3}\right)\right]$$

or

$$M = \frac{f_c bh}{2}\left[2d - \frac{h}{k}\left(1 + k - \frac{2}{3}\frac{h}{d}\right)\right] \tag{47}$$

Taking moments about the resultant of the compression and expressing f_s in terms of f'_c (Formula (39)) we have:

$$M = \frac{1}{3}\, bd^2 f'_c\left[np\;\frac{(1 - k)\;(3 - k)}{\dfrac{h}{d} - k} + \frac{h}{d}\left(\frac{h}{d} - k\right)\right] \tag{48}$$

and

$$f'_c = \frac{M}{bd^2}\,\frac{3\left(\dfrac{h}{d} - k\right)}{np\,(1 - k)\;(3 - k) + \dfrac{h}{d}\left(\dfrac{h}{d} - k\right)^2} \tag{49}$$

Also by substituting for f'_c the values from Formulas (43) and (39)

$$f_c = \frac{M}{bd^2}\,\frac{3k}{np\,(1 - k)\;(3 - k) + \dfrac{h}{d}\left(\dfrac{h}{d} - k\right)^2} \tag{50}$$

$$f_s = \frac{M}{bd^2}\,\frac{3n\,(1 - k)}{np\,(1 - k)\;(3 - k) + \dfrac{h}{d}\left(\dfrac{h}{d} - k\right)^2} \tag{51}$$

For a given bending moment, M, stresses may be found from the above formulas. Note that denominators in all equations are the same.

SHEARING STRESSES IN A BEAM OR SLAB

The bending of a beam produces a tendency of the particles to slide upon each other or shear. It is therefore necessary to study

(1) Vertical shearing stresses.

(2) Horizontal shearing stresses.

Vertical and Horizontal Shearing Stresses. Concrete is strong in direct shear (see p. 337) and capable of standing a working shearing

stress of at least 200 pounds per square inch, so that a concrete girder or beam or slab always has sufficent area of section to withstand this direct shearing stress. However, since the direct shearing stress is a measure of the diagonal tension (see p. 365), which is excessive when the direct shearing stress is comparatively low, it must always be computed in a beam or girder for use in the computation of diagonal stresses, as described on page 367.

The shear is a maximum at the support, where it is equal to the reaction. Maximum shears for various loads are given in the diagram (Fig. 151, page 505), in terms of the loads. While with uniform or symmetrical loading the reaction, and therefore the maximum shear, is one-half the total load upon the beam, it will be noticed from the diagram that where the end beams of continuous beams are freely supported, which is very nearly the case when a beam runs into a light wall girder, the shear at the first support away from the end may be 25 per cent greater than normal, and should be specially provided for in cases like a warehouse where the full live load is liable to be constantly maintained. A further study of the four diagrams (Figs. 151 to 154, pp. 505 and 508) will illustrate the cases where allowances should be made.

In case the concrete in a beam or slab has cracked vertically next to the support because of accident or poor design, the bearing value of the horizontal rods may have to be estimated.

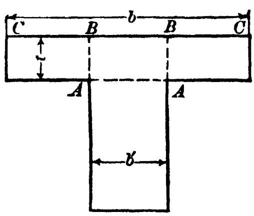


FIG. 102.—Section of a T Beam. (*See p.* 363.)

Longitudinal Vertical Shear in Flange of T-Beam. Vertical shear in a longitudinal direction is present in the wings of a T-beam due to the load upon a beam being maximum next to the flange, as shown by lines *BA* in Fig. 102, page 363. Results of tests are given on page 416.

The area of concrete in a solid horizontal floor slab is generally sufficient to take care of this shear, but the following method may be used for computing it if desired:

Let

v_h = unit horizontal shear at AA.

v_v = unit vertical shear at BA.

b' = breadth of stem.

b = breadth of flange.

t = thickness of flange.

The shear along the two planes BA may be considered as caused by the external forces acting not on the whole breadth, but only on the projecting flanges of the T-Beam BC.

Then it is readily shown* that

$$v_v = \frac{v_h\, b'\,(b - b')}{2\,t\,b} \qquad (52)$$

Although this vertical shear through the flanges is readily borne by the concrete, it is advisable, as stated on page 418, to place horizontal bars across the top of the beam, even if the bearing bars in the slab run parallel to the beam, in order to resist unequal bending moment which is liable to occur and to assure T-beam action.

Fillets at the angles between the flange and the beam, that is, between the slab and the beam, are not theoretically necessary, but they may be used for appearance sake and as an additional security in a deep beam with relatively shallow flanges or slabs. Small fillets are also advisable to aid in the removal of forms.

DIAGONAL TENSION

In a beam, besides direct horizontal tension and compression and direct horizontal and vertical shearing stresses, there exist also stresses acting in diagonal directions. The maximum diagonal stress composed of the tension and the shearing stresses is called diagonal tension.

In steel and other homogeneous beams diagonal stresses need no attention. In reinforced concrete, however, it has been shown in beams tested to destruction that, beside tensile cracks at the points of maximum moment, diagonal cracks, caused by diagonal tension, develop near the supports. (See tests on pp. 418 to 427.) These cracks have been often the cause of failure, frequently without warning, especially in beams reinforced with straight bars only or provided with insufficient web reinforcement. The need of low working stresses and effective web reinforcement is discussed in paragraphs which follow.

* The above principle may be expressed by the equation $v_v\, 2t = v_h b' \frac{b-b'}{b}$, which solved for v_v will give formula (52).

Diagonal Tension in Homogeneous Beams. The magnitude and inclination of the diagonal tension in homogeneous beams may be found from the following formula:

Let

f_d = diagonal tensile unit stress.
f'_c = horizontal tensile unit stress.
v = horizontal or vertical shearing unit stress.

Then*

$$f_d = \tfrac{1}{2} f'_c + \sqrt{\tfrac{1}{4} f'^2_c + v^2} \qquad (53)$$

The direction of this diagonal tension makes an angle with the horizontal equal to one-half the angle whose co-tangent is $\frac{1}{2}\frac{f'_c}{v}$.

From the formula it is evident, since the value of f'_c and v vary in different parts of the cross section of the beam, that the value of the diagonal tension and its angle of inclination also vary. At the bottom of the section where $v=0$, $f_d=f'_c$ and acts horizontally. At the neutral axis the direct tension, $f'_c=0$, which reduces the formula to $f_d=v$ and the angle of inclination to 45°.

Measure of Diagonal Tension for Reinforced Concrete Beams. In homogeneous beams, the diagonal forces can be determined easily by means of formula (53) above. In reinforced concrete, however, the diagonal stresses are indeterminate because, as seen from the formula, they depend upon the horizontal tensile stresses in concrete, f'_c. The action of concrete in tension is not dependable. It varies, also, for different stages of loading because for larger loadings concrete cracks, thus decreasing the tensile stresses carried by concrete. The tensile strength of concrete, which may be disregarded in figuring the moment of resistance of the beam, affects the magnitude of the diagonal tension to a great extent especially near the ends of simply supported beams where the stresses due to the bending moment are low and the stresses in concrete may not exceed its breaking strength in tension. While the exact determination of diagonal tension is impossible, tests show that **the shearing unit stress, figured as given on page 367, may be accepted as a convenient measure of diagonal tension.** That is, the diagonal tension may be assumed as proportional to the direct shearing stress so that, by adopting proper working stresses based on tests producing diagonal tension failures, formulas for shearing stresses may be used for diagonal tension. This measure has been universally accepted and, in subse-

* For derivation see Merriman's "Mechanics of Materials," 1905 edition, p. 265.

quent discussion, diagonal tension is expressed in terms of shearing stresses.

Diagonal Tension in Simply Supported Beams. In simply supported beams, diagonal tension cracks start at the bottom of the beam, not at the support where the shear is greatest, but far enough out for the tensile stresses due to the bending moment to break the concrete. Hence, the importance of using enough tensile steel, in addition to the web reinforcement, to keep these unit tensile stresses near the supports low. Tests of beams otherwise comparable in size, reinforcement, and loading, show that diagonal cracks that actually develop can be prevented by the use of an increased amount of horizontal steel near the support. Not more than two-thirds of the horizontal bars in simply supported beams should be bent up near the support.

Diagonal Tension in Continuous Beams. In continuous beams it is the top of the beam near the support that is in tension instead of the bottom. Accordingly, to prevent cracks, the steel should run well out on each side of the support before being bent down to carry the tensile stresses in the bottom of the beam near the center of the span.

The proportion of the bottom horizontal steel, therefore, that may be bent up in fixed and continuous beams is much larger than in simply supported beams and may even exceed two-thirds the total area of the steel in the center without increasing the danger of diagonal cracks. Enough must be left for all requirements of tension and compression produced by the bending moments.

Formulas for Shearing Stresses and Diagonal Tension. A convenient and safe method of determining the diagonal tension is by accepting for its measure the unit shearing stress as discussed on page 365.

Let

V = total shear at section considered. (Reaction minus the loads between the support and the section.)

v = horizontal (or vertical) shearing unit stress at section considered.

b = breadth of beam.

b' = breadth of web of T-beam.

jd = moment arm or distance between center of compression and center of tension (approximately, in a T-beam, distance between center of slab and steel).

Z = total shearing stress or diagonal tension in a given length of beam, s.

s = length of the portion of the beam considered.

The following general principles and formulas are discussed in paragraphs which follow.

(1) Horizontal (or vertical) shearing stress is zero at the top of the section and changes according to a parabola till it reaches its maximum at the neutral axis. (See Fig. 103).

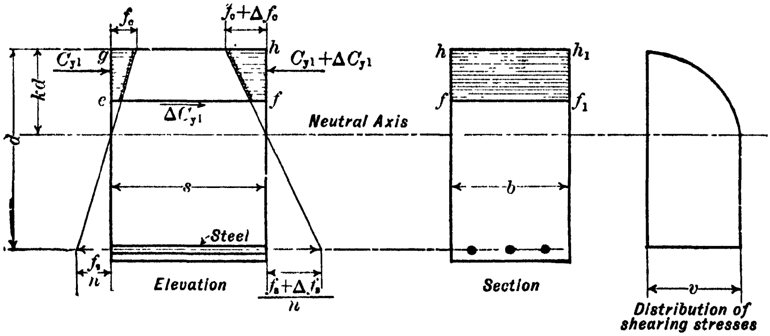


FIG. 103.—Horizontal and Vertical Shearing Stresses in Beam. (See p. 368.)

(2) If tension in concrete is neglected, the horizontal (or vertical) shearing stress is constant below the neutral axis.

(3) Total amount of horizontal shearing stress developed at any horizontal plane below the neutral axis in a distance, s, is

$$Z = \frac{Vs}{jd} \tag{54}$$

(4) Shearing unit stress, the measure of diagonal tension, is total horizontal shearing stress, Z, divided by the horizontal area, $b \times s$, that is $v = \frac{Vs}{jd} \div bs$.

Hence

$$v = \frac{V}{bjd} \quad (55) \qquad \text{For T-beams, } v = \frac{V}{b'jd} \quad (55a)$$

If the shear V changes in the distance, s, the same formulas may be used except that V in the formula is the average shear in that section.

(5) Vertical shearing unit stress is equal to the horizontal shearing unit stress, and acts at right angle to the plane of horizontal shearing stress. The distribution of vertical shearing stress over a vertical section is shown in Fig. 103.

(6) Diagonal tension may be expressed in terms of the shearing stress and the above formulas may be accepted as its measure.

(7) If the width of the section below the neutral axis is not constant, the shearing unit stress will vary with the width, b. The minimum b must be taken in figuring the maximum shearing unit stress and the maximum diagonal tension.

(8) In continuous T-beams, near the support, the maximum shearing unit stress will be in the stem right under the flange. The shearing stress and diagonal tension in the plane of tensile steel is small because the width, b, being the total width of the flange, is large.

The action of horizontal shearing stress is illustrated in Fig. 103. A portion of a beam between two vertical sections subject to bending stresses is represented. If the bending moment at the left is M_l, the shear, V, and the length of the section, s, then, from the principles of mechanics, the bending moment at the right is $M_r = M_l + Vs$.

Since the bending moment at the right is larger than the bending moment at the left, the unit compressive and tensile stresses at the right section are larger than at the left. Consider an arbitrary longitudinal plane, ef, above the neutral axis. The compressive stresses above this plane represented by the shaded portions of the triangles are at the left equal to C_{yl}, and at the right, $C_{yr} = C_{yl} + \Delta C_{yl}$. The difference between the two forces C_{yl} and C_{yr}, which act in opposite directions, is ΔC_{yl}. This tends to move the upper portion of the beam along the plane eff_1e_1, but is kept in equilibrium by the horizontal shearing resistance in the beam on that plane. The shearing unit stress is equal to ΔC_{yl} divided by the area, bs, of the plane, eff_1e_1.

At the top of the beam the value of ΔC_{yl} and also the total shearing stress is zero and increases steadily according to a parabola till it reaches its maximum at the neutral axis. There its value equals the difference between the total compression on the right and the total compression on the left. From the ordinary beam formulas, page 355, we know that the total compression may be found by dividing the bending moment by the moment arm; thus, at the left, the total compression is $C_l = \dfrac{M_l}{jd}$, and at the right, $C_r = \dfrac{M_r}{jd} = \dfrac{M_l}{jd} + \dfrac{Vs}{jd}$.

The difference between C_l and C_r is thus, $\dfrac{Vs}{jd}$. Therefore, the total amount of horizontal shearing stress at the neutral axis for the length,

s, and width, b, is $Z = \frac{Vs}{jd}$. This has to be resisted by the horizontal plane of the beam, bs, so that the shearing unit stress,

$$v \text{ is } \frac{Vs}{jd} \text{ divided by } bs, \quad \text{or} \quad v = \frac{V}{bjd}. \tag{56}$$

If there is no tension in concrete, the difference between the stresses acting above any plane located below the neutral axis is the same as the difference at the neutral axis. Consequently the total horizontal shearing stress is uniform at all planes below the neutral axis. As the shearing unit stress depends upon the width b, it is constant for rectangular sections, but varies with variable b.

At the plane of reinforcement the stresses in steel at the left are $T_l = \frac{M_l}{jd}$; and at the right, $T_r = \frac{M_r}{jd} = \frac{M_l}{jd} + \frac{Vs}{jd}$, and the difference, $T_l - T_r = \frac{Vs}{jd}$. This shows that the total horizontal shear or the tendency to move the upper portion of the beam is the same at the plane of the bars as at the neutral axis.

If there is tension in concrete, the total horizontal shearing stress, Z, on any plane below the neutral axis will be decreased by the difference in tension at the two vertical sections above that plane.

The increase in the stress in steel, equal to $\frac{Vs}{jd}$, between the two sections considered, must be transferred from the steel to the beam. Therefore, bond must exist between steel and concrete or else the upper portion of the beam will slide on the steel instead of increasing its stress. Tests of bond or resistance to slipping of bars are treated on page 429.

Diagonal Tension Acting on an Element of a Beam. Fig. 104 represents the stresses to which any element of a beam is subjected. In Fig. 104a is shown a rectangular element of the beam the sides of which are dx and dy. This element is kept in equilibrium by six forces: two forces $f'_c dy$ acting in opposite directions being either direct tension or compression; and four shearing stresses caused by the increment of the moment, as explained in the preceding paragraphs. The two horizontal shearing stresses form a couple, which is resisted by a vertical couple. The moments of the two couples are equal, wherefore the horizontal shearing unit stress must be equal to the vertical shearing unit stress.

If we consider any inclined plane by taking a triangle instead of a rectangle, as in Fig. 104b, we find that this triangle is kept in equilibrium by five forces: one of them is $f_c'dy$; three forces are shearing stresses on the three surfaces; and the last force is the diagonal tension, $f_d dz$. The magnitude of this force may be found from formula (53), page 365. For each case there is a certain inclination of the plane for which the diagonal tension is a maximum.

Fig. 104c represents a case when there is no direct tension or compression or $f_c' = 0$ and the length of sides are units. In this case, as is evident

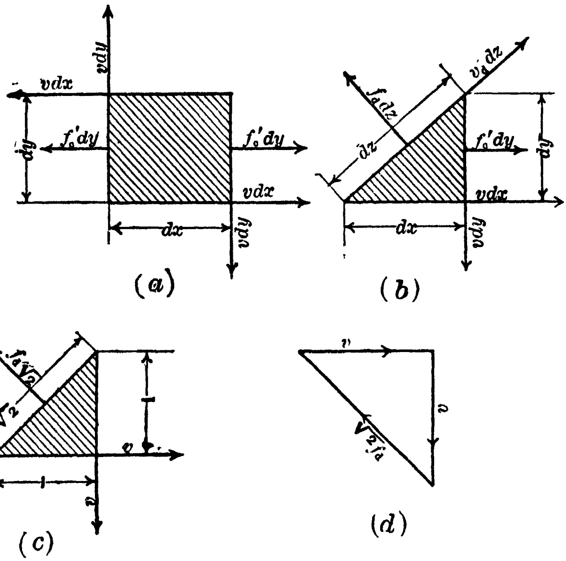


FIG. 104.—Stresses Acting on an Element of Beam. (*See p.* 369.)

from the force polygon in Fig. 104d, the magnitude of the diagonal force is $f_d\sqrt{2}$. The hypothenuse is $\sqrt{2}$; consequently the diagonal unit force is v, or the diagonal unit tensile stress equals the shearing unit stress. For reinforced concrete beams, the shearing unit stress is considered as the measure of the diagonal tension as to the magnitude, but not as to the direction, as seen from Fig. 104*b* to 104*d*.

Distribution of Diagonal Tension to Concrete and Stirrups. Tests prove that in beams with web reinforcement, both concrete and steel resist the diagonal tension found by formula (54), page 367. The relative portions of stress taken by the concrete and steel are somewhat indeterminate. Assumptions variously made are:

(1) Web reinforcement takes all the diagonal tension with no reliance on concrete. The web reinforcement therefore resists in the length s; $\frac{Vs}{jd}$.

(2) Web reinforcement takes two-thirds of the diagonal tension and the concrete the remainder. Web reinforcement resists in the length, s, the force $\frac{2}{3}\frac{Vs}{jd}$. Where the shearing unit stress does not exceed the allowable unit, v', all stress is taken by the concrete.

(3) Concrete resists a certain definite unit stress per square inch, v', the whole length of the beam, and the stirrups resist the remainder. Then in a length, s, the concrete resists $v'bs$, and the web reinforcement resists,

$$Z_1 = Z - v'bs = \frac{Vs}{jd} - v'bs = \frac{V - v'bjd}{jd}s.$$

The first assumption corresponds to that made in ordinary beam design where the tensile strength of the concrete is disregarded. Tests have shown, however, that the actual stresses in the stirrups are less than would be obtained with this assumption (see page 419).

The second assumption, that the web reinforcement takes two-thirds of the diagonal tension stress and the concrete the remainder, more nearly corresponds to actual conditions in a beam, and is therefore recommended for adoption.* At first thought it seems irrational to assume that concrete without stirrups is safe for, say 40 lb. per sq. in., whereas a stress of 45 lb. allows only 15 lb. for concrete, but it is recognized in reinforced concrete that as soon as the limit of safe strength of concrete is passed, a large proportion of the stress must be transferred immediately to the steel.

Area and Spacing of Vertical Stirrups. The area of steel and the spacing of stirrups may be found by placing the force to be resisted, as given above, equal to the working strength of the stirrups in tension.

Let

x = distance in feet from left support to point at which required spacing is desired.

x_1 = distance in feet from left support to point beyond which stirrups are unnecessary.

l = span of beam in feet.

w = uniform load in pounds per foot.

* Also recommended by the Joint Committee on Concrete and Reinforced Concrete.

V = total vertical shear in pounds at section x feet from left support.
v = total shearing unit stress at section in pounds per square inch.
v' = allowable shearing unit stress (or diagonal tension) on concrete alone.
A_s = cross-sectional area of all legs of a vertical stirrup in square inches. (In a U-stirrup this is the sum of the area of the two legs.)
f_s = allowable unit stress in stirrups in pounds per square inch.
jd = distance in inches from center of compression to center of horizontal reinforcement. (In a T-beam, this may be taken as distance between center of slab and steel; in a rectangular beam, as 0.87 of the total depth to steel.)
b = breadth of beam in inches.
b' = breadth of web in T-beam in inches.
s = spacing of stirrups in inches at a place x feet from left support.

Since A_s is the area of a stirrup resisting diagonal tension in a distance, s, and f_s is the tensile strength of steel, the strength of the stirrup in pull is $A_s f_s$. The area of stirrups and the spacing for different assumptions of distribution of diagonal tension between stirrups and concrete may be found as follows:*

(1) *Area and spacing if stirrups take all the diagonal tension.*

The diagonal tension to be resisted is $\frac{Vs}{jd}$. Hence $A_s f_s = \frac{Vs}{jd}$, and

$$A_s = \frac{V}{jdf_s} s \quad (57) \quad \text{and} \quad s = \frac{f_s jd}{V} A_s \qquad (57a)$$

(2) *Area and spacing if stirrups take two-thirds of the diagonal tension.*

Diagonal tension to be resisted is $\frac{2}{3}\frac{Vs}{jd}$. Hence $A_s f_s = \frac{2}{3}\frac{Vs}{jd}$, and we get by solving for A_s and s,

$$A_s = \frac{2}{3}\frac{V}{f_s jd} \cdot s \quad (58) \quad \text{and} \quad s = \frac{3}{2}\frac{f_s jd}{V} A_s \qquad (58a)$$

Formulas (58) and (58a) are recommended by the authors.

(3) *Area and spacing if concrete takes a definite amount of shear, v', and the stirrups, the remainder.*

The stress resisted by concrete in the distance, s, equals $v'bs$. As the total stress is $Z = \frac{Vs}{jd}$, the stirrups must carry the difference,

* The numbers of the formulas are changed from the second edition.

$$\frac{Vs}{jd} - v'bs, \quad \text{or} \quad \frac{V - v'bjd}{jd}s.$$

Equating this to the resistance of the stirrup, $A_s f_s$, and solving for A_s and s, we get

$$A_s = \frac{(V - v'bjd)}{f_s jd}s \quad (59) \quad \text{and} \quad s = \frac{f_s jd}{(V - v'bjd)}A_s \quad (59a)$$

In T-beams use width of web b' in place of b.

Uniformly Distributed Loading.* For uniformly distributed loading of w per lin. ft., the shear involving diagonal tension at any point distant from the support is $V = \frac{wl}{2} - wx = \frac{w}{2}\cdot(l - 2x)$, which, substituted above, gives:

(1) *If stirrups take all the diagonal tension.*

$$A_s = \frac{w(l - 2x)}{2f_s jd}s \quad (60) \quad \text{and} \quad s = \frac{2f_s jd}{w(l - 2x)}A_s \quad (60a)$$

(2) *If stirrups take two-thirds of the diagonal tension.*

$$A_s = \frac{w(l - 2x)}{3f_s jd}s \quad (61) \quad \text{and} \quad s = \frac{3f_s jd}{w(l - 2x)}A_s \quad (61a)$$

Formulas (61) and (61a) are recommended by the authors.

(3) *If concrete takes definite amount of shear, v', and stirrups, the rest.*

$$A_s = \frac{w(l - 2x) - 2v'bjd}{2f_s jd}s \quad (62) \quad \text{and} \quad s = \frac{2f_s jd}{w(l - 2x) - 2v'bjd}A_s \quad (63)$$

In T-beams use width of web b' in place of b.

Tables 9 and 10, made for case (2), where concrete is assumed to take one-third of the shear and stirrups two-thirds are given on page 585. These are recommended for general use.

Stirrups should be spaced by equation (58a) or (61a) up to a section where unit shear equals working shearing strength of concrete, bearing in mind, however, that the maximum spacing should not exceed three-fourths the depth of the beam. The distance from the support to the point where no stirrups are required, for uniform loading is†

* The numbers of the formulas are changed from the second edition.

† The diagram of shearing unit stresses is a triangle (Fig. 159, p. 526.) from which the distance x_1 may be obtained by the known rule $\frac{l}{2} \div \left(\frac{l}{2} - x_1\right) = v \div v'$. This equation solved for x_1 gives formula (64), page 374.

$$x_1 = \frac{l}{2}\left(1 - \frac{v'}{v}\right) \qquad . \qquad (64)$$

From the above formulas it is evident that the necessary spacing of stirrups is inversely proportional to the total shear V at any point and therefore is the smallest at the end of the beam and increases toward its middle.

Many constructors advise the insertion of occasional stirrups throughout the entire length of the beam even if they are not theoretically necessary.

For a small beam where the stirrups are spaced uniformly, for convenience, only the minimum value of s needs to be figured.

Usefulness of Web Reinforcement. Numerous tests have demonstrated that a beam properly reinforced with stirrups or bent bars sustains three or four times as much load as the same beam without web reinforcement. The same tests, however, show that the web reinforcement retards the appearance of first diagonal cracks only very little and that the web reinforcement does not get any stress until the first crack appears. It has been noticed* also that under working loads (that is, before the diagonal tension exceeds the tensile strength of the concrete) the beam acts similarly to a homogeneous beam, and as would be expected, the stress in the stirrups is sometimes compressive instead of tensile.

This is, nevertheless, no argument against the use of web reinforcement, because in beams without stirrups, final failure follows closely the appearance of the first crack, while with beams having web reinforcement, stirrups and bent bars represent a factor of safety which allows stressing of concrete in diagonal tension nearly to its ultimate strength without any danger to the stability of the structure. Under working loads the stirrups may not act, but in case of overstressing, due to faulty construction or to occasional excessive loading, the stirrups prevent the failure of the beam. The minute cracks that may open are not dangerous and in many cases are hardly visible.

Web Reinforcement for Continuous Beams. The formulas given above are based upon results obtained from the tests of simply supported beams. Their use for continuous beams is on the safe side.

In continuous beams, several conditions tend to prevent or at least to retard the formation of diagonal cracks. The compressive force, due to the reaction, tends to close the developed cracks. There exists also almost invariably some arch action, which decreases the direct and

* Bulletin No. 64, University of Illinois, January 13, 1913.

diagonal tension. In continuous T-beams, the horizontal shearing unit stress is zero at the bottom and increases till it reaches a maximum at the neutral axis. From there it is constant till it reaches the bottom of the flange. As the width of the flange is much larger than the width of the stem, the shearing unit stress in the flange is much smaller than in the stem. Diagonal cracks, therefore, tend to open in the portion between the neutral axis and bottom of flange, and larger unit stress is required to open them than in simply supported beams. As there have been comparatively few tests on continuous beams, the formulas for web reinforcement given above should be used.

Web Reinforcement for Cantilevers. The conditions affecting web reinforcement is the same for cantilevers as at the supports of continuous beams. In cantilevers supporting vertical loads, vertical stirrups must, therefore, be attached to the tension steel (at the top) and the free ends hooked in the compressive portion of the beam (at the bottom). In other cantilevers, the stirrups must be placed parallel to the direction of the force and attached in the manner suggested above.

COLUMN FORMULAS.

For reinforced concrete columns centrally loaded, the following formulas may be developed:

Let

f = average compressive unit stress upon the reinforced column, equal to the total load divided by the effective area.

f_c = average compressive unit stress upon the concrete of the column.

f'_s = average compressive unit stress upon the vertical steel in the column.

$n = \frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to modulus of elasticity of concrete.

P = load to be sustained by the column.

A = area of total effective cross-section of column (see pp. 289 and 558).

A_c = area of concrete in effective cross-section.

A_s = area of steel in cross-section.

$p = \frac{A_s}{A}$ = ratio of area of steel to total effective area of column.

Since, as is evident from tests, a reinforced concrete column under load acts as a unit, the deformation or shortening of steel in the column is the same as the deformation or shortening of the concrete.

From mechanics, $\dfrac{\text{stress per square inch}}{\text{modulus of elasticity}}$ = unit deformation, hence $\dfrac{f'_s}{E_s}$ = unit deformation of steel and $\dfrac{f_c}{E_c}$ = unit deformation of concrete.

The deformation of steel in a reinforced column is the same as the deformation of concrete and since $\dfrac{E_s}{E_c} = n$, we have:

$$\frac{f'_s}{E_s} = \frac{f_c}{E_c} \qquad \text{and} \qquad f'_s = nf_c$$

The stress in steel is therefore equal to the stress in concrete multiplied by the ratio of the moduli of elasticity, n.

If a column sustains a load P, stresses in steel and in concrete must be equal to the load. Hence: $P = f_cA_c + f'_sA_s$ or $P = f_cA_c + nf_cA_s$ Since $A_c = A - A_s$, we have $P = f_c\,[(A - A_s) + nA_s]$ Finally,

$$P = f_c\,[A + (n-1)\,A_s] \quad (65) \qquad \text{and} \qquad f_c = \frac{P}{A + (n-1)\,A_s} \quad (65a)$$

The area of steel, A_s, may be expressed in terms of A, by substituting $A_s = pA$, which changes the above formulas to

$$P = f_cA\,[1 + (n-1)\,p] \quad (66) \qquad \text{and} \qquad f_c = \frac{P}{A\,[1 + (n-1)\,p]} \quad (66a)$$

Knowing the stress, f_c, and the percentage, p, we may find the required area from

$$A = \frac{P}{f_c\,[1 + (n-1)\,p]} \qquad (67)$$

Knowing the stress, f_c, and the total area, the required area of steel, A_s, and the percentage may be found from

$$A_s = \frac{P - f_cA}{f_c(n-1)} \quad (68) \qquad \text{and} \qquad p = \frac{P - f_cA}{f_c(n-1)A} \quad (68a)$$

The average unit stress which is the total force, P, divided by the effective area, A,

$$f = \frac{P}{A} \qquad \text{and} \qquad A = \frac{P}{f}$$

The relation between f and f_c may be found by substituting for P in the above equation its value from formula (66), giving

$$f = f_c [1 + (n - 1) p] \tag{69}$$

Values of f for different percentages of steel are given on page 599.

Columns with Spiral Reinforcement. The ultimate strength of a column with spiral reinforcement depends upon (1) the amount of vertical steel, and (2) the amount of spirals. Therefore in formulas for the breaking strength of a spiral column the amount of spirals must be considered. In design, however, the elastic limit and not the breaking strength of the column is the determining value as explained on page 456. As this is not affected by the amount of spiral reinforcement, but by the amount of vertical steel only, the formulas given above for columns with vertical steel can be used and the difference in the two types taken care of in the assumed working unit stresses in the concrete. (See p. 561.)

MEMBERS UNDER FLEXURE AND DIRECT STRESS

The following formulas apply to cases in which members are subjected: (1) simultaneously to a bending moment and a direct thrust; (2) to an eccentric thrust. The first condition takes place, among others, in wall columns, which besides the vertical load must sustain a bending moment caused by a rigid connection between the beam and the column. The second condition occurs in arches when the line of pressure does not coincide with the neutral axis in which case the thrust acts on an eccentricity (see p. 718). A central load and a bending moment may be replaced by an eccentric thrust in which the eccentricity equals the bending moment divided by the thrust, and in turn the eccentric thrust can be replaced by a central load and a bending moment equal to the thrust multiplied by the eccentricity. Therefore the two cases will be treated at the same time because the method of determining stresses is exactly the same in both cases.

PLAIN CONCRETE SECTION UNDER DIRECT STRESS AND BENDING MOMENT

General Formula. For members subjected to a central load and a bending moment (or to an eccentric thrust) the stresses may be obtained by computing separately the stresses caused by the central load and by the bending moment. The sum of the results then gives the actual stresses.

Notation

Let

R = resultant of all forces acting on any section.
f_c = maximum unit compression in concrete.
f'_c = maximum unit tension in concrete or minimum compression.
N = thrust, a component of the forces normal to the section.
V = shear, the component of the force R parallel to the section.
b = breadth of rectangular cross section.
h = height of rectangular cross section.
e = eccentricity, that is, the distance from gravity axis to the point of application of the thrust which is the intersection of the line of pressure with the plane of the section.
M = bending moment on the section.
y = perpendicular distance from gravity axis to any point in the section.
I = moment of inertia of entire cross section of concrete about the horizontal gravity axis.
I_s = moment of inertia of cross-section of steel about the horizontal gravity axis.
A = total area of cross-section.
A_s = total area of section of steel.
y_1 = perpendicular distance from gravity axis of unsymmetrical section to outside fiber having maximum compression.
y_2 = perpendicular distance from gravity axis of unsymmetrical section to outside fiber having maximum tension or minimum compression.
f'_s = maximum unit compression in the steel.
f_s = maximum unit tension or minimum unit compression in the steel.
p = ratio of steel to total area of section; for rectangular sections p = ratio of steel area to bh.
n = $\frac{E_s}{E_c}$ = ratio of moduli of elasticity of steel and concrete.
k = ratio of depth of neutral axis to depth of beam h.
kh = distance from outside compressive surface to neutral axis.
d' = depth of steel in compression.
d = depth of steel in tension.
a = distance from center of gravity of symmetrical section to steel.
e_0 = value of eccentricity which produces zero stress in concrete at outer edge of rectangular section opposite to that on which thrust acts.
C_a, C_e = constants.

The stresses produced by a central load are uniformly distributed and are equal to $\frac{N}{A}$. The stresses produced by the bending moment, M, at any point at a distance, y, from gravity axis, as found by mechanics, equals $\pm \frac{My}{I}$. The sign depends upon whether the point is above or below the axis. The combined stresses, therefore, at any distance, y, from the axis of gravity equals the sum of the two above expressions, i.e. $\frac{N}{A} \pm \frac{My}{I}$. From the above, it is evident that the second term varies with the position of the point in relation to the axis of gravity. Therefore the stresses vary from a maximum at one edge to a minimum at the opposite edge.

Formulas for Rectangular Sections. Since in a rectangular section, $A = bh$, and $I = \frac{bh^3}{12}$, the above formula for stress at any point changes to $f_c = \frac{N}{bh} \pm \frac{12\,My}{bh^3}$. Since the bending moment, M, equals Ne, the above formula may also be written, $f_c = \frac{N}{bh}\left(1 \pm \frac{12ey}{h^2}\right)$. As a rule, we are concerned with maximum and minimum stresses which occur for $y = \pm \frac{h}{2}$. After substituting the value for y, we get

$$\text{Maximum compressive stress, } f_c = \frac{N}{bh}\left(1 + \frac{6e}{h}\right) \tag{70}$$

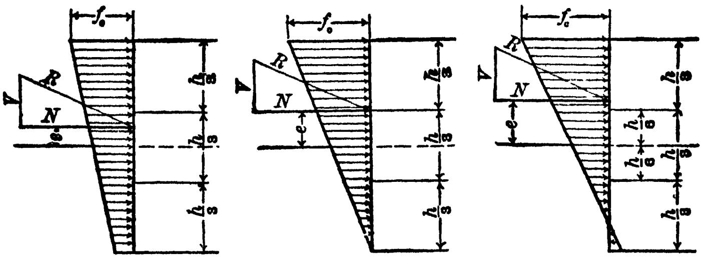
$$\text{Minimum stress, } f_c = \frac{N}{bh}\left(1 - \frac{6e}{h}\right) \tag{71}$$

The maximum stress is always compression. The minimum stress from Formula (71) may be either compression when $\frac{6e}{h}$ is smaller than unity, that is, when e is smaller than $\frac{h}{6}$. For $e = \frac{h}{6}$, or, when the force acts at the edge of the middle third of the section, the minimum stress equals zero, and the maximum stress equals double the stress caused by a central load of equal intensity. (See Fig. 105, p. 380.)

When e is larger than $\frac{h}{6}$, that is, if the load acts outside of the middle third, then the minimum stress is negative, i.e. the section is subjected

to tension. In such a case, this formula can be applied only when the material is capable of carrying tensile stresses.

If the material cannot resist tension, as in masonry foundations, or in concrete where tension exceeds the allowable stress, it is necessary



Force Within the Middle Third   Force at Edge of the Middle Third   Force Outside the Middle Third

FIG. 105.—Stresses Caused by Eccentrically Applied Thrust. (*See p.* 379.)

to assume that the pressure is distributed only over a section equal to three times the distance of the point of application on the load from the nearest edge. (See Fig. 106.) If that distance is g, the total effective width of the section is $3g$. Substituting in Formula (70), $e = \frac{g}{2}$, and $h = 3g$, we get for plain concrete and masonry

$$f_c = \frac{2N}{3bg} \qquad (72)$$

DISTRIBUTION OF STRESSES IN REINFORCED CONCRETE SECTIONS

General Formulas. The distribution of stresses over a reinforced concrete section caused by a central force and a bending moment can be determined by the following formulas.

As in column design (p. 376), the area of steel may be replaced by an area of concrete obtained by multiplying the steel area by n, the ratio of the modulus of elasticity of steel to the modulus of elasticity of concrete. This concrete should be placed at the same distance from the neutral axis as the steel area. The area of the transformed section, then, is $A + (n - 1)A_s$. The stresses may be obtained by determining separately the stresses caused by the central thrust and by the bending moment. The sum of the two stresses gives the actual stress.

The stress due to the central thrust equals the thrust divided by the

area, or $\frac{N}{A + (n - 1) A_s}$. The stress caused by the bending moment equals $\frac{My}{I_t}$, in which I_t is the moment of inertia of the transformed

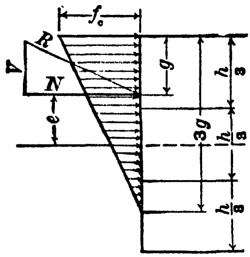


FIG. 106.—Stresses Caused by a Force Acting Outside the Middle Third of Plain Concrete Section. (*See p.* 380.)

section and is equal to $I + (n - 1) I_s$. Substituting in the above expression the value for the bending moment, $M = Ne$, and the value for the moment of inertia, the stress produced by the bending moment equals $\frac{Ney}{I + (n - 1) I_s}$. Therefore the unit stress in the concrete at any distance, y, from the gravity section is

$$f_c = \frac{N}{A + (n - 1) A_s} \pm \frac{Ney}{I + (n - 1) I_s} \tag{73}$$

It is evident that the stress is a maximum in fibers for which y is a maximum. The stress may be compression over the entire section, or compression over a portion of it, and tension over the remaining portion, depending upon the relative magnitude of the two expressions.

Stress in steel equals the stress in the concrete fiber, placed the same distance from the gravity axis as the steel, multiplied by n, the ratio of moduli of elasticity.

REINFORCED CONCRETE RECTANGULAR SECTIONS

Since in a rectangular section

$A = bh$, and $A_s = pbh$, and the moment of inertia of the composite section, $I + (n - 1) I_s = \frac{bh^3}{12} + (n - 1) pbha^2$, the unit stress in concrete at a distance, y, from the gravity axis is

$$f_c = \frac{N}{bh}\left(\frac{1}{1+(n-1)p} \pm \frac{12\,ye}{h^2 + 12\,(n-1)\,pa^2}\right) \tag{74}$$

The stress is compression when the result is positive, and tension when it is negative.

Maximum and minimum stresses are for outside fibers, for which the distance is $y = \frac{h}{2}$. The above formulas, therefore, change to

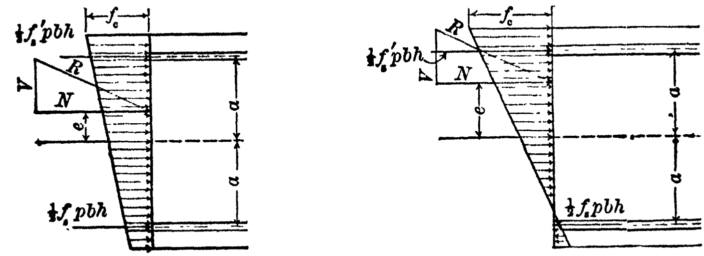
Maximum and minimum stresses in concrete,

$$f_c = \frac{N}{bh}\left(\frac{1}{1+(n-1)p} \pm \frac{6\,he}{h^2 + 12\,(n-1)\,pa^2}\right) = \frac{NC_e}{bh} \tag{75}$$

Maximum and minimum stresses in steel,

$$f_s = n\frac{N}{bh}\left(\frac{1}{1+(n-1)p} \pm \frac{12\,ae}{h^2 + 12\,(n-1)\,pa^2}\right) \tag{76}$$

Use Fig. 108, p. 383, to find the value of the parenthesis, or C_e, in formula (75) for the conditions given in the diagram.



Force Producing Compression upon the Whole Reinforced Section

Force Acting at a Distance Larger than e_0 from the Axis of Gravity of Reinforced Section

FIG. 107.—Stresses in Reinforced Concrete Section Caused by Eccentrically Applied Thrust. *(See p. 382.)*

Effect of Eccentricity. As in plain concrete sections, the location of the center of thrust determines the distribution of the stress, as evident from the above equations. If the thrust acts at the center of gravity, there is uniform compression over the whole section. As the center of thrust lies farther and farther from the gravity axis, the compression at the opposite surface decreases until it finally becomes zero, and then tension.

When the first term in the brackets of the above equation is greater

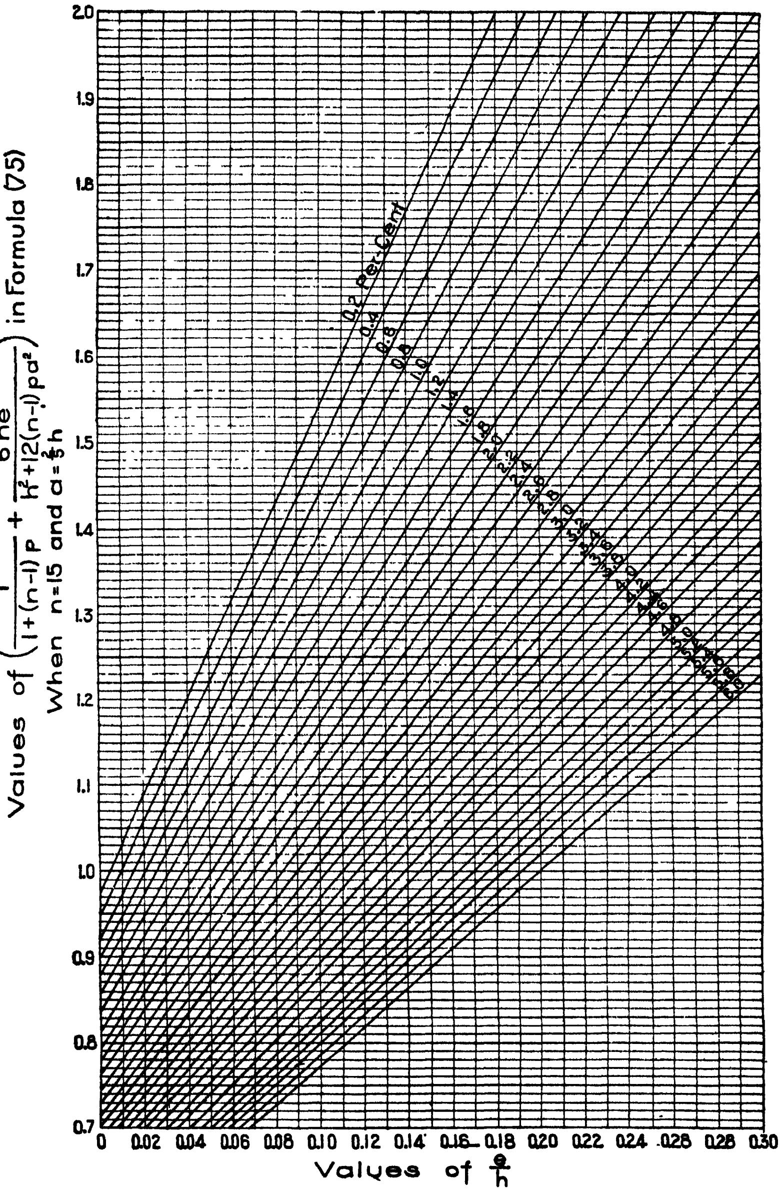


FIG. 108.—Diagram for Determining Compression and Eccentricity. (See p. 382.)

than the second, the minimum stress in the concrete will be compression. When the two terms are equal, the stress is zero in the outer edge of the concrete on the opposite side to that on which the thrust acts. When the second term is greater than the first, the result from the formula will be negative and the minimum stress will be tension.

If the tension determined by the above formula exceeds the allowable tension on concrete, the above formulas are not applicable and the formulas given on page 386 should be employed.

Thrust Applied So That the Compression at One Surface Becomes Zero. The eccentricity for which this occurs may be determined by equating the two terms in formula (75) and solving the resulting equation for e.

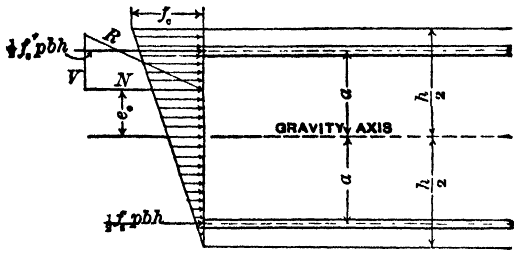


FIG. 109.—Stresses Caused by a Force Acting at a Distance Larger Than e_0 from the Axis of Gravity of Reinforced Section. (*See p.* 384.)

Using previous notation and also letting e_0 = value of e, which makes the stress zero, then

$$e_0 = \frac{h^2 + 12(n-1)pa^2}{1 + (n-1)p} \frac{1}{6h} \tag{77}$$

In the above case, the formula on page 382 changes to

Maximum unit compression in concrete,

$$f_c = \frac{2N}{bh(1 + (n-1)p)} \tag{78}$$

Maximum unit compression in steel,

$$f_s' = \frac{nN}{bh(1 + (n-1)p)}\left(1 + \frac{2a}{h}\right) \tag{79}$$

Minimum compression in concrete = 0

Minimum compression in steel is very small and does not need to be determined.

Distribution of Stress When One Surface Is In Tension. When the thrust is applied at a distance from the gravity axis greater than the eccentricity, e_0, derived by Formula (77), page 384, and the concrete is assumed to be unable to carry any tension, then Formulas (78) and (79), page 384 are no longer applicable, and the following method may be used in determining the stresses. In this method, the steel on the side opposite to that on which the thrust acts is assumed to carry all the tension stresses. Referring to Figure 110, page 385, and making the same assumptions as given in connection with simple flexure on page 351, we find the following relation between the stresses in steel and in concrete;

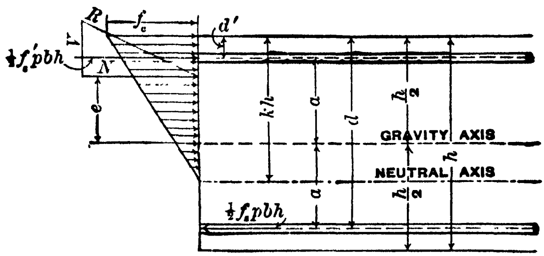


FIG. 110.—Stresses Caused by a Force Producing Compression and Tension upon a Reinforced Section, Tensile Strength of Concrete Neglected. (*See p.* 385.)

Unit compressive stress in the upper steel is

$$f_s' = nf_c\left(1 - \frac{d'}{kh}\right) \qquad \text{and} \qquad (80)$$

the unit tension in the lower steel is

$$f_s = nf_c\frac{d - kh}{kh} \qquad (81)$$

The stresses may be determined from the principle that for equilibrium the sum of the stresses acting on a section must equal the thrust, and that the bending moment of the external forces (which is the thrust multiplied by the eccentricity) equals the moment of resistance of the internal stresses. From the first principle, we have, since each steel area is $\frac{pbh}{2}$,

$$N = \frac{f_s'pbh}{2} + \frac{f_cbkh}{2} - \frac{f_spbh}{2} \qquad (82)$$

Substituting the values for f_s' and f_s from (80) and (81),

$$N = \frac{f_c bh}{2} \frac{k^2 + 2npk - np}{k} \tag{83}$$

The moment of the stresses about the gravity axis, obtained by taking the sum of the moments of all the stresses about the gravity axis, after eliminating f_s' and f_s by the use of Equations (80) and (81), is

$$M = f_c bh^2 \left(\frac{npa^2}{h^2 k} + \frac{k}{4} - \frac{k^2}{6}\right) \tag{84}$$

By equating the expressions in Formulas (83) and (84) to the known thrust and known bending moment, we get two equations from which the unknown values of k and f_c may be determined. This would mean, however, solving equations with the third power. In practice, the use of the curves given on page 387 and 388 will be found convenient.

Calling the quantity in brackets in Formula (84),

$$C_a = \left(\frac{npa^2}{h^2 k} + \frac{k}{4} - \frac{k^2}{6}\right), \text{ we may write}$$

$$M = C_a f_c bh^2 \tag{85}$$

from which

$$f_c = \frac{M}{C_a bh^2} \quad \text{and} \tag{86}$$

$$f_s = nf_c \frac{d - kh}{kh} \tag{87}$$

The value of C_a is dependent upon the known dimensions of the section and eccentricity; therefore, curves in Fig. 112, page 388, have been drawn to simplify the determining of the stresses.

Determining the Value of k. The value of k in Formula (80) or (81), page 385, can be obtained as follows:

Since the moment, M, equals Ne, the thrust multiplied by the eccentricity, Equation (83) multiplied by e may be equated to the formula for bending moment (84). The resulting equation is as follows:

$$k^3 + 3\left(\frac{e}{h} - \frac{1}{2}\right)k^2 + 6npk\frac{e}{h} = 3np\frac{e}{h} + \frac{6npa^2}{h^2} \tag{88}$$

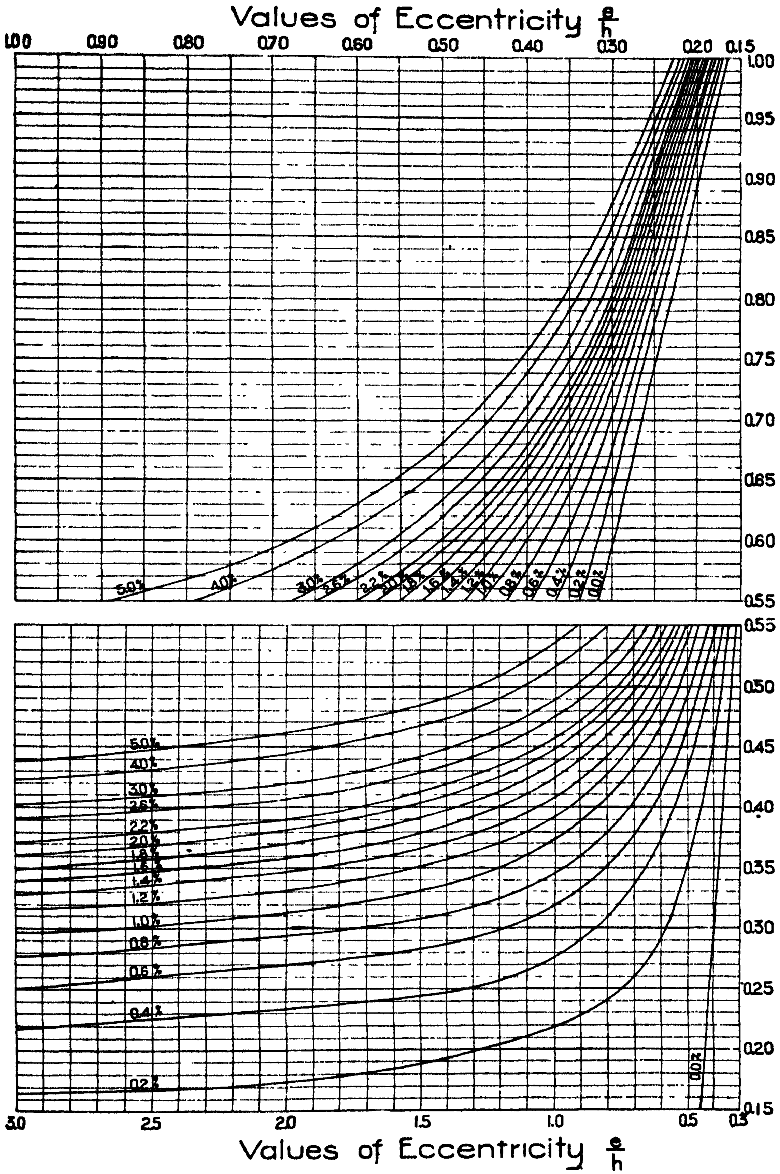


FIG. 111.—Diagram for Determining Depths of Neutral Axis for Different Eccentricities. Based on $n = 15$ and $2a = \frac{4}{5}h$. (*See p.* 389.)

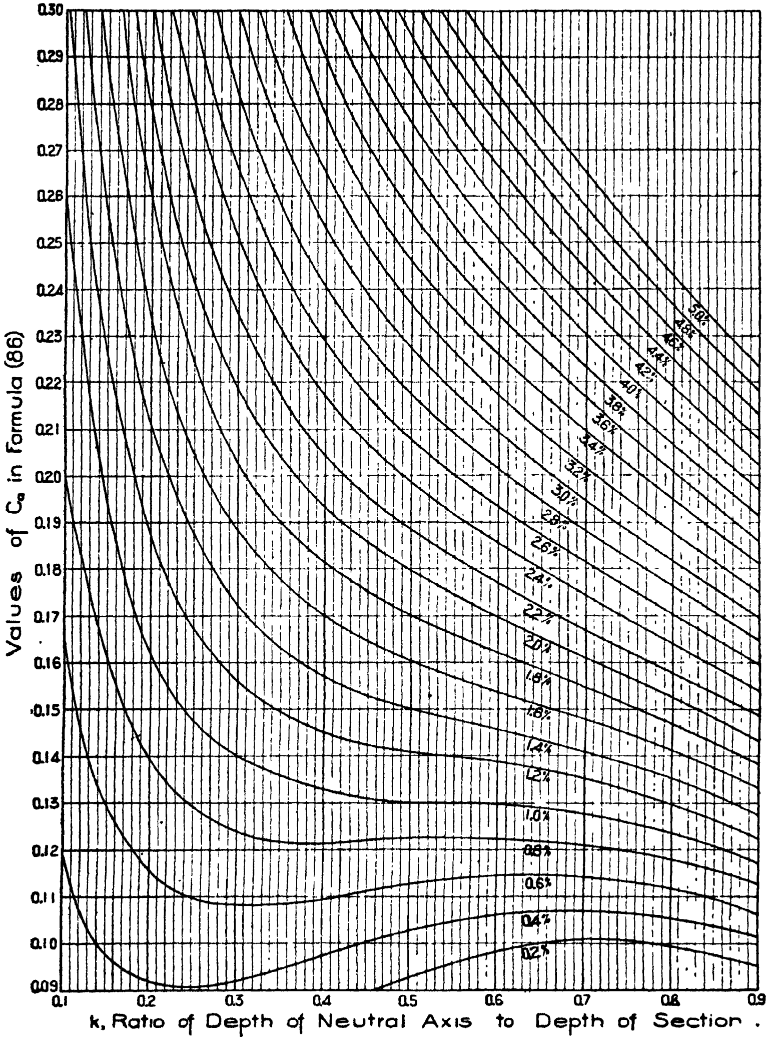


FIG. 112.—Diagram for Determining Constants C_a to be used in Formula (86). Based on $n = 15$ and $2a = \frac{4}{5}h$. (*See p.* 389.)

If the value of k must be determined directly, substitute $k = z - \left(\frac{e}{h} - \frac{1}{2}\right)$ when Equation (88) takes the form $z^3 + pz + q = 0$, and since by Cardan's formula,

$$z = \sqrt[3]{-\tfrac{1}{2}q + \sqrt{(\tfrac{1}{2}q) + (\tfrac{1}{3}p)^3}} \quad + \sqrt[3]{-\tfrac{1}{2}q - \sqrt{(\tfrac{1}{2}q)^2 + (\tfrac{1}{3}p)^3}}$$

the value of k may be computed. This follows the method suggested by Professor Mörsh in "Der Eisenbetonbau," 1906, page 111.

Curves for Determining Values of k and C_a. (Equations 86 and 88). The formulas for k and C_a are complicated and not adaptable for practice. To simplify the determining of the stresses, two sets of curves are given on pages 387 and 388: (1) curves in which the values of k can be found for given values of $\frac{e}{h}$ and ratio of steel, p; (2) curves from which values of C_a can be taken for any value of k and ratio of steel, p.

The curves for values of k were obtained by solving Equation (88) for $\frac{e}{h}$, using $n = 15$ and $2a = \frac{4}{5}h$. This gives

$$\frac{e}{h} = \frac{-k^3 + \frac{3}{2}k^2 + 14.4p}{3k^2 + 90pk - 45p} \tag{89}$$

From the above equation, curves for $\frac{e}{h}$ are readily drawn for different percentages of steel and varying values of k without solving the third power equation.

Determining of Stresses by Use of Diagrams. In finding the unit stresses for a given section having an eccentricity greater than e_0 (see p. 384) and containing a known quantity of steel, the following quantities would be known: breadth, b; depth, h; ratio of steel, p; ratio of elasticity, n; eccentricity, e; and moment, M. The method of procedure of finding stresses may then be as follows.

Determine $\frac{e}{h}$. Enter the bottom of Fig. 111, page 387, with this value of $\frac{e}{h}$ and find the k corresponding for the given percentage of steel. Then with this value of k enter Fig. 112, page 388, and find C_a. Apply Formula (86), page 386, where $f_c = \frac{M}{C_a bh^2}$.

Having found the unit stress in the concrete, the unit stresses in the steel may be determined from formulas (80) and (81), page 385.

FORMULAS FOR REINFORCED CONCRETE CHIMNEY AND HOLLOW CIRCULAR BEAM DESIGNS

Reinforced concrete chimneys may be regarded as vertical cantilever beams supported at the base. The loads to be provided for are (1) the weight of the chimney and (2) the wind pressure. Although the design is somewhat complicated by the fact that the beam is circular and hollow, the treatment is nearly identical with that of ordinary rectangular beams. In fact, the analysis which follows is based upon the several fundamental assumptions adopted in reinforced concrete beam design with only one additional assumption viz.: that, since the concrete is usually thin as compared to the diameter of the chimney, no appreciable error is involved in assuming all material as concentrated on the mean circumference of the shell. An analysis for shear is given on page 397. An example of chimney design and review is given in Chapter XXIII.

Although specially devised for a chimney, the formulas are applicable to any hollow beam.

The principles involved in the demonstration of the thickness of steel and concrete are taken by permission from the analysis by Messrs. C. Percy Taylor, Charles Glenday, and Oscar Faber.*

The principal formulas given below are quoted in the text, where the general subject of concrete chimneys is discussed, and tables are presented there with the values of constants for use in design.

NOTATION

W = weight in pounds of the chimney above the section under consideration.

M = moment in inch pounds of the wind about that section.

P = total compression in concrete.

T = total tension in steel.

$n = \frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete

f_c = maximum compression in concrete in pounds per square inch (measured at the mean circumference).

f_s = maximum tension in the steel in pounds per square inch.

D = mean diameter of shell in inches.

r = mean radius of shell in inches.

t = total thickness of shell in inches.

t_c = thickness in inches of concrete only.

* *Engineering* (London), Mar. 13, 1908

t_s = thickness in inches of an imaginary steel shell of mean radius r, and having a cross-sectional area equivalent to the total area of reinforcing bars.

A_s = total cross-sectional area, in square inches, of reinforcing bars in the section under consideration.

k = ratio of distance of neutral axis, from mean circumference on compression side, to diameter D.

j, z, C_P and C_T = constants for any given value of k. (Tables 1 and 2, pp. 665 and 666.)

jD = distance between center of compression and centre of tension.

zD = distance from center of compression to center of force due to weight.

Referring to Fig. 113, if f_c is the maximum intensity of stress in the concrete at the mean circumference on the compression side, then the intensity of compression in the steel at that point is nf_c. Since f_s is the maximum intensity of stress in the steel at the mean circumference on the tension side, then the variation of the stress in the steel, across the section cd, is represented by the straight line ab which cuts the line cd at e, thus locating the neutral axis or the line of zero stress. Having assumed a constant value for the modulus of elasticity of the concrete in compression, it therefore follows that, at any point of a given section, the stress in either the concrete or the steel is directly proportional to the distance of that point from the neutral axis.

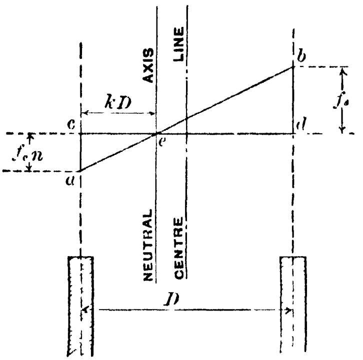


FIG. 113.—Resisting Forces in a Reinforced Chimney. (See p. 391.)

Calling kD the distance of the neutral axis from the mean circumference on compression side as shown in Fig. 113, we have by similar triangles

$$\frac{kD}{D} = \frac{nf_c}{f_s + nf_c}$$

whence

$$k = \frac{1}{1 + \frac{f_s}{nf_c}}$$

By this formula the position of the neutral axis may be determined for any combinations of f_c, f_s, and n.

If now, as shown in Fig. 114, α represents half the angle subtended at the center by the portion in compression, we have

$$\cos \alpha = (1 - 2k)$$

from which, for any given value of z, $\cos \alpha$ becomes known as well as α and $\sin \alpha$. Thus having located the neutral axis for any given combinations of f_c, f_s and n and bearing in mind that the stress at any point of the shell is proportional to the distance of that point from the neutral axis, it is now possible to determine the total force on the compression side, the total force on the tension side, and also the location of the center of compression and the center of tension.

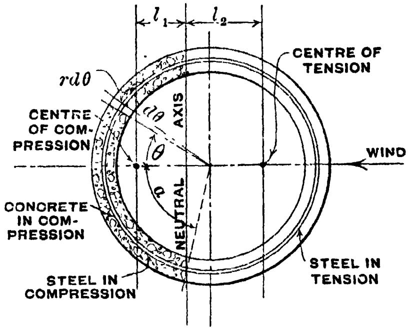


FIG. 114.—Distribution of Stresses in the Steel of a Reinforced Chimney. (*See p.* 392.)

Considering a small radial element subtending an angle $d\theta$, as shown in Fig. 114, we have in this element, since the length of an arc is its radius times the angle,

$$\text{area of concrete} = t_c r d\theta$$

$$\text{area of steel} = t_s r d\theta$$

The distance of the element from the neutral axis is $r(\cos \theta - \cos \alpha)$, while the distance from the neutral axis to the point of extreme stress f_c is $r(1 - \cos \alpha)$. Therefore the intensity of stress on this elemental area is

$$f_c \frac{r(\cos \theta - \cos \alpha)}{r(1 - \cos \alpha)} \text{ in the concrete}$$

and

$$f_c\, n \frac{r(\cos \theta - \cos \alpha)}{r(1 - \cos \alpha)} \text{ in the steel.}$$

Assuming these intensities at the mean circumference to represent the average for the entire element, we have the total force on the elemental area (concrete and steel)

$$d\,P = (t_c + n\,t_s)\,r\,d\,\theta\,\frac{f_c\,r\,(\cos\theta - \cos\alpha)}{r\,(1 - \cos\alpha)}$$

The total force P on the compression side of the section is therefore

$$P = (t_c + n\,t_s)\,2\int_0^{\alpha} \frac{f_c\,r\,(\cos\theta - \cos\alpha)}{(1 - \cos\alpha)}\,d\,\theta$$

Integrating this expression, gives

$$P = f_c\,r\,(t_c + n\,t_s)\,\frac{2}{(1 - \cos\alpha)}\,(\sin\alpha - \alpha\cos\alpha)$$

Since any given position of the neutral axis determines α, as shown above, this equation may take the form

$$P = C_P f_c r\,(t_c + n\,t_s) \tag{91}$$

in which C_P is a constant for a given position of the neutral axis. (See Table 1, page 665.)

Having determined the magnitude of P, its location, with respect to the neutral axis, may best be found by taking its moment about that axis and dividing by P, thus giving the distance from the neutral axis to the center of compression l_1, as shown in Fig. 114.

As before, the compressive force on an elemental area is

$$d\,P = (t_c + n\,t_s)\,r\,d\,\theta\,\frac{f_c\,r\,(\cos\theta - \cos\alpha)}{r\,(1 - \cos\alpha)}$$

The distance of this force from the neutral axis being $r(\cos\theta - \cos\alpha)$, we have as its moment about that axis

$$d\,M_c = (t_c + n\,t_s)\,r\,d\,\theta\,\frac{f_c\,r^2\,(\cos\theta - \cos\alpha)^2}{r\,(1 - \cos\alpha)}$$

while the moment of the total compressive force P is

$$M_c = (t_c + n\,t_s)\,2\int_0^{\alpha} r\,\frac{f_c\,r\,(\cos\theta - \cos\alpha)^2}{(1 - \cos\alpha)}\,d\,\theta$$

$$= (t_c + n\,t_s)\ \frac{2\,f_c r^2}{(1 - \cos\alpha)} \left[\int_0^\alpha \cos^2\theta\, d\,\theta \right.$$

$$\left. - 2\cos\alpha \int_0^\alpha \cos\theta\, d\,\theta + \cos^2\alpha \int_0^\alpha d\,\theta \right]$$

Integrating, we have

$$M_c = (t_c + n\,t_s) f_c r^2 \ \frac{2}{(1 - \cos\alpha)} \left[(\alpha \cos^2\alpha. - \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}\alpha\right)$$

Dividing M_c by P we have

$$l_1 = \frac{M_c}{P} = \frac{(\alpha\cos^2\alpha - \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}\alpha)}{(\sin\alpha - \alpha\cos\alpha)}\ r \qquad (92)$$

Following a similar method of procedure it is possible to determine the total tension and the location of the center of tension.

In accordance with our assumption that the concrete is to take no tensile stress it is evident that in considering the forces on the tension side of the section we are concerned merely with the steel. On the tension side a small element therefore has an area $= t_s\ r\ d\ \theta$

The intensity of stress on this element, being proportional to its distance from the neutral axis, is

$$f_s \frac{r\,(\cos\theta + \cos\alpha)}{r\,(1 + \cos\alpha)}$$

while the total tension on the small element is

$$d\ T = t_s\, r\, d\,\theta\, f_s\ \frac{(\cos\theta + \cos\alpha)}{(1 + \cos\alpha)}$$

The total force T on the tension side of the section is therefore

$$T = 2 \int_0^{(\pi-\alpha)} t_s\, r\, f_s \frac{(\cos\theta + \cos\alpha)}{(1 + \cos\alpha)}\, d\,\theta$$

Integrating, we have

$$T = f_s\, r\, t_s \frac{2}{(1 + \cos\alpha)}\ (\sin\alpha + (\pi - \alpha)\cos\alpha)$$

Since, as before, any given position of the neutral axis determines α, this equation may take the form

$$T = C_T f_s\, r\, t_s \qquad (93)$$

in which C_T is a constant for a given position of the neutral axis (see Table 1, page 665). By a method similar to that used in considering the force on

the compression side we may write the moment, about the neutral axis, of the force on a small element on the tension side as

$$d\,M_T = t_s\, r\, d\,\theta\, f_s \frac{r\,(\cos\theta + \cos\alpha)^2}{(1 + \cos\alpha)}$$

while the moment of the total tensile force T about this axis is

$$M_T = 2\int_0^{(\pi-\alpha)} t_s\, r\, f_s \frac{r\,(\cos\theta + \cos\alpha)^2}{(1 + \cos\alpha)}\, d\,\theta$$

Integrating, we have

$$M_T = t_s\, r^2 f_s \frac{2}{(1 + \cos\alpha)}\left[(\pi - \alpha)\cos^2\alpha + \tfrac{3}{2}\sin\alpha\cos\alpha + \tfrac{1}{2}(\pi - \alpha)\right]$$

Dividing M_T by T we have as the distance of the center of tension from the neutral axis

$$l_2 = \frac{\left((\pi - \alpha)\cos^2\alpha + \tfrac{3}{2}\sin\pi\cos\alpha + \tfrac{1}{2}(\pi - \alpha)\right)}{(\sin\alpha + (\pi - \alpha)\cos\alpha)}\, r \qquad (94)$$

From formulas (92) and (94) it is evident that the distance between the total force in compression and the total force in tension (i. e., $l_1 + l_2$) may, for any given position of the neutral axis, be expressed as a constant times the diameter D. Thus $l_1 + l_2 = jD$ as shown in Fig. 115. Likewise, as shown in Fig. 115, zD may represent the distance of the center of compression from the center of the chimney, z also being a constant for any given position of the neutral axis.

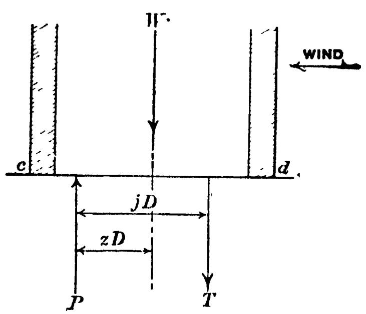


FIG. 115.—External and Internal Forces Acting upon a Chimney. (*See p.* 395.)

In a chimney the tensile and compressive stresses which we have been considering are produced by a combination of wind pressure and the weight of the chimney. Thus, on any horizontal section *cd*, as shown in Fig. 115, the forces external to that section are: the horizontal pressure of the wind, causing a moment M about the section, and a central vertical load W representing the weight of that portion of the chimney above the section under consideration. These forces are resisted, and held in equilibrium, by the forces P and T which represent the compressive and tensile stresses in the concrete and steel.

The system of forces as shown in Fig. 115 must be in equilibrium. Hence, taking moments about the force P, we may write

$$TjD = M - WzD$$

But

$$T = C_T f_s r t_s$$

Therefore

$$C_T f_s r t_s j D = M - WzD$$

Whence

$$rt_s = \frac{M - WzD}{C_T f_s j D}$$

The total area of steel $A_s = 2\pi r t_s$

Therefore

$$A_s = \frac{2\pi (M - WzD)}{C_T f_s j D} \tag{95}$$

From Table I, page 665, it may be seen that the constant j changes but slightly for a considerable variation in the position of the neutral axis. Taking $\frac{2\pi}{j} = 8$ for all cases, equation (95) may be

$$A_s = \frac{8 (M - WzD)}{C_T f_s D} \tag{96}$$

While this formula is not exact, the error involved is inappreciable for almost any case so that formula (96) may always be used instead of formula (95)

Applying now the condition that the summation of all vertical forces must be zero, we have

$$P - T = W$$

Substituting values of P and T as previously found, the equation becomes

$$C_P f_c r (t_c + n t_s) - C_T f_s r t_s = W$$

Transposing and solving for t_c we obtain

$$t_c = \frac{W + (C_T f_s - C_P f_c n) r t_s}{C_P f_c r}$$

The total thickness of the shell is

$$t = t_c + t_s$$

whence

$$t = \frac{W + (C_T f_s - C_P f_c n) r t_s}{C_P f_c r} + t_s$$

For convenience in use, after having determined A_s by the formula given above, by substituting $r = \frac{D}{z}$ and $t_s = \frac{A_s}{\pi D}$, this formula for t may best be written

$$t = \frac{2W + (C_T f_s - C_P f_c n) \frac{A_s}{\pi}}{C_P f_c D} + \frac{A_s}{\pi D} \tag{97}$$

In view of the fact that formulas (95), (96) and (97) contain the constants z, j, C_T and C_P, which, as has been shown, are dependent for their value solely upon the location of the neutral axis, it is evident that, for any specific values of f_c, f_s, and n, which in turn will determine the position of the neutral axis, the expressions for A_s and t will admit of a further simplification. For given values of f_c, f_s and n, the necessary thickness of shell and area of reinforcement may be expressed merely in terms of the moment of the wind M, the weight W, and the mean diameter D. The expressions, as given, however, seem best adapted to general use, and when supplemented by the tables given on pages 665 and 666, are rendered quite simple of solution for specific values.

In Table 2, page 666, are given values of k, the location of the neutral axis, for various combinations of f_c, f_s and n; while Table 1, page 665, gives the corresponding values of the constants C_P, C_T, z and j for various positions of the neutral axis.

Shear or Diagonal Tension. Having determined the necessary thickness of shell and vertical reinforcement, the size and spacing of the circular steel hoops must be considered. The external forces produce shear and diagonal tension which may be analyzed similarly to like stresses in rectangular beams, and the reinforcement necessary to resist the diagonal tension, which is a function of the vertical tension, may be determined. Usually this reinforcement is not so great as that which it is advisable to insert for the proper distribution of temperature stresses, but nevertheless it should be determined to be sure that it is sufficient in quantity.

The concrete should never be relied upon to carry any tension or vertical shear because the expansion from the heat may cause vertical cracks in the concrete. These need not be considered dangerous if sufficient horizontal reinforcement is provided any more than the vertical cracks in a brick or tile chimney. Considering the stresses due to vertical shear, it may be easily shown that at any horizontal section of a chimney the vertical shear per inch of height is the total horizontal shear on that section divided by the distance between centers of tension and compression, jD. With this as a

basis there may be developed a formula for practical use in determining the necessary area and spacing of horizontal steel hoops at any given section.

Thus let

h_l = height, in feet, of chimney above section under consideration.
F = effective wind pressure against chimney in pounds per square foot.
f_s = allowable tensile stress in pounds per square inch in steel hoops.
D = mean diameter of shell in inches.
p_0 = ratio of area of steel hoop to area of concrete.

At any horizontal section of a chimney the total shear on that section is equal to

$$\frac{D}{12} h_l F$$

while the maximum shear per inch of height is therefore

$$\frac{D}{12} \frac{h_l F}{jD}$$

Having seen that for all positions of the neutral axis j remains practically constant, and giving j an average value of, say, 0.783, the expression for the maximum vertical shear per inch of height becomes

$$0.106\ htF$$

while the shear or diagonal tension in one foot of height is $12 \times 0.106\ h_l F$.

The area of steel in one foot of height of chimney will be $12\ bp_o$ and the stress the hoops in this height are capable of sustaining on their two sections is

$$2 \times 12\ tp_0 f_s$$

Equating these we have

$$12 \times .106\ h_l F = 2 \times 12\ tp_0 f_s$$

whence

$$p_0 = \frac{h_l F}{18.8 f_s t}$$

This ratio of steel is for shear or diagonal tension only. To provide for temperature stresses or rather to distribute the strains so as to prevent the localization of cracks an additional amount of horizontal steel is needed. This may be provided for arbitrarily by assuming 0.25% steel or rather

0.0025 for temperature stress in addition to the steel for shear. Expressing this as a formula for ratio of steel gives

$$p_0 = \frac{h_t F}{18.8 f_s t} + 0.0025 \tag{98}$$

Small rods spaced 6 to 10 inches apart except in the upper part of the stack where the spacing may be greater are advised.

The spacing of hoops in many of the chimneys already built has been 18 inches to 36 inches, but as such chimneys have frequently cracked quite seriously, more recent designs have called for 8 or 9 inch spacing through the entire stack.

Design of Hollow Circular Beams. The analysis of a hollow circular reinforced concrete beam whose thickness, compared relatively with its diameter, is small, is similar in principle to that of a chimney. In this case the weight of the member acts in the same direction as the external forces, so that in formulas (96) and (97) W the weight in the axial direction, is zero. The forces of compression, P, and tension, T, are equal. The area of steel and the thickness of shell are therefore obtained from formulas (96) and (97), pages 396 and 397, by making $W = 0$.

Note on Slim Chimneys. Since, in designing a chimney the selection of certain allowable working stresses in the concrete and in the steel will fix the position of the neutral axis, it is evident that the ratio of these working stresses limits the compressive area of the section. Hence, for a very high chimney in which there is a large compression in the lower sections, it is possible that the selection of an ordinary working stress in the steel of 14000 or 16000 pounds per square inch together with the customary working stress in the concrete of, say, 500 pounds per square inch, would locate the neutral axis so near the compression side of the section as to make it impossible to obtain sufficient compression area to withstand the compressive forces without exceeding the allowable unit stress in the concrete.

If, therefore, the thickness of shell as computed from formula (97), page 397, should work out materially larger than the assumed thickness, recomputation should be made on the basis of a smaller working stress in the steel, thus changing the position of the neutral axis so as to allow a larger proportion of the section to carry compression. In such a case it may be necessary to make a series of trials with different working stresses in the steel until the computed thickness checks with the assumed thickness. In high chimneys of small diameter it may be impossible to utilize a working stress in the steel greater even than 7000 or 8000 pounds per square inch.

CHAPTER XXI

TESTS OF REINFORCED CONCRETE

The selected tests presented in this chapter were originally carried out to determine the principles of the theory and design of reinforced concrete. They are given here to illustrate the principles and conclusions presented in the preceding and the following chapters.

MODULUS OF ELASTICITY OF STEEL

The modulus of elasticity of steel varies from 28 000 000 pounds per square inch to 31 000 000 pounds per square inch; 30 000 000 is customarily taken as an average value, and is the value adopted in this treatise.

All Steel, irrespective of its Ultimate Strength, Elastic Limit or Chemical Composition, has Substantially the Same Modulus of Elasticity. It follows therefore from the principles of elasticity that the stretch under a given pull is independent of the character of the steel.

MODULUS OF ELASTICITY OF CONCRETE

For practical design it is recommended that the ratio of the modulus of elasticity of steel to that of concrete be taken at 15, corresponding to a concrete modulus of 2 000 000, for the 1:2:4 concrete used in ordinary practice.

Determination of Modulus of Elasticity. The modulus of elasticity, E, may be taken as the quotient of the stress per unit of area divided by the deformation (that is, the elongation or the shortening) in a unit length. In customary English units where the modulus is in pounds per square inch,

$$E = \frac{\text{stress per square inch}}{\text{deformation per linear inch}}$$

It is determined in the laboratory by measuring the deformation for the loads successively applied and plotting them as shown in Fig. 116. The curves in the diagram represent the deformations, at different stages of the loading, for a typical cylinder 8 inches in diameter by 16 inches high of extra strong 1:2:4 concrete, tested at the Structural Materials Testing Laboratories, United States Geological Survey, St. Louis, Mo., in 1907.

The set, which is the permanent deformation when the load is released, is not indicated in the diagram because the total deformation is that which must be used in reinforced concrete analysis.

The form of the deformation curve is approximately a parabola,* but the tests at St. Louis† indicate that for first-class concrete the modulus is

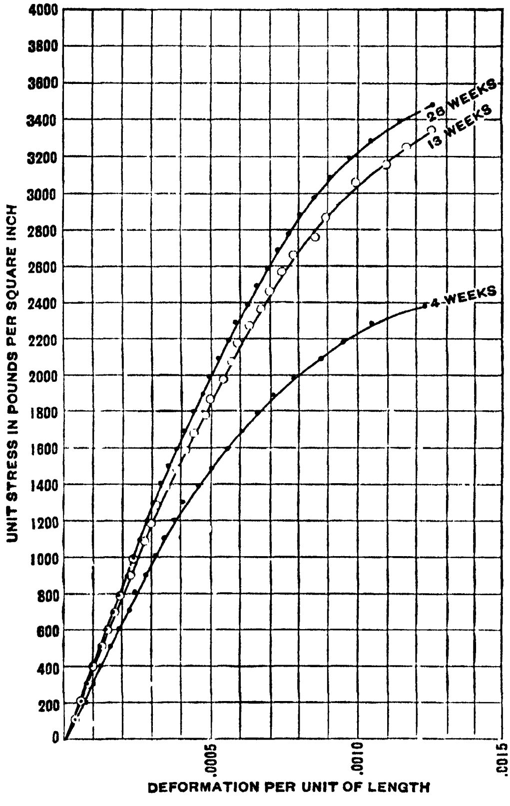


FIG. 116.—Stress Deformation Diagram, Limestone Concrete Cylinders of Medium Consistency and Extra Good Quality.‡ (See p. 400.)

nearly constant for about one-third of the ultimate strength. The modulus at this point is $\frac{800}{0.00025}$, or 3 200 000 pounds per square inch, in the four weeks old concrete tested.

* See discussion by Prof. Talbot in University of Illinois Bulletin, No. 10, Feb. 1, 1907, p. 21.
† Bulletin No. 344, U. S. Geological Survey, pp. 36–53.
‡ Bulletin No. 344, U. S. Geological Survey, p. 33.

Results of Tests. Numerous tests have been made to determine the modulus of elasticity of concrete which indicate as large a range in results obtained by different experimenters, even with concrete of the same proportions of cement to aggregate, as from 1 500 000 to 5 000 000 per square inch. The reasons for this are not yet fully determined; it has been conclusively proved, however, that the age of concrete, its richness and its density have undoubtedly a large influence on this variation.

The following table, compiled from various tests, may be of value as suggesting approximate values of the modulus for different proportions of concrete based upon the total deformation at one-third the crushing strength of cylinders at an age of thirty days. Two columns are given, one for ordinary wet concrete of medium quality, and one for concrete very carefully made with a dense mixture of mushy consistency and kept wet during hardening. The "ordinary" values are slightly below those which should be expected in practice on construction work.

The modulus of elasticity of concrete probably bears a definite relation to its ultimate strength, but the factors which enter into this relation probably will never be determined exactly. Plotting the results of a large number of tests made at the Watertown Arsenal, at the Government Laboratory at St. Louis, and at many of the colleges, indicates an approximate ratio of 1 300 between the modulus of elasticity and the ultimate strength.

Moduli of Elasticity of Concrete of Different Proportions. Approximate Average Values. *(See p. 402.)*

	PROPORTIONS.	ORDINARY WET CONCRETE.		EXCEPTIONALLY STRONG CONCRETE.	
		Crushing Strength at 30 days. lb. per sq. in	Modulus of Elasticity lb. per sq. in.	Crushing Strength at 30 days. lb. per sq. in.	Modulus of Elasticity lb. per sq. in.
Broken stone or gravel concrete	1 : 1½ : 3	2300	2 500 000	2800	3 600 000
	1 : 2 : 4	1700	2 000 000	2500	3 200 000
	1 : 2½ : 5	1500	1 800 000	2200	2 800 000
	1 : 3 : 6	1300	1 600 000	1900	2 500 000
	1 : 4 : 8	900	1 300 000	1500	2 000 000
	1 : 2 : 5	700	900 000	1000	1 300 000

NOTE—A modulus of 2 000 000, corresponding to a ratio of 15, is recommended for general use for 1 : 2 : 4 concrete and a ratio of 12 for 1 : 1½ : 3 concrete.

Tests of Mortar Prisms. Elastic properties of prisms of neat Portland cement and cement mortar, from tests made by Mr. Howard* at the Watertown Arsenal, are presented in the following table:

Elastic Properties of Cement and Mortar Prisms 6 by 6 by 18 inches.

Watertown Arsenal. (*See p.* 403)

Brand of Cement	Composition		Age	Modulus of elasticity between loads per square inch of			Permanent sets after loads per square inch of			Compressive strength per square inch
	Cement	Sand	Days	100 and 600 lb.	100 and 1 000 lb.	1 000 and 2 000 lb.	600 Inch	1 000 Inch	2 000 Inch	lb.
Alpha	Neat	0	7	7 143 000	5 000 000	8 333 000	0.	0.	0.	4 783
			7	4 167 000	3 600 000	3 448 000	0.	0.	.0002	5 000
Alpha	1	1	15	3 125 000	2 812 000	2 326 000	−.0002	−.0002	.0007	3 846
			36	2 381 000	2 500 000	2 041 000	0.	.0002	.0012	4 763
			36	2 632 000	2 727 000	3 030 000	.0001	.0002	.0010	4 948
Alpha	1	2	15	1 724 000	1 475 000		.0005	.0023		1 376
			36	2 273 000	2 195 000	1 538 000	.0001	.0006	.0040	2 184
			38	2 778 000	2 812 000	2 325 000	0.	.0004	.0020	2 755

Gaged length, 10 inches

Modulus of Elasticity in Beams vs. Columns. The modulus of elasticity in beams as determined by measurements and computations by Professor Talbot is approximately the same or possibly slightly lower than in columns.

Effect of Consistency of Concrete upon the Modulus of Elasticity. An excess of water in the concrete not only decreases the strength (see page 317), but also affects the deformation curve so as to show a more variable modulus near the beginning of the test. The moduli of concrete of different consistencies and at different ages are shown in the tables from tests of the authors on following page. The specimens were 12-inch cubes.

Relation of Stress Deformation Curve to the Theory of Beams. The theory of beams is worked out under the assumption that a section plane before bending remains plane after bending so that the deformation or stretch at any point in the compressive portion of the beam is proportional to the distance of this point from the neutral axis. According to this assumption the distribution of stresses is also proportional to the distance from the neutral axis so long as the modulus of elasticity is constant. This distribu-

* Tests of Metals, U. S. A., 1898.

tion may be then represented by a straight line as shown in Fig. 98, p. 353. When, however, the modulus of elasticity changes, Hook's law—that stress is proportional to deformation—is no longer applicable, since the intensity of stress is no longer proportional to the distance from the neutral axis but changes according to the relation of the moduli of elasticity at different loadings, and the line representing the distribution becomes a curve.*

Modulus of Elasticity of Concrete of Different Consistencies.† *Proportions by Volume* 1 : 2½ : 4½

BY TAYLOR AND THOMPSON. (*See p.* 403.)

Approximate age in months.	Dry.		Medium.		Very Wet.	
	Compressive strength. Pounds per sq. in.	Modulus at ⅓ ultimate strength. Pounds per sq. in.	Compressive strength. Pounds per sq. in.	Modulus at ⅓ ultimate strength. Pounds per sq. in.	Compressive strength. Pounds per sq. in.	Modulus at ⅓ ultimate strength. Pounds per sq. in.
1	4370	4 050 000	3360	4 500 000	2110	2 100 000
2	5430	4 050 000	3940	4 550 000	2770	3 400 000
6	5170	5 255 000	5170	3 760 000	3350	2 880 000
17	5510	3 920 000	4720	3 750 000	2430	2 080 000

Since the modulus is nearly constant within the working limits the authors have adopted the straight line theory of distribution of stress as simplest and most practical.‡

Formerly the parabolic distribution of pressure in concrete above the neutral axis was used in preference to the straight line theory because it corresponds somewhat more nearly to actual test. The two theories, however, require practically identical percentages of steel and the only difference is in the determination of the unit stress in the concrete. When using the parabola theory, about 15% lower compressive stress in the concrete must be used than when figuring by the straight line theory to obtain similar results. For example, 650 pounds per square inch safe compression by the straight line theory corresponds to about 565 pounds per square inch by the parabola theory.

* A comprehensive analytical discussion of the effect of a varying modulus of elasticity upon the pressure in a beam under different loadings is presented by Prof. Talbot in Journal Western Society of Engineers, Aug. 1904.

† "The Consistency of Concrete," by Sanford E. Thompson, American Society for Testing Materials. Vol. VI, 1906.

‡ It is also recommended by the Joint Committee, 1916.

TESTS OF RECTANGULAR BEAMS.

The most important determinations from the tests of beams are:

Stress at which first cracks appear and the corresponding stretch in concrete;

Location of neutral axis;

Relation of ultimate compressive fiber stress to strength of concrete in compression;

Distribution of stresses;

Relation of bending moment to moment of resistance based on stresses in steel;

Effect of the percentage of reinforcement;

Effect of mix of concrete and age.

The results and conclusions are given on the following pages. Formulas for design based on the tests may be found in Chapter XXII, pages 481 to 484.

Phenomena of Loading Rectangular Beams. During loading of reinforced concrete beams, three stages can be distinguished: first stage, before the appearance of the first crack; second stage, after first crack is developed, but before either of the materials passes its elastic limit; third stage, after the elastic limit has been passed.

First Stage. Before the appearance of the first crack, a reinforced concrete beam behaves similarly to a homogeneous beam. Compression is resisted by concrete and tension is resisted by concrete and steel in proportion to their moduli of elasticity. The position of the neutral axis nearly coincides with the center of gravity of the section obtained by replacing the steel by concrete of an area equal to the area of steel multiplied by the ratio of moduli of elasticity. At an elongation or stretch of concrete equal to the ultimate stretch of plain concrete, first cracks appear in the beam. At first they are not visible to the naked eye and do not extend up to the reinforcement. This is called the first stage.

Second Stage. At increased loads the number of cracks increases. They widen and move up toward the center of the beam. The larger the amount of reinforcement, the larger the number of cracks and smaller their width, as illustrated in Figure 117, page 406. In this stage, tension is resisted by steel and by the portion of concrete between the end of the crack and the neutral axis.

The cracks never extend way up to the neutral axis (which rises as the cracks develop), because, since the deformation increases from zero at the neutral axis to its maximum at the level of the steel, there must

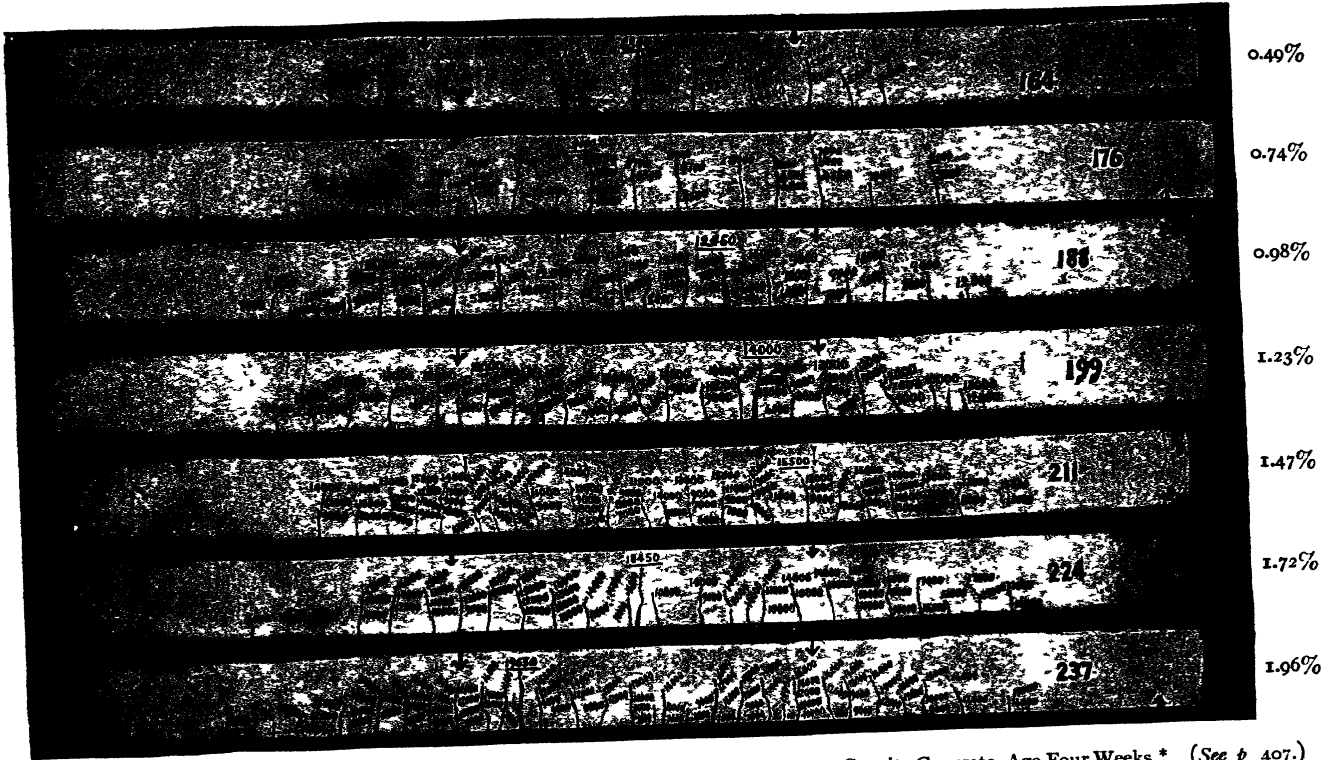


Fig. 117.—Variation in Position of Cracks with Variation in Percentage of Reinforcement, Granite Concrete, Age Four Weeks * (*See p* 407.)

* U S Bureau of Standards, Technologic Paper No 2 1912, p 97 Tests by Messrs Richard L Humphrey and L H Losse

be a portion of concrete where the deformation is smaller than the ultimate stretch for the concrete. There must be, therefore, a fiber which is just at the point of breaking and, above it, a portion of concrete carrying tensile stresses. The portion of the tensile stress carried by concrete decreases with the increase in the load. For equal intensity of loading, the stress carried by concrete is larger for smaller percentages of steel. For large loads and large percentages of steel, the amount of this stress in concrete is negligible, and as explained in the chapter on Theory, page 351, it is disregarded in designing reinforced concrete beams. In analyzing tests, however, it is necessary to take the tension in concrete into account since its existence explains why the moment of resistance, based on the stress in steel and figured by formula $M = A_s f_s jd$, is smaller than the bending moment, and why the actual stresses in steel obtained from deformations are smaller than the theoretical stresses. (See Fig. 119, page 413.) The amount of tension carried by the concrete may be estimated by comparing the moment of resistance, based on the actual stresses in steel, with the bending moment.

Third Stage. Beams Failing by Tension in Steel. When the steel reaches its elastic limit, one or two of the cracks, which were small up to this point, begin to open and extend towards the top. This is shown by the loads underlined in Figure 117, page 406. The deflection increases appreciably as the cracks widen and extend toward the top (the neutral axis rising), the compressive area becomes smaller, and finally the beam fails by totally destroying the compressive area. This condition is brought about by a small addition to the load at which the steel passes the elastic limit. The passing of the elastic limit of steel marks, therefore, the failure of the beam. **Ultimate strength of steel is never reached, and is, therefore, of no consequence in reinforced concrete design.**

Beams Failing by Compression. For beams failing by crushing of concrete, the third stage is marked by cracks in the top of the beam which appear after the elastic limit of the concrete in compression has been reached. At increased load, wedge-shaped pieces of concrete spall off and the beam fails.

Appearance of First Crack and Corresponding Stretch in Concrete. Numerous tests prove that the appearance of first cracks in reinforced concrete corresponds to about the same stretch as the appearance of cracks in plain concrete. This stretch may be taken approximately as 0.00012 of its length (corresponding to 3 600 lb. per sq. in. in the steel) for 1 : 2 : 4 stone concrete, and 0.00018 (corresponding to 5 400 lb. per

sq. in. in the steel) for cinder concrete.* The cracks, however, at this stretch, as discussed below, are very minute and not visible to the naked eye.

The conclusion of Mr. Considère in France, as the result of his tests, that the stretch of concrete when reinforced was 0.002 of its length, or twenty times the stretch of concrete without reinforcement, has been disproved by further experiments. Professor Turneaure,† in testing moist beams, observed, at about the same stretch at which first cracks developed in plain concrete beams, dark marks which he called water marks. Part of these water marks developed later into actual cracks. Professor Bach‡ investigated the subject further and came to the conclusion that water marks are places where adhesion between particles of concrete became loosened just previous to formation of cracks. In plain concrete, each water mark develops into a crack. In reinforced concrete, on the other hand, only a part of the water marks actually open because the steel strengthens these weakened spots and retards somewhat the appearance of actual cracks or prevents their formation altogether.

Professor Bach's Tests. Professor Bach's tests‡ in Stuttgart, summarized below, present the relation between actual and computed tensile stresses in concrete under different conditions. All of the values are high as Bach evidently worked with a stronger concrete than the same proportions give ordinarily. It must be noted further, as has been emphasized elsewhere, that the actual stresses, low at this stage as compared with the computed stresses, do not affect the accuracy of the ordinary formulas for practical design as this does not increase the factor ot safety nor the load at elastic limit. (See p. 412.)

Influence at First Crack of Richness of Mix. The increase in strength with richness of mix is shown in the table on page 409.

Here, as in the tables on the pages that follow, is shown, not merely the increase in strength with the richer proportions but also the effect of the concrete in reducing the stress in the steel because of its own strength in tension at early periods of the loading. At the period indicated, which is that of the first crack in the concrete, it is seen that the computed stress in steel is almost 3½ times the actual stress. As is shown later, this ratio decreases until at the actual breaking load in tension they nearly agree.

* Technologic Paper No. 2, U. S. Bureau of Standards, 1912, p. 39.

† Proceedings American Society for Testing Materials, 1904, p. 498.

‡ Bach-Spannungen unmittelbar vor der Rissbildung. Deutscher Ausschuss, Heft, 24, 1913.

Actual and Computed Stresses at First Crack for Different Proportions of Concrete.
(*See p.* 408.)

Age of beams at test, 45 days; aggregates, Rhine sand and gravel; ratio of steel, $p = 0.0056$. Wet storage.

Compiled from tests by C. BACH.

Proportions	Strength of Plain Concrete. Lb. per sq in.		Tensile Stresses at First Crack. Lb. per sq in		
			In Concrete.	In Steel.	
	Compressive	Tensile.	f'_c	Actual Stresses f_s	Computed by Formula $f_s = \frac{M}{A_s jd}$
1 : 3 : 4	2 100	108	290	3 900	13 400
1 : 2 : 3	3 750	270	380	5 150	17 200
1 : 1.5 : 2	4 400	330	485	6 600	23 000

Tensile stresses, f'_c, and actual stresses, f_s, are figured by formulas on page 362, where the tensile stresses in concrete are taken into account. The stresses computed by formula, $f_s = \frac{M}{A_s jd}$, on the other hand, are figured neglecting the tensile value of concrete.

Influence of Storage. The tensile stress in concrete, f'_c, was smaller for beams stored dry than for beams kept wet, the difference amounting on an average to about twenty per cent. Concrete stored dry tends to shrink, causing initial tensile stresses in concrete because free movement of concrete is prevented by the adhesion of concrete to steel. Concrete kept wet, on the other hand, tends to expand, which, prevented by the steel, causes initial compressive stresses in concrete. When loaded, the initial tensile stresses increase tension on the section while initial compressive stresses decrease it. To concrete in building construction the values for dry storage are applicable because, even if the concrete is kept wet during construction, in course of time it will dry out and the ultimate amount of shrinkage will be substantially the same as if it were held in dry storage. (See p. 261.)

Influence of Percentage of Steel. Professor Bach's tests* show that in concrete beams of the same proportions the actual unit stresses in concrete and steel at first crack are constant irrespective of the percentage of steel in the beam. The theoretical stresses in steel at the first crack, however, figured by the ordinary formulas neglecting the

* Bach-Spannungen unmittelbar vor der Rissbildung. Deutscher Ausschuss, Heft 24, 1913.

tensile resistance of concrete, vary with the percentage of steel. Similar results, as shown in the table below, were obtained in the tests by the Bureau of Standards carried on by Mr. Richard L. Humphrey and Mr. Louis H. Losse.*

Actual and Computed Stresses with Different Percentages of Steel. (*See p.* 410.)

Experimenters.	Proportions of Concrete.	Age. Days.	Tensile Stresses in Steel at First Crack. Lb. per sq. in.			
			Actual Stress Lb. per sq. in.	Computed by Ordinary Formula.		
				p = .005	p = .01	p = .02
Bach	1 : 1.5 : 2	45	6 600	25 000	13 000	8 000
Bureau of Standards	1 : 2 : 4	28	4 200	25 000	13 000	8 000

Influence of Consistency. A wet consistency reduces the strength. At an age of 45 days for 1 : 2 : 3 concrete, $p = 0.0056$, and for percentages of water by weight, varying between 6.8% and 10.0%, the tensile stress, f_c', at first crack ranged from 395 lb. to 310 lb. per sq. in., while the compressive strength of the same concrete ranged from 3800 lb. to 2360 lb. per sq. in., and the tensile strength in direct pull, from 485 lb. to 245 lb. per sq. in.

Influence of Age. Increase in strength with age, as determined by Bach, is shown in the table below.

Actual and Computed Stresses at Different Ages. (*See p.* 410.)
Proportions of concrete, 1:2:3. Ratio of steel, $p = 0.0056$.
Compiled from tests by C. BACH.†

	Stresses at First Crack. Lb. per sq. in.			
	Age.	Age.	Age.	Age.
	28 Days.	45 Days.	6 Months.	1 Year.
f_c' Actual stresses in concrete	360	380	466	495
f_s Actual stresses in steel. . .	4 900	5 100	6 300	6 700
f_s Computed stresses in steel	16 400	17 500	21 700	23 000

The theoretical stresses, f_s, are the stresses in steel figured by Formula (9), p. 355.

Position of Neutral Axis. The position of the neutral axis in reinforced concrete beams varies with the percentage of steel and the strength

* Technologic Paper No. 2, U. S. Bureau of Standards, p. 39.
† Bach-Spannungen unmittelbar vor der Rissbildung. Deutscher Ausschuss, Heft 24, 1913.

of concrete and also with the intensity of the loading. For beams with large percentages of steel, the initial position of the neutral axis is lower than for beams with smaller percentages of steel. With the same steel and stronger concrete, the neutral axis is higher than with a weaker concrete.

In any beam the neutral axis at the beginning of the loading nearly coincides with the center of gravity of a section in which the steel is considered as replaced by an area of concrete equal to the area of steel times the ratio of the moduli of elasticity. With the progress of the loading, it moves upward. In Figure 118, page 411, is given the typical movement of the neutral axis during loading for beams with different percentages of steel. As is evident from the figure, the position of the neutral axis for different loadings was determined by plotting at proper

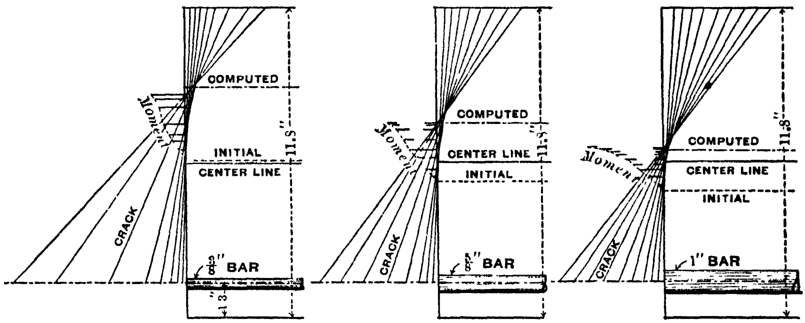


FIG. 118.—Change in Position of Neutral Axis During Loading for Different Percentages of Steel.* (*See p.* 411.)

levels the deformation of the upper concrete fiber and the deformation of steel. The intersection of the line, obtained by connecting the two points, and the vertical section of the beam gives the position of the neutral axis. For usual percentages of steel the distance from the compressive side of the beam under working loads is three-tenths to four-tenths of the depth. The formula for location of neutral axis is given on page 354.

Stresses in Steel for Varying Intensity of Load. Figure 119, page 413, gives the typical deformation of steel and of the upper fiber of concrete in inches per inch of length in beams with different percentages of steel, based on the tests of Messrs. Humphrey and Losse.† The deformation

* Bach "Biegeversuche mit Eisenbetonbalken," Berlin, 1907, pages 7 and 8.

† Technologic Paper No. 2, U. S. Bureau of Standards, 1912, p. 39.

curve for steel is not a straight line but a composite curve, the shape of which varies with the percentage of reinforcement. The deformation and, therefore, the stresses in steel at the first stage of the loading, that is, before the first crack, are comparatively small and proportional to the load, so that the deformation curve for this stage is almost a straight line. At deformation equal to the ultimate deformation in plain concrete beams, cracks in concrete open, and the tensile stresses borne by it are transferred to the steel, causing an abrupt change in the steel deformation curve. As is evident from the change in deformation, as shown in the diagram, the change in the direction of the curve is much larger for smaller percentages of steel because the amount of tensile stress, constant for beams of same cross-sections, which is transferred from the concrete to the steel, is distributed over a smaller amount of steel so that the increment in the unit stress in steel is larger.

On the deformation diagram the load at first crack is marked by the change in deformation from a straight line to a curve.

After the first crack, a large proportion of the total tensile stresses is carried by the steel. The concrete, however, still carries a small proportion dependent in amount upon the percentage of reinforcement in the beam. Because of these stresses carried by the concrete, the deformation in steel at different intensities of loading does not vary proportionally to the load. It is absolutely necessary that this be taken into account when analyzing results from tests not carried to the breaking point, for instance, in tests of completed buildings.

The actual stresses obtained in steel computed from deformation are smaller, for reasons indicated above, than the computed stresses for the same load. With small percentages of steel, concrete carries a considerable portion of the stresses up to the breaking point of the beam. (This is shown by the deformation curve for 0.49% of reinforcement.) For the larger percentages of steel, the dash line on the diagrams, which indicates the theoretical deformation of the steel obtained from Formula (9), page 355, strikes the actual deformation curve at the deformation corresponding to a stress of 39 000 pounds to 43 000 pounds per square inch. This indicates that near the elastic limit the actual stresses agree very well with the theoretical stresses.

The diagrams on page 413 give a comparison of actual stresses with theoretical stresses computed by the ordinary formulas. From this it is seen that if a beam or slab is designed by formulas on page 482 for an allowable unit stress in steel of 16 000 lb. per sq. in., the actual stress for the design load is smaller than 16 000 lb. and varies with the per-

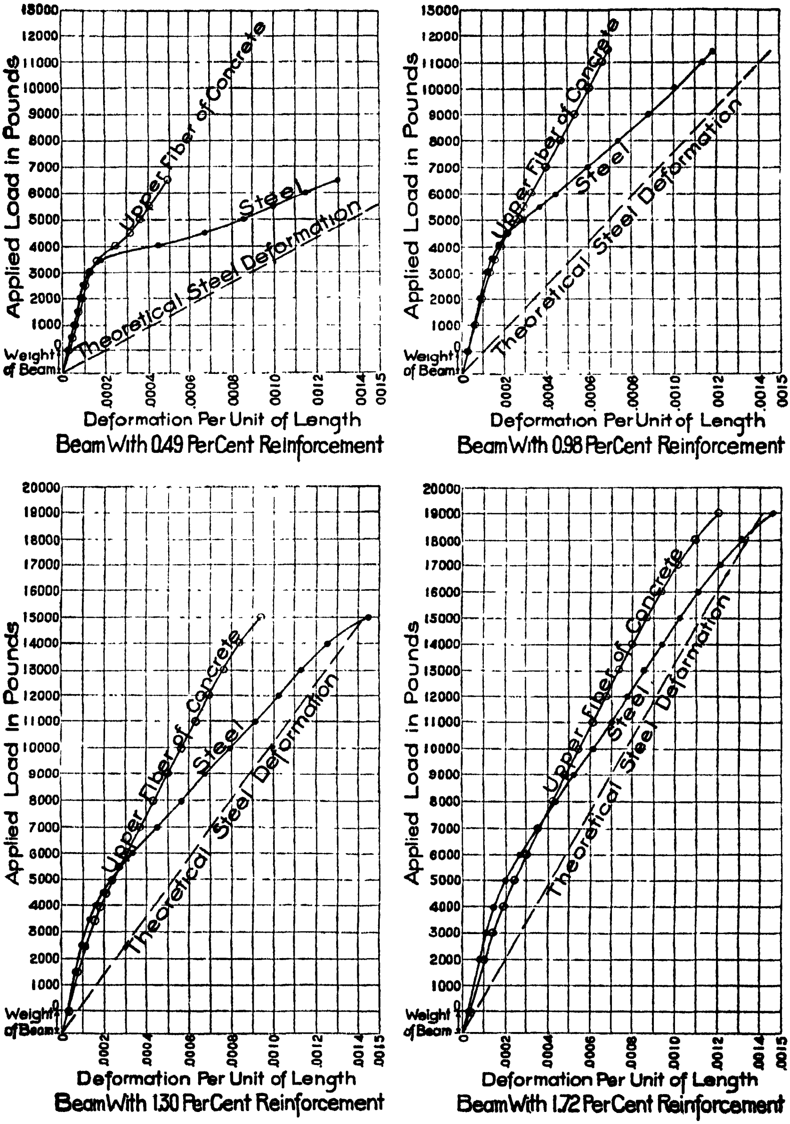


FIG. 119.—Deformations in Steel and Concrete Due to Loading.* (*See p.* 411.)

* Technologic Paper No. 2, U. S. Bureau of Standards, 1912, p. 39.

centage of tensile reinforcement. This does not, however, increase the factor of safety of the beam because, irrespective of the magnitude of the actual stress at the design load, double the load will bring the actual stress to about 32 000 pounds. It does, however, influence the formation of cracks so that the cracks do not appear at nearly so early a stage as would be expected from the ordinary formulas. In actual construction, tensile resistance of concrete cannot be counted upon as it is often destroyed either by shrinkage due to hardening or by temperature changes.

Formulas on page 482, therefore, although they do not represent the actual conditions of stresses at the design load, give the required factor of safety and are recommended for use in design.

The behaviour of a beam reinforced with steel in two layers is the same as of a beam with one layer. The steel in two layers is less effective than the same amount of steel placed at the lower level because the upper layer nearer the neutral axis is effective only in proportion to the ratio of its distance from the neutral axis to the distance of the lower layer. If, therefore, the upper layer is eight inches from the neutral axis and the lower, ten, the effective area is the area of the lower layer plus four-fifths of the upper layer.

Compressive Stresses in Concrete. As is evident from the deformation curves (Fig. 119, p. 413) the compressive stresses in concrete in the first stage are proportional to the loads. After the formation of the first crack, the deformation curve undergoes an abrupt change after which it is again almost a straight line.

The theoretical stresses computed from ordinary formulas for percentages of steel above one per cent agree fairly well with stresses computed from the deformation curves in beams and compared with deformation curves for cylinders of the same concrete. For small percentages of steel the computed stresses are smaller than the actual. This, however, does not affect the design because in such a case the beam would fail by tension so that the actual concrete stresses are unimportant.

Tests with Compressive Failure. Tests*† show that ultimate fiber stress determined from deformations is larger than the crushing strength of a cube of the same concrete so that it is safe to use larger stress for extreme fiber than for direct compression. The same phenomenon is observed in tests of T-beams. (See table on p. 416.) The number of tests, however, is not sufficient to draw a definite conclusion as to the ratio of crushing strength to ultimate fiber stress.

* University of Wisconsin, Bulletin No. 175, November, 1907.
† Dr. E. Mörsch, "Der Eisenbetonbau," 4th Edition, p.167.

TESTS OF T-BEAMS.

The discussion of the phenomena of loading and the movement of the neutral axis given for rectangular beams and the results given in connection with the appearance of first cracks on page 405 apply also to T-beams. The initial position of the neutral axis, however, will be different in a T-beam than in the rectangular beams and depends upon the relative dimensions of the flange and the stem and the percentage of reinforcement.

It must be remembered in applying to T-beams the discussion of influence of percentage of steel upon the appearance of first crack given for rectangular beams, that the percentage of steel must be figured for the width of beam equal to the width of the stem.

T-beams may fail by either tension in steel, compression in concrete, diagonal tension, or bond. The tests discussed below are grouped according to the cause of failure.

Tensile Failures of T-Beams. Professor Talbot's test of T-beams* consisted of nine beams. *Dimensions:* total length, 11 ft.; test span, 10 ft.; depth to steel, d = 10 in.; height, h = 12 in.; thickness of slab, t = 3¼ in.; breadth of stem, b' = 8 in.; width of flange, b = 16, 24, and 32 in. (three beams of each width). *Concrete,* 1 : 2 : 4 by volume. *Steel:* the amount of reinforcement varied from 0.92% to 1.1% of the area of enclosing rectangle, bd. Longitudinal reinforcement: ¾-inch plain round bars with yield point of 38 300 lb. per sq. in., and ¾-inch corrugated square bars with yield point of 53 800 lb. per sq. in., with ½ in. U-shaped stirrups (corrugated square) spaced 6 in. apart in the outside thirds of beam.

All beams failed by tension. Stresses in steel at maximum load, figured by Professor Talbot by formula, $f_s = \dfrac{M}{0.86\ A_s d}$, agree well with stresses at yield point of the steel. Calculated stresses ranged, for plain bars, from 37 600 to 41 500 lb. per sq. in., with an average of 39 800 lb., and for corrugated bars, from 55 700 lb. to 64 300 lb., with an average of 55 700 lb. per sq. in.

No beam failed by diagonal tension, although the maximum shearing unit stress from formula, $v = \dfrac{V}{b'jd}$, reached the value of 605 lb. per sq. in. The web reinforcement, therefore, proved to be adequate. The total diagonal tension, considered as resisted by the stirrups only, would

* University of Illinois Bulletin No. 12, February 1, 1907.

produce a theoretical stress in stirrups of 55 500 lb. per sq. in., or higher than the elastic limit of stirrup steel. Judging from the size of the diagonal cracks, the actual stress in stirrups was much below the elastic limit, which indicates that a part of the diagonal tension is carried by concrete, justifying the recommendation on page 371 allowing one-third of the total diagonal tension to be considered as resisted by concrete with the remainder carried by the steel.

Tests of T-Beams to Determine the Effective Width of Flange. The following test was made at the testing laboratory in Stuttgart,* with a number of beams of the same span, cross-section, and amount of steel, but varying widths of flange. Three beams of each type were tested. Loads were applied at one-third points.

Beams 2, 3, and 4 failed by crushing of concrete in the flange. The failure in concrete occurred in about the same way as in cubes, by splitting of wedge-shaped pieces of concrete. The shortening of the flanges of beam No. 3 was uniform throughout the width of the flange during the whole progress of the test. For Beam No. 4, there was a difference in shortening of only 8% for loads near the crushing strength of concrete.

T-Beam Tests to Determine Effect of Width of Flange. (*See p.* 416.)

Compiled from tests by C. Bach.*

Proportions of concrete, 1 : 3 : 4 by volume, with $9\frac{1}{2}\%$ of water by weight. Elastic limit of steel, 48 000 lb. per sq. in. Age of test, 45 days.

Common dimensions: Total length, 10.89 ft.; testing span, 9.84 ft.; breadth of stem $b'=7.08$ in.; total depth, $h=9.84$ in.; depth of steel, $d=8.66$ in.; thickness of flange, $t=2.36$ sq. in.; 2.36 in. fillets at juncture of slab and stem.

Steel: 4-1.17 in. round bars; area of steel $A_s=4.38$ sq. in.; $\frac{1}{4}$-in. round U stirrups spaced 3 in. on centers in outside thirds.

No. of Beam.	Width of Flange. in.	Ratio of Projections of Flange to Thickness of Slab.	Maximum Load. Lb.	Stresses at Maximum Load. Lb. per sq. in.		Strength of Cubes. Lb. per sq. in.	Ratio of Compressive Fiber Stress to Strength of Cubes.‡
				f_s†	f_c†		
2	7.1	0.0	16 900	10 050	2 200	1 580	1.39
3	20.0	5.0	31 500	18 700	1 900	1 580	1.20
4	29.1	9.5	47 200	28 100	2 200	1 750	1.26
5	39.4	13.6	56 800	33 750	2 140	1 800	1.19
5a	39 4	13.6	35 300	21 000	1 340	1 620	0.83

† Based on Formulas (18) and (19), p. 357.

‡ Ratios would have been still larger if oblong cylinders had been tested instead of cubes.

Beams 3, 4, and 5 were provided in outside thirds of beam with $\frac{1}{4}$-in. round cross bars spaced 6 inches on centers. Beam 5a had no cross bars.

* C. Bach Mitteilungen uber Forschungsarbeiten aus dem Gebiete des Ingenieurwesens, Heft 90 and 91.

Beams No. 5 and No. 5a were built alike except that cross bars were spaced about 6 inches apart in Beam No. 5, while No. 5a had no cross bars. Both of these beams failed by shearing off of the flanges, but there was great difference in action and in maximum load between the two attributable entirely to the effect of the cross bars. For Beam No. 5 with cross bars, the stresses and the shortening of the flange were uniform over the whole width of the flange up to a load of about 44 000 lb. when the first longitudinal crack appeared at the junction of the slab and the stem. This crack was followed by a number of similar cracks which finally caused failure. After the crack occurred, the shortening at the edges as compared with that at the stem decreased, and at maximum load, it amounted only to half of the shortening at the stem. Both edges of the slab, or flange, of the T-beam bent down or deflected along the entire length of beam: the maximum total deflection at the middle of the edge was 0.012 inch.

Beam No. 5a, without cross bars, failed from the same causes as Beam No. 5, except that the first crack occurred at a smaller load, and failure caused by the separation of the stem and the flanges followed closely the appearance of the first crack. This proved conclusively that it is advisable to place reinforcement across every beam which is expected to act as a T-beam so as to insure it against the separation of the flange and the stem.

The compressive stresses in the flange which must be transferred to the stem cause shearing stresses along the juncture of the stem and the flange. The magnitude of the unit shearing stress is proportional to the amount of compression carried by the flange and inversely proportional to the thickness of the flange. For method of determining the shearing stresses, see page 362.

The following conclusions can be drawn from the tests:

(*a*) Maximum load is increased materially by introduction of cross bars.

(*b*) Maximum load can be still further increased by fillets. Beams with fillets, making a 30° angle with the horizontal and having a depth of $\frac{3}{4}$ of the depth of slab, withstood a 20% larger load than beams without fillets. The effectiveness of fillet did not increase with the increase of size of the fillet.

(*c*) No appreciable difference is shown in deformation in flange at the edge and at the stem for 39.4-inch flange (projections 13.6 times depth of slab).

(*d*) Maximum load for beams with 60-inch flange (projections 21

times depth of slab) is only 5% larger than for 39.4-inch flange (projections 13.6 times depth of slab).

From the above conclusions, it is evident that:

(1) **In computing effective strength, the use of a width of flange equal to six times the thickness of slab on each side of the stem is conservative.**

(2) **In ordinary cases, no fillets are required.**

(3) **Cross bars on the top of T-beams are required to insure T-beam action.**

TESTS OF REINFORCED CONCRETE BEAMS TO DETERMINE EFFECT OF DIAGONAL TENSION

Most comprehensive tests, illustrated and described below, were made by Professor Bach in years 1908 and 1912,* comprising 64 sets of beams divided into two groups. In Group I, (see Fig. 120, p. 420,) the load is applied at the one-third points, and in Group II, (see Fig. 121, p. 422,) at eight points, uniformly spaced. Both groups include beams with no web reinforcement, with stirrups, with bent bars, and with stirrups and bent bars. In Group I the span is 9.8 ft.; in Group II it is 13.1 ft. Other dimensions were† h = 15.7 in.; d = 13.9 in.; b' = 7.9 in. (except as noted); t = 3.9 in.; and b = 19.7 in. in Group I, and 23.6 in. Group II. The amount of longitudinal steel was practically the same in all beams (see summary), the only variable being the arrangement of web reinforcement.

Concrete: 1 : 2 : 3 by volume; 9% of water by weight. Aggregates: Rhine sand and gravel. Age of specimens, 45 days. Average strength of cubes, 3 440 lb. per sq. in. *Steel:* yield point varied from 45 400 lb. to 51 000 lb. per sq. in. for Group I, and from 44 000 lb. to 63 000 lb. per sq. in. for Group II.

In both groups there are:

(*a*) Beams without stirrups with widths of stem, b', 5.9 in., 7.9 in., and 11.8 in.

(*b*) Beams with U-shaped 0.275-in. round stirrups, spaced 3.8 in. apart and with widths of stem varying as in the previous case.

(*c*) Beams without stirrups, horizontal steel provided with hooks at ends.

* Deutscher Ausschuss für Eisenbeton, Heft X, XII, XX.

† For notation, see p. 353.

(*d*) Beams with U-shaped stirrups varying in diameter from 0.2 in. to 0.39 in. and spacing varying from 2 in. to 7.9 in.

(*e*) Beams with bent bars of different arrangement with and without stirrups.

In Group II, the web reinforcement was designed by Formula (60), page 373, for loads producing in tensile steel a stress of 14 200 lb. per sq. in. The figures on page 420 and 422 give arrangement of reinforcement; maximum loads carried by the beam; cross section of horizontal steel; cross section of bent-up bars; maximum shearing unit stress, v; stress in steel at maximum load, f_s; stress in stirrups figured with assumption of stirrups taking the total amount of diagonal tension.

The following conclusions can be drawn:

General. (1) Tests show that it is possible to provide sufficient web reinforcement in the shape of stirrups, bent bars, or combinations of the two to develop the maximum carrying capacity of the beam whether governed by horizontal steel or by crushing strength of concrete.

(2) For beams with and without stirrups failing by diagonal tension, the maximum load increases in direct proportion with the width of the stem (Group I (a) and (b)). This proves that diagonal tension is resisted partly by concrete and partly by reinforcement, else in beams with stirrups failing by diagonal tension the width of stem would be of no influence on the ultimate load.

(3) The stresses in web reinforcement are smaller than obtained by assuming the total diagonal tension from Formula (54), page 367 to be resisted by steel only. The two above conclusions justify the recommendation that, in designing, one-third of the diagonal tension be considered as resisted by concrete. (See also page 416.)

(4) Hooks at ends of horizontal bars largely increase the strength of beams by preventing slipping of the bars. The increase may reach 50% or more. (See also page 438.)

Stirrups. (5) Stirrups increase the capacity of the beam, as is evident from comparison of Beam 51 and Beam 52, on page 422. For equal spacing, the ultimate load increases with the increase of the diameter of the stirrup, and for equal diameters of stirrups, it increases with the decrease of spacing.

(6) Stirrups of small diameter spaced closely are more effective than large diameters with correspondingly larger spacing. Stirrups in tests were most effective with a spacing equal to one-third of the depth of

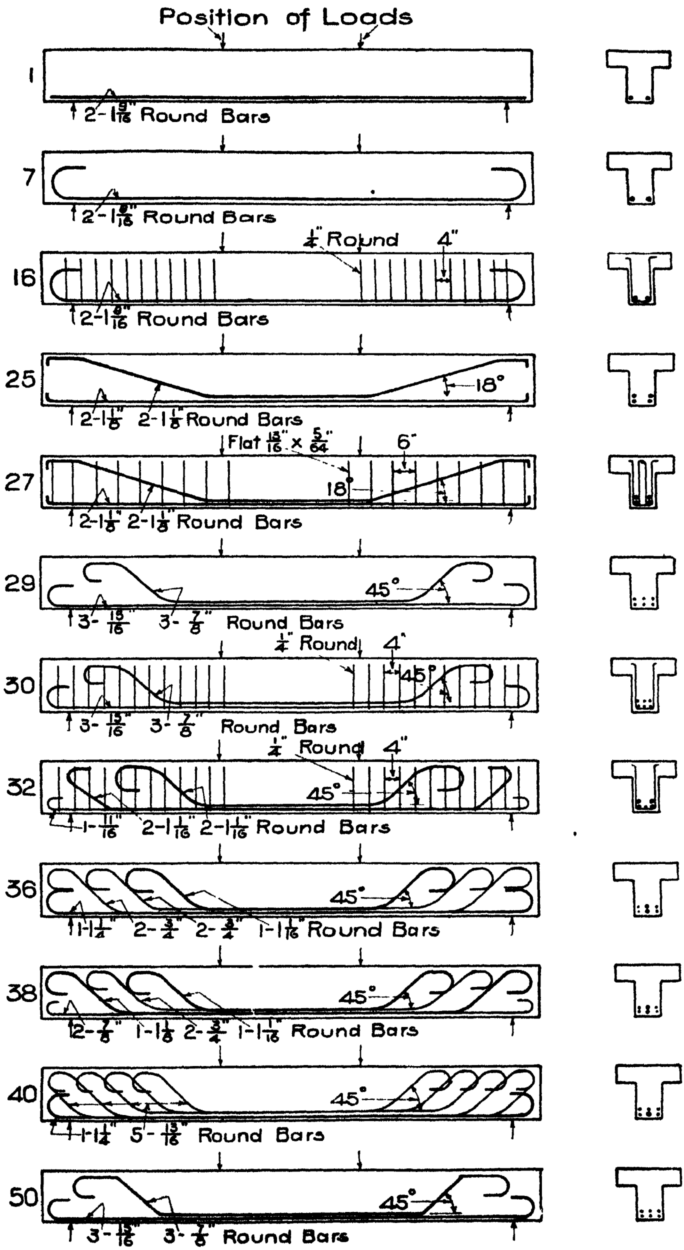


FIG. 120.—Effect of Diagonal Tension. Design and Loading of Test Beams. Group I. (*See p.* 418.)

the beam; i.e., for this spacing, the increase in ultimate load per pound of steel in stirrups was a maximum.

(7) Only about one-third of the total depth may be counted upon in developing bond in stirrups, because with the progress of the diagonal cracks, which may reach to about one-third of the depth of the beam from the top, only the portion of concrete above the crack is effective. Slip in stirrups under load by actual measurement was largest for the stirrups intersected by the crack near the top of the beam. It is advisable, therefore, to hook the ends of the stirrups.

(8) The stirrups influence the bond of the horizontal steel, which increases with the decrease of spacing of stirrups.

Tests of Beams to Determine the Efficiency of Web Reinforcement. (*See p.* 420.)

Compiled from Tests by C. BACH*

Number of Beam.	Area of Cross Section of Bars.† Horizontal (Total)	Area of Cross Section of Bars.† Bent.	Ultimate Load.	Stresses at Ultimate Load. lb. per sq. in. f_c	Stresses at Ultimate Load. lb. per sq. in. f_s	Stresses at Ultimate Load. lb. per sq. in. v	Increase in Ultimate Load by Hooking of Horizontal Bars.	Deflection. Ultimate Load.	Deflection. Half Ultimate Load.	Cause of Failure. (See note.)
	sq. in.	sq. in.	lb.				Per cent.	in.	in.	
				Beams Loaded at Two Points						
1	3.91		35 000	990	14 630	185		0.09	0.03	D
7	3.91		54 300	1 580	22 580	286		0.19	0.06	D
16	3.91		88 000	2 430	36 130	451		0.35	0.11	D
25	3.83	1.92	75 900	2 210	32 980	407	22	0.28	0.09	D
27	3.83	1.92	98 600	2 860	42 760	528	20	0.45	0.14	D
29	3.91	1.78	92 400	2 680	39 500	501	19	0.33	0.13	D
30	3.91	1.78	106 900	3 090	45 480	567		0.47	0.15	T
32	3.91	3.57	98 600	2 940	42 350	529		0.41	0.14	D
36	3.91	2.64	101 200	2 940	43 960	535	12	0.42	0.15	D
38	3.91	2.73	108 900	3 160	45 830	573	11	0.47	0.16	T
40	3.95	2.71	100 100	2 880	41 690	529	5	0.41	0.14	D
50	3.87	1.77	81 800	2 440	35 340	438		0.36	0.11	D

NOTE: D = diagonal tension failure
B = bond failure
T = tension failure
DB = diagonal tension and bond failure

* Deutscher Ausschuss für Eisenbeton, Heft X, XII, XX, 1908 and 1912.

† Areas of bars are converted directly from the metric dimensions. Diameters in Fig. 120 are approximate to nearest sixteenth inch.

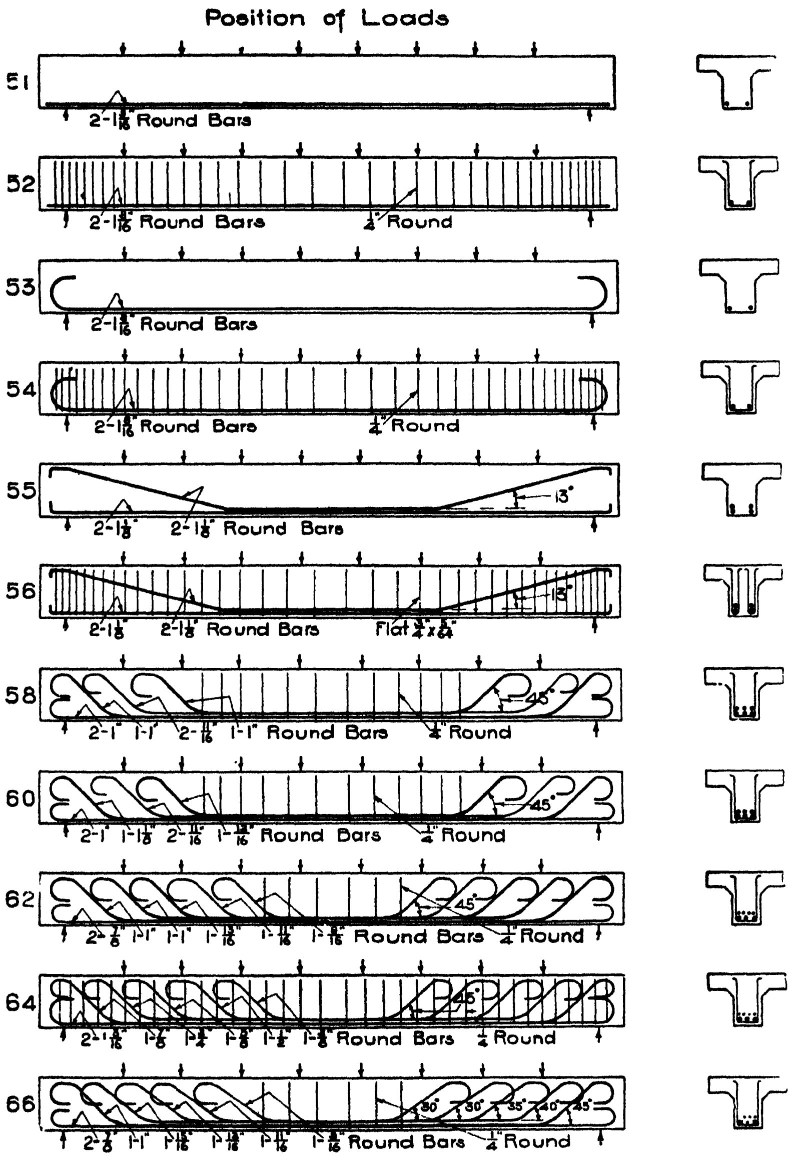


FIG. 121.—Effect of Diagonal Tension. Design and Loading of Test Beams. Group II. (*See p.* 418.)

Bent Bars. (9) Bars bent at one point only are more effective when bent at about 45° than if bent flatter at about 18° with the horizontal. Beam 29, on page 421, resisted 20% larger load than Beam 25, although the area of bars bent at 45° was 1.8 sq. in., against 1.98 sq. in. bent at 18° in Beam 25. No marked difference was found in strength for beams with bars bent at 30°, 40°, and 45° respectively.

(10) Bent bars as well as stirrups are effective reinforcement for diagonal tension. Compare Beams 30 and 38, page 420, both of which failed by tension in steel.

(11) The strength of beams with bars having sharp bends was smaller than for beams with a circular bend with a radius equal to about 12 diameters.

(12) It is evident from comparison of the stresses at ultimate loads with the elastic limit of steel that almost all the beams with bent bars failed by tension in longitudinal steel.

Tests of Beams to Determine the Efficiency of Web Reinforcement. (*See p.* 422.)
Compiled from Tests by C. Bach.*

Number of Beam.	Area of Cross Section of Bars †		Ultimate Load.	Stresses at Ultimate Load. lb per sq. in			Increase in Ultimate Load by Hooking of Horizontal Bars.	Deflection.		Cause of Failure. (See note.)
	Horizontal (Total)	Bent	lb.	f_c	f_s	v		Ultimate Load.	Half Ultimate Load.	
	sq. in.	sq. in					Per cent.	in.	in.	
Beams Loaded at Eight Points										
51	3.91		46 900	1 310	22 420	252		0.22	0.10	DB
52	3.89		67 300	1 870	32 020	360		0.37	0.15	DB
53	3.91		51 300	1 420	24 390	273		0.24	0.09	D
54	3.97		93 900	2 552	43 250	492		0.73	0.22	
55	3.81	1.91	73 300	2 150	36 890	404		0.50	0.20	D
56	3.80	1.91	100 300	2 920	50 300	549		1.01	0.27	T
58	3.86	2.32	95 300	2 750	46 690	516	6	0.74	0.25	T
60	3.91	2.37	95 300	2 760	46 220	518	11	0.64	0.24	T
62	3.94	2.74	99 400	2 860	48 010	539	17	0.88	0.26	T
64	3.89	1.70	106 200	2 950	50 450	566	18	0.88	0.26	T
66	3.91	2.73	101 900	2 900	49 180	552		0.75	0.26	T

Note: D = diagonal tension failure   T = tension failure
B = bond failure   DB = diagonal tension and bond failure

* Deutscher Ausschuss für Eisenbeton, Heft X, XII, XX, 1908 and 1912.

† Areas of bars are converted directly from the metric dimension. Diameters in Fig. 121 are approximate to nearest sixteenth inch.

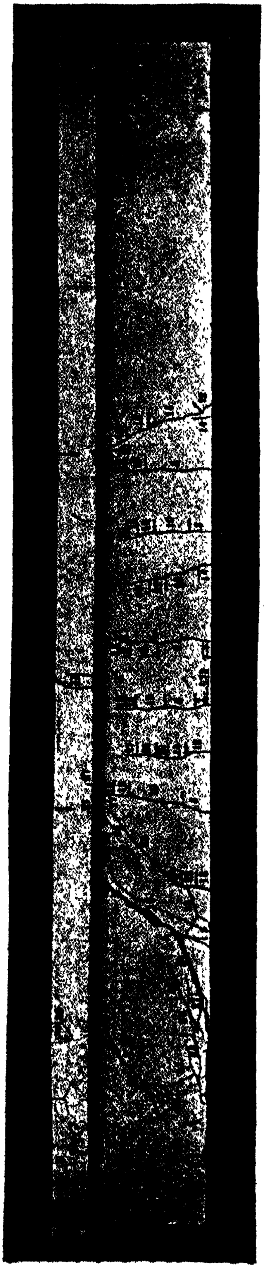


FIG. 122.—Typical Diagonal Tension Failure. (*See p.* 425.)
Tests by PROF. BACH.

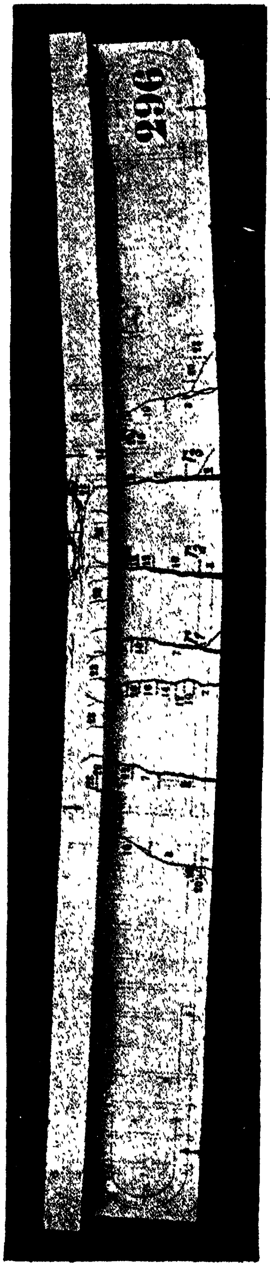


FIG. 123.—Typical Tension Failure. (*See p.* 425.)
Tests by PROF. BACH.

BEHAVIOR OF REINFORCED CONCRETE BEAM FAILING BY DIAGONAL TENSION UNDER LOAD

The difference between the intensity of loading at first diagonal crack and the ultimate loading for beams without web reinforcement depends upon the strength of the concrete. Lean, or green concrete beams fail with little or no warning, so that the load at first diagonal crack coincides with the breaking load, while in richer and stronger concrete beams, diagonal cracks are visible for some time before final failure occurs.

Figure 122, page 424, shows a reinforced concrete beam of 1 : 2 : 3 concrete, 45 days old, with no stirrups, after failure by diagonal tension and slipping of the bar. At a load of 14 700 lb., the first crack developed in the middle portion of the beam which was loaded at one-third points. At 17 640 lb., a diagonal tension crack developed in the out-

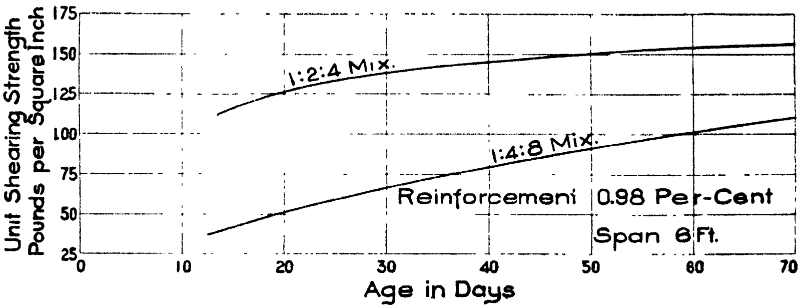


FIG. 124.- Effect of Age Upon Web Resistance. (*See p.* 426.)
Tests by PROF. TALBOT.

side third just beyond the load. This crack increased with increased intensity of load with an inclination toward the load. At 22 000 lb., the crack extended up almost to the bottom of the flange. The diagonal tension cracks were much larger than the tensile crack in the middle portion. At 22 000 lb., small horizontal cracks developed at the level of the horizontal bar. At further loading, more additional horizontal cracks appeared than all the previous horizontal cracks combined and formed at the failure which took place at the loading of 28 600 lb., a continuous crack extending from the support to the load, as is shown in the figure.

Fig. 123, page 424, shows a typical tension failure of a beam of similar dimensions as shown in Fig. 122, page 424, provided with stirrups where the cracks are confined to the center of the beam. These tests are both selected from the series by Bach.

BEAMS WITHOUT SHEAR REINFORCEMENT

The maximum unit shearing stress at which beams without web reinforcement fail by diagonal tension depends primarily upon the richness of the concrete and the age, and in smaller degree upon the percentage of steel and the ratio of depth to length of span. The last two items can be neglected in ordinary design. Since diagonal tension failure in beams without web reinforcement is sudden, a large factor of safety is advisable. (See page 374.) In all cases quoted below unit shearing stresses were computed by formula (55a), page 367.

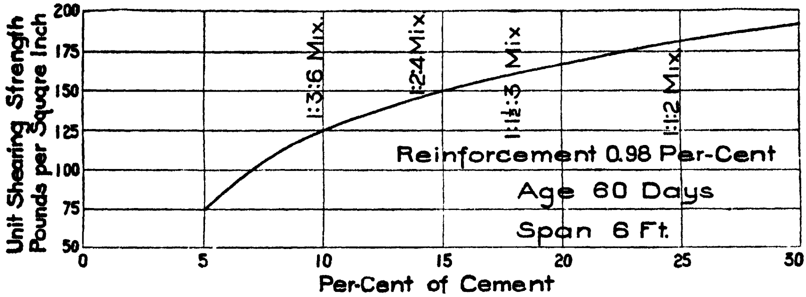


FIG. 125.—Effect of Proportion of Concrete Upon Web Resistance. (*See p.* 426.) Tests by PROF. TALBOT.

Effect of Age upon Web Resistance. The effect of age is of great importance in determining the time for removal of forms and the age at which concrete can be loaded. It is illustrated in Fig. 124, page 425, taken from tests made by Professor Talbot at the University of Illinois.*

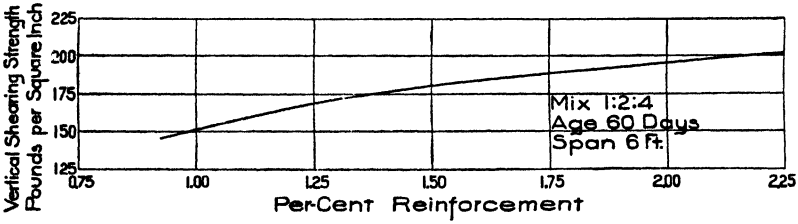


FIG. 126.—Effect of Percentage of Horizontal Steel upon Web Resistance. (*See p.* 427).
Tests by PROF. TALBOT.

Effect of Richness of Mixture of Concrete upon Web Resistance. Figure 125, page 426, taken from Professor Talbot's tests shows the increase of

* Bulletin No. 29, January 4, 1909.

web resistance with the amount of cement in concrete. The increase is quite marked although somewhat less than the increase in compressive strength for the same cause.

Effect of Percentage of Horizontal Steel upon Web Resistance. As is evident from Fig. 126, page 426, the percentage of steel has a marked effect on web resistance which can be attributed to two causes. First, for smaller percentages of steel, the deformation is larger with consequently increased tendency of concrete to crack. Second, with larger percentages of steel, the tensile stresses developed near the support are smaller, consequently the appearance of the tension cracks which later develop into diagonal tension cracks is retarded.

Ratio of Maximum Shearing Unit Stress Involving Diagonal Tension to the Modulus of Rupture of a Plain Beam and to the Compressive Strength. In beams without web reinforcement, from tests by Professor Talbot* the ratio of maximum vertical shearing unit stress in beams failing by diagonal tension to modulus of rupture averages 0.5, and to the compressive strength, of 8 by 16-inch cylinders† averages 0.09.

BEAMS REINFORCED FOR TENSION AND COMPRESSION

Tests prove conclusively the effectiveness of steel as compression reinforcement.

Professor M. O. Withey's Tests.‡ The series of 1906 consisted of eight beams, 12 feet long; breadth = 8 inches; height = 11 inches; depth to steel = $9\frac{3}{4}$ inches, with 2.9% tensile reinforcement and varying amounts of compressive reinforcement. The web reinforcement consisted of three bars bent up in two different places at a very flat angle.

The results of the tests, although interesting, do not bring out fully the value of steel as compressive reinforcement because all beams failed by diagonal tension, with the exception of the beam without compressive reinforcement, which failed in compression. Notwithstanding this, however, the maximum load of the beam without compression steel was 22 000 lb., while the maximum load for the beam with compressive reinforcement was 29 000 lb.

Series of 1907 consisted of four beams similar in design to the beams previously described except that they were provided at each end with 10-$\frac{1}{4}$-in. round stirrups. All the beams failed in tension at an average

* University of Illinois, Bulletin No. 29, January 4, 1909.

† In determining this ratio the authors have converted the results found on cubes to a cylinder basis (see p. 344).

‡ Bulletins of the University of Wisconsin, No. 175 and 197, Series of 1906 and 1907.

load of 34 000 lb., showing an increase of 55% over the beam without

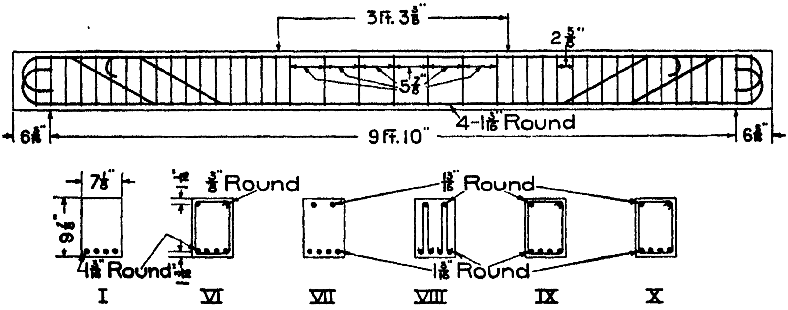


FIG. 127.—Dimensions of Beams, Stuttgart Tests. (*See p.* 428.)
By PROF. C. BACH.

compression reinforcement. Still, because of the tension failure, the full value of compression reinforcement was not demonstrated.

Bach's Stuttgart Tests.* Bach's tests of beams with compressive steel consisted of six types of beams, the dimensions and arrangement of reinforcement of which is shown in Fig. 127. The results of the tests are given on page 428. The reinforcement of beams VII, VIII, and IX is alike except that beam VII in the middle portion has no stirrups while beams VIII and IX have stirrups of the shapes shown in the draw-

Test of Beams with Compression Reinforcement. (*See p.* 428.)

Concrete: 1 : 3 : 4 by volume. Aggregates: Rhine sand up to 0.27 in. diameter, and Rhine gravel up to 0.79 in. diameter; 9.5% of water by weight. Age of test, 45 days.

Compiled from Tests by C. BACH.

Specimens.	Maximum Load. lb.	Computed Unit Stresses in Steel and Concrete at Maximum Load Based on $n = 15$			Compressive Strength of Cubes. lb. per sq. in.	Ratio Computed Unit Fibre Stress to Strength of Cubes.
		Unit tensile stress in steel f_s lb. per sq. in.	Unit compressive stress in steel f'_s lb. per sq. in.	Unit stress in concrete f_c lb. per sq. in.		
I........	16 860	11 220		2 220	1 590	1.40
VI........	20 650	14 390	28 680	2 390	1 490	1.60
VII........	27 500	17 750	29 200	2 500	1 480	1.69
VIII........	29 000	18 720	29 800	2 670	1 610	1.66
IX........	28 600	18 480	31 620	2 590	1 520	1.70
X........	36 000	23 200	36 800	3 240	1 590	2.04

* Mitteilungen uber Forschungsarbeiten aus dem Gebiete des Ingenieurwesen, Heft 90 and 91.

ing. The amount of the compressive reinforcement in beam X is the same as in beams VII and VIII, but the steel is of higher elastic limit.

Beams VI, VII, VIII, and IX failed by compression. Beam X, in which the bond strength of the compressive steel was exceeded, failed by splitting the concrete. This latter beam shows a considerable increase in strength over beams with the same amount of reinforcement because of the use of compression steel with high elastic limit. The table on page 428 gives the ultimate loads, the stresses in steel and concrete at the ultimate load under the assumption of $n = 15$, the strength of cubes and ratio of strength of cubes to figured stress in concrete in the beam. In Beams VII to IX, the compression steel reached its elastic limit first, and for the farther loading kept the same stress till the elastic limit of concrete was reached. In Beam X, on the other hand, the elastic limit in the concrete was reached first, and after this, stresses due to the additional loading were carried by the steel only until both materials reached the elastic limit. This points to the adjustment between compressive stresses in steel and concrete after one of the materials passes its elastic limit. The same phenomenon was observed in the test of reinforced concrete columns.

From inspection of the table, it is evident that for beams with compression steel, the theoretical unit stresses in the concrete itself computed at the ultimate test load by formulas on page 360, and on the basis of $n = 15$, are much larger than the similar unit stresses at which the beams without compression reinforcement failed. Since the same concrete was used in all cases it is rational to assume that this extra stress must be attributed to compressive steel. This shows that the compressive steel carries stresses larger than would be expected from the formulas, and that its actual effect is greater than the theoretical effect. It is especially noticeable in Beam X, for which the theoretical maximum fiber stress was 3 240 lb. per sq. in., while the crushing strength of the concrete was 1 590 lb. per sq. in.

The above tests prove conclusively that compressive steel may be relied upon to strengthen the compressive zone of a beam, and that its effect is even larger than would be expected from the formulas.

TESTS OF BOND BETWEEN CONCRETE AND STEEL

Bond between concrete and steel, or the resistance to withdrawal of steel imbedded in concrete may be divided into two elements: (1) grip caused by shrinkage of concrete; (2) frictional resistance caused by the unevenness of the surface of the bar. Both elements act together until

the bar begins to slip. Then the grip is destroyed and frictional resistance alone resists the pull.

In deformed bars, the two elements are aided by the pressure or bearing of the projections on the concrete, but this does not come into play until after the first slip.

The pull-out tests are treated separately from the bond tests in beams because the action of bond stresses in the two cases is different.

PULL-OUT TESTS

Pull-out test specimens consist of bars imbedded in blocks. The load is applied at the free end of the bar and is resisted by the resistance to the withdrawal of the steel imbedded in the block.

In practice similar conditions occur in end anchors for fixed or cantilever beams where the concrete at the support corresponds to the block in the pull-out tests. The maximum stress in steel at the edge of the support, which is transferred to the support by bond, corresponds to the applied force in pull-out tests.

In computation the bond stresses are considered as uniformly distributed over the whole surface of contact between steel and concrete. (See Formula 36, p. 534.) Actually, however, the bond stresses vary from a maximum at the edge of the support to a minimum within the support. In many cases, in fact, the bar begins to slip at the place of application of the force before the bond resistance of the whole bar comes into play. Therefore, ordinarily the portion of the bar near the point of support offers frictional resistance only, while the farther end of the bar offers grip and frictional resistance. The variation in magnitude of bond stresses along the length of imbedded bar depends upon the length of imbedment. Hence in basing allowable unit stresses on the tests, the effect of the ratio of the imbedded length to the diameter of bar must be taken into account.

When a bar imbedded in concrete slips, the movement of the free end is somewhat greater than of the imbedded end, the difference being equal to the deformation of the imbedded portion of the bar under stress.

Effect on Bond Strength of the Ratio of Length of Imbedment to Diameter of Bar. The average bond resistance considered as distributed uniformly over the total surface area of imbedment is smaller for long imbedments than for short imbedments. At the University of Illinois.*

* University of Illinois Bulletin No. 71, December 8, 1913, p. 39.

in tests by Mr. Duff A. Abrams for 1¼-inch plain round bars imbedded in 1 : 2 : 4 concrete, 74 days old, the average bond resistance for 6-inch imbedment (4.8 diameter of the bars) was 420 lb. per sq. in., while for 24-inch imbedment (19.2 diameters), it was 328 lb. per sq. in. Similar results were obtained by Prof. C. Bach.*

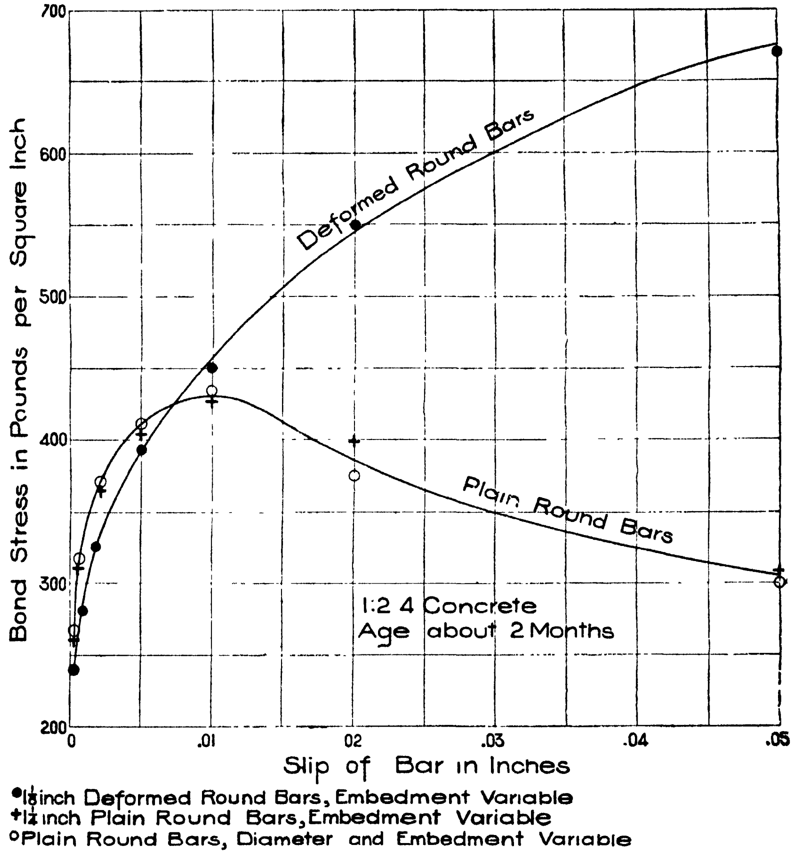


FIG. 128.—Relation of Bond Stress to Slip of Bar During Progress of Loading.†
(*See p.* 432.)

Method of Determining Bond Resistance. In computing the bond resistance of a bar the ratio of the length of imbedment to diameter of bar, and not the length of imbedment, is the determining item. The

* C. Bach, Zeitschrift des Vereines Deutscher Ingenieure, 1911, S. 859.
† University of Illinois Bulletin No. 71, December 8, 1913, p. 29.

required length of imbedment increases in direct proportion with the increase of the diameter of bar. Thus a 25-inch imbedment is sufficient for a ½-inch bar because the ratio of the length to diameter is 50. It would not be large enough for a one-inch bar because the ratio then is only 25. (See p. 539.)

Bond Resistance for Different Slips. Fig. 128, p. 431, shows the relation between the bond stresses and slips for plain and deformed bars during the progress of loading. As is evident from this diagram, for plain bars initial slip occurred at 260 lb. per sq. in., or at about 60% of the maximum bond resistance. After the maximum bond resistance, which corresponds to a slip of 0.01 inches, was reached, the resistance to withdrawal decreased. After a slip equal to five times the slip at maximum resistance has taken place, only 70% of the maximum load is required to produce further slipping. The curves for the deformed bars are discussed on page 434.

Effect of Surface Condition and Shape of Bars. The following conclusions may be drawn from Abrams' tests.

The bond resistance of square bars is only 75% of the bond resistance of plain round bars.

Rusted bars (with no scale) give bond resistance 15% higher than similar bars with ordinary milled surface.

The bond resistance of T-bars per unit of area decreases with the increase in size. For 1:2:4 concrete, imbedment 8 inches, and age 70 days,* the maximum bond resistance of 1-inch round plain bar was 370 lb. per sq. in.; of 1-inch T-bar, 310 lb. per sq. in.; and 2-inch T-bar, 220 lb. per sq. in.

Influence of Age and Mix. The following table gives the effect of age and mix on bond of ¾-inch plain round bars and of ¾-inch corrugated square bars.

Influence of Freezing. In Abrams' tests, specimens made out-doors in freezing weather, where they probably froze and thawed several times during the period of setting and hardening, were almost devoid of bond strength.

Ratio of Compressive Strength to Bond Resistance. The ratio of bond strength at first slip to compressive strength of 8 by 16-inch cylinder is about 0.13, and of the maximum bond strength, 0.19.

These ratios were determined by Mr. Abrams from tests on specimens varying in age from 2 days to 2½ years, and proportions from

* University of Illinois Bulletin No. 71, December 8, 1913, p. 40.

1 : 1 : 2 to 1 : 5 : 10. These values agree very well with the results obtained by other experimenters.

*Maximum Bond Stress and Bond Stress at 0.001 Inches Slip for Varying Proportions and Ages.** *(See p. 434.)*

By D. A. Abrams

Size of Bar.	Age.	Stress in Pounds per Square Inch of Surface of Bar. Proportions. 1:1:2.† Maximum.	1:1:2.† At 0.001-Inch Slip	1:1½:3. Maximum.	1:1½:3. At 0.001-Inch Slip.	1:2:4. Maximum.	1:2:4. At 0.001-Inch Slip.	1:3:6. Maximum.	1:3:6. At 0.001-Inch Slip.	1:4:8. Maximum.	1:4:8. At 0.001-Inch Slip.
¾-inch plain round	2 days	141	107	159	123	123	89	53	32	27	17
	4 days	197	156	231	195	153	110	77	43	49	32
	7 days	246	202	300	250	226	158	165	112	54	32
	28 days to 32 days	393	300	546	457	404	288	241	130	149	120
	60 days to 65 days	530	399	554	492	452	363	311	227	190	135
	120 days to 132 days	666	479	667	538	603	469	536	398	210	172
	16 months	779	656	896‡	875	841‡	800	372	333	373	253
¾ inch corrugated square	2 days	231	96	205	92	219	97	157	44	64	13
	4 days	368	176	258	115	305	129	239	74	110	23
	7 days	419	171	330	140	459	187	286	104	133	35
	28 days to 32 days	828	344	560	281	641	306	462	179	273	97
	60 days to 65 days	1 132	498	1 053‡	536	854	434	623	280	391	139
	120 days to 132 days	1 153	599	1 070	564	1 079	576	746	326	470	159
	16 months	1 535	892	......	....	.....	....	....	....	728	322

* University of Illinois Bulletin No. 71, December 8, 1913, pp. 82–83.

† The reason for relatively low strength of 1:1:2 concrete and ¾-inch bars is unexplained and may be an erratic result.

‡ Bars stressed to or beyond yield point.

Deformed Bars. Results of pull-out tests with deformed bars are given* on pages 431, 433 and 435. The first slip for the deformed bars occurs at about the same stress as for plain bars.

After the first slip, the projections help to resist farther slipping. Considering all the bond stresses except those resisted by frictional resistance taken by the projections, the bearing stresses on concrete for some types of deformed bars at large slips are very large, reaching in some cases 14 000 lb. per sq. in. of the area in contact. This high compressive stress on concrete explains the splitting of the blocks in pull-out tests. Since the allowable working stresses are only a fraction of the ultimate bond stress the bearing stresses on projections always are within safe working limits.

The maximum bond stresses, being accompanied by large slips, cannot be utilized in construction, where only a very small slip is permissible, consequently the working bond stresses must be based on stresses at a slip not exceeding 0.01 inches rather than on ultimate bond strength. The factor of safety for deformed bars based on this slip, however, may be made smaller than for plain bars since the high ultimate bond strength and the existence of mechanical bond reduce the danger of actual bond failure. This is of special importance during construction when comparatively green concrete may be called upon to support a considerable construction load.

Allowable Working Stresses. Allowable working unit stresses based on the tests are given on page 573.

BOND STRESSES IN BEAMS

The method of computing bond stresses in reinforced concrete beams is given on page 533. Although the formulas do not represent the actual conditions in a beam, as explained below they form a proper basis for design with values for working stresses based on tests and figured for the same assumptions.

The computed maximum bond stresses in a beam occur at points of maximum shear. With uniform loading, this is at the supports and decreases uniformly to zero at the center of the beam. In beams loaded at one-third points, maximum bond stresses act in the outside thirds and are zero in the central portion of the beam.

Phenomena of Bond Tests. The bond stresses in beams are caused by the change from point to point, i.e., the increase, in the stresses in

* University of Illinois Bulletin No. 71, December 8, 1913.

the longitudinal steel. This increase in stress in steel as computed is proportional to the amount of increase in the bending moment, and therefore, proportional to the vertical shear. Actually, however, the change in stress in steel is affected by the presence of tensile stresses

Distribution of Bond Stress in Reinforced Concrete Beams. (*See p.* 436.)

Beams 8 by 12 in. in section and 10 in. deep to center of reinforcing bar. Loaded at the one-third points of a 10-ft. span.

All beams failed by excessive tensile stress in the reinforcing bars.

Compiled from Tests by DUFF A. ABRAMS*

Beam No.	Size and Kind of Bar.	Age at Test	Applied Load on Beam.	Average Computed Bond Stress	Observed Bond Stress.	
					Over Region Just Outside of Load Points.†	Near Ends of Beam.‡
			lb.	lb. per sq in.	lb. per sq. in.	lb. per sq. in.
1055.6	One 1-in. Plain Round	2 yr.	2 000	38	100	16
			4 000	76	125	34
			6 000	114	191	36
			8 000	152	226	64
			10 000	190	201	117
			11 700	222	165	238
1055.3	One 1-in. Plain Round	2 yr.	2 000	38	48	15
			4 000	76	75	54
			6 000	114	155	95
			8 000	152	141	100
			10 000	190	200	130
			10 700	203	140	156
1049.3	One 1⅛ in. Corrugated Round	13 mo.	2 000	34	80	20
			4 000	68	137	45
			6 000	102	226	95
			8 000	135	285	135
			10 000	170	250	150
			12 000	204	315	150
			14 000	236	350	225
			16 000	270	385	260
			18 000	306	400	290
			20 000	338	450	315
			21 000	355	200	360
			21 900	370	....	390

* University of Illinois Bulletin No. 71, December 8, 1913, p. 193.

† These stresses are, in general, the average bond stresses developed over a length of about 12 in. in the portion of the beam about 4 to 16 in. outside the load points.

‡ The average observed stress over a length of 0 to 15 in. at the ends of the beam.

in concrete, the amount and the proportion of which to the total tensile stresses is different in different parts of the beam. The effect is smaller near the point of maximum tensile stresses (where the concrete is cracked), and larger near the support where concrete may carry stresses even at maximum load. The increment of stresses in steel is not proportional to the shear; the bond stresses which are caused by that increment are, therefore, not proportional to the shear.

The table on page 435 gives observed bond stresses and computed bond stresses for varying intensities of loading for a beam loaded at one-third points. Since the shear between the support and the point of application of the load is constant, the computed bond stresses are con-

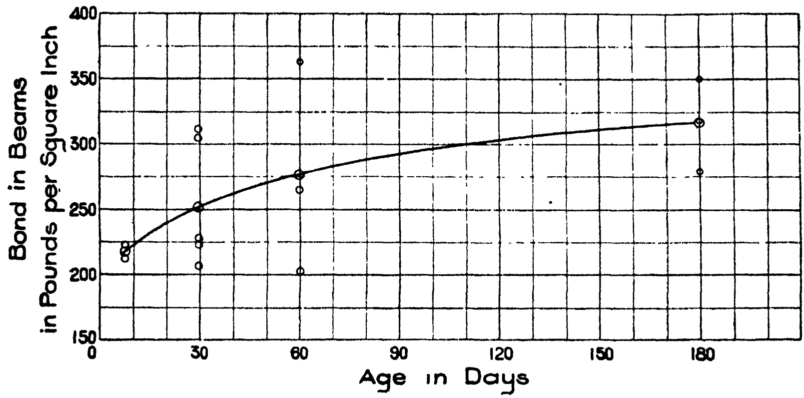


FIG. 129.—Effect of Age on Bond in Beams. (*See p.* 437.)
Tests by PROF. WITHEY.

stant. The observed bond stresses, however, near the support are smaller than just outside of the points of application of the load until the steel reaches the elastic limit, after which a readjustment takes place and the bond stresses become equalized. From the table, it is evident that in beams 1049.3, for example, the observed bond stress just outside of load points for a load of 16 000 lb. is larger than the average computed bond stress for the ultimate load, i.e. 21 900 lb. This explains why for beams failing by bond the average computed bond stress at the ultimate load based on Formula (36), page 534, is smaller than the maximum bond strength in pull-out tests.

The bond stresses given in subsequent discussion are those obtained by Formula 36, page 534.

Effect of the Distance of the Load from the Support on the Bond Stresses. As may be inferred from the discussion of the phenomena

of the bond stresses, the average maximum bond stresses are larger as the load is placed nearer the support. Prof. C. Bach* in tests of beams of 1:2:3 concrete at age of 45 days, finds values of ultimate bond strength for distances 9.8 inches, 19.7 inches, and 29.5 inches from the support to average 507, 325, and 308 lb. respectively.

Effect of Age on Bond. Fig. 129 on page 436, from Professor Withey's tests at the University of Wisconsin† shows the increase of strength with age.

Professor Bach found for 1:2:3 concrete the following bond strength:

Ages	28 days.	45 days.	6 months.	One year.
Beams kept moist, lb. per sq. in.	278	308	393	435
" " dry, " "	271	319	356	363

He suggests the following formula for increase in bond strength with age:

$$u = 745\left(1 - \sqrt[6]{\frac{1}{15A + 1}}\right)$$

Where

u = unit bond strength in lb. per sq. in.

A = age in months.

Effect of Mix of Concrete. From tests it is evident that the richness of mortar in concrete affects the bond strength considerably. The quality of stone is of little effect provided pockets around the reinforcement are prevented. The table below gives values for bond strength for concrete of different proportions.

Bond Strength in Beams for Different Proportions of Concrete. (*See p.* 437.)

Beams, 5 in. by 5 in. by 5 ft. 0 in. long. Reinforcement, 3-$\frac{3}{8}$-in. round bars. Lower bars imbedded in concrete for length of 10 inches at both supports. Beams tested on 5-foot span. Compressive tests on separate specimens.

Compiled from Tests by MORTON O. WITHEY‡

Mix.	Age. days.	Coarse Aggregate.	Average Bond. lb. per sq. in.	Compressive Strength. lb. per sq. in.
1 : 2 : 4	60	Limestone	276	1 790
1 : 2 : 4	60	Gravel	275	2 200
1 : 3 : 6	60	Limestone	216	830
1 : 2½	60		267	1 600

* Widerstand Einbetonierten Eisens Gegen Gleiten. Einfluss der Haken, von C. Bach and O. Graf page 18.

† University of Wisconsin, Bulletin No. 321, October, 1909, p. 27. ‡ Ibid. 28.

Hooks as End Anchorage. The requirements of a properly constructed hook are: (1) it should permit the stressing of the steel to its elastic limit without appreciable movement; (2) the bearing stresses on the concrete must be within a safe limit. Since the allowable bearing stresses on concrete depend upon the properties of the concrete, the factor of safety against crushing must be the same as that used in determining the allowable fiber stresses in concrete. Tests show that the crushing strength of concrete when confined is much larger than the crushing strength of cubes or cylinders. Hence, the safe bearing stress of the hook on the concrete should be based on the crushing strength of confined concrete. In comparing, therefore, the relative efficiency of hooks, their bearing area is of first importance.

When used for end anchorage, hooks which allow stressing the steel to elastic limit, but which at the same time split or crush the concrete, have not the required factor of safety as far as concrete is concerned because at working stresses the concrete would have only a factor of safety of two instead of four as required by rational design.

Tests* made for the Eastern Concrete Construction Company at the Massachusetts Institute of Technology determined the capacity of the hook, but did not determine the load at which the first movement of the hook took place.

In all the tests, ¾-inch round bars were imbedded in blocks 12 inches square and 15 inches long to a depth of 12 inches with additional bends of different lengths. Right-angular bends and semi-circular bends on a 3-inch diameter were tested. Several specimens of each type were tested, the results of which were extremely uniform.

The following conclusions may be drawn from the tests.

(1) A 4-inch right-angular bend in a ¾-inch round bar (5 diameters) combined with 12-inch imbedment (16 diameters) is sufficient to stress the steel to its elastic limit. This hook, however, crushed the concrete and split the block, therefore it does not give the required factor of safety against crushing of concrete. A longer bend does not increase the security because the bearing stress is not appreciably reduced.

(2) A semi-circular bend with a diameter four times the diameter of the bar is more effective than the square bend and is preferable because the bearing stresses on concrete can be kept within working limits.

Action of Hooks in Beams. Beams in which longitudinal steel is provided with hooks show a much larger load carrying capacity than similar beams with ends of bars straight. Tests at age of 45 days by

* Concrete Plain and Reinforced, Second Edition, p. 466.

Professor Bach on beams of 1:2:3 concrete, 12 inches square and 6-foot span reinforced with one 0.98-inch diameter round bar provided with three different kinds of hooks, gave the carrying capacity of the beam without hooks as 14 330 lb.; with right angle hook, 24 250 lb.; with 45° hook, 25 800 lb.; and with circular hook, 28 060 lb. The beam with rectangular hooks failed by straightening the hook.

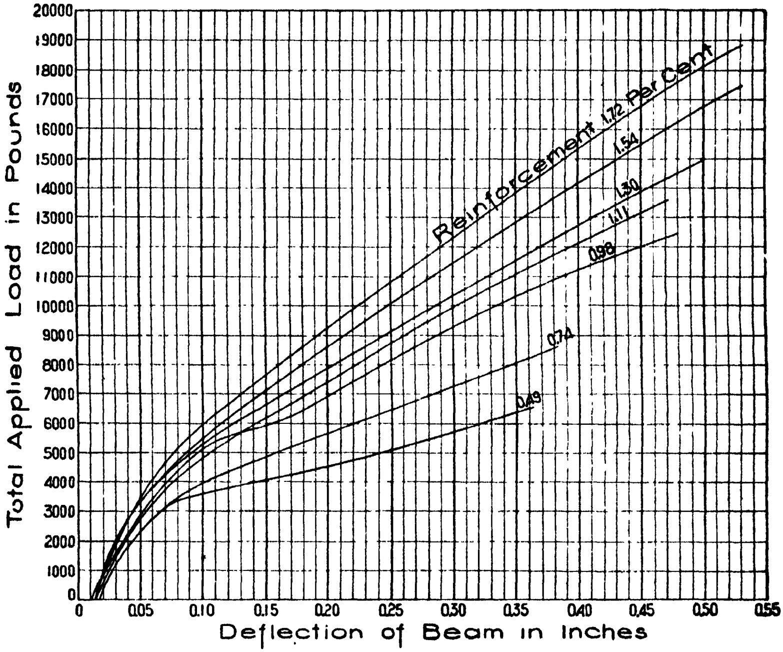


FIG. 130.—Deflection of Beams with varying Percentage of Steel. (*See p.* 441.)
Tests by RICHARD L. HUMPHREY and L. H. LOSSE.

SPLICES OF TENSILE REINFORCEMENT IN BEAMS AT POINTS OF MAXIMUM STRESS

Tests have been made by H. Scheit and O. Wawrzinio k* to determine the effectiveness of different methods of splicing steel at the point of maximum stress. The beams were 12 inches square of spans 6½ feet and 10 feet, reinforced with one-inch bar. They were tested with two symmetrical loads spaced 3 feet 3 inches apart for the shorter beams, and 5 feet apart for the longer beams.

* Deutscher Ausschuss fur Eisenbeton, Heft 14, 1912.

Straight splices were made with a lap of 10, 20, and 30 diameters respectively for the short beams, and 40, 50, 60, 70, and 80 diameters for the long beams; in hooked splices, the hooks consisted of a semicircle with an inside diameter of 5 diameters of the bar and an extra length of 6 diameters of the bar parallel to the bar, and the bars were lapped 10 inches, 20 inches, and 30 inches respectively.

Results. For straight splices, the best results were obtained with a splice of 50 diameters with which the elastic limit of steel was reached.

Hooked splices proved very effective. Even a 10-diameter lap (the smallest lap used) in combination with a hook, as described above, was sufficient to provide the same carrying capacity as the beam without the splice.

DEFLECTION

The deflection in reinforced concrete depends primarily upon the ratio of the depth of the beam, or slab, to the span. It also depends upon the percentage of tension and compression reinforcement, and in T-beams, upon the width of the flange.

Influence of Percentage of Steel upon Deflection. For equal depths and widths, the deflection of beams increases with any decrease in the

Deflection of T-Beam with Varying Widths of Flanges. (*See p.* 441.)

Span of beams, 9.84 feet; reinforcement, four $1\frac{3}{16}$-inch round bars; load applied at one-third points.

By C. Bach.*

Total Load.	Deflection in inches.			
	Rectangular Beam 7.1 × 9.84 in.	T-Beam, depth 9.84 in.; width of stem, 7.1 in. Width of Flange in Inches.		
		18.9	29.5	39.4
lb.	in.	in.	in.	in.
8 800	0.106	0.071	0.047	0.042
17 600	0.376	0.177	0.110	0.097
26 500		0.368	0.188	0.161
35 300			0.290	0.235
44 100			0.467	0.351
52 900				0.544

* Mitteilungen uber Forschungsarbeiten aus dem Gebeite des Ingenieurwesen, Heft 90 and 91.

tensile steel. Fig. 130, page 439, shows the deflections of beams 13 feet long, 8 by 11 inches in cross section, tested on a 12-foot span, by two equal loads applied at one-third points. The test was made by Messrs. Richard L. Humphrey and L. H. Losse.* The deformations in steel and concrete for the same beams are shown on page 413.

Influence of Width of Flange upon Deflection. In Bach's tests† to determine the effect of width of the flange, the results given in the table on page 440 were obtained. It will be seen that although the percentage of steel based on the area of the stem was the same in all cases, the deflection for equal loads is smaller for beams with larger widths of flange.

Deflection of Beams with Compression Steel. (*See p.* 441.)

All beams, 7.1 × 9.8 inches; span, 9.84 feet; tensile reinforcement, four $1\frac{3}{16}$-inch round bars; load applied at one-third points.

By C. Bach.‡

Total Load.	Deflection in Inches. Compression Steel in Percent.			
	0	0.4	1.58	1.58§
lb.	in.	in.	in.	in.
4 400	0.046	0.042	0.038	0.037
8 800	0.119	0.102	0.086	0.084
13 200	0.256	0.184	0.143	0.139
17 600		0.298	0.210	0.203
22 000			0.287	0.276

Influence of Compressive Steel upon Deflection. The table above gives deflection of beams without compressive reinforcement and with different percentages of compressive reinforcement. From the figures, it is evident that for equal percentage of tensile reinforcement the deflection decreases with the increase of compression reinforcement.

TESTS OF CONTINUOUS BEAMS

Since in concrete construction beams are usually continuous over several supports, it is of the greatest importance to determine by tests whether this continuity can be relied upon.

* Technologic Paper No. 2, U. S. Bureau of Standards, June 27, 1911.
† Mitteilungen uber Forschungsarbeiten aus dem Gebiete des Ingenieurwessens, Heft 90 and 91.
‡ Mitteilungen uber Forschungsarbeiten aus dem Gebiete des Ingenieurwesen, Heft 90 and 91.
§ High elastic limit steel used.

Tests of Continuous Beams by Prof. H. Scheit and Dr. Ing. E. Probst.* These tests included the concrete beams shown in Figs. 131 to 133, pages 442 to 444. Two beams of each type were tested to destruction.

The spans and the reinforcement for the loaded spans were the same for all beams. The complete series included beams as follows:

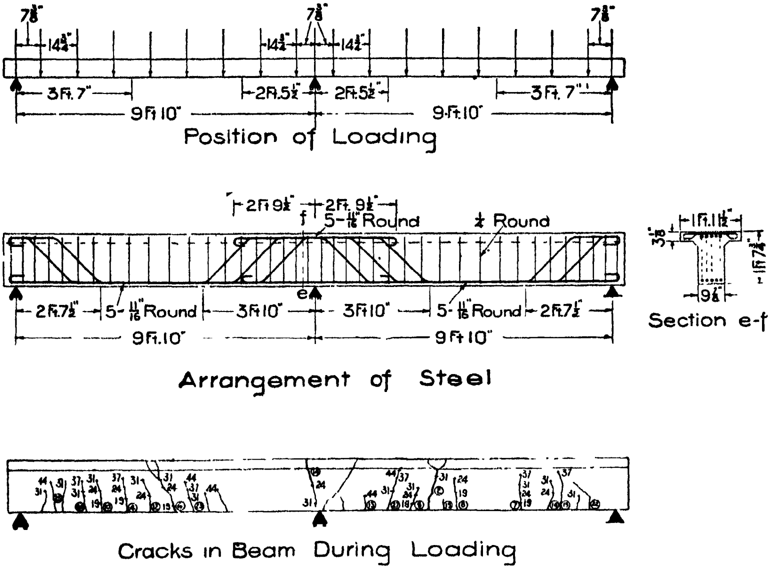


FIG. 131.—Continuous Beam of Two Spans; Type 2. (*See p.* 443.)

Type 1, simply supported beams.
Type 2, continuous beams over two spans.
Type 3, continuous beams over three spans, end spans loaded.
Type 3a, beams similar to Type 3, but supported by columns; loading same as Type 3.
Type 4, beams of five spans, alternate spans loaded.

Figs. 131 to 133 show the cracks, and Fig. 132, Type 3, the deflection at different loads. Results of tests are discussed below:

Comparison of Theoretical Deflection with Actual Deflection. To determine the efficiency of continuous beams, the ratios of deflections of continuous beams to those of simple beams obtained from tests were compared with theoretical ratios for homogeneous beams.

For continuous beams of two spans, the observed ratio of deflections

* "Untersuchungen an durchlaufenden Eisenbetonkonstruktionen," Berlin, 1912.

was between 0.42 and 0.45, while the theoretical ratio was 0.40. For beams continuous over three spans with end spans only loaded, the observed ratio was 0.69 against the theoretical ratio of 0.74.

For beams of Type 3 of the same design as the one above but in which the ends were connected with columns, the ratio varied from 0.34 to 0.37. From the theoretical figures, it appears that as far as deflection is concerned, this type is almost midway between a beam fixed

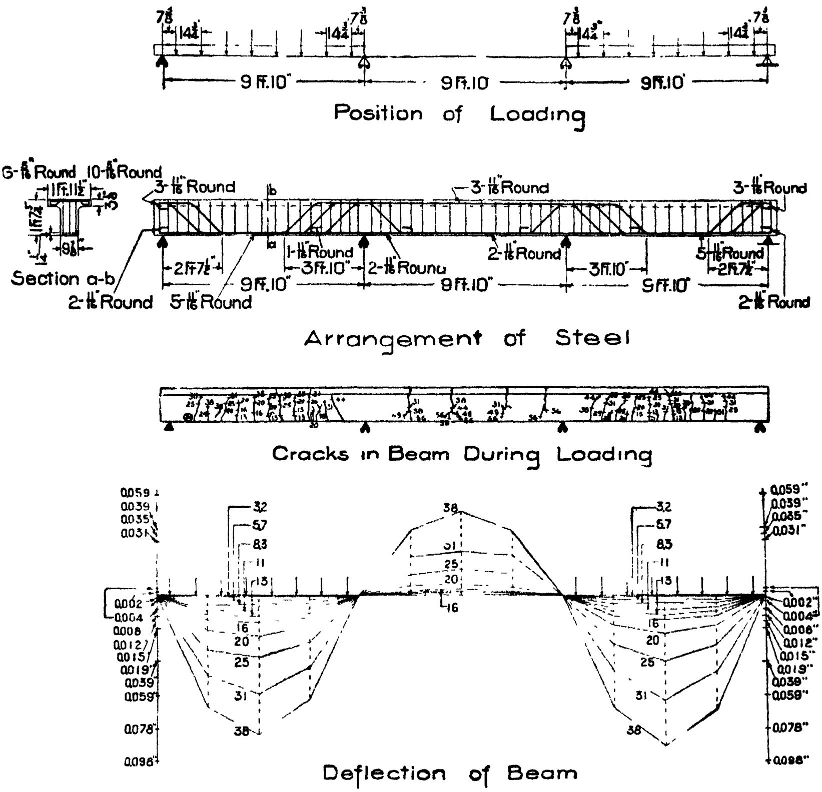


FIG. 132.—Continuous Beam of Three Spans; Type 3. (*See p.* 444.)

at one end, for which the ratio is 0.40, and a beam fixed at both ends, for which the ratio is 0.20.

Type 2. The beams continuous over two spans failed at the support at an average load of 14,240 lb. per lin. ft. The theoretical negative bending moment at the support is $-\frac{wl^2}{8}$. The stress in steel at the

support for the maximum load figured on the basis of the above bending moment is about 64 000 lb. per sq. in. It is evident that this stress is much higher than the elastic limit of the steel used in the test, which shows that the assumed theoretical bending moment coefficient is too large. Based on the moment of resistance for the yield point of steel, we get a bending moment coefficient of 10 instead of the theoretical 8. The point of inflection was found to coincide with the theoretical point of inflection. (See Fig. 131, p. 442.)

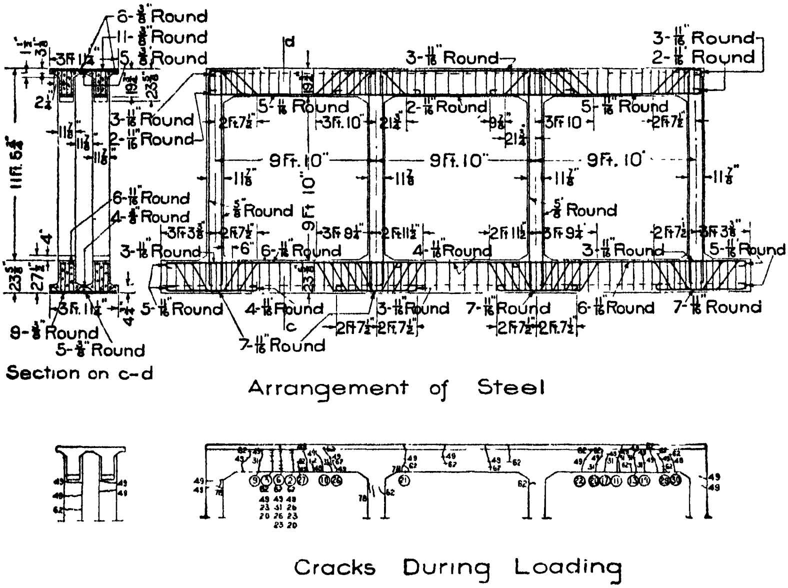


FIG. 133.—Continuous Beams of Three Spans; Type 3a. (*See p.* 445.)

Type 3. In the beams continuous over three spans, the first cracks appeared at the bottom of the beam in the central portion of the loaded span at a load of 2 565 lb. per lin. ft. In the unloaded span, the first crack appeared at the top of the beam at a load of 5 600 lb. per lin. ft. The sequence of other cracks and the loads at which they appeared is evident from the illustration. The failure at a load of 12 600 lb. per lin. ft. was caused by passing of the elastic limit of steel in the loaded spans.

The deflection diagram, Fig. 132, page 443, gives a positive proof of

continuous action of the beam. As is evident from the diagram, the deflection in the end span is positive, while in the center span, it is negative. The computed stresses and bending moments at the different loads agree quite closely with the measured stresses.

The measured compressive stress for the maximum load in the middle span (which was not loaded) was found to be 1 500 lb. per sq. in., which is almost identical with the theoretical stress. The cracks in the unloaded span, which are uniformly distributed over its whole length, furnish a conclusive proof of continuous action. If no provision had been made for the negative bending moment in the unloaded span, failure would have been certain.

Type 3a. In the beam continuous over three spans and monolithic with columns, as shown by Fig. 133, page 444, the connection between beams and columns was not rigid, as would be used in rigid frames, but was built as in ordinary building construction. The beams were of exactly the same design as in Type 3. The comparison, therefore, gives the effect of the connection of the beam with the column. The first cracks in the loaded span appeared at a load of 4 590 lb. per lin. ft., and in the unloaded span, at a load of 10 935 lb. per lin. ft. The corresponding figures in Type 3 were 3 565 lb. per lin. ft. and 5 670 lb. per lin. ft. At a load of 13 905 lb. per lin. ft., the first cracks appeared at the top of the end column, and at 16 740 lb. per lin. ft., a crack appeared at the top of the middle column. The beam failed at 17 620 lb. per lin. ft. by steel passing the elastic limit. At the time of failure, cracks were observed in the compressive part of the interior column.

After the tests, cracks were found at the bottom of the columns located in reverse position to the cracks at the top. During test, not only the beams but also the columns deflected, which shows that the whole construction acted as a unit. The deflection in the beams was smaller than in Type 3, as explained before.

As was expected, the moment of resistance at the ultimate load does not agree with the bending moments for continuous beams based on the assumption of free ends. The construction must be considered as a frame. Mr. Probst finds that the positive bending moment coefficient in the loaded span was 12.02, which agrees very closely with the bending moment coefficient computed by him by the rigid frame method.

Type 4. The cracks in the T-beams continuous over five spans indicate clearly that the beams acted as continuous. From the comparison of the moment of resistance of the beam at the maximum load, with the theoretical bending moment obtained from ordinary continu-

ous beam formulas, we find a very close agreement, which proves that even for five spans a continuous beam acts as continuous.

TESTS OF SLABS WITH CONCENTRATED LOADS

Tests to determine what width of slab, supported at the two ends, may be considered as carrying a concentrated load, were made by Prof. C. T. Morris* for the Ohio State Highway Department, from which he draws the following conclusions:

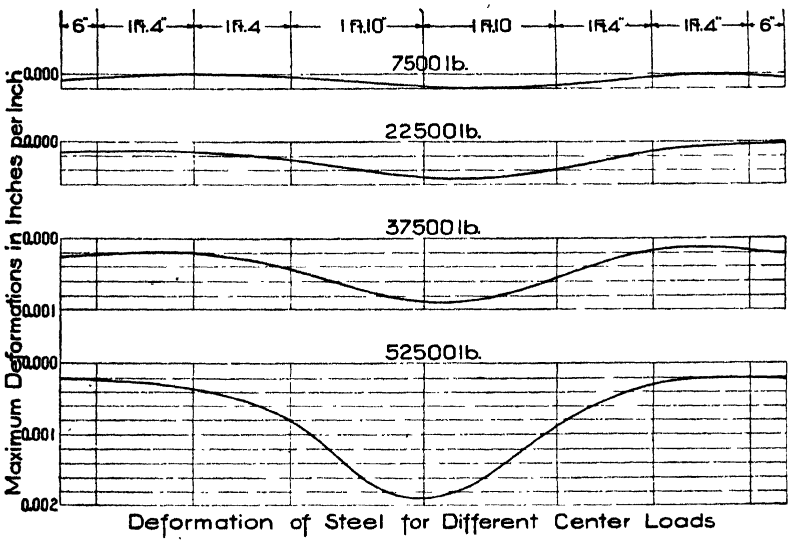


FIG. 134.—Deformation of Steel in Slab Along Section Parallel to Supports. (*See p.* 447.)

(1) The effective width is affected very little by the percentage of transverse reinforcement (parallel to support).

(2) The effective width decreases in a small degree as the load increases.

(3) The effective width in percentage of the span decreases as the span increases.

(4) The following formula, in which e is the effective width in feet and S is the span in feet, gives a safe value of effective width where the total width of the slab is greater than $1.35\,S \times 4$ ft.

$$e = 0.6\,S + 1.7 \text{ ft.}$$

* State of Ohio, Highway Department, Bulletin No. 28, September, 1915.

The effective width of a slab, *e*, in this formula, is that over which a single concentrated load may be considered as uniformly distributed on a line parallel to the supports.

(5) Thickness of slab shows small effect on the distribution of load.

Test slabs were 4 inches and 7 inches thick, with widths for 3½-foot spans of 1 foot, 3½ feet, and 7 feet; for 5-foot spans of 1 foot, 5 feet, and 10 feet; and for 7-foot spans of 1 foot and 7 feet. Main reinforcement for slabs consisted of 1.04% of steel, while transverse reinforce-

FIG. 135.—Appearance of Slab After Failure. (*See p.* 447.)

ment varied from 0.20% to 0.78% of steel. The tested slabs were supported on steel I-beams. Fig. 134, page 446, shows the deformation of steel across a section taken in the center of the slab parallel to the supports and Fig. 135, page 447, shows a 3½-foot slab, 7 feet wide, after failure.

TESTS OF SLABS TO DETERMINE DISTRIBUTION OF LOAD TO JOISTS

The tests show that if a continuous concrete slab is supported by several parallel joists of any material, and a concentrated load is placed

directly above one joist, the load is distributed by the rigidity of the slab to several joists (see Fig. 136.) The distribution depends upon the ratio of the thickness of the slab to the span.

The laboratory test. in question, consisted of slabs, 6, 7, and 8 inches thick, supported on three lines of 10-inch 25-pound I-beams as joists spaced 3 feet 6 inches on centers. (See Fig. 137, p. 449.) The span of the joists was 12 feet, and they were supported on other I-beams, which in turn rested on concrete pedestals, similarly as in bridge construction. The load was placed right over the middle joist in its center.

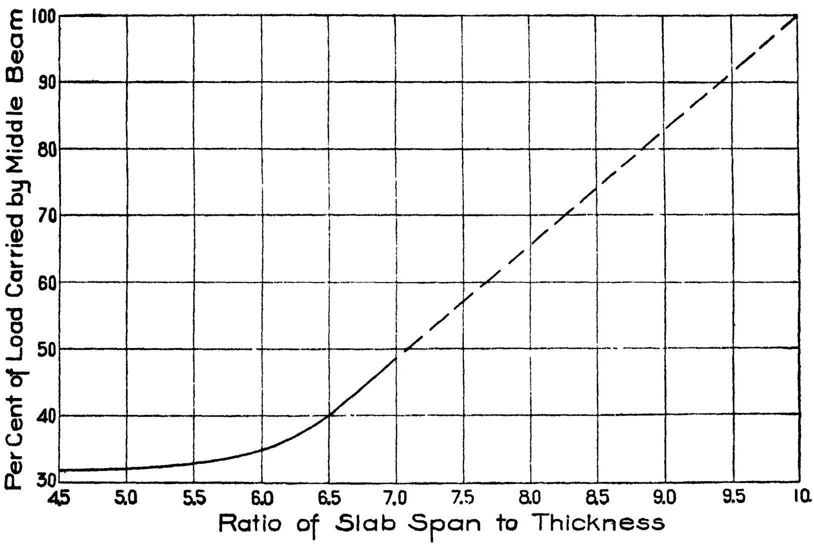


FIG. 136.—Distribution of Slab Load to Three Parallel Supporting Joists. (*See p.* 448.)

The following conclusions may be drawn from the above tests.

(1) The percentage of reinforcement in the slab has little or no effect upon the load distribution to the joists, so long as safe loads on the slab are not exceeded.

(2) If the span is ten times the thickness of the slab, or more, total load must be considered as carried by the joist under the load. The amount of load distributed by the slab to other joists than the one immediately under the load, increases with the thickness of the slab.

(3) The outside joists should be designed for the same total live load as the intermediate joists.

(4) The axle load of a truck may be considered as distributed uniformly over a 12-foot width of roadway.

Fig. 138, page 450, shows the elongation in extreme fiber of the steel beam and deflection for the middle beam and for outside beams.

Similar results would be obtained with concrete joists. The per-

FIG. 137.—Relation of Slabs and Joists in Tests. (*See p.* 448.)

centages of the load carried by the different joists are given in table, page 450.

The conclusions apply only to cases in which ratio of span to thickness of slab does not exceed 10. The largest ratio used was 7, but the results may be exterpolated.

Equivalent Uniform Loads on Joists for a Total Concentrated Load of 20 000 Lbs.

Computed from Elongations of Lower Fiber

By Prof. C. T. Morris.

Number of Slab	Thickness.	Reinforcing.	Equivalent Uniform Loads.				Percentage of Load Carried by		
			Side Beam.	Middle Beam.	Side Beam.	Sum.	Side Beam.	Middle Beam.	Side Beam
A1	6 in.	¾%	4 480	9 990	4 800	19 270	22.4	50 0	24.0
A2	6 in.	¾%	4 360	10 350	4 490	19 200	21.8	51.7	22.5
B1	7 in.	¾%	4 430	6 220	4 540	15 190	22.2	31.1	22.7
C1	8 in.	¾%	4 480	4 970	4 440	13 890	22.4	24.9	22.2
D1	6 in.	1%	4 480	9 030	4 890	18 400	22.4	45 2	24.5
D2	6 in.	1%	3 960	10 350	4 020	18 330	19.8	51.8	20.1
E1	7 in.	1%	4 790	7 050	4 540	16 380	24.0	35.3	22.7
F1	8 in.	1%	4 530	7 290	4 510	16 400	22.7	36.4	22.6
F1	8 in.	1%	5 000	6 520	4 020	15 540	25 0	32.6	20.1

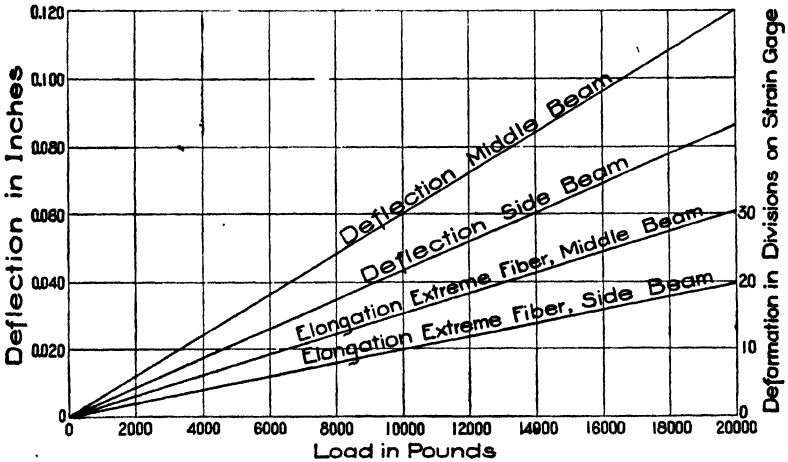


Fig. 138.—Elongation in Extreme Fiber of the Steel Beam and Deflection of the Middle Beam and of Outside Beams. (*See p.* 449.)
Value of One Division on the Strain Gauge is 0.00019 inch.

TESTS OF PLAIN CONCRETE COLUMNS

Professor Talbot* made a very comprehensive series of tests, the results of which are given in the table on page 451. Conclusions, confirmed also by experimenters abroad, are as follows:

(1) **Manner of Failure.** Plain columns fail either by shearing at a

* University of Illinois Bulletin No. 20, 1908.

Tests of Plain Concrete Columns (*See p.* 450.)

Materials· Portland cement, Wabash River sand, crushed limestone. Columns, 12 in. diameter, round, 10 ft. long

BY ARTHUR N. TALBOT.

Number of Specimens Tested.	Proportions of Concrete.	Average Age. days	Average Ultimate Unit Strength. lb. per sq. in.	Maximum Ultimate Unit Strength. lb. per sq. in	Variation in Per Cent from Average. %	Minimum Ultimate Unit Strength. lb. per sq. in.	Variation in Per Cent from Average %
2	1 : 1½ : 3	64	2 300	2 480	8	2 120	− 8
7	1 : 2 : 4	65	1 740	2 210	27	1 165	−33
2	1 : 3 : 6	61½	1 033	1 110	7	955	− 7
2	1 : 4 : 8	63	575	575	....	575	....
6	1 : 2 : 4	192	2 025	2 680	32	1 770	−13
2	1 : 2 : 3¾	14 mo.	2 710	2 770	2	2 650	− 2

diagonal plane of fracture, or by crushing, when the material is shattered and cracked longitudinally. The diagonal shearing failure almost always occurs suddenly and with little, or no warning, while the compressive failure is more gradual.

(2) **Effect of Richness of Concrete.** The strength of columns increases in nearly the same proportion, i.e., almost as a straight line, with the increase of the proportion of cement to total dry material used. (See pp. 312 and 316.)

(3) **Modulus of Elasticity and Poisson's Ratio.** The modulus of elasticity of the columns is almost constant for the first one-third of the strength of the concrete. Beyond this point the modulus decreases till it reaches at the ultimate load about one-half of its initial value. The Poisson's ratio, or the ratio of the lateral to the longitudinal deformation (see p. 339) was found for 1 : 2 : 4 concrete to be between 0.10 and 0.17 up to a load of about one-half the ultimate. It increases with the load, reaching probably 0.25 at the ultimate load.

(4) **Effect of Repetition.** Repetition has no effect on deformation for loads up to one-half of the breaking strength of the column. For higher loads, the deformation increases after repeated applications of the load. After ten repetitions of a load three-fourths the normal breaking strength, for example, the deformation was increased by 25%.

It must be noted, moreover, that the suddenness of failure of plain concrete is increased by the length of the column. This absolutely excludes plain concrete columns from structures where they are apt to

be exposed to shock or to secondary stresses due to bending, as in building construction.

Concrete vs. Brick Columns. Tests carried out by the U. S. Bureau of Standards on columns of common, hard, and vitrified brick laid with lime and cement mortar, indicate that the strength varies with quality of brick and mortar, while large and small columns show about the same unit stresses.

A series showing the strength of piers of common, hard and vitrified brick, laid with different mortars, is given in the following table. The lime mortar specimens showed a nearly entire lack of carbonation on the interior. Three piers of each kind of brick and mortar were made with headers every other course, every fourth course and every seventh course, but this variable appeared to have no effect. Bricks were laid flat.

Two large size columns 48 inches square and 12 feet high, of common,

*Compressive Strength of Brick Piers**

Tests by the U. S. Bureau of Standards. (*See p.* 452.)

Dimensions 30 inches square by 10 feet high

Kind of Brick.	Mortar.	Age. Months.	Compressive Strength. lb. per sq. in.
Common	1:3 lime	4	170
	1:3 cement	1	575
Hard	1:6 lime	4	910
	1:3 (15% lime, 85% cement)	1	1465
	1:3 cement	1	1650
Vitrified	1:6 lime	4	1360
	1:3 (15% lime, 85% cement)	1	2900
	1:3 cement	1	2780

* *Engineering News*, August 5, 1915, p. 242.

hard burned brick, one laid in 1:1 cement mortar and one in 1:3 lime mortar, were tested by the Bureau and crushed at 2 920 and 760 pounds per square inch respectively.†

Tests made at the Watertown Arsenal and quoted by the Committee of the American Society of Civil Engineers on the Compressive Strength of Cement‡ give the ultimate strength of common brick piers about

† James E. Howard, *Engineering Record*, March 22, 1913, p. 332.

‡ Transactions American Society of Civil Engineers, Vol. XV, p. 717. and Vol. XVIII, p. 264.

eighteen months old as ranging from 800 to 2 400 pounds per square inch, the results for brick laid with lime mortar averaging nearer the lower figure, and those for 1 : 2 Portland cement mortar nearer the higher figure.

The unit stresses allowed by the New York Borough of Manhattan Building Code, 1916, for brickwork are,

Brickwork in·	lbs. per sq. in.
Portland cement mortar	250
Natural cement mortar	210
Lime cement mortar	160
Lime mortar	110

The first value is but little more than one-half that recommended for good 1 : 2 : 4 Portland cement concrete on page 573.

TESTS OF COLUMNS REINFORCED WITH VERTICAL STEEL

Tests prove positively that in reinforced concrete columns, steel and concrete are effective in resisting the load carried by the columns. As explained in the Theory Chapter, page 376, the stress in steel equals the stress of concrete multiplied by the ratio of moduli of elasticity. This fact also is borne out by the tests.

Mr. Spitzer* in Austria and Professor Withey† at the University of Wisconsin observed that near ultimate load, adjustment between steel and concrete takes place so that finally the failure occurs by both of the materials passing the elastic limit simultaneously. This adjustment may be explained by the following consideration.

After either of the two materials reaches its elastic limit, any increase in stress in that material tends to cause very large deformations, which the other material, being still within elastic limit, cannot undergo. Therefore, this other material takes all the stresses due to any increase of the load till it finally reaches its elastic limit, and the column fails.

The table on page 455 gives results of tests of full sized columns made at the Watertown Arsenal.

Manner of Failure. Contrary to expectations, in most cases failure in columns occurs near the top or bottom instead of at the center. This has been explained as probably due to greater porosity of concrete.

In most columns, hair cracks appeared at 85% to 90% of the maxi-

* Mitteilungen Uber-Versuche ausgefuhrt vom Eisenbeton-Ausschuss des österreichischen Ingenieur- und Architekten-Vereins, "Versuche mit Eisenbetonsaulen," Heft 3.

† University of Wisconsin Bulletin No. 466, December 1911.

mum load. In some cases, however, the failures were sudden. In a number of cases, concrete split at the column reinforcement after its elastic limit was reached. The splitting effect is caused by the lateral deformation of steel which exerts pressure on the concrete, which, if sufficient, breaks the concrete. The steel, therefore, should be placed at a sufficient distance, say at least one inch, from the face of the concrete. With proper protection there is no danger of buckling till after the elastic limit of the steel is reached.

From Spitzer's tests, it would appear that columns with steel placed well within the cross-section of column are somewhat stronger than with steel placed according to the usual custom. In practice, however, columns are apt to be subjected to eccentric loading; therefore, the placing of steel in abnormal positions must be discouraged.

Factor of Safety. A column with vertical steel only is liable to fail without notice when its ultimate strength is reached. The ultimate strength of columns even if built under the same conditions is more variable than steel columns. Therefore the commonly accepted factor of safety is larger than used in steel columns. When designed according to formulas given on page 562 with allowable unit stresses on page 573, columns with vertical steel only are very reliable.

Modulus of Elasticity for Reinforced Columns. The modulus of elasticity varies for different intensities of loading and for different mixes of concrete. In selecting the ratio of moduli of elasticity to be used in design, it is proper to be guided more by the required factor of safety than by the actual modulus of elasticity at any particular stage of the loading. From the tests thus far made, the moduli of elasticity given by the Joint Committee with the suggested working stresses (see p. 573) seem to give the required factor of safety.

Rich Versus Lean Mix. As evident from the table on page 455, cement is very good reinforcement for the column as the increase in strength is much larger than the additional cost of cement.

Influence of Bands. In tests of Mr. Spitzer* of columns having different spacing of bands, those in which the spacing was equal to, or smaller than the diameter of the column gave somewhat greater strength than columns in which the spacing exceeded the diameter of column.

Influence of the Percentage of the Steel. The tests show clearly that the effect of reinforcement in columns is the same whether the percentage is large or small. In all cases the steel takes a stress equal to the stress in concrete times the ratio of moduli of elasticity.

* See footnote on page 453.

Watertown Arsenal Tests. The table on page 455, from tests by Mr. James E. Howard at the Watertown Arsenal, gives the relation of actual tests to theoretical computations based on a ratio of elasticity of 15. It is noticeable that the actual strength is almost always more than the theoretical, and this is especially the case with the leaner mixtures because the modulus of elasticity of the leaner concrete is lower, and therefore the ratio of 15 is very conservative.

An excellent analytical treatment of columns reinforced with vertical steel is given by Professor Talbot in one of his University Bulletins.* The problem is discussed briefly by one of the authors in a paper before the Boston Society of Civil Engineers.†

Many of the tests at the Watertown Arsenal, for example, were made with vertical bars imbedded in columns 12 ins. square and 8 ft. long, with absolutely no bands or horizontal steel of any kind placed around these vertical bars to hold them in place; that is, the bars 8 ft. in length were placed in the four corners of the column—in some tests only 2 ins. from the surface—and simply held in place by the 2 ins. of concrete itself.‡ There was no sign whatever of buckling until

Strength of Plain vs. Vertically Reinforced Concrete and Mortar Columns. Columns 12″ × 12″. Height 8 feet. Age of Mortar and Concrete 6 months. Watertown Arsenal. (See p. 453.)

Proportions: Cement.	Proportions: Sand.	Proportions: Stone.	Plain Concrete or Mortar Columns Actual Strength lb. per sq. in.	Reinforced Columns: Reinforcement. Description.	Reinforced Columns: Reinforcement. Ratio Area Steel to Area Column.	Reinforced Columns: Actual Strength lb. per sq. in.	Reinforced Columns: Computed Strength based on col. (4) and a ratio of $n = 15$ lb. p. sq. in.	Reference to "Tests of Metals" U. S. A.
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	2	0	3070	8-¾″ round bars	0.029	4200	4290	1905 p. 377
1	3	0	2380	8-¾″ round bars	0.029	3840	3320	1905 p. 377
1	4	0	1520	8-¾″ round bars	0.029	3380	2120	1905 p. 377
1	5	0	1080	8-¾″ round bars	0.029	2810	1510	1905 p. 377
1	5	0	1080	13-¾″ round bars	0.046	3900	1780	1905 p. 377
1	1	2§	1720	4-¾″ twisted bars	0.014	2890	2060	1904 p. 386
1	2	3§	1769	4-¾″ twisted bars	0.014	2010	2100	1904 p. 386
1	2	4	1413	4-0″ 0.74″ × 0.74″ trussed bars	0.014	1900	1689	1906 p. 538
1	2	4§	1710	4-¾″ twisted bars	0.014	1990	2050	1904 p. 386
1	2	4‖	2400	8-¾″ twisted bars	0.029	3700	3360	1907 p. 242
1	3	6‖	1450	8-⅝″ corr. bars	0.019	2290	1840	1904 p. 379 1906 p. 535

* University of Illinois, Bulletin No. 12, Feb. 1, 1907.

† Sanford E. Thompson in Journal Association Engineering Societies, June 1907, p. 316.

‡ Test of Metals, U. S. A., 1905, p. 344.

§ ½″ to 1½″ pebbles.

‖ Age 17 months 22 days.

the compression was so great that the elastic limit of the steel was passed, when of course nothing further could be expected of it.

TESTS OF SPIRAL COLUMNS

In analyzing results from tests of spiral columns, it is necessary to examine not only the strength, but also the deformation, or change in length. In building construction, because of the dependence of the different members upon each other, it is advisable to permit a shorten-

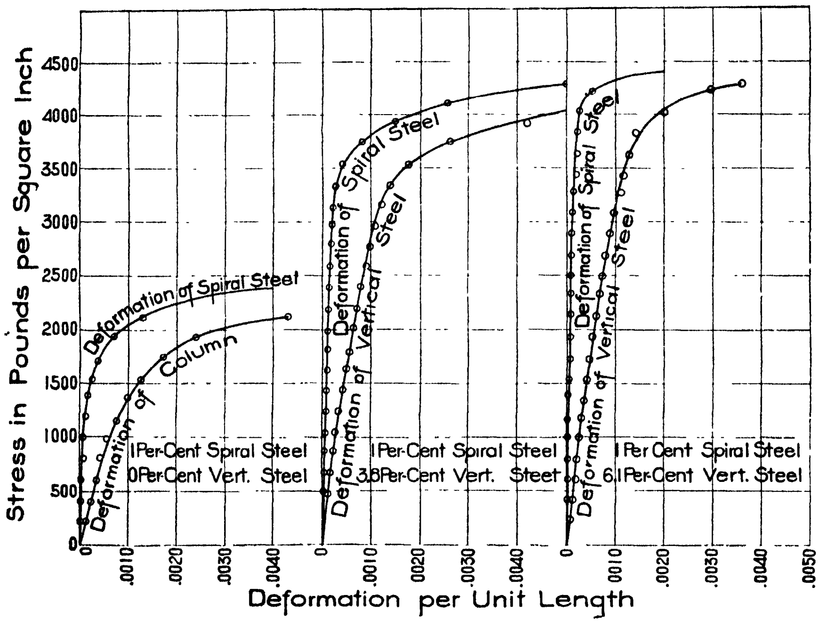


FIG. 139.—Deformation Curves for Spiral Columns with Varying Amount of Vertical Steel * (*See p.* 457.)

Concrete 1 : 2 : 3½; 1% Spiral High Carbon Steel; 0% Vertical Mild Steel.
Concrete 1 : 2 : 3½; 1% Spiral High Carbon Steel; 3.8% Vertical Mild Steel.
Concrete 1 : 2 : 3½; 1% Spiral High Carbon Steel; 6.1% Vertical Mild Steel.

ing, or deformation, of not over 0.007 per unit of length. The factor of safety, therefore, must be based on the load causing a certain deformation rather than on ultimate strength. Although the strength of column beyond the yield point is not available in ordinary construction, the greater ultimate strength, ductility, and uniformity in strength

* University of Wisconsin Bulletin No. 466, December, 1911, pp. 92, 93, 94.

of spiral columns reduces the danger of sudden failure, and larger unit stresses may be allowed. In practice it is more rational to increase working unit stress in concrete than to compute the stress by a formula which takes into account the steel in the spiral. (See p. 560.)

Early European tests of spiral columns were made on short columns and without deformation diagrams, and as a result, very high working stresses based on the ultimate strength were recommended. More recent tests, notably by Talbot and Withey in the United States, showed the excessive deformation of spiral columns at high stresses.

Column Tests by Prof. M. O. Withey. The tests at University of Wisconsin, Series of 1910, consisted of four series, two of which will be considered below; namely, series 1, columns with varying percentage of longitudinal and lateral reinforcement, and series 2, columns with varying proportions of concrete. The table on page 458 gives the general results of the tests.

The following general conclusions may be drawn from the tests:

1. The cheapest way of increasing the strength of a column is by using a rich mix; but toughness is sacrificed to strength obtained in this way.

2. Spiral reinforcement greatly increases toughness and ultimate strength of a column, but does not raise the yield point. (See columns M and O, page 458.) The strength beyond the yield point can not be utilized in building construction; hence, the amount of steel for spirals should be made only large enough to produce required ductility and raise the factor of safety against failure. In practice, 1% of spiral reinforcement seems to be sufficient.

3. Longitudinal steel increases the stiffness of the column and raises the yield point.

4. Stress in steel at the yield point of columns is practically the same for all mixes of concrete and only a little below the yield point of the vertical steel. (See table, page 458.) This phenomenon may be explained by the fact that for leaner concrete the ultimate strength is smaller, but the deformation at the yield point larger than for rich mixes. For rich concrete, the stress in concrete at the yield point is larger, but the ratio of the moduli of steel to concrete decreases. Since the stress depends upon the product of stress in concrete times the ratio of the moduli of elasticity, the two values simply adjust themselves so that the product is the same in all cases.

5. Columns loaded eccentrically give results which agree closely with the formula given on page 382, as is evident from Fig. 140, page 459, in which the straight lines represent figured stresses in steel and con-

TESTS TO DETERMINE THE EFFECTS OF VARYING THE PERCENTAGE OF VERTICAL AND SPIRAL REINFORCEMENT IN REINFORCED CONCRETE COLUMNS. (See p. 457.)

By Morton O. Withey.*

Average values given. Columns C-1, 2, 3, 4, D-1, 2, 3, 4, 120 inches long; all others 102 inches.

Column Number.	Mix.	Age. days	Ratio Length of Column to Diameter $\frac{l}{d}$	Area of Core. A sq. in.	Reinforcement. Vertical Bars, Round.	Reinforcement. Spiral. Wire.	Reinforcement. Spiral. Pitch.	Per cent Reinforcement. Vertical.	Per cent Reinforcement. Spiral.	Maximum Load. P lb.	Ultimate Strength. $\frac{P}{A}$ lb. per sq. in.	Load at Yield Point. P_1 lb.	Ratio of Stress at Yield Point to Ultimate Strength. $\frac{P_1}{P}$	Stresses at Yield Point. In Core. $\frac{P_1}{A}$ lb. per sq. in.	Stresses at Yield Point. In Steel. Vertical. lb. per sq. in.	Stresses at Yield Point. In Steel. Spiral. lb. per sq. in.	Stresses at Yield Point. In Concrete. f_c lb. per sq. in.	Compressive Strength of Cylinders. C lb. per sq. in.	Ratio Strength of Cylinders to Strength in Concrete at Yield Point. $\frac{C}{f_c}$	Elastic Properties. Modulus of Elasticity at ¼ Ultimate Strength. lb. per sq. in.	Elastic Properties. $\frac{E_s}{E} = n$ at ¼ Ultimate Strength.	Elastic Properties. $\frac{E_s}{E} = n$ at Yield Point.	Elastic Properties. Poisson's Ratio at ¼ Ultimate Strength.
Series I.																							
W-1-3	1: 2 : 4	52	8.2	86.6	0	0		0	0	225 500	2 600									3 500 000	8.6		0.114
H-1, 2	1: 2 : 3½	57	8.5	78 5		No. 7	2″	0	0 50	175 500	2 235	145 500	0 83	1 855		10 050	1 855	1 750	0.94	2 350 000	13 0	27 0	0.127
G-1, 2	1: 2 : 3½	53½	8.5	78 5	8 ½″	No. 7	2″	2.00	0 50	259 500	3 300	213 000	0 83	2 710	40 500	12 000	1 940	1 760	0 91	3 350 000	11.0	21.0	0.110
I-1, 2	1: 2 : 3½	57	8.5	78 5	8 $\frac{11}{16}$″	No. 7	2″	3.78	0.50	327 000	4 160	273 000	0 84	3 470	42 000	6 450	1 960	2 180	0 13	3 400 000	13.0	22.0	0.110
J-1, 2	1: 2 : 3½	58	8.5	78 5	8 ⅞″	No 7	2″	6.11	0 50	402 750	5 120	333 000	0 83	4 240	40 650	6 150	1 860	2 055	1.11	4 350 000	11.5	22.0	0.088
L-1, 2	1: 2 : 3½	57½	8.5	78.5	0	No. 7	1″	0	1 00	207 450	2 640	108 000	0 52	1 370		4 800	1 370	1 770	1 29	2 100 000	14.5	19.5	0.146
K-1, 2	1: 2 : 3½	57	8.5	78.5	8 ½″	No. 7	1″	2 00	1.00	306 750	3 905	205 500	0 67	2 615	39 450	7 050	1 865	1 995	1 07	2 800 030	13.5	21.5	0.123
N-1, 2	1: 2 : 3½	57½	8.5	78.5	8 $\frac{11}{16}$″	No. 7	1″	3 78	1.00	329 250	4 195	265 500	0 81	3 375	38 700	6 900	1 985	1 800	0.90	3 800 000	11.0	20.0	0.130
M-1, 2	1: 2 : 3½	57½	8.5	78.5	8 ⅞″	No 7	1″	6 11	1 00	368 250	4 685	295 500	0 80	3 760	37 650	6 450	1 565	1 685	1.09	3 425 000	18.5	24.0	0.101
P-1, 2	1: 2 : 4	58½	8.5	78.5	8 1″	No. 7	1″	8.00	1 00	543 900	6 925	445 500	0 82	5 660	39 150	8 100	2 750	2 365	0.86	5 300 000	9.6	14.5	0.121
O-1, 2	1: 2 : 4	57	8.5	78.5	8 ⅞″	¼″	1″	6 11	1 96	516 750	6 580	348 000	0 68	4 430	36 900	8 400	2 310	2 480	1.08	4 650 000	9.9	16.0	0.131
R-1, 2	1: 2 : 4	53	8.5	78.5	8 1″	¼″	1″	8 00	1 96	547 500	6 965	408 000	0 74	5 190	37 500	8 250	2 385	2 390	1.005	5 250 000	9 7	16.0	0.128
Q-1, 2	1: 2 : 4	53	8.5	78.5	8 1⅛″	¼″	1″	10.12	1 96	556 500	7 090	453 000	0.82	5 765	37 200	6 750	2 230	2 305	1.03	5 500 000	10.8	17.5	0.094
C-1-4	1: 2 : 4	63¼	10 0	78.5		¼″	1″	0	2 00	316 750	4 030	180 500	0.58	2 333			2 333	2 200	0 95				
D-1-4	1: 2 : 4	60¼	10.0	78.5	9 ⅝″	¼″	1″	3 50	2 00	373 250	4 750	280 500	0 75	3 575	40 500		2 223	2 252	1.02				
Series II.																							
Z-1, 2	1: 3 : 6	48½	8.3	82.6	8 ⅞″	No. 7	1″	5.83	1 00	443 500	5 370	264 500	0.60	3 200	36 900	8 250	1 115	1 770	1.59	3 950 000	12.80		0.085
X-1, 2	1: 1¾: 3½	56½	8.3	83.6	0	No. 7	1″	0	1 00			338 000		4 100		8 400	4 100	4 760	1.16	5 050 000	5.95		0.175
Y-1, 2	1: 1¾: 3½	61	8.3	83.6	8 ⅞″	No. 7	1″	5.75	1 00			470 000		5 700	30 600	6 300	4 160	4 120	0 99	6 700 000	5.70		0.190
S-1, 2	1: 1 : 2	57	8.5	78 5	0	No. 7	1″	0	1 00	460 500	5 855	318 000	0.69	4 050		7 350	4 050	4 070	1 01	4 125 000	7.30		0.135
T-1, 2	1: 1 : 2	56½	8.5	78.5	8 ⅞″	No 7	1″	6 11	1 00	513 000	7 290	453 000	0 79	5 760	37 050	7 350	3 725	4 400	1.18	5 175 000	8.45		0.140
V-1, 2	1: 1.29	56½	8.5	78.5	0	No. 7	1″	0	1.00	420 000	5 340	280 500	0 67	3 570		6 750	3 570	4 880	1.37	3 400 000	8.80		0.135
U-1, 2	1: 1.29	56½	8 5	78.5	8 ⅞″	No. 7	1″	6.11	1 00	640 500	8 150	468 000	0.73	5 950	36 450	6 750	3 970	4 555	1.15	5 250 000	8.40		0.125
AA-1, 2	1: 1.33	57	8.3	8[illegible].6	8 ½″	No. 7	1″	1.86	0 99	617 500	7 300†	489 000		5 780	34 750	7 050	5 185	6 787	1.30	5 625 000	5.9c		0.190
AB-1, 2	1: 1.33	59½	8.3	83.6	8 $\frac{11}{16}$″	No. 7	1″	3.55	1 00	617 500	7 390†	514 000		6 150	35 250	7 700	5 090	6 387	1.25	5 950 000	5.95		0.185
AC-1, 2	1: 1.33	53½	8.3	84.6	8 ⅞″	No. 7	1″	5.69	0.99	617 500	7 480†	542 000		6 590	36 000	6 600	4 760	6 075	1.28	5 150 000	6.40		0 180
AD-1, 2	1: 1.33	195	8.2	85 6	8 ⅞″	No. 7	1″	5.62	0 98											5 450 000	7.55		0.135

* University of Wisconsin Bulletin No. 466, Engineering Series, Vol. 7, No. 1.

† Column did not fail.

crete by Formulas (75) and (76), page 382, while the dots and circles show the actual stresses obtained from deformation.

Action of Columns under Test. Up to the point of breaking strength of plain concrete, the action of the columns with spirals was the same as for columns with vertical steel only. The observed stress in spirals was from 6 000 to 8 000 lb. per sq. in. For spiral columns with vertical steel, the deformation curve continues as a practically straight line to the yield point of the column. The yielding is indicated by scaling

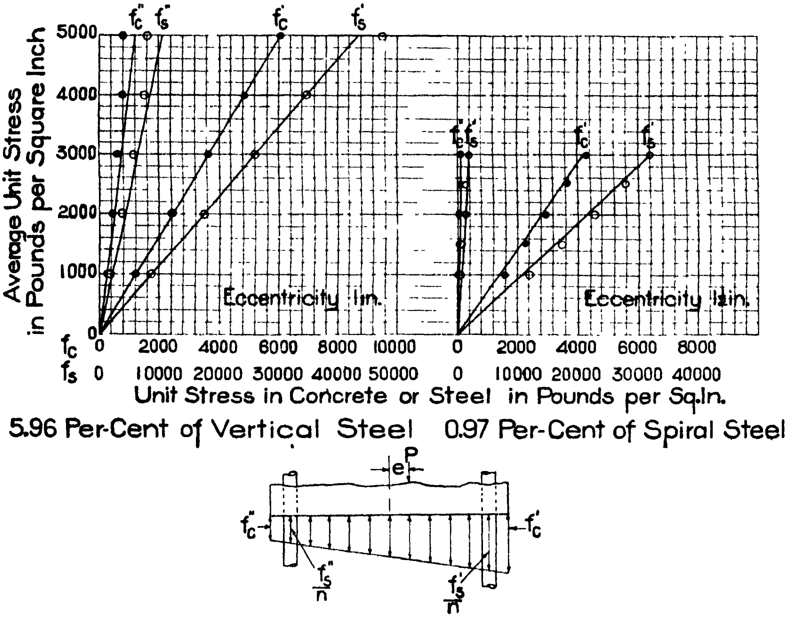


FIG. 140.—Comparison of Theoretical and Actual Stresses for Eccentric Loading.*
(*See p.* 457.)

off of the protective shell and by an increase of ratio of lateral and longitudinal deformation to the applied load. The yield point is more marked for columns with large percentage of reinforcement (see Fig. 139, p. 456). For spiral columns without vertical steel, the deformation diagram is a curve without a marked yield point, so that the yield point is only distinguishable by scaling of the shell.

After the yield point has been passed, the disintegration of the shell progresses very rapidly. The ratio of shortening due to the applied

* University of Wisconsin Bulletin No. 466, December, 1911, p. 71.

load becomes larger and final failure takes place by buckling of the column, or, in columns with a small amount of lateral steel, by breaking of the spirals.

Stresses in Steel and Concrete. The table on page 458 gives the stresses in steel and concrete at yield point and at maximum load. The stresses in vertical steel were obtained from the deformation by using a modulus of elasticity of 30 000 000. The remainder of the load assumed as carried by the concrete, and divided by the area of the core, gave the unit stress in the concrete. In figuring the stress in concrete, the area of the core was used in preference to the total area of the column, because at yield point and at maximum load, either a part or the whole of the outside shell is destroyed and is then ineffective for carrying the load.

The stress in spirals obtained from lateral deformation is very small at the yield point of the column, which corroborates the statement that up to yield point* the spirals do not affect the column appreciably.

The table is of interest in giving a comparison of the stresses in steel with the stresses in concrete for different mixes and also in giving the values of the ratio of moduli, n. Although the value of this ratio was variable, the stress in steel at the yield point of the column was about the same for all columns, which seems to show that in a column under load an adjustment of stresses takes place. From the table, also, it is evident that the stress in concrete at the yield point was the same irrespective of the amount of vertical reinforcement.

An interesting experiment was tried in connection with tests by Wayss and Freytag.† The columns after reaching the maximum stress were again loaded after nine months and after one year, and showed a large increase of strength over the original maximum strength, in some cases reaching 50% increase. After these two loadings, the column spirals were removed and the core tested again. In no case was the core after removing the spirals disintegrated; in fact, each core showed a considerable strength, which tends to disprove a contention previously held by several authorities that the concrete in hooped columns becomes disintegrated after a certain point in loading is reached and is simply prevented from flowing by the hoops.

TESTS OF SQUARE COLUMN WITH RECTANGULAR BANDS

Tests by Wayss and Freytag† consist of eleven types of 12-inch columns, square and rectangular in cross-section. These tests prove

* See also Professor Talbot's Bulletin No. 20, University of Illinois.

† Mörsch, 4th edition, p. 117.

that the bands in square columns are not very effective. The outside shell started breaking off as soon as concrete reached its maximum crushing strength.

TESTS OF COLUMNS WITH STRUCTURAL STEEL REINFORCEMENT

The size of column can be reduced by the use of structural shapes, rigid enough to serve as a structural steel column, imbedded in concrete. Below are given results from tests of columns with two types of structural steel which proved very reliable. The results must not be considered as applying to all conditions and must be used with caution where the structural members differ materially from those in the tests.

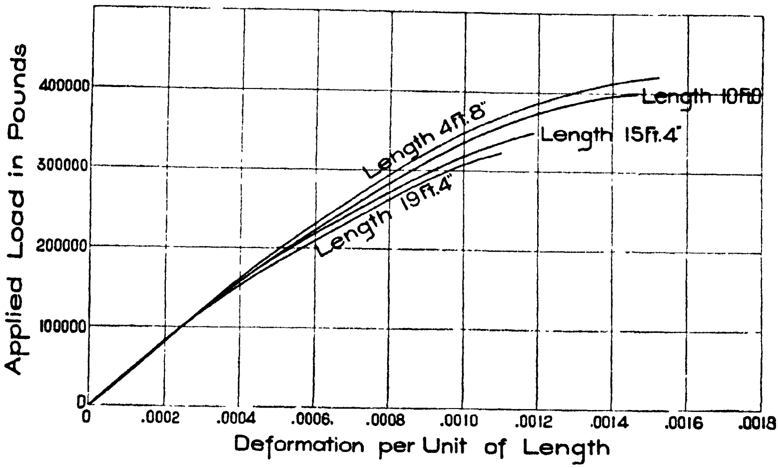


FIG. 141.—Average Deformations of Plain Steel Columns.* (*See p.* 463.)

Talbot-Lord Tests of Columns.† The Talbot-Lord tests consisted of thirty-two columns divided into four groups:

1. Plain steel columns.

2. Core type columns, i.e.; columns in which the portion within the structural steel members was filled with concrete.

3. Fireproofed columns, i.e.; core type columns having a 2-inch protective covering.

* University of Illinois Bulletin No. 56, 1912, p. 19.
† University of Illinois Bulletin No. 56.

Tests of Steel Columns, Reinforced with Concrete

BY TALBOT AND LORD* (See Fig. 143, page 464.)

Age of concrete, 59 to 61 days.

P. S. = Plain steel column of 8–3″ × $2\frac{1}{2}$″ × $\frac{5}{16}$″ angles with no concrete core.

C. T. = Core type column of 8 – 3″ angles with concrete core.

F. = Fireproofed column same as core type with 2″ of extra concrete outside of structural steel. No spiral.

S = Spiraled column same as core type with spiral steel and concrete core filled out to conform with diameter of spiral.

Column No.	Type of Column.	Area of Cross-Section. Sq. in	Mix of Concrete.	Length of Column. ft in.	Ratio of Length of Column to Least Radius of Gyration. l/r	Total Load in lb.			Stresses in lb. per sq. in.	
						Column Load.	Load Considered Carried by Steel.	Load Considered Carried by Concrete.	In Steel.	In Concrete.
8 902	P. S.	13	0	2–0	6.1	487 300	......	....	37 500	.....
8 905–6	P. S.	13	0	4–8	14.4	444 600	....	....	34 200	.....
8 907–8	C. T.	120	1 : 2 : 4	4–8	.	589 500	444 600	144 900	34 200	1 355
8 910-11-14	P. S.	13	0	10 0	30.8	420 000	....	.. ....	32 400	.....
8 912–13	C. T.	120	1 : 2 : 4	10–0	...	547 350	418 000	129 350	32 150	1 210
8 915–16	P. S.	13	0	15–4	47.2	372 000	.... ..	....	28 600	.....
8 917–18	C. T.	120	1 : 2 : 4	15–4		500 350	372 000	128 350	28 600	1 200
8 920–21	P. S.	13	0	19–4	59 5	359 600	. . .	. .... .	27 650	.....
8 922–23	C. T.	120	1 : 2 : 4	19–4	. .	493 450	359 600	133 850	27 650	1 250
8 925–26	C. T.	120	1 : 1 : 2	10–0	....	645 500	418 000	227 500	32 150	2 125
8 927–28	C. T.	120	1 : 3 : 6	10–0	..	523 250	418 000	105 250	32 150	985
8 930–31	F.	213	1 : 2 : 4	10–0	.	633 200	418 000	215 200	32 150	1 075
8 933	S†	153	1 : 2 : 4	10–0	...	600 000	Applied 5 times; not broken.			
8 934	S†	153	1 : 2 : 4	10–0	...	856 000	Not broken near ultimate.			
8 935	S†	153	1 : 2 : 4	10–0	....	600 000	Applied 3 times; not broken.			
.... .	. ...	..	...	....	....	830 000	Second test; near ultimate.			
8 936	S†	153	1 : 2 : 4	10–0	....	625 000	Not broken.			
8 937	S‡	153	1 : 2 : 4	10–0	...	830 000				
.. ..	....	.....	.....	....	....	714 000	Test with spiral and outside concrete removed.			

† 0.75 % of spiral reinforcement.
‡ 1.00 % of spiral reinforcement.

* University of Illinois Bulletin No. 56, 1912, pp. 14 and 15.

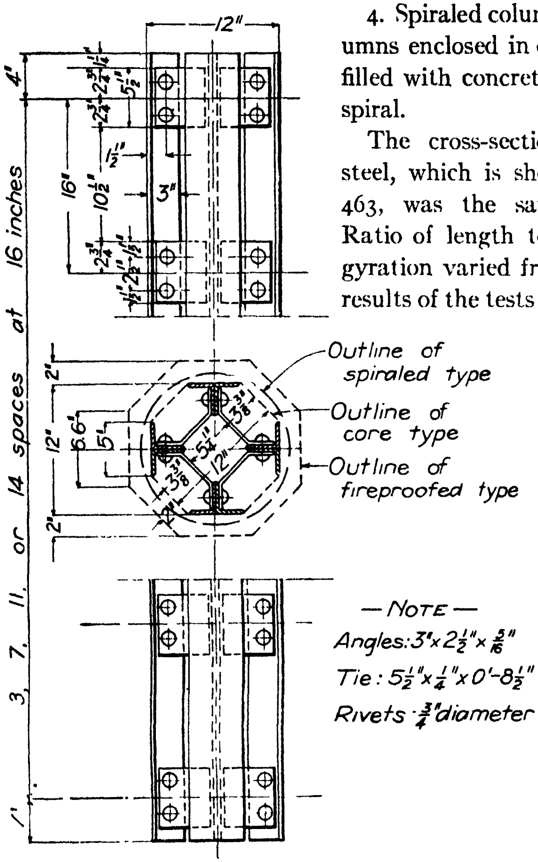


FIG. 142.—Dimensions of Test Columns.* (*See p.* 463.)

4. Spiraled columns, i.e.; core type columns enclosed in close fitting spiral and filled with concrete to outer surface of spiral.

The cross-section of the structural steel, which is shown in Fig. 142, page 463, was the same for all columns. Ratio of length to minimum radius of gyration varied from 6.1 to 59.5. The results of the tests are given on page 462.

Plain Steel Columns. No bending was visible to the eye at the maximum load. After the maximum load was passed, bending developed very gradually. The average deformations per unit of length are shown in Fig. 141, page 461.

The effect of the length of the column was more marked at high than at low loads. According to Professor Talbot, the ultimate stress in column for different ratios of $\frac{l}{r}$ may be represented by a straight line formula, $\frac{P}{A} = 36\,500 - 155\frac{l}{r}$.

Core Type Columns. The columns of core type were very tough and failure slow. For short columns, the failure was caused in most cases by crushing of the concrete; for longer columns, by bending and crushing of concrete. No bending visible to the eye was observed until maximum load was reached. The effect of mixture of concrete on the

*University of Illinois Bulletin No. 56, Fig. 1, p. 4.

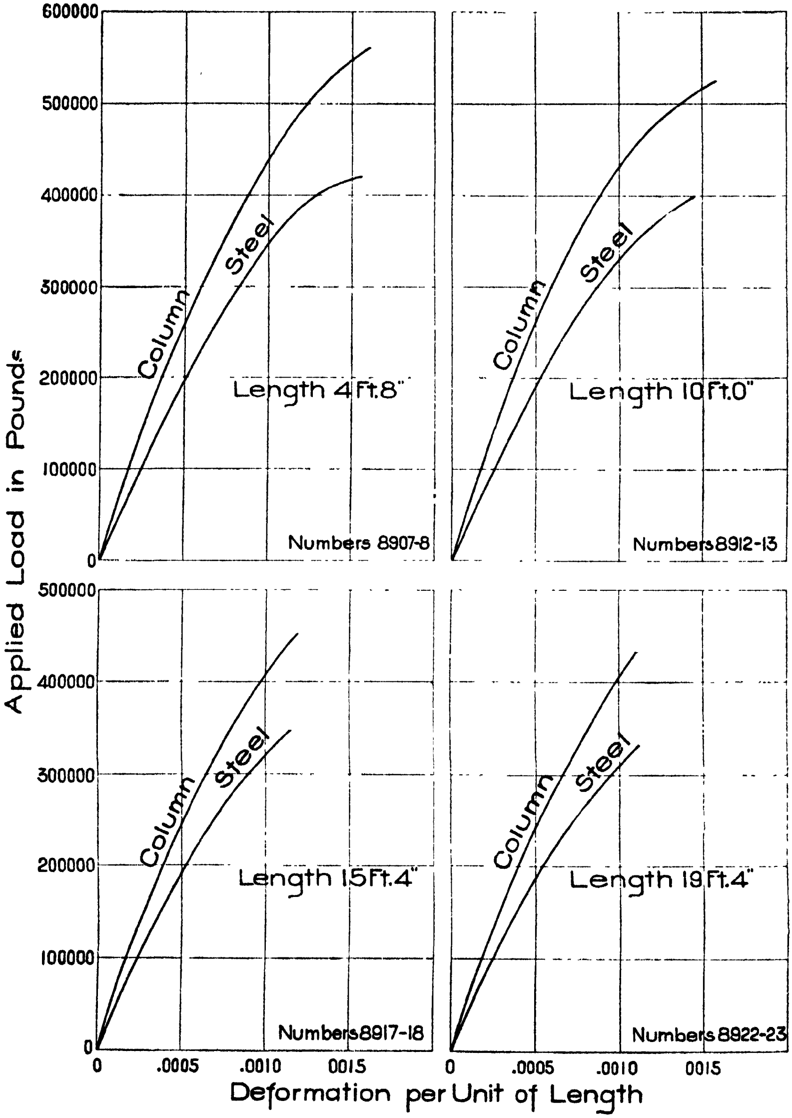


FIG. 143.—Load Deformation Diagrams for Plain Steel Columns and Corresponding Core Type Columns.* (*See p.* 465.)

NOTE:—Curves labelled *Column* refer to core type; those labelled *Steel* refer to plain steel type. See also Fig. 141.

*** University of Illinois Bulletin No. 56, March, 1912, p. 24.**

strength was small, because the strength of the column was governed by the steel rather than by the concrete.

In Fig. 143, page 464, are shown deformations per unit of length of core type column. Deformations of plain steel columns are also shown for comparison. (See also Fig. 141.)

The relative loads carried by the concrete and by the steel, respectively, are given in the table on page 462. The amounts were determined by assuming that for equal deformations the structural shapes in the core type column carried the same load as a similar plain steel column and the balance was carried by the concrete.

As a result of this test, the ultimate strength of the core type column may be considered as consisting of the strength of the plain steel column plus the strength of the concrete core figured with a unit stress equal to the strength of concrete in cylinders.

Fireproofed Columns. The behavior of fireproofed columns of the core type was about the same as that of the column without fireproofing. The concrete shell outside of the structural shapes, however, remained intact until the ultimate deformation of the column was nearly reached, but its effective unit strength was lower than the unit stress of the concrete core, probably because the shell failed before the maximum stress in steel and concrete core was reached.

Even if the protective covering is not relied upon as adding to the strength of the column, it is advisable to tie it by means of hoops or spirals of large pitch so as to prevent spalling in case of fire.

Spiraled Columns with Structural Steel. The core type column with surface spirally reinforced exhibited larger strength and toughness than similar columns without spiral. The thin protective cover remained intact. The strength of the columns exceeded the capacity of the Illinois and Lehigh University testing machines, which is 830 000 lb.

Because of the large deformation, the increase of strength afforded by the spiral is not available in ordinary building construction; hence a large percentage of spiral is not justifiable. One per cent of spiral reinforcement, however, makes the column tougher and safer and also prevents the outer shell from spalling. Since the danger of sudden failure is removed, such columns may be designed with somewhat higher working loads than allowed for the fireproofed type.

Tests by Professor Withey.* The structural steel reinforcement consisted of four angles 2 in. x 2 in. x $\frac{3}{16}$ in. placed in four corners of square column. The out to out dimensions of the steel core were 8 in. square. The result of this test agrees with the tests previously described.

* University of Wisconsin Bulletin No. 300, "Tests of Plain and Reinforced Concrete Columns."

Recommendations for the Design of Fireproofed Columns. Tests by Professor Talbot and Professor Withey show that in columns with structural steel the protective cover does not fail until the column reaches its maximum load. In practice, the protective cover is not considered as adding to the strength of the column. Therefore, in designing this type of column, the recommendations given in connection with core-type columns may be used. Similar results were obtained by Emperger and Spitzer.*

TESTS OF LONG COLUMNS

Tests by Spitzer on columns, 9.8 by 9.8 in. cross-section, 9.9 ft., 14.8 ft., and 23 ft. long respectively, with ratio of length to the least diameter of 12, 18, and 28, show no appreciable difference in strength and no buckling in any of the columns.

To determine the effect of slenderness, Professor Bach† compared the strength of 4 ft. long column (ratio of slenderness 4.3) with that of a 29.5 ft. column (ratio of slenderness 32), and found the strength of the longer column to be 0.75 of the strength of the short column.

As a result of this test, Professor Bach suggests the following formula for the strength of a long column in terms of the strength of a short column, which in turn may be assumed equivalent to the strength of 8 by 16-inch cylinders.

$$f_c' = f_c \frac{1}{1 + 0.0072 \dfrac{Al^2}{I}}$$

Where
f_c = allowable working unit stress, lb. per sq. in., for short columns.
A = cross section, sq. in., of column.
l = length of column in feet.
I = moment of inertia inch units of the cross-section of the column.

RESISTANCE OF CONCRETE AND REINFORCED CONCRETE TO TWISTING

Tests‡ made by C. Bach and O. Graf in Stuttgart, Germany, to determine the resistance of concrete and reinforced concrete to twisting,

* Mitteilungen über-Versuche ausgeführt vom Eisenbeton-Ausschuss des österreichischen Ingenieur- und Architekten-Vereins, "Versuche mit Eisenbetonsaulen," Heft 3.

† Knickungsversuche mit Eisenbetonsaulen Zeitschrift des Vereins Deutscher Ingenieure, Prof. C. Bach, 1913, p. 1969.

‡ "Versuche uber die Widerstandsfaehigkeit von Beton und Eisenbeton gegen Verdrehung" by C. Bach and O. Graf, Berlin 1912, Heft 16.

consisted of two groups of specimens: (1) plain concrete with square, rectangular, circular, and circular ring cross-sections; and (2) square and rectangular reinforced concrete with varied amounts and dispositions of reinforcement. The length of the specimens was 6.4 ft., and the cross section was 11.8 inches square, 8.3 by 16.6 inches rectangular, and 15.8 inches diameter for circular reinforcement.

Method of Testing. The specimens were tested by applying twisting moments at the ends. An initial twisting moment of 21 670 inch pounds was applied first and the instrument read. The load was then increased, until failure, in increments of 21 670 inch pounds; but after each reading, and before raising the total load by this increment, the load was reduced to the original 21 670 inch pounds so that the increments as made actually consisted of multiples of 21 670 inch pounds.

Resistance to Twisting of Plain Concrete. In all plain concrete specimens, the failure occurred in the center of the specimen by cracking at 45°. Failure always followed closely the appearance of the first crack. In specimens with rectangular cross-sections, the first crack started on the wide face.

The ultimate torsional unit stresses are given in the table below and were figured by the following general formulas for homogeneous beams.

Ultimate Torsional Unit Stresses. (*See p.* 467.)

Concrete, 1:2:3 by volume. Aggregates, Rhine sand from 0 to ¼-inch diameter and Rhine gravel from ¼ inch to ¾ inch. Average compressive strength of 12-inch cubes at 45 days, 3 540 lb. per sq. in. Age of specimens at test, 45 days.

Compiled from Tests by C. Bach and O. Graf.

Cross Section	Values of Torsional Unit Stress.		
	Lb. per sq. in.	In Terms of Tensile Strength	In Terms of Compressive Strength.
Square	432.33	1.62	0 12
Rectangular	462.20	1.75	0.13
Circular	364.05	1.38	0.10
Circular Rings	243.18	0.92	0.07

Let f_t = ultimate torsional unit stress in concrete in lb. per sq. in.

M_t = the torsional moment in inch pounds.

b = the short side of the section in inches.

h = the long side of the section in inches.

For rectangular and square sections,

$$f_t = \left(3 + \frac{2.6}{0.45 + \frac{h}{b}}\right)\frac{M_t}{b^2h} \tag{1}$$

For circular sections

$$f = \frac{16}{\pi}\left(\frac{M_t}{d^3}\right) \tag{2}$$

Resistance to Twisting of Reinforced Concrete Specimens. Longitudinal reinforcement has very small influence on torsional resistance. For specimens reinforced with 1.13% and 2.26% of straight bars, the increase of torsional resistance was only 9% and 14%. More marked was the influence of inclined bars. In specimens with 1.13% of reinforcement where the bars were inclined at 12°, the ultimate resistance was increased by 27%.

The best reinforcement for specimens subject to twisting consists of stirrups, or spirals, inclined at 45°, because the cracks due to twisting open at 45° and therefore the spiral resists the twisting stresses directly. In specimens with 2.26% of longitudinal reinforcement and with stirrups, 0.276 in. diameter, spaced 3.93 inches on centers, the bending moment at first crack was 31% larger than at ultimate failure for plain specimens. The ultimate bending moment for this specimen was 66% larger than the ultimate bending moment for plain specimens.

Specimens reinforced with 2.26% of longitudinal steel and spirals, 0.276 in. diameter, arranged parallel to each side and inclined at 45°, with a pitch of 3.7 inches, showed an increase of 55% at first crack, and 134% at the ultimate bending moment over plain specimens.

TESTS OF REINFORCED CONCRETE BUILDINGS UNDER LOAD

One of the most important developments in testing within the last few years is the testing of complete structures under load. Deflection tests of engineering structures have been customary to determine whether the structure can safely carry the load for which it has been built. Such tests, however, are of little scientific use as they do not give the stresses in the structure. Sometimes they are even misleading because with small deflection there may exist stresses in certain parts of the structure much higher than allowable. Weakness of details also and the effect of continuity cannot be determined from such tests.

The recent tests on completed structures are much superior to the

old deflection tests as they measure not only deflection of the structure, but also the stresses in various parts of the members. These tests were inaugurated by Prof. Arthur N. Talbot of the University of Illinois, with the assistance of Messrs. A. R. Lord and W. A. Slater. The first building tested in this way was the Deere and Webber building in Minneapolis in October and November, 1910. Following, these several other tests were made under the auspices of the Reinforced Concrete Committee of the American Concrete Institute.

The instruments used in such tests are (1) Extensometer for measuring the stretch or compression of the materials, (2) Deflectometer, for measuring deflection.

The extensometer consists of a framework (which in the best extensometers is made of invar steel to prevent appreciable changes in length due to the changes of temperature), two movable legs attached to it provided with sharp points, and of means for measuring accurately any changes in distance between the points. In order to find the stretch in steel, the bar to be tested is uncovered in two places a few inches apart, then small holes, called gauge holes (0.055 in. in diameter) are drilled. An observation on the gauge line is taken before the structure is loaded and then at each increment of the load. The difference between the original reading and the reading at any load gives the stretch of the steel due to that load. The stress is then found from the known relation between the deformation and the stress.

The compression in concrete is measured by making small holes in the concrete, inserting metal plugs, and then marking the gauge holes in these plugs in a similar manner as was done for the steel. Readings and stresses are obtained in the same way as for steel. Since concrete flows gradually under heavy loads the readings must be made immediately after each loading. (See p. 339.)

For measuring deflections, a rigid scaffold is built right under the members to be tested. In the place in which a deflection reading is desired, a steel plate is fastened by plaster of Paris to the under side of the beam, or slab. On a vertical line below this steel plate a steel rod is fastened to the scaffold. Before beginning the test and at different stages of loading, the deflectometer is placed between the plate and the rod below, readings are taken and the difference between the original reading and the readings under the load give the deflection

For loading the panels, there may be used: (*a*) brick, (*b*) cement in sacks, (*c*) loose sand in boxes or in sacks, and (*d*) pig iron. In making

tests, care always must be taken that the material does not arch itself. The whole floor cannot be covered with the load because there must be left places uncovered in which measurements are taken, also there must be aisles left to make the points accessible.

It is important that the test load should cover a sufficient floor space to insure that certain parts of the floor resist nearly the full load which, in the calculations, they are considered to take.

Wenalden Building Test.* The floor panels in this building are 15 feet by 20 feet. The slab is $3\frac{7}{8}$ inches thick; the girders, placed between columns in the short direction, $7\frac{1}{2}$ inches by $20\frac{7}{8}$ inches, reinforced with four $\frac{7}{8}$-inch square bars in the middle. The longitudinal beams, 5 feet apart on centers, are $6\frac{1}{4}$ inches by $18\frac{7}{8}$ inches, reinforced with four $\frac{3}{4}$-inch square bars in the middle. Half as much steel was used over the supports as in the center.

The floor was designed for a live load of 200 pounds per square foot, and the total test load was made 400 pounds per sq. ft. and placed in layers of 80 pounds per sq. ft. A set of observations was taken after every additional loading.

The measurements were taken in steel at the support and at the center of the span, also measurements of stresses in concrete at the support and at the center of the span.

This test proved conclusively that the beams and the girders act as continuous ones. While the stresses in steel in the center and at the support were not excessive, the highest stress being 17 000 lb. for the total test load, the stresses in concrete at the supports of the beams were high and in some places even reached a stress of 2 200 lb. per sq. in. Even under the working load, stress in concrete was 1 150 lb. per sq. in. The compressive stresses in the center of the beam were low, and it appeared from the test that the total slab acted as a compressive flange of the T-beams. It must be noted that in this case the overhang of the flange was 7 times the thickness of the slab, while in practical designs, we consider only an overhang of 6 times the thickness of the slab as effective in taking compression. The total compression in a beam, figured with the assumption of a straight line distribution of stress and no tension in concrete, was much larger than the total tension, showing that either arch action existed in the beam or considerable tension was carried by concrete. The difference was especially large at the supports where the tension must have distributed itself over the entire slab.

* University of Illinois, Bulletin No. 64, January 13, 1913.

Test Cracks. Tensile cracks were observed in the middle portion of the bottom of the beams. They formed at the same stress in steel as is usually found in the laboratory. Diagonal cracks developed in the girder which carried a very large shear ($V = 40\ 000$ lb. and $v = 360$ lb.), just outside the junction with intermediate beams. The cracks were inclined at about 45°. They did not close entirely after removal of the load. It is supposed that the restraint at the ends prevented fuller development of the cracks. (See also p. 442 on formation of cracks in continuous T-beams.)

Deflections. The deflections offered further proof of the continuity of the beams, in the middle panel being much larger for one panel loaded than for three panels loaded, as would be expected from a continuous beam. With three spans loaded, deflection of intermediate beam was 0.09 inch, and for one span loaded was 0.15 inch.

Turner-Carter Building Test. The panels in this building are 17 feet 4 inches by 19 feet 6 inches. The girders are placed in the short direction and their dimensions are 10 by 24 inches, with two 1-inch square and three $\frac{7}{8}$-inch square bars at the middle, and two 1-inch square bars over the support. Beams, 7 by 18 inches, reinforced with one 1-inch square bar and two $\frac{7}{8}$-inch square bars at the middle, and one 1-inch square bar (plus ten $\frac{3}{8}$-inch round bars in the slab) over the support, are placed between the columns and at one-third points of the girder. The thickness of slab is 4 inches.

The structure was designed for a live load of 150 lb. per sq. ft. and the beams and girders were figured as simply supported, but reinforcement was supplied for continuity. The test load was 300 lb. per sq. ft. or double the designed load.

Results of Test. The beams and girders acted as continuous. The stresses in steel in the beams were comparatively low, the maximum observed for the test load being 11 000 lb. The stresses in concrete, however, at the end of the beam reached 1 100 lb. per sq. in. At the middle the compression in concrete reached only 350 lb. per sq. in., which shows that the compression there must have distributed itself over a large portion of the slab. In the girders the tensile stresses at the middle reached only 8 000 lb. per sq. in. At the supports no measurements were taken because the steel was not accessible. The compressive stress at the end of the beam in the bottom was 900 lb. per sq. in. and was very low at the center in the top surface.

In both beams and girders the total compression was much larger than the total tension, a condition that was found in the previous test.

As far as observation shows, the entire slab acted as compression flange of the T-beam.

General Conclusions. In drawing conclusions from tests on completed structures it must be remembered that although the stresses in steel are low it does not indicate a large factor of safety. The conditions are the same as were explained in connection with laboratory beam tests (see p. 412) in which the stresses at half the maximum load were small, while the maximum load stressed the steel to the elastic limit. The results of such tests must be used with caution.

TESTS OF OCTAGONAL CANTILEVER FLAT SLABS

An interesting test of cantilever flat slabs supported on a central column, as shown in Fig. 144, page 473, was made by Mr. Edward Smulski under the supervision of Sanford E. Thompson.

Eight specimen slabs were made: octagonal in shape, 6 feet 6 inches in small diameter, and with an octagonal column head in the center built monolithic with the slab and having an inside diameter of 2 feet. The slab was 4 inches thick. The reinforcement of Specimens 1 to 4 arranged as shown in Fig. 144, differed in the diameter of bars used for the five outside rings, as shown in the table on page 475. Specimens 5 and 6 were similar to 3 except that 10 and 5 radials respectively were used instead of 20. Specimen 7 was reinforced by four layers of bars running in four directions, each layer consisting of nine $\frac{5}{16}$-inch round bars. Specimen 8 was reinforced with steel in top and bottom; the tensile reinforcement consisted of two layers placed at right angles, with twelve $\frac{3}{8}$-inch round bars per layer, and the compressive reinforcement consisted of two layers with eight $\frac{3}{8}$-inch round bars per layer.

Purpose of Test. The purpose of the test was to compare the effectiveness of circumferential with band reinforcement and to determine the most effective distribution of steel between rings and radials.

Materials of Construction. Concrete in proportions 1 : 2 : 4 was used. The compressive strength of 6-inch cubes, tested at 52 days, was 2 100 pounds per square inch. Reduced to 8 x 16-inch cylinders and to 28 days, the strength of the concrete was about 1 400 pounds per square inch, or lower than first-class 1 : 2 : 4 concrete (see p. 310).

Plain round bars with an average elastic limit of 35 000 pounds per square inch were used.

Method of Testing. In testing, the slabs were placed on a wooden column resting upon a base which distributed the load to the soil. The

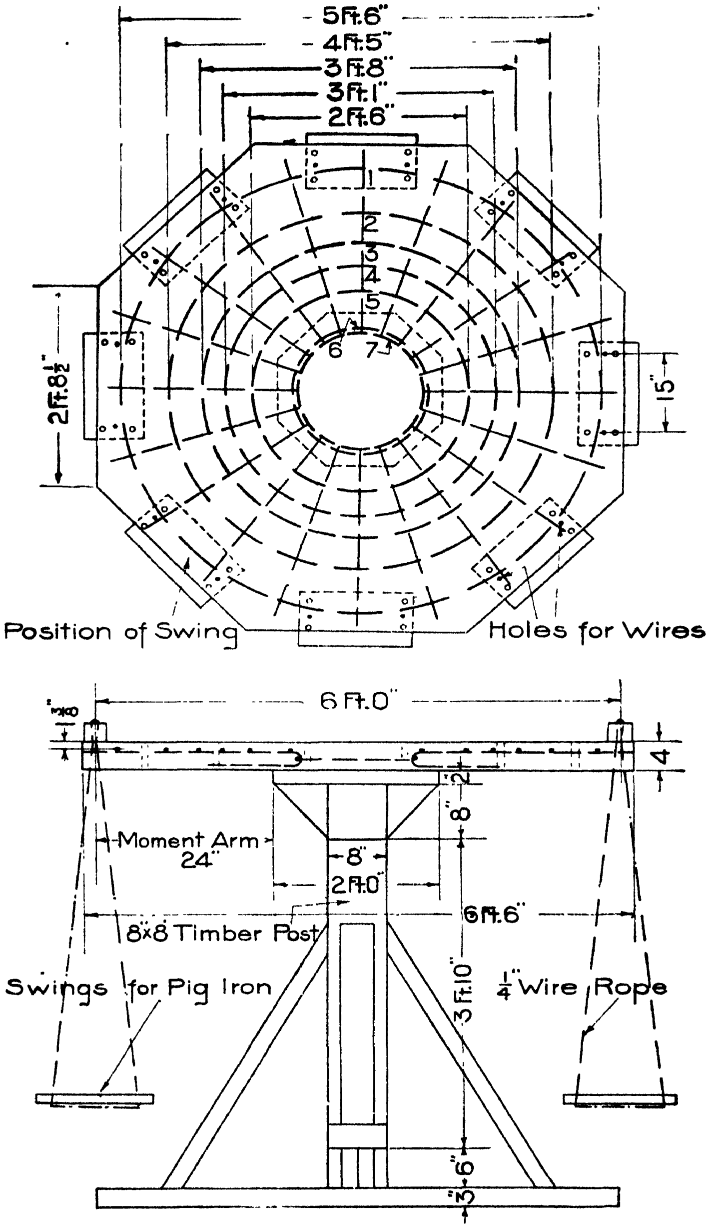
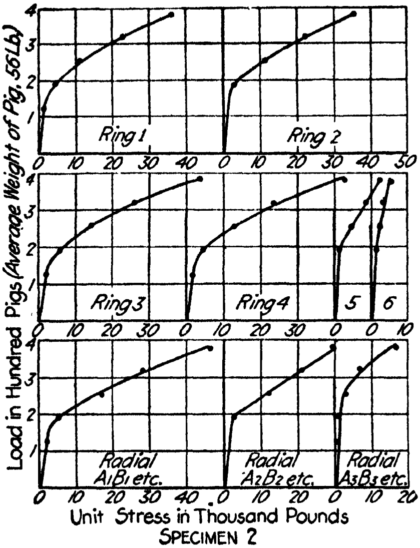


FIG. 144.—Cantilever Slab and Loading Platform. (*See p.* 472.)



load, consisting of pig iron averaging 56 pounds per pig was placed on swings arranged along the circumference of the cantilevers as shown in Fig. 144, page 473. By this method the point of application of the load, and therefore the moment arm, was positively fixed. Furthermore, actual conditions occurring in a continuous flat slab floor were substantially reproduced, the stresses in the cantilever corresponding to those produced by the negative bending moment at the column in a floor.

Deformation Readings. Deformations in steel, due to the loading, were measured by a Berry extensometer on 8 inch gage lines. For this purpose gage holes about $\frac{1}{32}$-inch diameter were drilled in the steel. Each ring and each bar was provided with at least four gage lines to eliminate the possibility of erratic results. Average stresses were plotted in a deformation diagram. Fig. 145, page 474, shows the deformations at different loadings for Specimens No. 1 and 2 and Fig. 146, page 475, for Specimen 7. The curves for Specimen 8 are substantially like Specimen 7.

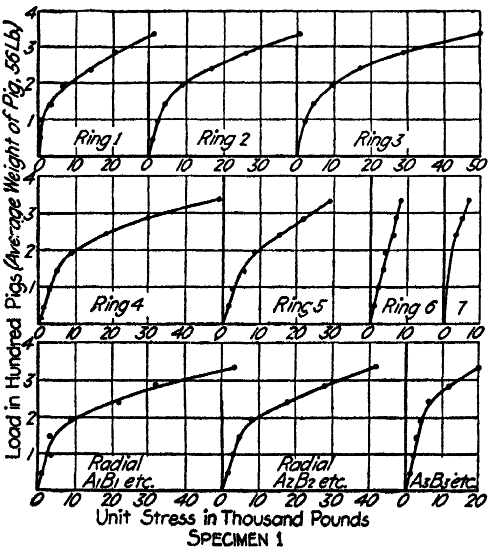


FIG. 145.—Deformation Diagrams for Slab Specimens. No. 1 and 2. (*See p.* 474.)

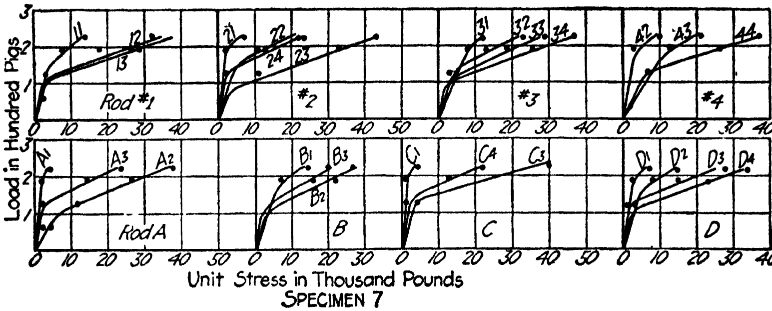


Fig. 146.—Deformation Diagrams for Slab Specimen No. 7. (*See p.* 474).

Results of the Tests. The results of the tests are shown in the table on page 475, which gives the dimension of specimens, total loads, and load per pound of tensile steel. The measured stress and the load at first visible crack also are given in the table, from which it is evident that the first visible cracks occurred at about two-thirds of the load at the elastic limit. Judging from the stress diagrams, hair cracks invisible to the eye must have appeared at a smaller load corresponding to the break in the deformation curve.

Summary of Results of Tests of Octagonal Cantilever Flat Slabs.

Octagonal slab 6 ft. 6 in inside diameter; column head 2 ft. diameter; 1: 2: 4 concrete; mild **steel.**
Specimens Nos. 1 to 4, Radial; Specimen 7, 4-way; Specimen 8, 2-way.
All slabs 4 inches thick.

Specimen No.	Age days	Weight of tensile steel. lb.	Number and diameter of round bars in inches: Radial bars	Outside rings	Straight bars	Area of radial or straight bars around circumference, sq. in.	Area of effective rings, sq in.	Average effective depth, in.	Total load at elastic limit, lb.	Load per pound of tensile steel Col. 10 ÷ Col. 3, lb.	Load at first visible crack, lb.	Measured stress in steel at first visible crack, lb. per sq. in.
1	47	42 6	20 − 3/8	5 − 5/16	.....	2 2	0.368‡	2 56	18 800	442	12 400	14 000
2	44	50.2	20 − 3/8	5 − 3/8	....	2 2	0 484‡	2 62	22 500	448	15 000	15 000
3	42	71.7	20 − 3/8	5 − 1/2	.. ...	2 2	0 901‡	2 60	*32 000	447	21 200	16 000
4	35	101.2	20 − 3/8	5 − 5/8	. ....	2.2	1.38	2 50	†42 500	421	26 600	15.000
7	51	60.0	.. ...	....	36 − 5/16	5 5	....	2.25	12 600	210	8 950	15 000
8	50	56.0	Tension / Compression		24 − 3/8 / 16 − 3/8	5.3	....	2 25	12 600	225	8 900	15 000

* Estimated from stress diagram. Broken by accident at 29 500 lb., before elastic limit was reached.
† Estimated from stress diagram. Elastic limit not reached at maximum applied load.
‡ Only part of the area of Ring 5 was considered as effective because it was placed too near the column head and therefore carried smaller stress than the other rings.

The first crack, at first hardly noticeable, extended all the way around the circumference of the column head several inches from its edge. For additional loading, the crack opened slowly and additional circumferential and radial cracks appeared. The test was discontinued after the steel had reached the elastic limit with the exception of Specimen 4, in which the elastic limit of the slab was not reached on account of the difficulty of applying further loading. No cracks developed within the column head although the radials were stressed to elastic limit and the hooked portions did not bear against the center ring. Of interest is the fact that the cracks in Specimens 7 and 8, reinforced with bands of bars, were also radial and circumferential.

Splicing of Rings. From the stress diagrams, it is noticeable that in Specimens 1 to 3, the outside Rings 1 to 4, and in Specimen 4, Rings 1 to 3, were equally effective in resisting the bending moment, the stresses at different loads being almost equal. The stress in Ring 5 was smaller than in the other rings, which can be accounted for by the fact that the ring was placed too near the column head. The stresses in Rings 6 and 7 within the column head are very small, showing that very little stress is transferred by the radials to the center rings. Evidently most of it is transferred to concrete by bearing. All rings were spliced with a 50-diameter lap. During testing, special attention was paid to the behavior of the steel at the splices and it was found that the elastic limit was reached without any movement being observed at the splices.

Conclusions. (1) First crack occurred at substantially the same measured stresses in the steel, irrespective of the arrangement and amount of reinforcement. The load at first crack increased with the increase of reinforcement.

(2) The actual load sustained in all specimens is larger than would be expected from ordinary methods of computation, proving the effect of Poisson's ratio. The reduction of bending moment coefficients suggested for flat slabs on page 547 is justified.

(3) The relative effectiveness of the various arrangements of steel can be obtained by comparing the load per pound of tensile steel which for specimen 1 to 4 varied between 420 pounds and 450 pounds and for specimens 7 to 8 between 210 and 225 pounds.

(4) In specimens reinforced by rings the stresses were uniformly distributed over all rings. (See stress diagrams, p. 474).

(5) The lap of 50 diameters of a plain bar was sufficient to develop the elastic limit of the rings.

CHAPTER XXII

REINFORCED CONCRETE DESIGN

In this chapter are given the definite principles and rules used in the design of reinforced concrete structures. The matter is based on the two preceding chapters of which the first goes much more fully than the present chapter into the fundamental theory of reinforced concrete, giving formulas and their derivations for rectangular beams, T-beams, beams with steel in top and bottom, columns, and members under direct compression and flexure, while the second describes the tests which verify both theory and rules for design. In the present chapter are taken up the working formulas which are necessary in actual design. Before using these final formulas, the designer should become acquainted with the derivations already given so as to have a thorough understanding of the subject.

The formulas and recommendations are grouped under headings and sub-headings for convenient reference. At the end of this chapter are tables and diagrams for use in design. Many of these are copied from office standards of the authors.

RATIO OF MODULI OF ELASTICITY

As seen from the tests, pages 400 to 404, the value of the modulus of elasticity of concrete depends upon the quality of the aggregates used, the consistency, and the age. It varies also for different stages of the loading, but may be considered constant within working limits. The modulus of elasticity of steel being practically constant (see p. 400), the ratio of moduli of steel to concrete, n, changes in direct proportion with the change of the modulus of concrete.

In computations, it is advisable to vary the ratio according to the ultimate strength of the concrete. The ratios, n, recommended* for use are:

(*a*) For concrete having a crushing strength of 2 200 lb. per sq. in. or less, a value of 15.

(*b*) For concrete having a crushing strength between 2 200 and 2 900 lb. per sq. in., a value of 12.

(*c*) For concrete having a crushing strength exceeding 2 900 lb. per sq. in., a value of 10.

* These values agree with the recommendations of the Joint Committee, 1916.

The value of 15 has been adopted in the British, German, and Austrian rules up to 1916. The French rules for 1907 authorize a range from 8 to 15 according to conditions. For determining deflection of beams when using formulas which do not take into account the tensile strength developed in the concrete, a ratio of 8 may be used.

The effect of the ratio of moduli on the stresses in beams may be seen from the formulas and also the tables. For a given beam with a definite amount of steel, the use of higher ratio of moduli lowers the position of the theoretical neutral axis, and for a given bending moment decreases the stresses in concrete and increases the stresses in steel, the latter, however, in much smaller proportion. For the same unit stresses and bending moments, but different ratios of moduli, the beam designed for the larger ratio will have a smaller depth, but at the same time a larger amount of steel. Therefore, in beam design, if a concrete richer than ordinary is used, the question of economy must be carefully considered.

Modulus of Elasticity in Tension. But few tests of modulus of elasticity of concrete in tension have been made, but these indicate* that the value is probably the same as the modulus in compression.

QUALITY OF REINFORCING STEEL

The 1914 Specifications of the American Society for Testing Materials require the following properties for reinforcement:

Tensile Properties of Concrete Reinforcement Bars

Properties Considered.	Plain Bars.			Deformed Bars.			Cold-twisted Bars.
	Structural-Steel Grade.	Inter-mediate Grade.	Hard Grade.	Structural-Steel Grade.	Inter-mediate Grade.	Hard Grade.	
Tensile strength, lb. per sq. in........	55 000 to 70 000	70 000 to 85 000	80 000 min.	55 000 to 70 000	70 000 to 80 000	80 000 min.	Recorded only.
Yield point, min., lb. per sq. in.....	33 000	40 000	50 000	33 000	40 000	50 000	55 000
Elongation in 8 in., min., per cent...	1 400 000† / Tens. str.	1 300 000† / Tens. str.	1 200 000† / Tens. str.	1 250 000† / Tens. str.	1 125 000† / Tens. str.	1 000 000† / Tens. str.	5

† Deduct 1 per cent for each increase of 1/8-inch above 3/4-inch diameter, or for each decrease of 1/16-inch below 7/16-inch diameter.

It is generally recognized in reinforced beam design that the yield point of the steel should be considered as the point of failure of this

* Prof. W. K. Hatt, Journal Association Engineering Societies, June 1904, p. 32.

material. Tests show that when the metal reaches its yield point, the beam sags, and this deflection, due to the stretch of the steel and in some cases to the slipping of the steel because of its reduced cross-section is likely to produce crushing in the concrete.

Many engineers do not approve of the use of high steel because of its brittleness when of poor quality, and the danger of sudden accident, and because of the fact that it is prohibited in ordinary structural steel work. Brittleness in steel, however, is less dangerous in reinforced concrete than in many classes of structural steel work because the concrete protects it from shock, and also because smaller sections of steel are used in concrete beams than in steel beams, and the large and irregular shapes of the latter render them much more sensitive to irregular cooling during the process of their manufacture.

Mild steel, that is, ordinary market steel, is manufactured and sold under such standard conditions that for unimportant structures it often may be used without other test than the bending test given on page 480. High steel, on the other hand, must be thoroughly tested. When tested, however, it is entirely safe and to be preferred to mild steel. The objection to it for reinforced concrete is based largely upon the use of a poor quality of material and the extra cost. Another objection which has been raised is that before the elastic limit is reached, the stretch in the high steel may produce excessive cracking in the concrete in the lower portion of the beam, and thus expose the steel to corrosion. The mere fact that cracks are visible does not prove that they are dangerous, because the steel is always designed to take the whole of the tension. Mr. Considère's and Professors Talbot's and Turneaure's tests indicate that there is no dangerous cracking even with high steel until the yield point of the steel is reached.

Tests made in Europe in 1907 (see p. 292) prove quite conclusively that the cement protects the steel from ordinary and even extraordinary corrosive action until the elastic limit of the steel is nearly reached. In cases where very minute cracking of the concrete may cause anxiety (even although not dangerous), the steel, whatever its quality, should not be stressed beyond the ordinary limits of, say, 16 000 pounds per square inch.

A yield point in steel of 30 000 pounds per square inch corresponds to a stretch of 0.0010 of its length and a yield point of 50 000 to a stretch of 0.00167.

If steel could be made with a high modulus of elasticity it would be particularly serviceable for reinforced concrete, because the higher the modulus of elasticity of a material the less is the deformation under any given

loading. Unfortunately, however, all steel, whether high or low in carbon, has substantially the same modulus of elasticity (30 000 000 lb. per sq. in.).

It may be stated, then, that high carbon steel, say, 0.56% to 0.60% carbon, of the quality used in the United States for making locomotive tires, is better than mild steel for reinforced concrete provided the steel is well melted and rolled, and is comparatively free from impurities, such as phosphorus.* However, a high carbon steel, unless limited by chemical analysis, and made under careful inspection, is in danger of being more brittle than low carbon steel. Its use, therefore, should be limited strictly to work important enough to warrant the ordering of a special steel and the taking of sufficient trouble on the part of the purchaser to insure strict adherence to the specifications. Since manufacturers cannot always be depended upon to exactly follow specifications of this nature, it is necessary that an inspector be sent to the works either by the dealer or the purchaser.

Bending Test for Steel. The most important test in the specifications is the bending test and **no steel which fails to pass this bending test should be used under any circumstances.** The bending test of the 1914 Specifications of the American Society for Testing Material is as follows: Test specimens for bending shall be bent cold to the following angles without fracture on the outside of the bent portion:

Bend-Test Requirements.

Thickness or Diameter of Bar.	Plain Bars.			Deformed Bars.			Cold-twisted Bars.
	Structural-Steel Grade.	Intermediate Grade	Hard Grade.	Structural-Steel Grade	Intermediate Grade.	Hard Grade.	
Under $\frac{3}{4}$ in	180 deg. $d = t$	180 deg. $d = 2t$	180 deg. $d = 3t$	180 deg. $d = t$	180 deg. $d = 3t$	180 deg. $d = 4t$	180 deg. $d = 2t$
$\frac{3}{4}$ in. or over...........	180 deg. $d = t$	90 deg. $d = 2t$	90 deg. $d = 3t$	90 deg. $d = 2t$	90 deg. $d = 3t$	90 deg. $d = 4t$	180 deg. $d = 3t$

EXPLANATORY NOTE: d = the diameter of pin about which the specimen is bent;
t = the thickness or diameter of the specimen.

Steel with high elastic limit, whether due to high carbon or to manipulation in manufacture, should be purchased with these reservations even if the working stress is to be no higher than is used with mild steel, say, 16 000 pounds per square inch, because it is liable to be brittle. In case a lot of steel has been delivered without previous test by the purchaser, one bar

* In Bessemer steel, phosphorus should be not over 0.10 per cent and in open hearth steel not over 0.05 per cent. In hard steel, manganese should be between 0.40 and 0.80 per cent, and sulphur should be not over 0.06 per cent.

selected at random in every 100 should be subjected to this test and if it fails to pass, the portion from which it is taken should be rejected.

FORMULAS FOR DESIGN OF RECTANGULAR BEAMS

From the discussion on page 350 it is evident that a beam must have breadth and depth sufficient to prevent excessive compression in the concrete in the top of the beam and enough steel to take all the pull without exceeding the working stress of the steel. Rules for this are given in the simple formulas which follow. The steel must also have sufficient bond (see p. 533) and in most cases inclined or vertical reinforcement is required as treated in connection with diagonal tension, pages 516 to 533. Continuous beams also require reinforcement over the supports, as described in pages 496 to 499.

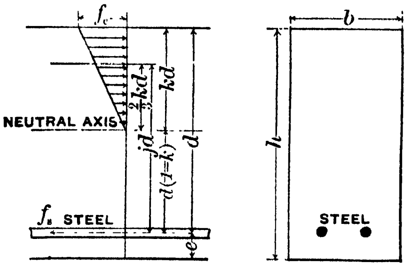


FIG. 147.—Resisting Forces in a Reinforced Concrete Beam. (*See p.* 482)

Considering the design of a simple beam, let

d = distance from outside compressive fiber to center of gravity of steel.
b = breadth of rectangular beam or breadth of flange of T-beam.
p = ratio of cross-section of steel in tension to cross-section of beam, bd.
A_s = cross-section of steel in tension.
M = moment of resistance or bending moment in general.
C = constant in table, page 483.

Having computed the maximum bending moment due to the loads (see p. 510) the breadth of the beam, b, is assumed and then the depth of the beam, d, and the amount of steel are found from the following formulas:

$$d = C\sqrt{\frac{M}{b}} \quad (1) \quad \text{and} \quad A_s = pbd \quad (2)$$

The constants C and p may be taken from Table on page 483, selecting values corresponding to the working stresses in steel and concrete and to their ratio of elasticity.

Substituting in (1) and (2), for C and p, the values corresponding to 650 lb. per sq. in. in concrete and 16 000 lb. per sq. in. in steel:

$$d = 0.096\sqrt{\frac{M}{b}} \quad (3) \quad \text{and} \quad A_s = 0.0077\ bd \qquad (4)$$

Equations (1) and (3) give the minimum allowable depth for assumed working stresses. Sometimes for construction reasons, it is necessary to use a larger depth than obtained by these formulas. For such cases a smaller amount of steel is permissible and may be obtained from equation

$$A_s = \frac{M}{jdf_s} \qquad (4a)$$

For ordinary cases the value of j may be taken as $\frac{7}{8}$.

Example 1: What depth of beam and what area of steel are required, for a freely supported beam having a span of 18 feet using 1:2:4 concrete, with a load of 600 pounds per running foot?

Solution: Bending moment, M, for $\frac{wl^2}{8}$ is $\frac{600 \times 18 \times 18 \times 12}{8} = 291\,600$ inch pounds. Assuming a breadth of 8 inches and using formula (3)

$$d = 0.096\sqrt{\frac{291\,600}{8}} = 18.3 \text{ inches}$$

With 2 inches of concrete below the steel, the total depth of beam is thus 20.3 inches.

The area of steel from formula (4) is $A = 0.0077 \times 8 \times 18.3 = 1.13$ square inches, thus (from table page 574) requiring four $\frac{5}{8}$-inch round bars, or their equivalent.

The steel and concrete stresses, f_s and f_c and ratio, p, are interdependent and for any values of f_s and f_c there is always a corresponding value of p. See formula (5), page 354.) With f_s and f_c given, the corresponding ratio, p, must never be exceeded, else, if the stress in the steel is maintained, the stress in the concrete would be increased beyond the permissible values given on page 573.

If it is necessary to use a larger ratio of steel than the value p corresponding to the required stresses and at the same time maintain the stress f_s the excess steel must be balanced by compression steel as discussed on page 493.

Example 2: To the stresses f_s = 16 000 and f_c = 650 corresponds a ratio of steel p = 0.0077. If a ratio of say p = 0.01 is used and the steel is stressed to 16 000 pounds per square inch the corresponding stress in concrete would be 770 instead of 650 pounds per square inch. To maintain with a ratio p = 0.01 the stress in concrete f_c = 650 pounds per square inch without adding compression steel it would be necessary to limit the stress in steel to only 13 000 pounds per square inch. This shows that with too large a ratio of steel the full working value of steel cannot be utilized and therefore the beam is not economical.

The following table gives the values of constants for selected stresses.

Constants in Beam and Slab Design

For use in beam formula $d = C\sqrt{\frac{M}{b}}$ and in slab formula $d = C_1\sqrt{M}$ (*See p.* 482 and 485). For additional values see table on p. 596.

Proportion.	Ultimate Strength	n	Allowable Unit Stresses lb per sq in Steel f_s	Allowable Unit Stresses lb per sq in Concrete f_c	k	j	p	C	C_1
1:1 :2	3 000	10	16 000	975	0.378	0 874	0 0115	0 079	0 023
1:1½:3	2 500	12		810	0.378	0 874	0.0096	0.086	0.025
1:2 :4	2 000	15		650	0 378	0 874	0.0077	0.096	0.028
1:2½:5	1 600	15		520	0.327	0.892	0.0053	0 115	0 033
1:3 :6	1 300	15		420	0.282	0 906	0.0037	0.137	0.040
1:1 :2	3 000	10	18 000	975	0 351	0.883	0 0095	0 081	0 024
1:1½:3	2 500	12		810	0.351	0 883	0 0079	0 089	0 026
1:2 :4	2 000	15		650	0.351	0 883	0 0063	0 099	0 029
1:2½:5	1 600	15		520	0.302	0 900	0.0044	0 119	0.035
1:3 :6	1 300	15		420	0 259	0 914	0.0030	0 142	0 041

Depths and Loads for Different Bending Moments. The depth may be obtained in terms of the unit load, if desired, by substituting for M in formula (1) its value in terms of the load and the span. This may be readily transposed also to give the load, w, which a given beam will carry.

Formulas To Review A Beam Already Designed. To review a beam already designed, the following formulas may be used, the derivation of which is given on page 354.

Let

f_c = compressive unit stress in concrete in pounds per square inch.

f_s = tensile unit stress in steel in pounds per square inch.

b = breadth of beam in inches.

d = depth of beam from compressive surface to center of steel in inches.
k = ratio of depth of neutral axis to depth of beam d.
j = ratio of distance between the centers of compression and tension to depth of beam, d.
$jd = d\left(1 - \frac{k}{3}\right)$ = distance between the centers of compression and tension.
A_s = area of cross-section of steel in square inches.
p = ratio of cross-section of steel to cross-section of beam above center of gravity of steel.
M = bending moment in inch-pounds.
n = ratio of modulus of elasticity of steel to concrete.

Then

$$p = \frac{A_s}{bd} \quad (5) \qquad k = \sqrt{2pn + (pn)^2} - pn \quad (6) \qquad j = 1 - \frac{k}{3} \quad (6a)$$

$$f_s = \frac{M}{A_s jd} \quad (7) \qquad f_c = \frac{2M}{bd^2 jk} \quad (8)$$

The value of p is figured first, then k and j computed or taken from Table on page 482, and substituted in equations (7) and (8).

For rectangular beams designed with stresses ordinarily used, the moment arm, jd, is about $\frac{7}{8}\,d$ and the above formulas may be expressed as

$$f_s = \frac{M}{0.87 A_s d} \quad (7a) \qquad f_c = \frac{6M}{bd^2} \quad (8a)$$

Neither the allowable tension in steel nor the allowable compression in concrete should be exceeded. Tables for determining the dimensions and loading of rectangular beams are given on pages 576 to 578, and the methods of practical computation and details of design are illustrated in Example 8, page 553. T-beams are treated on page 487.

The selection of bending moments to use in design of continuous beams is treated on page 510.

DESIGN OF SLABS

A slab, so far as computation is concerned, is a rectangular beam. The dimensions and stresses, therefore, can be obtained by the formulas given for rectangular beams.

The bending moment is figured for a definite width of slab so that the formula for depth of slab can be simplified by combining the selected

value of b = 12 inches with the constants given for rectangular beams, changing formulas as given below. In the formula for required area of steel, A_s, it is most convenient to assume a width of slab, $b = 1$ inch. The formula then gives the area of steel per inch of width of the slab, and the spacing of the bars can be readily determined by dividing the cross sectional area of a bar by the determined area per inch of width of slab.

Using notation on page 484, and making

M = bending moment in inch-pounds per foot of width of slab,
A_s = area of steel in square inches per inch of width,
C_1 = constant based on these units,

the formulas (1) and (2), change to

$$d = 0.29\, C\sqrt{M} = C_1\sqrt{M} \quad \text{(inches)} \tag{9}$$

$$A_s = pd \quad \text{(per inch of width)} \tag{10}$$

The table on page 483 gives the values of constants, C_1, for concrete of selected proportions.

For 1 : 2 : 4 concrete, adopting stresses in this table, the formulas become

$$d = 0.028\sqrt{M} \quad \text{(inches)} \tag{11}$$

$$A_s = 0.0077\, d \quad \text{(per inch of width)} \tag{12}$$

in which d is found from equation (9). If larger depth of slab is used than required by this formula, the area of steel may be found from

$$A_s = \frac{M}{12jdf_s} = \frac{M}{10.5\, f_s d} \quad \text{(per inch of width)} \tag{13}$$

The use of these formulas is illustrated in Example 8, page 553.

Table 7 on page 582 gives dimensions and reinforcement for slabs for different live loads based on stress in concrete, f_c = 650, stress in steel, f_s = 16 000, and ratio of elasticity, n = 15.

Slabs which are continuous over the supports, such as those in a floor or in a buttressed retaining wall, must be designed with provision for the negative moment at the supports. For uniformly loaded spans continuous over two or more intermediate supports, a moment $M = \frac{1}{12} wl^2$ may be used both in the centers of the spans and also at the supports, while for end spans a moment $M = \frac{1}{10} wl^2$ is necessary.

Moments at Support. To provide for the moments over supports some designers bend up all the bars near the $\frac{1}{4}$ point, but a better way,

to be sure that no point in tension is unprovided with steel, is to bend up one-half, two-thirds or three-quarters of the bars and run them over the supports allowing the remainder to continue at the bottom of the slab. To provide the rest of the steel at the support, the bars in the adjoining span can be carried back over the support. Where the bars are so long as to extend over several spans, they can be arranged to break joints at different places, and so keep as much steel over top of supports as at center of span.

The bend in the bars should be near the $\frac{1}{4}$ points in the span, and usually at an angle of about 30 degrees with the horizontal. Too sharp an angle may tend to crack the slab, while, on the other hand, they must be brought to the top of the slab far enough from the support to properly provide for the negative moment.

Tables for determining dimensions and loading of slabs can be found on pages 579 to 582, and examples and details of design are given on pages 552 to 557.

Cross Reinforcement of Slabs. Cross reinforcement, that is, bars at the bottom of slab at right angles to the principal bearing rods, is customarily used to prevent shrinkage and temperature cracks. The amount of steel to use for this usually is selected somewhat arbitrarily, a cross-sectional area of bars equivalent to 0.2 per cent. to 0.3 per cent. ($p = 0.002$ to 0.003) of the cross-section of the floors being the usual practice.

Reinforcement over Girders. The top of the slab over a girder or beam which is parallel to the principal reinforcement bars should be reinforced transversely not only for stiffening the T-beam (see p. 418) but also to provide for the negative bending moment produced with the bending of the slab next to the beam or girder. This reinforcement is also necessary even when the beam is simply a small stiffener. (See p. 491.)

Computing Ratio of Steel. The ratio of steel in a slab is most readily found by dividing the cross section of one bar by the area between two bars, this area being the spacing of the bars times the depth of steel below top of slab. For example, a slab with steel 4 inches below the top and $\frac{1}{2}$ inch round bars spaced 6 inches apart has a ratio,

$$p = \frac{0.196}{4 \times 6} = 0.0082, \text{ or } 0.82 \text{ per cent steel.}$$

Square and Oblong Slabs Supported by Four Beams. When a slab is supported by four beams and its length does not exceed $1\frac{1}{2}$ times its

width, the loads will be carried by the slab to all four beams, and therefore the slab must be reinforced in two directions, as shown below.

The following table gives the ratio of the unit load, w_B, carried by the short span for different ratios of $\frac{L}{B}$.*

Ratio of Load Carried by the Shorter Span. (*See p.* 487.)

Ratio of Length to Breadth of Slab.	Ratio of Load Carried by the Shorter Span.	Ratio of Length to Breadth of Slab.	Ratio of Load Carried by the Shorter Span.
1.00	0.50	1.30	0.80
1.05	0.55	1.35	0.85
1.10	0.60	1.40	0.90
1.15	0.65	1.45	0.95
1.20	0.70	1.50	1.00
1.25	0.75		

It must be noticed that the shorter span carries the larger proportion of the load.

After the proportion of the load is determined, the bending moments are found as for slabs reinforced in one direction and the dimensions or stresses are found by the ordinary formulas. The thickness of the slab, of course, is governed by the larger bending moment of the two.

DESIGN OF T-BEAM

The formulas given below are sufficient to design or review a T-beam for a given bending moment.

To Design a T-Beam. Design the slab. Determine width of flange (see p. 488). Determine bending moment and end shear. If headroom is not limited, determine most economical depth (see p. 490). (In designing a number of similar beams the economical depth needs to be figured only for one beam, and estimated for the remainder.) Before

* The load carried in either direction can be determined from the following formulas.

Let

w_B = unit load carried by the short span.

w_L = unit load carried by the long span.

L = length of slab.

B = width of slab.

Then

$$w_B = \frac{L}{B} - 0.5 \tag{14}$$

and

$$w_L = 1.5 - \frac{L}{B} \tag{15}$$

selecting final depth of beam and breadth of stem, b', see that the compression in concrete and the shear do not exceed the allowable working stress (pages 588 and 489). Figure amount of tension steel (formula (20) p. 491). For large beams, a saving in steel may be effected by using the more exact formulas. In such case, preliminary A_s for k and z may be found from formula (20) and final A_s from formulas (21) to (26) page 357.

To Review a T-Beam. Dimensions are given and the stresses are to be determined. Determine width of flange. Find compressive stresses in concrete by use of table on page 588.

The stresses in steel may be found from formula (20) page 491.

If desired, k and z may be determined from formulas (15) and (16) page 356, and then f_c and f_s from (18) and (19) page 357.

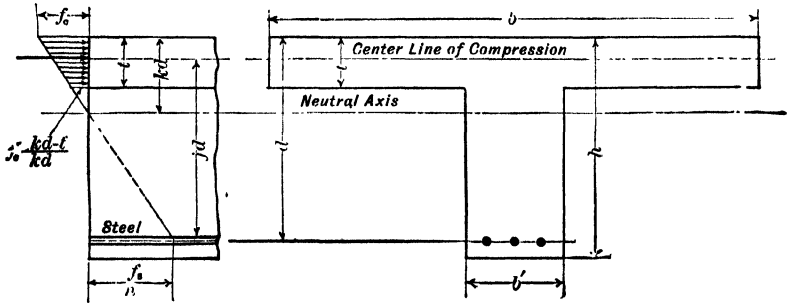


FIG. 148.—Section of T-Beam. (*See p.* 488.)

Width of Flange. The width of the slab, b, to use for the flange of the T-beam in compression is selected somewhat arbitrarily. In no case, of course, can it be taken greater than the distance between beams. The Joint Committee has recommended the following rules, which are approved by the authors, for the width of slab to be considered effective:

(a) It shall not exceed one-fourth of the span length of the beam;

(b) Its overhanging width, on either side of the web, shall not exceed six times the thickness of the slab.

(c) It must not exceed the distance between beams.

This practice is conservative. (See tests, pages 415 to 418.)

Beams in which the T-form is used only for the purpose of providing additional compression area of concrete should preferably have a width of flange not more than three times the width of the stem and a thickness of flange not less than one-third of the depth of the beam.

Cross-section of Web as Determined by the Diagonal Tension. The width of the web of a T-beam is governed by the layout of the tension bars (see p. 537) and by the shearing stresses (see p. 515).

The area of the web required for shear involving diagonal tension, using notation on page (491) and letting $V=$ total vertical shear, and $v=$shearing unit stress may be found from the formula (see also formula (32a) p. 517).

$$b'\left(d-\frac{t}{2}\right)\geqq\frac{V}{v} \tag{16}$$

That is, the area of web at any point in the beam (considering this up to the middle of the slab) must not be less than the total shear divided by the maximum allowable unit shear for the reinforced beam.

The vertical unit shearing stress (used as measure of the diagonal tension) in a beam effectively reinforced with bent bars or stirrups, or both, is limited by the Joint Committee to 120 pounds per square inch for ordinary concrete having a compressive strength (in cylinders) of 2 000 pounds per square inch at 28 days. See Example 8, page 553.

Minimum Depth of T-Beam. The minimum depth is the depth at which concrete and steel are stressed simultaneously to their working limits. It is governed by **the compression in the flange which must not exceed the working compressive stress in the concrete.** Greater depth than the minimum is generally used for economy. A smaller depth gives excessive compressive stresses. For an example, see page 587.

To find the minimum depth the rectangular beam formula, (1) page 481, may be used where the depth of the beam is not greater than four times the thickness of slab, using in this formula the breadth of the flange, b for the breadth of the beam. For ratios of depth of T-beam to thickness of slab larger than four, the rectangular beam formula gives unsafe results and the following formula must be used. (See page 554, and Tables 11 and 12, pages 586 and 587.)

$$\text{Minimum } d=\frac{MC_d}{jf_s bt} \tag{17}$$

Minimum Depth at the Support for Continuous Beams. At the support a continuous T-beam becomes practically a rectangular beam with steel in top and bottom. The minimum depth for known stresses f_s and f_c, bending moment M, and the selected ratio* of compressive to tensile steel, may be found from formula (18), illustrated in example 3, page 490. For the definition of p_1 see page 492.

$$\text{Minimum } d=\sqrt{\frac{M}{jbp_1 f_s}} \tag{18}$$

*The use of a ratio greater than 1.0 should be avoided because of the cost, and the difficulty of placing and keeping the steel in position during construction.

Example 3: Given: $M = 1\ 200\ 000$ inch pounds, $f_c = 750, f_s = 16\ 000, n = 15$. Find the minimum depth if it is desired to limit compressive steel to one-half the tensile steel, so that $\frac{p'}{p_1} = 0.5$.

Solution: Assume $a = 0.06$. From Table 14, page 589, in the section for $f_s = 16\ 000$ and $f_c = 750$, find, in the column for $a = 0.06$, the value of $p' = 0.007$ corresponding to $p_1 = 0.014$. This satisfies the requirement that $\frac{p'}{p_1} = 0.5$. Assuming $j = 0.89$ (it is not necessary to be very exact in the choice of j), we find the minimum depth to be

$$\text{minimum } d = \sqrt{\frac{M}{jbp_1f_s}} \doteq \sqrt{\frac{1\ 200\ 000}{0.89 \times 10 \times 0.014 \times 16\ 000}} = \sqrt{602} = 24.6 \text{ inches.}$$

The exact value of j from Formula (27), page 496, or Diagram 1, page 593, is 0.885. This does not change the minimum depth and the approximate value is near enough for the purpose. The exact value of a is $\frac{1.5}{24.6} = 0.061$ also near enough to the approximate value.

Economical Depth for a T-Beam. Usually a greater depth than the minimum is desirable for economy, because deepening the beam reduces the area of steel proportionally. Professors Turneaure and Maurer* analyze the depth for maximum economy and suggest from this the most economical values.

Using the notation given on page 491 and

r = ratio of cost of cubic foot of steel in place to cubic foot of concrete in place, the economical depth is

$$d - \frac{t}{2} = \sqrt{\frac{r\ M}{f_s b'}} \qquad (19)$$

From this formula the most suitable depth may be selected after two or three trial computations for different widths of stem. The ratio of costs, r, ranges between 37 and 75. For concrete† in place at 20 cents per cubic foot, and steel† in place at 3 cents per pound, or 1470 cents per cubic foot, the ratio of costs is 74, while for concrete at 40 cents per cubic foot and steel at 3 cents per pound, this value will be reduced to 37. In calculations where no unit costs are given, a value of 60 may be selected for r.

The depth of the T-beam should not be made too great in proportion to the breadth of stem. Many designers make the ratio of the depth of a T-beam to its width of web between 2 and 3. For very deep and large beams a ratio of 4 may be accepted; while, if head room is limited, the depth of the beam fixed and the width of stem determined by area required for shear, the ratio may be even less than 2.

* Turneaure and Maurer's "Principles of Reinforced Construction," Second Edition, p. 238.

† The cost of concrete need not include cost of form construction since a variation in depth affects this but slightly. To the actual unit cost of steel add 25 to 40 per cent for laps and stirrups.

Another plan sometimes followed in studying designs is to make the depth of T-beam an arbitrary ratio to its span. Comparison of a number of representative designs shows an average ratio of span to depth of beam between 10 and 12, which suggests the approximate rule to make the depth in inches equal to the span in feet.

Sectional Area of Steel in a T-Beam. The area of cross-section of steel in tension may be obtained very closely by the following formula:

Let

b = breadth of flange of T-beam.
b' = breadth of web of T-beam.
d = depth of T-beam in inches.
t = thickness of flange in inches.
j = ratio of lever arm of resisting couple to depth, d.

Then

$$A_s = \frac{M}{jdf_s} \tag{20}$$

The value of j may be taken from the following table:

Value of j for Different Ratios of Depth of Beam to Thickness of Slab, $\frac{d}{t}$. (*See p.* 491.)

$\frac{d}{t}$	10	9	8	7	6	5	4	3
j	0.95	0.94	0.94	0.93	0.92	0.91	0.89	0.88

Sometimes jd is assumed to equal $\frac{7}{8}d$, or $d - \frac{t}{2}$, but the error is larger than in the above table.

More exact values of j may be found from table on page 588, or by the use of formulas on page 357.

Details of Design. The design of a T-beam must also be studied for diagonal tension reinforcement (see p. 516), bond of steel to concrete (see p. 533), and especially for the design at the support, which must be adapted to the negative bending moment (see p. 496).

To insure proper T-beam action and prevent cracks through the slab adjacent to the stem, reinforcement at right angles to the beam or girder is necessary. When the principal slab reinforcement is parallel to the girder, additional short bars should be placed at the top of the slab transversely over the girder and extending well into the slab.

The example on page 553 illustrates the use of the formulas and principles of design. The selection of bending moments is treated on page 510.

DESIGN OF BEAMS WITH STEEL IN TOP AND BOTTOM

In the chapter on theory, page 358, are given formulas for beams with steel in top and bottom. They are, however, too complicated to use conveniently in ordinary design. A simplified method, devised by Mr. Edward Smulski, of adapting the exact formulas to practical use is given below. By this method the whole process of design is clear and can be readily followed. It can be used without the aid of tables since the constants, k, j, and p, for the customary unit stresses can be easily remembered. The method is of such importance in relieving one of the most complicated problems in reinforced concrete design that it will be demonstrated by two examples.

Determination of Ratio of Compressive Steel, p', for Given Dimensions of Beam and Area of Tensile Steel.

If the ratio of tension steel in a beam is larger than the limiting value p, corresponding to the allowable working stresses f_s and f_c, the required ratio of compression reinforcement may be determined as follows:

Let

p_1 = ratio, in beams with steel in top and bottom, of cross-section of steel in tension to cross section of beam, bd.

p = ratio, in beams with tensile steel only, of cross-section of steel in tension to cross-section of beam, bd.

p' = ratio of cross-section of steel in compression to cross-section of beam, bd.

k = ratio of depth of neutral axis to depth of steel in tension.

a = ratio of depth of compressive steel to depth of beam.

Formula 33, page 359, for p_1 consists of two terms. The first term, $\frac{k^2}{2n(1-k)}$, is identical with formula (6), page 354, for the ratio, p, corresponding to the allowable stresses f_s and f_c given on page 482 for beams without compression steel so that formula (33) becomes

$$p_1 = p + p' \frac{k-a}{1-k}$$

From which we get

$$p' = (p_1 - p) \frac{1-k}{k-a} \qquad (21)$$

This equation gives the required ratio* of compressive steel if certain

* That is, ratio of cross section of steel to cross-sectional area of beam disregarding any projecting flanges which are in tension.

values of f_s and f_c are to be maintained and the ratio of tensile steel, obtained from calculation, is larger than it would be in a similar beam without compressive reinforcement. In the above formula values of p_1, p, k and a are known. Formula (21) has been used in making out Table 14 on pages 589 to 591, from which values of p' may be taken directly. The use of the formula is explained by Example 4.

Example 4. Find required amount of compressive steel for a T-beam at the support where $b = 12$ inches, $d = 22$ inches, $A_s = 3.5$ square inches for tensile stresses, $f_s = 16\,000$ and $f_c = 750$. Distance from top of beam to center of compressive steel, $d_1 = 1\frac{1}{2}$ inches and $a = \frac{1.5}{22} = 0.068$.

Solution. At the support the T-beam changes into a rectangular beam (see p. 496). By dividing area of steel by bd, ratio of tensile steel, $p_1 = \frac{3.5}{12 \times 22} = 0.0133$. From table on page 596 the ratio of steel for beams without compressive steel, for stresses, $f_s = 16\,000$, and $f_c = 750$, is $p = 0.0097$ and $k = 0.414$. Since ratio of steel used is larger than p, compressive steel is required as given below.

Ratio of required compressive steel,

$$p' = (0.0133 - 0.0097)\,\frac{1 - 0.414}{0.414 - 0.0682} = 0.0061.$$

Therefore, 0.6% of compressive steel is required to keep compressive stress in concrete within working limits of 750 lb. per sq. in. if the stress in steel is 16 000 lb. per sq. in.

From Table 14 on pages 589 to 591, the required ratio of compressive steel for a given ratio of tensile steel may be taken direct. This makes the design of a beam with steel in top and bottom very simple. The tensile steel may be determined approximately by assuming $j = 0.89$. After obtaining the required compressive steel, the value of j may be corrected by referring to the diagram on page 593.

Method of Design of Beams with Steel in Top and Bottom. As explained on page 358, the compression is made up of compression in steel and compression in concrete, but tension is in tensile steel alone. Since total tension must equal total compression, a certain part of the tensile steel balances the compressive stress in concrete, while the remainder of the tensile steel must balance the compressive stress in the compression steel. The moment of resistance of the beam with compressive steel may be divided into (a) moment of resistance of beam without compressive reinforcement; and (b) moment of resistance of the couple formed by the compression steel and the portion of the tension balanced by this steel. The moment of resistance of the simple beam is obtained by multiplying the compression in concrete by the distance

from the center of gravity of the triangle to the center of tensile steel. By multiplying the compressive stresses in steel by the distance from center of compressive steel to center of tensile steel, we get the additional moment of resistance produced by the use of extra tensile steel and the compressive steel. This principle may be used to simplify the designing of beams with steel in top and bottom.

Let

A_{s_1} = area of cross-section of tensile steel in beam under consideration.

$A_s = pbd$ = area* of tensile steel in simple beam for which steel and concrete are stressed to given allowable stresses, f_s and f_c.

p = ratio* of cross-section of steel in tension to cross-section of beam above this steel, in beam with tensile steel only.

k = ratio of depth of neutral axis to depth of steel in tension.

$A_{s_2} = A_{s_1} - A_s$ = extra tensile steel in beam with steel in top and bottom to be balanced by compression steel.

A'_s = area of compression steel required to balance extra tensile steel.

M_1 = moment of resistance or bending moment in general.

M = moment of resistance of beam without compressive reinforcement.

The moment of resistance of beam without compressive reinforcement and with the required area of tensile steel as given on page 354 is

$$M = f_s A_s jd \text{ (see p. 484) in which } A_s = pbd.$$

If the dimensions b and d of a beam are limited and it is called upon to resist a bending moment M_1 larger than the moment of resistance given above of a simple beam, corresponding to the allowable unit stresses, the following method may be used in determining the required amount of tensile and compression reinforcement.

The total area of tensile reinforcement is

$$A_{s_1} = pbd + A_{s_2} \tag{22}$$

The difference between M_1 and M must be resisted by a couple formed by stresses in extra tensile steel, $A_{s_2} = A_{s_1} - A_{s}$ and by stress in compression steel, A'_s.

The arm of this couple is $d\,(1-a)$.

Therefore

$$M_1 - M = A_{s_2}\, d\,(1-a)\, f_s \tag{23}$$

* The limiting value for which no compression steel is required.

from which

$$A_{s_2} = \frac{M_1 - M}{d\,(1 - a)\,f_s} \tag{24}$$

Substituting the above in formula (22) total area of tensile steel is

$$A_{s_1} = pbd + \frac{M_1 - M}{d\,(1 - a)\,f_s} \tag{25}$$

The area of compression steel is

$$A'_s = A_{s_2}\,\frac{1 - k}{k - a}* \tag{26}$$

The above method is illustrated by the following example:

Example 5. Given size of beam, b = 10 in. and d = 18 in. as determined by architectural requirements; bending moment, M = 500 000 in. lb.; allowable unit stresses f_s = 16 000 lb. per sq. in., f_c = 750 lb. per sq. in. Find required amount of tensile and compressive steel.

Solution. For f_s = 16 000 lb.; f_c = 750 lb.; from table on page 596, k = 0.414; j = 0.862; Ratio and area of tensile steel for simple beam, p = 0.0097; therefore $A_s = 10 \times 18 \times 0.0097 = 1.75$ sq. in.

Moment of resistance of beam without compression reinforcement, therefore, would be

$M_1 = A_s jdf_s$ = 435 000 in. lb.

By comparing this moment with the bending moment of 500 000 in. lb., we find a difference of $M_1 - M$ = 500 000 − 435 000 = 65 000 in. lb.

This must be provided for by extra tensile steel and by the corresponding amount of compressive steel. Since center of compressive steel is distant $ad = 1\frac{1}{2}$ in. from the top of beam, distance between tensile steel and compressive steel is $d\,(1 - a) = 18 - 1\frac{1}{2} = 16\frac{1}{2}$ in. This is moment arm of couple composed of compressive stress in steel and tensile stress of extra amount of tensile steel. Required extra amount of tensile steel, A_{s_2}, therefore, can be found by dividing the extra bending moment by $d\,(1-a)\,f_s = 16\frac{1}{2} \times 16\,000$ lb.; hence $A_{s_2} = \frac{65\,000}{16.5 \times 16\,000} = 0.25$ sq. in. The total tensile steel, therefore equals

$$A_{s_1} = A_s + A_{s_2} = 1.75 + 0.25 = 2.0 \text{ sq. in.}$$

From formula (26), required amount of compressive steel is

$$A'_s = A_{s_2}\,\frac{1 - k}{k - a} = 0.25\,\frac{1 - 0.414}{0.414 - 0.084} = 0.44 \text{ sq. in.}$$

* The tensile stresses resisted by the extra tensile steel, As_2, equals $As_2 f_s$ and the compressive stresses resisted by compression steel, A'_s, equals $A'_s f'_s$. For equilibrium, $As_2 f_s = A'_s f'_s$ or $A'_s = As_2 \frac{f_s}{f'_s}$. Since ratio of the compressive unit stress in steel to the tensile unit stress in steel, $\frac{f'_s}{f_s} = \frac{k - a}{1 - k}$ (see formula (29) p. 359), $A'_s = As_2 \frac{1 - k}{k - a}$.

Moment Arm. The ratio of moment arm to depth of beam can be determined from the formula

$$j = \left(1 - \frac{k}{3}\right) + \frac{p'}{p_1}\left(\frac{k}{3} - a\right)\frac{k-a}{1-k} \qquad (27)$$

in which k is the ratio of depth of neutral axis and $1 - \frac{k}{3}$ is the ratio of moment arm for simple beams depending upon the ratio of stress, $\frac{f_s}{nf_c}$. The diagram on page 593 gives values of j for different ratios of $\frac{f_s}{nf_c}$ and $\frac{p'}{p_1}$. In practice a value for j of 0.89 may be assumed and the amount of tensile steel determined from $A_s = \frac{M}{jdf_s}$. After obtaining the required compressive steel the value of j may be corrected by referring to the diagrams.

DETAILS OF CONTINUOUS BEAMS AT THE SUPPORT.

Design of a Continuous Beam at the Supports.* The formulas just given and the diagram on pages 594 and 595, for beams with steel in top and bottom are of the greatest practical use in designing continuous beams at the supports.

In the past too little attention has been paid to the details of reinforced concrete beams at the supports, with the result that a number of reinforced concrete structures have been built with beams and girders containing insufficient steel at the top of the beam over the supports to take the pull caused by the negative bending moment, and insufficient area of concrete to take the compression.† These beams not only fail to have the required factor of safety, but frequently even the working loadings cause cracks that are always unsightly and sometimes dangerous. Moisture also is liable to penetrate the open cracks and rust the steel.

Just as much care, therefore, is necessary in designing the reinforced concrete beam at the supports as in the middle of the span Not only the tensile stresses in the steel but also the compressive stresses in the concrete must be figured.

The tendency to overstress the concrete at the supports is due to the T-beam design. In a T-beam, at the center of the span the portion of the slab forming the flange is available for taking compression. At the

* See "Bending Moments in Continuous Reinforced Concrete Beams" by Sanford E. Thompson, in *Engineering Record*, June 7, 1913, p. 639.

† See p. 470.

supports, however, where the bending moment is negative the flange is in the tensile part of the beam and the compressive area consists merely of the part of the stem below the neutral axis. Usually it is advisable to increase the compressive area by using compressive steel.

Allowable Working Stresses at the Support. As is evident from the curves of bending moments (pp. 505 to 508), negative bending moment decreases very rapidly so that only a very short section of a beam is under maximum stress, thus permitting at the supports the use of a higher compressive working stress than is used in the center of the beam. This increase may be taken at 15%, which makes the allowable working stress at the support 750 pounds per square inch for 2 000 pound concrete, when the working stress in the center of the beam is 650 pounds.

Even with this allowance, however, it is often impossible, without special provisions, to keep the stresses within working limits.

Compression Steel at the Support. Frequently additional compressive area is provided by steel in the compressive part of the beam in which case the beam must be considered and computed as reinforced at the top and bottom. The effectiveness of steel in compression has sometimes been questioned, but the results of tests with reinforced concrete columns, as well as with beams with steel in top and bottom (see p. 427), have proved conclusively that compression steel not only takes its share of the stress but also stiffens the beam.

In a reinforced concrete beam, part of the steel usually is bent up, and the rest, frequently one-half, is carried horizontally to the supports, and may be utilized as compressive reinforcement. As the bars get their maximum compression at the edge of the support they must be extended beyond it a sufficient length to develop the stress by bond (see p. 539). In some cases where a large amount of compression steel is required, the bars may be run over the support into the adjoining span a sufficient length not only to provide anchorage, but also to increase the compressive reinforcement there. The compressive reinforcement in such case consists of the horizontal steel from both spans. To facilitate erection it is desirable to limit the amount of compression steel to the amount of tensile steel used in the center of the beam. Therefore the depth of the beam should be selected so as not to require a larger value of p' than p (see p. 489).

The stresses and the necessary amount of steel may be figured from formulas on pages 494 to 496. Table 14 on page 589, and the diagrams on pages 594 and 595.

Depth of Beam Increased by a Haunch. Another method of providing additional compression area is by increasing the depth of the beam at the support by a flat haunch. The increased depth may be obtained by assuming a new depth and then, with the aid of the diagrams on pages 594 and 595, determining whether the stresses do not exceed the specified limit. This is illustrated in the example on page 556.

Under ordinary conditions the computation need be made only at one point, that is, next to the support, since the point to end the slope can be readily figured from the following formula:

For a uniformly loaded beam, let

M_b = negative bending moment next to the support.
M_r = moment of resistance of the inverted T-beam without the haunch, governed by the concrete.
x = length of haunch.
l = span of beam.

Then
$$x = \frac{l}{5} \frac{M_b - M_r}{M_b} \quad \text{(approximately)*} \qquad (28)$$

An illustration of the use of this formula is given in Example 8, page 556.

Tension Steel at the Support. For reinforced concrete beams designed for a negative bending moment at the support equal to the positive bending moment at the center of the span, there are two common ways of arranging the details so as to bring the same amount of tensile reinforcement at both places.

The first method, most commonly used, is to bend about one-half of the horizontal steel, carry it at an angle to the top of the beam, and then horizontally towards and over the support. When bending the bars, determine, from the diagrams, pages 535 to 536, from the positive bending moment where the bars may be bent up and from the negative bending moment where they are required over the support. Then bend the bars in places where they would be most useful as web reinforcement, keeping in mind, however, that their main purpose is to provide negative moment reinforcement. The bent bars are carried across the support a sufficient distance to provide anchorage by bond and also to serve as tension steel in the adjacent span. If this amount of steel

* This formula is based upon the fact that the point of zero moment is at approximately $\frac{1}{5}$ of the span, and from the curves of bending moment on p. 505, it is evident that the variation in the moment between the support and the $\frac{1}{5}$ point is very nearly a straight line. Hence the difference between the bending moment and the moment of resistance is in approximately the same ratio to the bending moment as is the ratio of the distance from the point where the haunch is needed to the point of zero bending moment. When the point of zero moment is not approximately at $\frac{1}{5}$ span the fraction may be altered accordingly.

is not sufficient, which may happen when the adjoining spans are not of the same strength, additional short bars may be introduced. This arrangement has the advantage that all the bars are easily kept in place, and besides, the bent bars serve as diagonal tension reinforcement.

In the second method, which is sometimes employed, all bottom bars are carried straight. Some of them are carried the whole length of the beam to provide compressive reinforcement, and the rest are cut short at places where, due to the decrease of the bending moment, as determined from page 536, a smaller amount of steel is sufficient. The bars to be cut must be extended, however, beyond the theoretical points and must be hooked at the ends or otherwise anchored to prevent slipping. Only a small proportion of the steel, preferably one bar at a time, should be cut so as not to transfer suddenly a large stress to the remaining bars. In the top of the beam over the support, a separate set of straight bars is introduced, which extends both ways from the support into the beams a distance equal to about $\frac{1}{4}$ of the span. Although this method has the advantage that just enough steel is used in the center and at the supports, and the labor of bending bars is saved, it is not recommended because it is difficult in construction to avoid displacement of bars.

Anchoring of Bars at the Supports. In fixed and cantilever beams, the maximum tensile stress in reinforcing bars exists at the point of support, and the steel must be anchored in the support sufficiently to develop this stress. This may be done by extending the steel over the support a sufficient length to develop the stress by bond (see p. 539). However, where this is impossible, as at the wall line in a building or in a retaining wall, the anchorage should be provided by a hook, preferably curved, of sufficient strength to develop the stress (p. 540). The hook must be placed deep enough to avoid danger of breaking or shearing off of the concrete.

EFFECT OF VARYING MOMENT OF INERTIA UPON THE BENDING MOMENT

As the bending moment in a continuous beam depends upon its moment of inertia, it will vary with the variation in the moment of inertia. In a reinforced concrete beam, the moment of inertia is very seldom constant for the whole length of a continuous beam. Especially is this true of T-beams and beams provided with haunches.

A thorough study by the authors of different conditions, however, shows that a variation in the moment of inertia in a beam, as great as is possi-

ble in ordinary design, causes a variation in bending moment of less than 10 per cent. Under most conditions, the variation is even smaller. Consequently a continuous beam may be designed safely with the bending moment recommended on page 510.

SPAN OF A CONTINUOUS BEAM OR SLAB

It is customary to consider the span of a continuous beam or slab as the distance between the centers of its supports. In general this is the simplest plan to follow and one which is always on the side of safety. If the support is exceptionally wide, as when a slab runs into a wide beam, or a beam or girder into a large column, the length of the span may be considered arbitrarily as the clear span plus the depth of the beam or slab.

DISTRIBUTION OF SLAB LOAD TO THE SUPPORTING BEAMS

If slabs are reinforced in both directions, the loads carried to the beams supporting them will not be uniformly distributed over the length of the beam, but may be assumed to vary in accordance with the ordinates of a triangle.

Assuming that the slab transmits a load to its nearer support, we have the following formulas for determining the moment to use in computing the long and the short supporting beams.

Let

l_l = the longer span of a rectangular slab in feet.
l_s = the shorter span of the slab in feet.
w = load per linear foot of beam if the slab is considered as supported by longer beams only.
M_l = bending moment in foot pounds of longer beam.
M_s = bending moment in foot pounds of shorter beam.

Then the moments of the two beams, assuming them as freely supported, are found by the application of simple mechanics, to be

$$M_l = \frac{1}{8} w l_l^2 \left(1 - \frac{1}{3}\frac{l_s^2}{l_l^2}\right) \quad (29) \qquad \text{and} \quad M_s = \left(\frac{1}{8} w l_s^2\right)\frac{2}{3} \quad (30)$$

For continuous or fixed beams the fraction $\frac{1}{8}$ may be changed to $\frac{1}{10}$ or $\frac{1}{12}$

Formula (30) does not apply to girders supporting one or more beams. This case is treated under the heading which follows.

The following example illustrates the use of the formulas:

Example 6: What will be the bending moments in the two continuous beams supporting an oblong panel the length of which is one and one-quarter the breadth and which is reinforced so as to transmit its load both ways?

Solution: Using $\frac{1}{12}wl^2$ for the continuous beams instead of $\frac{1}{8}wl^2$ and substituting:

$$\text{Moment in longer beam, } M_l = \frac{1}{12}wl^2\left(1 - \frac{1}{3}\frac{(\frac{4}{5}ll)^2}{ll^2}\right) = \frac{59}{900}wl_l^2$$

in terms of the longer span, and

$$\text{Moment in shorter beam, } M_s = \frac{1}{12}\cdot wl_s^2\,\frac{2}{3} = \frac{1}{18}wl_s^2$$

DISTRIBUTION OF BEAM AND SLAB LOADS TO GIRDERS

When one or more beams run into a girder, the load upon the girder consists of the concentrated live and dead loads from the beams acting at their points of intersection with the girder, the uniformly distributed weight of the girder itself, and the unsymmetrically distributed weight of a small portion of the floor slab, with its live load, which bears directly upon the girder.

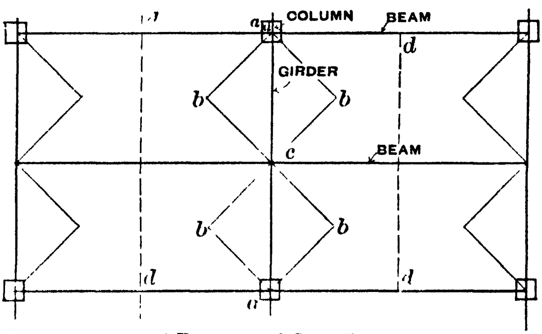


FIG. 149.—Distribution of Beam and Slab Loads to Girder. (*See p.* 501.)

To avoid the computation of several moments, a series of studies have been made by the authors for different conditions, and it has been found that the maximum bending moment of a girder may be obtained without appreciable error by considering, as a uniformly distributed load, the weight of the girder plus the weight of slab and its live load, for an area whose length is the length of the girder and whose width is the average length of the beams running from each side into the girder. The sum of these loads divided by the length of the girder gives a uniformly distributed load for which the ordinary formula may be used.

Thus in Fig. 149, instead of computing the moment on the girder as the

sum of the moments produced by loads of the triangles, *a b c*, plus the concentrated loads from the beams at *c*, the entire load *d d d d* may be considered as uniformly distributed over the girder in the length *a a*.

With only one condition is there an appreciable variation from the exact maximum moment, and this is a case where two beams run into a girder at the one-third points. Here the maximum moment obtained by the uniformly distributed method gives slightly too conservative results, and may be reduced by 10%.

Moments in a girder other than the maximum must be computed for individual conditions.

BENDING MOMENTS AND SHEARS

Bending moments and shearing forces have to be computed so frequently in reinforced concrete design that the more common rules and formulas are given here and, besides this elemental matter, diagrams are presented for estimating the moments and shears in various kinds of loading, and recommendations are made for the computation of bending moments in design. Shear and diagonal tension in beams are taken up at length.

Rule to Find Reactions at Supports. The reaction at a support must be found in order to determine the bending moment. The sum of the upward forces, which in ordinary beams are the reactions at the supports, is equal to the sum of all the downward vertical forces or loads. In a simple beam supported or fixed at the two ends, the reaction at either end is found by taking moments of all forces about the other support and solving for the reaction desired.

Expressed as a formula, if

R = desired reaction.
P = any vertical load.
l = span.
x = distance of load from the support at which the reaction is desired.
Σ = sum, using $-$ for downward and $+$ for upward forces, then

$$R = \frac{\Sigma P\,(l - x)}{l} \qquad (31)$$

Example 7: In Fig. 150, where there is a uniform load over the entire span and also concentrated loads $P_1 = 200$ and $P_2 = 350$ at the ⅓ points, what is the left reaction?

Solution: $R = \dfrac{(200 \times 8) + (350 \times 4) + (100 \times 12)6}{12} = 850$ pounds.

The determination of reactions and moments of continuous beams is referred to on page 510.

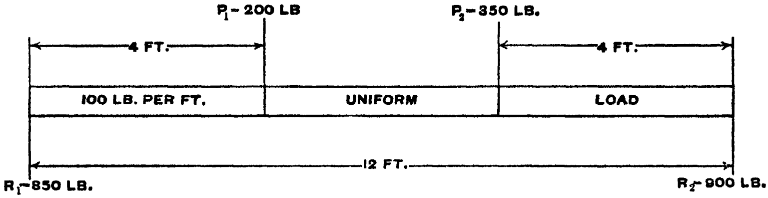


FIG. 150.—Beam Loaded with Distributed and Concentrated Loads. (*See pp.* 509 *and* 510.)

Rule to Find Bending Moment at Any Point in a Beam. Consider either side of the vertical section passing through the point and disregard the other side. Multiply each load and reaction by its average distance from the section and add the products, taking loads acting downward as negative and those acting upward as positive.

This sum is the bending moment at the section.

Moments in English measure are usually taken in inch-pounds. Hence, the distance must be in inches and the weights in pounds.

Example 8: In Fig. 150 what is the bending moment in inch-pounds at the middle of the span?

Solution: $M = R_1\,(6 \times 12) - P_1\,(2 \times 12) - (100 \times 6) \times 3 \times 12 = 34\,800$ inch pounds.

Rule to Find Shear at any Point in Beam. Consider either side of the section passing through the point and disregard the other side. Add the loads and reactions, taking the loads acting downward as negative and those acting upward, such as a reaction, as positive. The sum is the shear at the section.

Example 9: In Fig. 150 what is the shear at the left support and at the center?

Solution: $R_1 = 850$ pounds at left support and at the center the shear is $R_1 - P$—$(100 \times 6) = 50$ pounds.

Table of Common Bending Moments and Shearing Forces. The following table for convenient reference gives values of the shearing forces and bending moments for common cases. The values for external forces are independent of the structure of the beam.

*Bending Moments and Shearing Forces.**

Description	Loading.	Section Considered.	Shearing Force. At distance x from support.	Shearing Force. Greatest.	Bending Moment. At distance x from support.	Bending Moment. Greatest.
Beam fixed at one end, unsupported at other.	At end		W	W	$W(l-x)$	Wl
	Uniform.		$\frac{W}{l}\left(l-x\right)$	W	$\frac{W}{2l}\left(l-x\right)^2$	$\frac{Wl}{2}$
Beam supported at both ends.	At middle.	Between support and middle.	$\frac{W}{2}$	$\frac{W}{2}$	$\frac{W}{2}x$	$\frac{Wl}{4}$
		Beyond middle.	$-\frac{W}{2}$		$\frac{W}{2}\left(l-x\right)$	
	Uniform.		$\frac{W}{l}\left(\frac{l}{2}-x\right)$	$\frac{W}{2}$	$\frac{W}{2l}\left(lx-x^2\right)$	$\frac{Wl}{8}$
	At distance a from support.	Between support and load.	$\frac{W(l-a)}{l}$	$\frac{W(l-a)}{l}$	$\frac{W(l-a)}{l}x$	$\frac{Wa(l-a)}{l}$
		Beyond load.	$-\frac{Wa}{l}$	$\frac{Wa}{l}$	$\frac{Wa}{l}\left(l-x\right)$	

* W = total load; l = span; x = distance of section considered from support. If moment is in inch pounds, l and x must be in inches and W in pounds. If load is distributed so as to be in terms of weight per unit length, substitute wl for W in the formulas.

Table of Moments of Inertia. The table on page 509 gives the moment of inertia for beams of a few sections which might be used in concrete construction. The reinforcement, if any, may be considered as replaced by an area of concrete which is the area of the steel times the ratio of elasticity, n, and is located at the same distance from the neutral axis.

SHEAR AND BENDING MOMENT DIAGRAMS

The diagrams in Figs. 151 to 154 pages 505 to 508 give bending moments and shears for beams continuous over two, three, and four spans. Beams with free ends and with fixed ends are shown separately. In using the diagrams with fixed ends it must be remembered that in practice beams are hardly ever positively fixed at the wall supports. (See p. 513.) Positions of the loads were selected giving absolute maximum bending moments in the center of the spans and at the supports

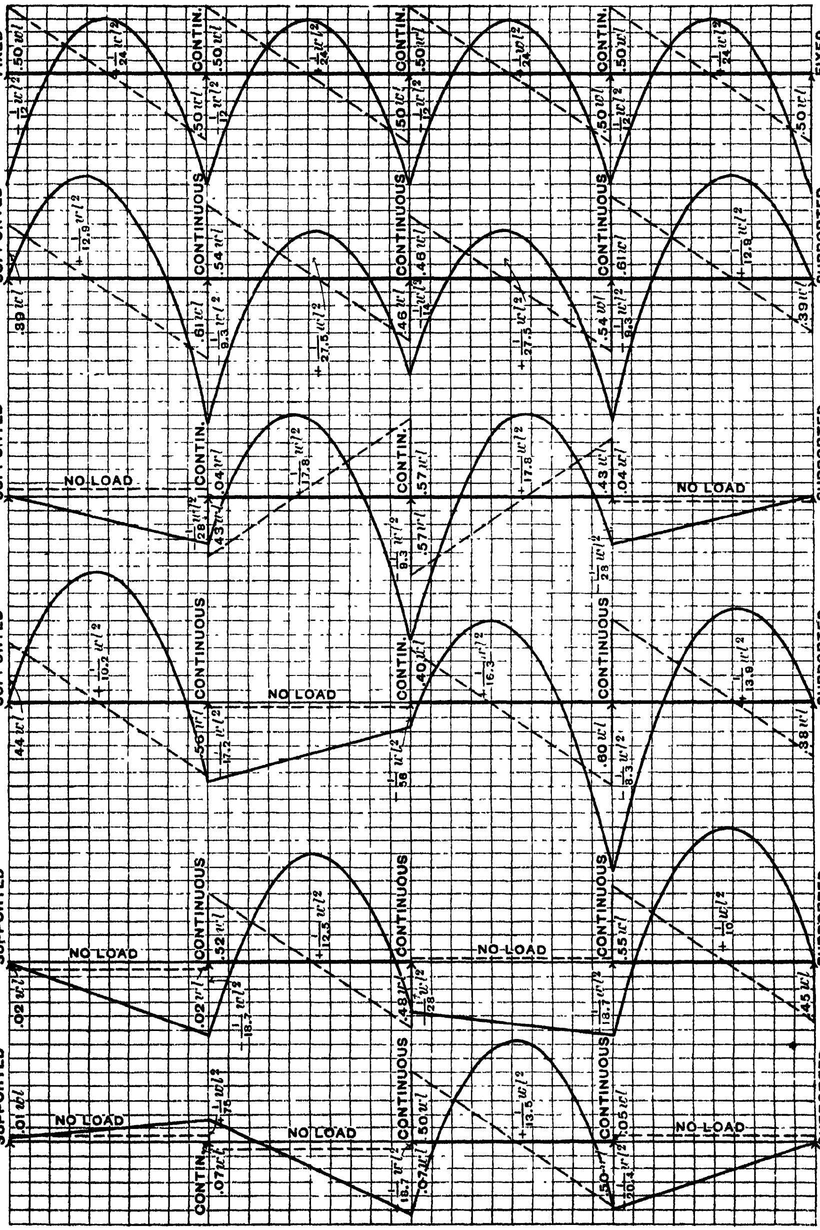


FIG. 151.—Bending Moments and Shears for Continuous Beams, Distributed Loads. (See p. 504.)

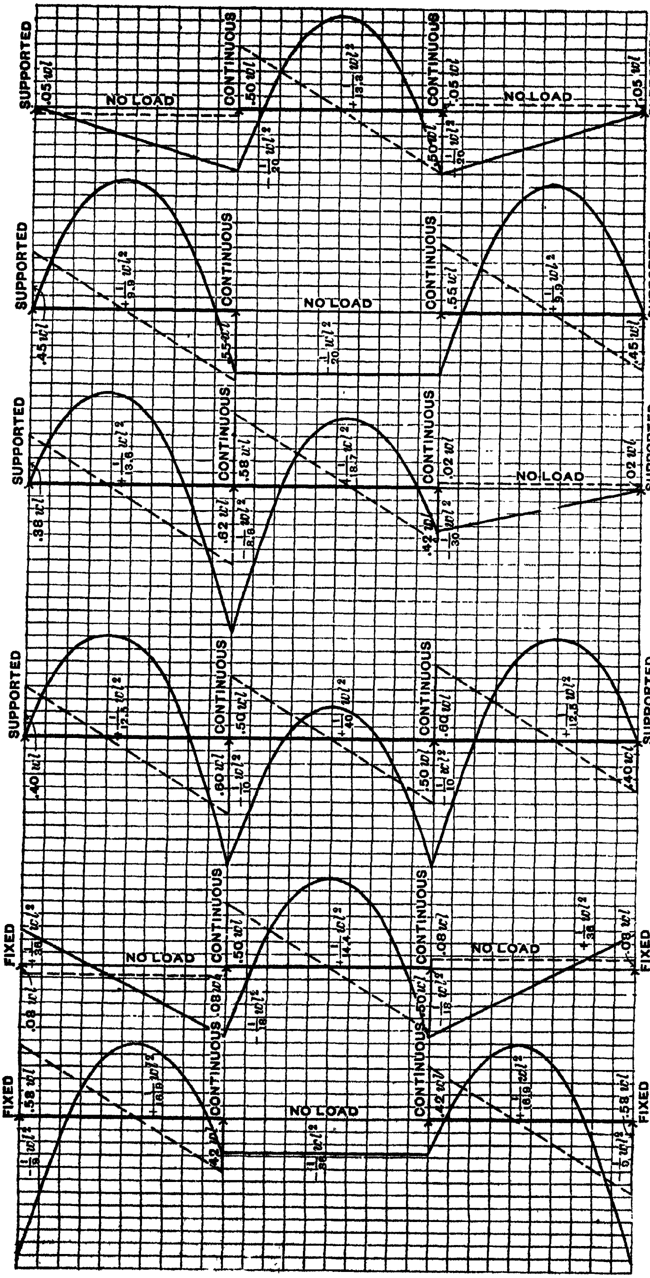


FIG. 152.—Bending Moments and Shears for Continuous Beams, Distributed Loads. (See p. 504.)

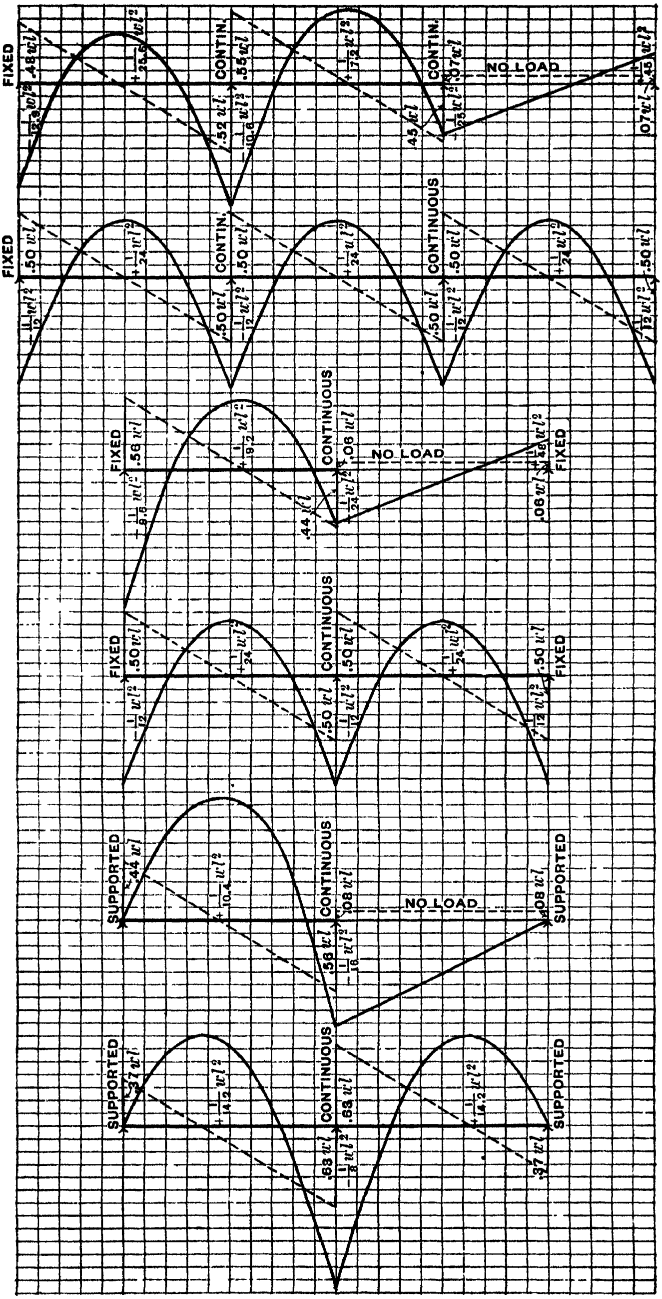


FIG. 153.—Bending Moments and Shears for Continuous Beams, Distributed Loads. (*See p.* 504.)

respectively. The cases chosen are sufficiently representative to be used without appreciable error as maximum and minimum values for beams of any number of spans and any distribution of uniform loading.

As stated with the diagrams, the curves are all drawn to scale on cross-section ruling so that proportionate values may be read. The loads are given in terms of w, the load per unit of length. The horizontal scale has ten divisions per span so that the moments and shears can be read-

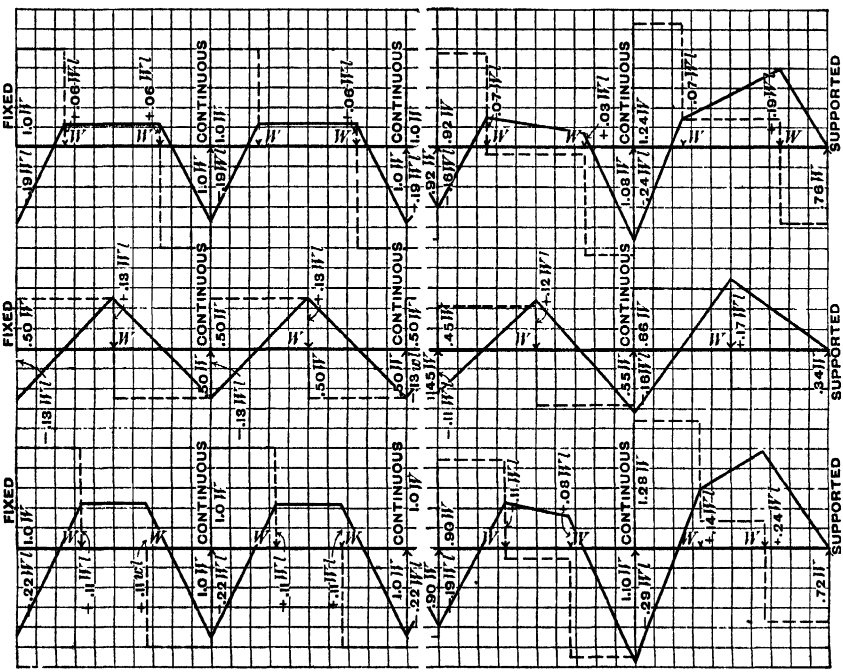
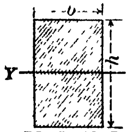
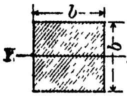
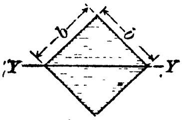
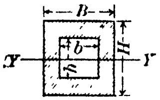
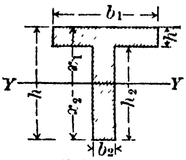
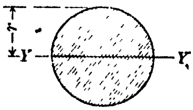
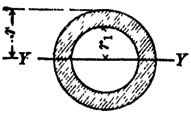
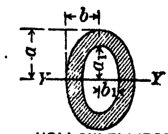


FIG. 154.—Bending Moments and Shears for Continuous Beams, Concentrated Loads. (*See p.* 508.)

ily estimated at any point. The values which are printed for the bending moments are in common fractions for convenience of comparison, although in order to scale them they must be changed from the common fraction to a decimal. Each vertical division for the bending moment scale represents 0.01 and for the shear scale represents 0.1. Bearing this in mind, the bending moment and shear can be scaled at any part of the span.

Concentrated loads are treated in Fig. 154, the loads being located

Moments of Inertia. (*See p.* 504.)

Figure.	Area A	Moment of Inertia. I	Distance of Neutral Axis from most Strained Fiber* Y
 RECTANGLE	bh	$\frac{bh^3}{12}$	$\frac{h}{2}$
 SQUARE	b^2	$\frac{b^4}{12}$	$\frac{b}{2}$
 SQUARE	b^2	$\frac{b^4}{12}$	$\frac{1}{2} b\sqrt{2}$
 RECTANGULAR CELL	$BH - bh$	$\frac{1}{12}(BH^3 - bh^3)$	$\frac{H}{2}$
 T SECTION	Area of flange + area of web $= A_1 + A_2$	$\frac{A_1h_1^2 + A_2h_2^2}{12} + \frac{A_1A_2(h_1+h_2)^2}{4(A_1+A_2)}$	$x_1 = \frac{h}{2} - \frac{A_1h_2 - A_2h}{2(A_1 + A_2)}$ $x_2 = \frac{h}{2} + \frac{A_1h_2 - A_2h}{2(A_1 + A_2)}$
 CIRCLE	πr^2	$\frac{\pi r^4}{4}$	r
 HOLLOW CIRCLE	$\pi(r^2 - r_1^2)$	$\frac{\pi(r^4 - r_1^4)}{4}$	r
 HOLLOW ELLIPSE	$\pi(ab - a_1b_1)$	$\frac{\pi a^3b}{4} - \frac{\pi a_1^3b_1}{4}$	a

*Applicable only to homogeneous (not to reinforced) beams

at the quarter points, middle points, and third points respectively. This diagram is of special use in studying girders supporting cross beams. The stresses are computed for a beam of four spans and as the curves are symmetrical at each end, the diagram is broken in two, one-half being shown with fixed end and the other half with end supported. The results with a larger number of spans will not be appreciably different.

The vertical scale for concentrated loading is 0.05 per division for bending moments and 0.2 per division for shears.

The concentrated loads are given in terms of W, the load which is concentrated at each point.

The continuous beam is statically indeterminate, so that the moments and reactions have to be found by the theory of flexure, using the formula of three moments first evolved by Clayperon.*

In applying this to the various cases, the assumption is made that the moment of inertia of the beam is constant throughout its length. While this is not strictly true, extensive studies of various cases in reinforced concrete show that a large change in the moment of inertia makes a very small change in the bending moment, so that the relations are substantially correct until a member enters a much larger member. (See p. 499.)

BENDING MOMENTS TO USE IN DESIGN OF REINFORCED BEAMS

Tests in the laboratory and on actual structures show that a reinforced concrete beam built continuous over several supports and properly reinforced for the positive as well as the negative bending moment acts as continuous and may be designed as continuous.

The bending moment for any span of a continuous beam depends not only upon its length and loading but also upon the number of spans in the beam, their relative lengths, and the condition of loading in the remaining spans. In building construction the loadings are indefinite so that instead of refined and laborious computations the following rules are recommended as safe for ordinary beams designed for uniformly distributed loading.

Let

M = bending moment in inch-pounds.

w = load uniformly distributed in pounds per inch of length.

l = length of beam in inches.

Then

(a) For beams continuous over two or more intermediate supports,

* See Lanza's "Applied Mechanics."

the bending moment at the center and at the support for interior spans, shall be taken at $\frac{wl^2}{12}$, and for end spans at $\frac{wl^2}{10}$ for the center and the adjoining support, for both dead and live loads.

(*b*) For beams continuous for two spans only the bending moment at the central support shall be taken as $\frac{wl^2}{8}$ and near the middle of the span, as $\frac{wl^2}{10}$.

When the end of the above beam is monolithic with the column and provided with negative bending moment reinforcement, the bending moment at the central support may be taken as $\frac{wl^2}{10}$ and near the middle of the span as $\frac{wl^2}{12}$ instead of the bending moments specified above.

(*c*) At the ends of continuous beams running into a column provision shall be made for the negative bending moment, the amount of which will depend upon the condition of fixedness.

In any case provision should be made for at least a bending moment $\frac{wl^2}{20}$. In ordinary cases $\frac{wl^2}{16}$ should be used. Where beams run into very heavy columns or piers, use $\frac{wl^2}{12}$. (See also p. 513.)

(*d*) For spans of unusual length, more exact calculations shall be made. Also for beams designed for special concentrated or moving loads, as in bridges, the bending moment shall be determined separately for the dead load and the most unfavorable position of the live load.

Even if the center of the span is designed for a larger bending moment than is recommended, the negative bending moment shall not be taken at less than the values called for above.*

The above bending moments are recommended only for cases where spans are equal or nearly equal. Where the difference in length of spans equals about 60 % of the longest span, special computations should be made.

For a beam consisting of 3 spans in which the two end spans are equal and the center span is a fraction (between 0.25 and 0.40) of the end span, the following bending moments should be used:†

* In the Turner-Carter test on the completed building large compressive stresses developed at the support although the beams were designed as freely supported. (See page 471.)

† See "Design and Construction of the Massachusetts Institute of Technology" by Sanford E. Thompson, Journal American Concrete Institute, July 1915, pp. 383 and 389.

Center of the end span	$\frac{wl^2}{11}$
Central support	$-\frac{wl^2}{11}$
Center of the center span,	
Positive bending moment	$\frac{wl^2}{60}$
Negative bending moment	$-\frac{wl^2}{20}$

In the above formulas l is the length of the end span.

The negative bending moment reinforcement in the center span must extend over the whole span. The amount, however, can be varied according to a straight line from a maximum at the support to a minimum in the center; otherwise, the negative bending moment from the long span would be transferred to the column.

The value of $\frac{wl^2}{12}$ for the bending moment for interior spans in building construction has been widely adopted in Continental Europe. However, it is absolutely necessary, when designing by this formula, that the beam be really continuous both in design and construction; that the stresses due to negative bending moment at the support be provided for; that the steel be accurately located; and that, to obtain the best workmanship, the concrete be laid by a responsible builder and superintended by a man experienced in concrete construction.

An examination of the bending moment diagrams (Figs. 151 to 153) will show that under these conditions the value is conservative. For uniformly distributed load extending over all spans, the positive bending moment does not, except in the end spans, exceed $\frac{wl^2}{24}$ and for the most unfavorable panel loading it is for the middle of a span $\frac{wl^2}{12.5}$.

Many of the building laws in the United States, to provide for the possibility of poor construction or unforeseen conditions, give the more conservative figure, $M = \frac{wl^2}{10}$. For this reason and also because other assumptions may be provided for by multiplying by a decimal, this value is used in many of the tables in this book, and in fact it is advised for constructors who are not thoroughly familiar with reinforced concrete.

The same diagrams all show that the negative bending moments

are usually greater than the moments at the middle of spans. However, partial floor loading greatly reduces the negative moment, and as a live load is scarcely ever uniform over two full panels, it is considered safe to use the same value for negative moment as is used for the positive moment in the center of the beam, that is $M = \frac{wl^2}{12}$.

At the end support, the beam, if it runs into a column or heavy wall girder, may be practically "fixed" and thus require top reinforcement for the negative moment, $\frac{wl^2}{12}$, and the bars running into the support must be bent or otherwise anchored. Sometimes, if the slab has cross reinforcement running into the wall girder, it may be assumed to assist in connecting the beam and girder.

DESIGN OF WALL COLUMNS AND END BEAMS

In ordinary practice, beams and columns are built practically monolithic. Consequently, a bending moment is developed in the end beams at the wall support and is transferred to the wall columns, which must be designed accordingly. The following recommendations are from a more complete discussion* "Design of Wall Columns and End Beams," by Edward Smulski, where the theory is developed at length and tests are cited.

End Beams. In a properly designed beam built monolithic with the end column at the wall support, the positive bending moment in the center and the negative bending moment at the interior and exterior supports may be taken as given in the table on page 515.

For ordinary conditions, the bending moment at the wall column will be in the vicinity of $\frac{wl^2}{16}$. When a beam frames into a large column, the value of the negative bending moment approaches the limiting value of $\frac{wl^2}{12}$. On the other hand, if a deep beam is supported by a slender column, the restraint offered by the column is very small.

Wall Columns. The wall column is ordinarily subjected to stresses caused by the superimposed load and by the bending moment transferred from the beam to the column. In some cases additional bending moment in the columns is developed by an eccentrically applied

* See "Design of Wall Columns and End Beams" by Edward Smulski, Journal American Concrete Institute, July 191[illegible].

spandrel load. This may be either of the same sign as the bending moment from the end beam, in which case it should be added to it, or of opposite sign, when it should be subtracted in determining the resultant moment.

The bending moment in the wall column due to the end beam may be determined as follows:

For top story columns consider the bending moment in the end beam at the wall column (see table p. 515) to be resisted by the column just below the beam. This bending moment is negative, i.e., it produces tension on the outside face of the column.

For lower stories consider half of the bending moment to be resisted by the column above the beam and the remaining half by the column below the beam. The bending moment above the beam is positive and produces tension at the inside column face but below the beam it is negative as before.

Sometimes, to reduce the size of the top column, a joint is made at the under side of the beam, and the column reinforcement at the outside face is stopped there so as not to transmit any bending moment from the beam to the column. In such cases, however, the load from the beam must be considered as applied at the middle third of the column section.

Allowable Unit Stresses. The maximum allowable combined stresses in the columns due to flexure and direct stress may be assumed much larger than the allowable stresses for direct compression only, because the stresses are in the nature of fiber stresses, that is, they are not uniformly distributed over the whole cross-section, but are a maximum at the edge and decrease according to a straight line. The bending moment also decreases rapidly, and therefore the maximum stress acts only in a short length and on a small part of the cross-section. The same unit stresses, therefore, may be allowed as in a beam at the support, i.e., 15% above the regular fiber unit stress, with the understanding, however, that the uniform stress caused by the superimposed load alone does not exceed the allowable unit stress or axial loading.

To Design End Beams and Wall Columns. The table on page 515 gives values of the coefficient α in the formula $M = \alpha w l^2$.

The coefficients in the table were computed by the theory of least work and are given for different conditions occurring in practice. As is evident from the table, the coefficients depend upon the ratio of the moment of inertia of the beam and the column and also upon the ratio of the length of beam to the height of column.

Design the end beam at the wall support for total Bending Moment "At Wall Column."

In the top story column consider Bending Moment "At Wall Column" as resisted by the column just below the beam.

In lower stories consider half of this bending resisted by column above and half by column below the beam.

In the table below let

I_1 = moment of inertia of beam. l = length of beam.
I = moment of inertia of column. h = height of column.
$\frac{I_1}{I}$ = ratio of moments of inertia $\frac{l}{h}$ = ratio of length to height.

w = uniform load per linear foot of beam.

Moment Coefficients α for End Beams Connected With Wall Columns.

To be used in Formula $M = \alpha w l^2$

By Edward Smulski.

$\frac{I_1}{I}$	Ratio of Length of Beam to Height of Column, $\frac{l}{h}$									
	1		2		3		4		5	
	Bending Moment in End Beam.		Bending Moment in End Beam.		Bending Moment in End Beam.		Bending Moment in End Beam.		Bending Moment in End Beam.	
	At Wall Column.	At Center and at Interior Column.	At Wall Column.	At Center and at Interior Column.	At Wall Column.	At Center and at Interior Column.	At Wall Column.	At Center and at Interior Column	At Wall Column.	At Center and at Interior Column.
0.5	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{14}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
1	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{14}$	$\frac{1}{12}$	$\frac{1}{14}$	$\frac{1}{12}$	$\frac{1}{13}$	$\frac{1}{12}$
2	$\frac{1}{21}$	$\frac{1}{11}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{15}$	$\frac{1}{12}$	$\frac{1}{14}$	$\frac{1}{12}$
4	$\frac{1}{36}$	$\frac{1}{10}$	$\frac{1}{23}$	$\frac{1}{11}$	$\frac{1}{20}$	$\frac{1}{11}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{17}$	$\frac{1}{12}$
10	$\frac{1}{72}$	$\frac{1}{10}$	$\frac{1}{41}$	$\frac{1}{10}$	$\frac{1}{32}$	$\frac{1}{11}$	$\frac{1}{27}$	$\frac{1}{11}$	$\frac{1}{25}$	$\frac{1}{11}$

Note—Bending moment in beam at wall column and interior column, negative.
Bending moment in center of beam, positive.
Bending moment in column below beam, negative—above beam, positive.

VERTICAL AND HORIZONTAL SHEARING STRESSES

Concrete is strong in direct shear (see p. 337) and capable of standing a working shearing stress of at least 200 pounds per square inch, so that a concrete girder or beam or slab always has sufficient area of section to withstand this direct shearing stress. However, since the direct shearing stress is a measure of diagonal tension (see p. 516), which is excessive when the direct shearing stress is comparative low,

it must always be computed in a beam or girder for use in the computation of diagonal stresses, as described on page 517.

The shear is a maximum at the support, where it is equal to the reaction. Maximum shears for various loads are given in the diagram (Figs. 151 to 154, pages 505 to 508), in terms of the loads. While with uniform or symmetrical loading the reaction, and therefore the maximum shear, is one-half the total load upon the beam, it will be noticed from the diagram that where the end beams, in a series of continuous beams, are supported, which is very nearly the case when a beam runs into a light wall girder, the shear at the first support away from the end may be 25 per cent greater than normal, and should be specially provided for in cases like a warehouse where the full live load is liable to be constantly maintained. A further study of the four diagrams (Figs. 151 to 154, pages 505 to 508) will illustrate the cases where allowance should be made.

In case the concrete in a beam or slab has cracked vertically next to the support because of accident or poor design, the bearing value of the horizontal rods may have to be estimated.

Diagonal Tension. In addition to vertical and horizontal shear, concrete beams must be reinforced for diagonal tension, which in beams not properly designed causes diagonal cracks near the support. Stirrups and bent up bars are both effective in carrying diagonal tension. Since these cracks frequently start from tensile cracks forming along the bottom of simply supported beams near the support, it is necessary to bend up only a part of the tensile reinforcement. It is bad practice to bend up more than two-thirds of these bars. Similarly in continuous beams the tensile steel at the top over supports must be run well out on each side of the support. For more complete discussion of shearing stresses and diagonal tension, see Chapter XX, page 362.

Formulas for Shearing Stresses and Diagonal Tension. A convenient and safe method of determining the diagonal tension is by accepting for its measure the shearing unit stress, as discussed on page 365.

Let

V = total shear at section considered. (Reaction minus the loads between the support and the section.)

v = horizontal (or vertical) shearing unit stress at section considered.

b = breadth of beam.

b' = breadth of web of T-beam.

jd = moment arm or distance between center of compression and center of tension (approximately, in a T-beam, distance between center of slab and steel).

Z = total shearing stress or diagonal tension in a given length of beam, s.

s = length of the portion of the beam considered.

The following general principles and formulas are discussed on pages 362 to 375.

In the upper part of the beam the horizontal (or vertical) shearing stress varies according to a parabola from zero at the top to a maximum at the neutral axis. Below the neutral axis it continues constant (neglecting the tensile strength of the concrete) to the bottom of the beam. (See Fig. 103, page 367.)

The formulas for the unit shearing stress, the measure of diagonal tension, are

$$v = \frac{V}{bjd} \qquad (32) \qquad \text{For T-beams } v = \frac{V}{b'jd} \qquad (32a)$$

The vertical and horizontal shearing unit stresses at any point are equal.

If the width of the section below the neutral axis is not constant, the shearing unit stress will vary inversely with the width, b. The minimum b must be taken in figuring the maximum shearing unit stress and the maximum diagonal tension.

In continuous T-beams, near the support, the maximum shearing unit stress will be in the stem right under the flange. The shearing stress and diagonal tension in the plane of tensile steel is small because the width, b, being the total width of the flange, is large.

Distribution of Diagonal Tension to Concrete and Stirrups. Both concrete and web reinforcement carry diagonal tension stress and as a result of tests it may be assumed for design purposes that web reinforcement takes two-thirds and the concrete one-third (see p. 371). Hence, in the distance s, the web reinforcement resists the force $\frac{2}{3}\frac{Vs}{jd}$. Where the unit stress does not exceed the allowable limit, v', all stress is taken by the concrete. Other assumptions are discussed on page 371.

Area and Spacing of Vertical Stirrups. The area of steel and the spacing of stirrups may be found by placing the force to be resisted, as given above, equal to the working strength of the stirrups in tension.

Let

x = distance in feet from left support to point at which required spacing is desired.
x_1 = distance in feet from left support to point beyond which stirrups are unnecessary.
l = span of beam in feet.
w = uniform load in pounds per foot.
V = total vertical shear in pounds at section x feet from left support.
v = total shearing unit stress at section in pounds per square inch.
v' = allowable shearing unit stress (or diagonal tension) on concrete alone.
A_s = cross-sectional area of all legs of a vertical stirrup in square inches. (In a U-stirrup this is the sum of the area of the two legs.)
f_s = allowable unit stress in stirrups in pounds per square inch.
jd = distance in inches from center of compression to center of horizontal reinforcement. (In a T-beam, this may be taken as distance between center of slab and steel; in a rectangular beam, as 0.87 of the total depth to steel.)
b = breadth of beam in inches.
b' = breadth of web in T-beam in inches.
s = spacing of stirrups in inches at a place x feet from left support.

Since A_s is the area of all legs of a stirrup resisting diagonal tension in a distance, s, and f_s is the tensile strength of steel, the strength of the stirrup in pull is A_sf_s. The area of stirrups and the spacing may be found as follows:

The diagonal tension to be resisted is $\frac{2}{3}\frac{Vs}{jd}$. Hence, $A_sf_s = \frac{2}{3}\frac{Vs}{jd}$, and we get by solving for A_s and s,

$$A_s = \frac{2}{3}\frac{V}{f_sjd}s \qquad (33) \qquad \text{and } s = \frac{3}{2}\frac{f_sjd}{V}A_s \qquad (33a)$$

These formulas are recommended by the authors, but formulas for the area of stirrups and spacing, for other assumption of distribution of diagonal tension between stirrups and concrete are given on page 372.

Uniformly Distributed Loading. For uniformly distributed loading of w per lin. ft., the shear involving diagonal tension at any point distant from the support is $V = \frac{wl}{2} - wx = \frac{w}{2}(l - 2x)$, which, substituted above, gives:

$$A_s = \frac{w(l - 2x)}{3f_s jd} s \qquad (34) \qquad \text{and} \qquad s = \frac{3f_s jd}{w(l - 2x)} A_s \qquad (34a)$$

In T-beams use width of web b' in place of b.

Tables 9 and 10, on page 585 are recommended for general use.

Stirrups should be spaced by equation (33a) or (34a) up to a section where the unit shear equals the working shearing strength of concrete, bearing in mind, however, that the maximum spacing should not exceed three-fourths the depth of the beam.* The distance from the support to the point where no stirrups are required, for uniform loading is†

$$x_1 = \frac{l}{2}\left(1 - \frac{v'}{v}\right) \qquad (35)$$

From the above formulas it is evident that the necessary spacing of stirrups is inversely proportional to the total shear V at any point and hence is smallest at the end of the beam and increases toward its middle.

Many constructors advise the insertion of occasional stirrups throughout the entire length of the beam even if they are not theoretically necessary.

For a small beam where the stirrups are spaced uniformly, for convenience, only the minimum value of s needs to be figured.

Bent-Up Bars and Inclined Stirrups. For bent-up bars and inclined stirrups where the angle of inclination is 38° to 45°, the same formulas as given above may be used with the modification that $0.7\,V$ is substituted for V. The areas and spacing may also be found as above and then the area of stirrups multiplied by 0.7, or the spacing may be divided by 0.7.

Usefulness of Web Reinforcement. Numerous tests have demonstrated that a beam properly reinforced with stirrups or bent bars sustains three or four times as much load as the same beam without web reinforcement. The same tests, however, show that the web reinforcement retards the appearance of first diagonal cracks only very little and that the web reinforcement does not get any stress until the first crack appears. It has been noticed‡ also that under working loads (that is, before the diagonal tension exceeds the tensile strength of the concrete)

* The Joint Committee, 1916, reccomends a spacing for stirrups of one-half the depth of the beam and for bent bars, of three-fourths of the depth.

† The diagram of shearing unit stresses is a triangle (see Fig. 160) from which the distance x_1 may be obtained by the known rule $\frac{l}{2} \div \left(\frac{l}{2} - x_1\right) = v \div v'$. Solved for x_1 this gives the above.

‡ Bulletin No. 64, University of Illinois, Jan. 13, 1913.

the beam acts similarly to a homogeneous beam, and as would be expected, the stress in the stirrups is sometimes compressive instead of tensile.

Web reinforcement should always be used whether or not diagonal stresses are likely to occur under working loads. In beams without stirrups, final failure follows closely the appearance of the first crack, while with beams having web reinforcement, stirrups and bent bars represent a factor of safety which allows stressing of concrete in diagonal tension up to its ultimate strength. Under working loads the stirrups may not act, but in case of overstressing, due to faulty construction or to occasional excessive loading, the stirrups prevent the failure of the beam. The minute cracks that may open are not dangerous and in many cases are hardly visible.

Allowable Unit Stresses. The following unit stresses are recommended for concrete testing 2 000 lb. per sq. in. in compression at the age of 28 days.

(*a*) In a beam with straight bars only, the maximum shearing unit stress being the measure of diagonal tension) as determined from formula (32) must not exceed 40 lb. per sq. in.

(*b*) In fully reinforced beams, the shearing unit stress (being the measure of diagonal tension) may be increased to three times the shear allowed with straight bars only, that is, to 120 lb. per sq. in. One-third of this may be allowed on the concrete, and web reinforcement provided for the remainder.

These values must be used in determining the smallest allowable cross-sectional area of a beam.

In important beams and in beams subject to dynamic forces, as in bridges, it is advisable to place stirrups the whole length of the beam even if not required by the formulas.

Web Reinforcement for Continuous Beams. The formulas given above are based upon results obtained from the tests of simply supported beams. Their use also for continuous beams is recommended.

Methods of Web Reinforcement. Web reinforcement may be in the form of (*a*) vertical stirrups; (*b*) inclined stirrups or bent-up bars; (*c*) a combination of vertical or inclined stirrups and bent-up bars.

The most appropriate reinforcement to resist diagonal tension stresses would be steel bars placed in the direction of the stress. The inclination of the diagonal tension varies, however, in different parts of the beam, being steep near the end and flattening out toward the center of the beam. Since it is impracticable to vary the inclination of the stirrups,

when inclined stirrups are used, they are placed usually at about 40° with the horizontal. Although theoretically, inclined stirrups are superior to vertical stirrups, in practise they are less adapted for use as web reinforcement because it is difficult to attach them properly to the horizontal bars, and also difficult to keep them in place during construction.

Bent-up bars, if properly distributed and inclined, may be used as web reinforcement. They have an advantage over inclined stirrups because of a rigid connection to the tension steel. Adequate anchorage, however, must be provided at the top. In practice the number of

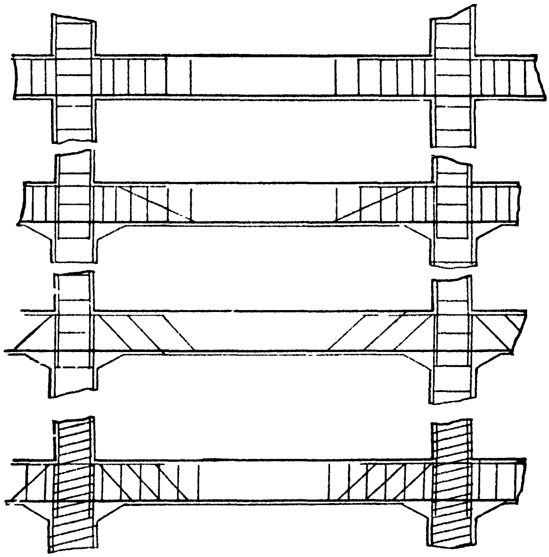


FIG. 155.—Reinforcement of a Continuous Beam. (*See p.* 522.)

bent bars is apt to be small and therefore the web reinforcement is concentrated at a few points instead of distributed through the length of the beam. In such cases bent-up bars alone do not form adequate web reinforcement and stirrups must be used in addition.

Vertical stirrups, although not placed in the direction of the stress, resist diagonal tension effectively. They are secure against slipping in a horizontal direction and are easy to keep in position during construction. Their number and spacing can be varied to suit conditions.

The most effective web reinforcement consists of a combination of bent-up bars and vertical stirrups, provided they are designed so as to give proper distribution throughout the length of the beam.

Types of Shear Reinforcement. Fig. 155 illustrates different types of diagonal tension reinforcement, showing beams reinforced with stirrups alone, with bent bars, and with a combination of bent bars and stirrups. The method of providing for the negative bending moment over the support is also indicated.

Fig. 156, page 522, shows different types of stirrups.

Illustration of Action of Web Reinforcement. Fig. 157 illustrates the action of vertical and inclined stirrups in a simply supported beam. Stirrups do not act until minute cracks open. After a crack forms, as in the figure, the reaction and the shear, V, tend to open the crack and cause failure of the beam. This tendency is resisted by the stirrups

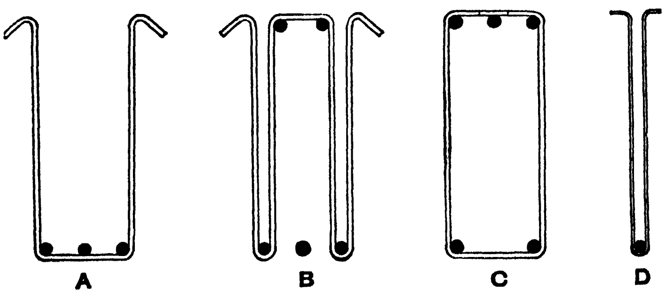


FIG. 156.—Types of Stirrups. (*See p.* 522.)

acting in tension. Figure 157 also represents what would happen if there was no web reinforcement. Fig. 158 represents the action of stirrups in a continuous beam. It may be noticed that the stirrup gets its stress at the top instead of at the bottom.

Design of Web Reinforcement. Slabs and rectangular beams can be designed so that the diagonal tension does not exceed 40 lb. In T-beams, however, it is always advisable from an economical standpoint to make the stem as narrow as possible, and therefore it is usually necessary to strengthen the web by some kind of web reinforcement. Web reinforcement is especially essential where the beams are subjected to dynamic action caused by moving loads, as in bridge design.

General principles of the design of web reinforcement are as follows:

(1) The web reinforcement must be securely wired to the tensile reinforcement since it receives its maximum stress there. (See p. 523.)

(2) In continuous beams, it must be remembered, the stirrups near

the support receive their stress at the top of the beam, where the tension steel is located, instead of at the bottom of the beam. (See p. 524.)

(3) Web reinforcement must be anchored in the compressive part of the beam to prevent its pulling out under stress. Anchorage may be obtained preferably by a curved hook, or if feasible, by a sufficient length of imbedment in the concrete, which may be obtained from formula (39), page 539, taking six-tenths of the length of one leg as effective in bond. Diameters to use are discussed on page 525.

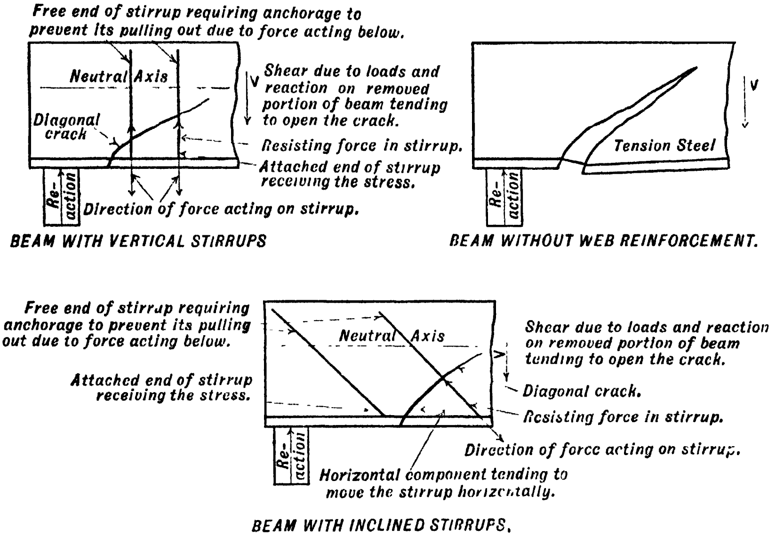


FIG. 157.—Action of Vertical and Inclined Stirrups in Simply Supported Beams. *(See p. 522.)*

(4) Bars should be bent at an angle with the horizontal not greater than 45° nor less than 30° to be considered effective web reinforcement.

(5) The spacing of stirrups, obtained from formulas (33a) and (34a), pages 518 and 519, to be effective, must not exceed three-quarters the depth of beam.

To Design Web Reinforcement. Determine the maximum total shear, V (see p. 516) and from this the shearing unit stress, v. See that v does not exceed the maximum allowable stress.* (See p. 573.)

*The Joint Committee, 1916, recommends for bent bars, or for stirrups simply looped about the longitudinal reinforcement, stresses ½ less than normal.

Vertical or Inclined Stirrups. If vertical or inclined stirrups only are used, determine the maximum diameter of stirrup. (See p. 525.)

Select the diameter and shape of the stirrup so that the minimum spacing is not too small (preferably not less than six inches), and the total number of stirrups in a beam not too large. Remember that the maximum spacing of stirrups in the part of beam where stirrups are required must not exceed three-quarters of the depth of beam. Use the same size of stirrups and the same design for the whole length of the beam and if possible for all similar beams in the entire structure, as a variety of designs may lead to errors and confusion.

Common sizes of stirrup bars are $\frac{5}{16}$-inch, $\frac{3}{8}$-inch, $\frac{7}{16}$-inch, and $\frac{1}{2}$-inch diameter in the shape of a U with the free ends hooked.

For uniformly loaded beams, vary the spacing as given on p. 526. The number and the spacing of stirrups for different conditions may be taken from the table on page 585.

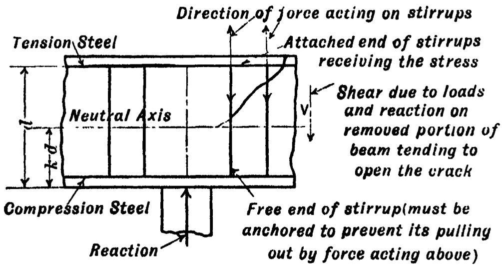


FIG. 158.—Action of Stirrups in Continuous Beams. (*See p.* 522.)

For concentrated loads, a graphical method of determining the spacing of stirrups is the most satisfactory. (See p. 528.)

Vertical Stirrups and Bent-Up Bars. Determine the maximum total shear, V, and shearing unit stress, v, as in previous case. (See p. 517.)

Determine the number of bars to be bent and the places where the bends can be made. (See p. 534.)

Select proper diameter of stirrup as suggested in previous case.

For uniformly distributed loading, either make the spacing of the stirrups constant and vary the spacing of the bent-up bars, or make the spacing of bent-up bars constant and vary the spacing of stirrups.

Remember that if bent-up bars can not be bent in places where they can resist diagonal tension or are bent in one or two places only, their full value as web reinforcement must not be counted upon. Bent-up bars may be considered as effective web reinforcement for a distance from the point of bending-up equal to three-fourths of the depth of the beam.

Maximum Diameter of Round or Square Stirrups with Straight Ends. (*See p.* 525.)
At least 50% Larger Diameters may be used with Hooked Ends.

DEPTH OF BEAM, d. INCHES.	VERTICAL STIRRUPS. Allowable bond unit stress. lb. per sq. in.		INCLINED STIRRUPS. Allowable bond unit stress. lb. per sq. in.	
	$u = 80$	$u = 100$	$u = 80$	$u = 100$
10	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$
15	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{8}$
20	$\frac{1}{4}$	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{7}{16}$
25	$\frac{5}{16}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{1}{2}$
30	$\frac{3}{8}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{5}{8}$
35	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{11}{16}$	$\frac{3}{4}$
40	$\frac{1}{2}$	$\frac{9}{16}$	$\frac{3}{4}$	$\frac{13}{16}$

Diameter of Stirrups. The diameter to select for stirrups is governed by the limiting spacing of the stirrups as given in the preceding paragraphs, by the bond of the stirrup prongs, and by convenience in selecting and placing the reinforcement. The effective length of the stirrup prong should be taken less than the total length because of the slight change in the intensity of shear below the neutral axis and because also a lower bond strength may be expected there.

Tests by Prof. Talbot indicate that it is safe to use up to at least six-tenths of the total length of the stirrup in figuring the bond.

The maximum diameter of stirrups with straight free ends which can be used by these assumptions without danger of slipping, as determined by the bond, is given in the table above. The unit stress in stirrups is assumed at 16 000 pounds per square inch.

For plain bars, the bond unit stress of 80 pounds per square inch may be accepted (see p. 567.) For deformed bars, this may be increased to 100 to 150 pounds per square inch according to the character of the bar.

It is evident from the above table that the diameters that can be used with straight free ends are smaller than practicable in most cases. Consequently, stirrups should be made with hooked ends.

Tests (p. 438) indicate that a right-angle bend of 5 diameters or a semicircular bend of similar length is sufficient to stress the steel to its elastic limit provided the hook is well imbedded in the concrete so that it cannot kick out. As a more conservative recommendation for practice, stirrups ranging from $\frac{5}{16}$ inch for beams 10 inches deep up to $\frac{3}{4}$-inch for 40-inch beams are advised with intermediate sizes for intermediate depths.

GRAPHICAL METHOD OF SPACING STIRRUPS

By the graphical method, lay out the length of the beam to scale and plot the shearing unit stresses (which are accepted as measures of diagonal tension) as ordinates in the respective points of the beam. The diagram of shearing unit stresses is similar to the shear diagram so that one can be used for the other by changing the scale.

Uniformly Distributed Loading. The shear is the maximum at the support and zero at the center so that the figure representing the shear (and also shearing stresses) will be a triangle as shown in Fig. 159.

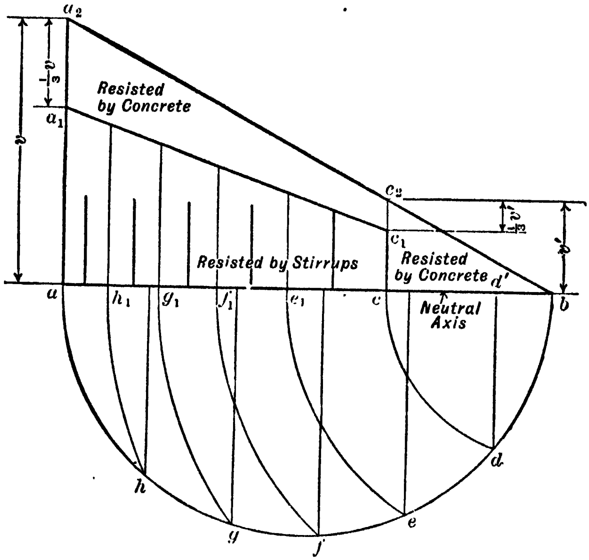


FIG. 159.—Spacing of Vertical Stirrups for Uniform Load. (*See p.* 526.)

Vertical Stirrups. Draw shear diagram Fig. 159. Determine section where total diagonal tension is resisted by concrete. To the right of c_2 all diagonal tension is carried by concrete. To the left, two-thirds is resisted by web reinforcement, and one-third, by concrete.

Mark off the diagonal tension resisted by concrete $a_1a_2c_2c_1$.

Find the total amount of diagonal tension equal to the area, aa_1c_1c times the width of beam, b.

Required number of stirrups equals the total diagonal tension divided by safe strength of one stirrup in pull, A_sf_s.

Divide area aa_1c_1c into the required number of equal divisions in the following manner:

Draw a half circle, taking ab as the diameter. With b as a center, and bc as radius, draw an arc till it intersects with the half circle at d. Erect a vertical dd'. Divide the distance, ad', into the required number of equal parts. From the points of division, drop verticals till they intersect with the circle, thus obtaining points e, f, g, h. With b as a center, draw arcs till they intersect with the line ab at e_1, f_1, g_1, and h_1. Verticals erected in the last mentioned points divide the trapezoid into the required number of parts. The stirrups are placed in the center of gravity of the divisions.

Analytically the same results may be obtained by using Table 10, p. 585.

Inclined Stirrups or Bent Bars. The diagonal tension to be resisted and the spacing may be determined by drawing a line from the center

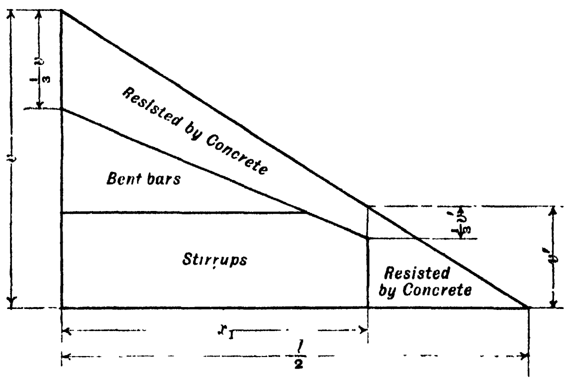


FIG. 160.—Spacing of Stirrups and Bent Bars for Uniform Loading. (*See p.* 528.)

of the span at the angle of inclination of the stirrups and projecting half of the span on this line. With this new line, a_1b, as a basis, one may proceed in exactly the same manner as in the previous case. It will be found, however, that for the same conditions the points of spacing of inclined stirrups on the neutral axis will coincide with the points obtained by the method suggested for vertical stirrups. The same method, therefore, may be used for inclined stirrups as for vertical stirrups. The points of division plotted on the neutral axis will give the point of intersection of inclined stirrups with the neutral axis. It must be remembered that the actual diagonal tension to be resisted by the inclined stirrups is seven-tenths of that to be resisted by vertical stirrups. For inclined stirrups, therefore, the area aa_1c_1c (Fig. 159) times the width of beam, b, must be multiplied by 0.7.

Combination of Stirrups and Bent Bars. In Fig 160 we obtain the trapezoid to be resisted by the web reinforcement in the same way as explained in connection with Fig. 159. Knowing the number and the strength of the bent bars in pull, we may mark off at the top a triangle of diagonal tension that can be resisted by the bent bars. The bottom part of the trapezoid then represents the diagonal tension to be taken by stirrups. The stirrups may be spaced uniformly, and the spacing of the bent bars may be easily determined by dividing the top triangle into the required number of parts, as explained in the previous example. If the bent bars cannot be bent in the required places, or if they are bent

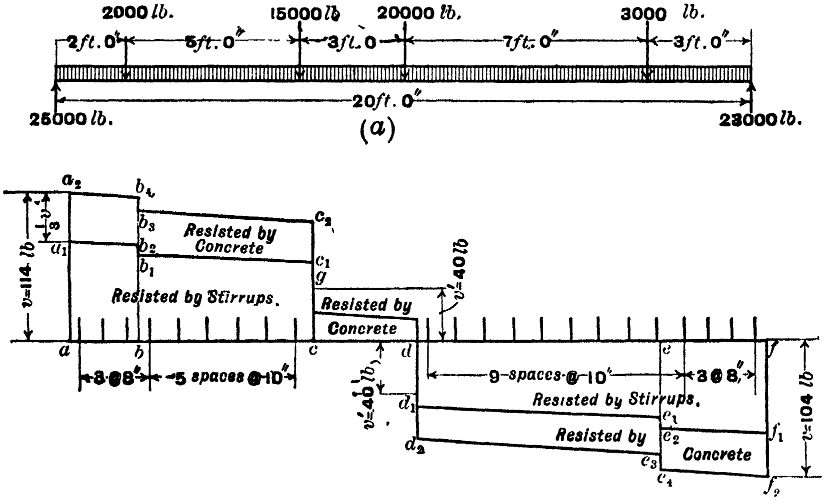


FIG. 161.—Spacing of Stirrups for Concentrated Loads. (*See p.* 529.)

in one place only, their full value in resisting diagonal tension cannot be counted on and stirrups must be used instead.

Concentrated Loads. For concentrated loads, the shear diagram (being a measure of diagonal tension) obtained by plotting the shear on the length of the beam will not always be a regular figure, as it depends upon the number of concentrated loads and their position.

One Load Concentrated in the Center. In this case the shear will be uniform for the whole length of the beam (except the small difference in shear caused by the dead load of the beam). After determining the shearing unit stress and selecting the kind of web reinforcement, determine the spacing by dividing the tensile value of stirrup by the shearing

unit stress times the width of the beam. The spacing then will be uniform throughout the beam.

Loads at Third Points. In this case, as in the last, the spacing of stirrups is uniform. The stirrups, however, will extend only from the support to the load. No stirrups are theoretically necessary in the middle third of the beam.

Concentrated Loads, Irregularly Spaced, with Uniform Load. Draw the shear diagram, Fig. 161, page 528, (as the measure of diagonal tension) and mark off the amount of shearing stress that can be taken by the concrete. Then from the figure, starting at the support, determine and mark off the area that can be resisted by one stirrup, and place the first stirrup in the center of gravity of that area. Next mark off the area for the second stirrup and place the stirrup and proceed till the total area is provided for. This method although the simplest that can be devised, is quite laborious. With some practice, however, it is possible to divide the shear diagram into equal areas without much figuring, as illustrated on page 528. After the diagram is drawn, it is easy to find the total amount of shearing stress and the number of stirrups required. Then with the diagram as a guide, the stirrups may be placed by inspection.

To illustrate the method more clearly the detailed computations are given for the number and the spacing of stirrups for a beam 20 feet long, the dimensions of which are $b = 10$ in., $d = 25$ in., $jd = 22$ in., and the loading as shown in Fig. 161.

Find first the reaction and then the total shear at points *a*, *b*, *c*, and *d*. Dividing the total shear by bjd, which in this case is 10x22 = 220, we get the shearing unit stresses at the respective points as follows:

Point	Total shear. Left	Right	Shearing Unit Stress. Left	Right
a................		25 000		114
b................	24 200	22 000	110	100
c................	20 200	5 200	92	23
d................	4 000	−16 000	18	−73
e................	−18 800	−21 800	− 86	−99
f................	−23 000		−104	

Lay out the shear diagram as in Fig. 161.

Find section where concrete can resist the total diagonal tension by drawing a horizontal line for $v' = 40$ lb. At the left end of the beam, this line strikes the outline at *g*. To the right of section *g* and to the left of *d*, all diagonal tension is resisted by concrete, and to the left of point *g* and to the right of *d*, one-third is resisted by concrete and two-thirds by web reinforcement.

Mark off the one-third area resisted by concrete.

Total amount of diagonal tension, the measure of which are the shearing stresses, for left end of beam equals the areas aa_1b_2b and bb_1c_1c times width, $b=10$ inches.

$$\text{Areas } aa_1b_2b + bb_1c_1c = \frac{2}{3}\left(\frac{114+110}{2}\right)\times 24 + \frac{2}{3}\left(\frac{100+72}{2}\right)\times 60 =$$
$$1\ 792 + 3\ 440 = 5\ 232 \text{ lb.}$$

Multiplying by $b=10$, we obtain 52 320 lb.

Using $\frac{1}{2}$-inch stirrups with two legs, area $A_s=2\times 0.196=0.392$ and

$$A_sf_s=0.392\times 16\ 000=6\ 272 \text{ lb.}$$

$$\text{Number of stirrups } N_s = \frac{52\ 320}{6\ 272} = 8.3 \quad \text{Use 9 stirrups.}$$

By trial, starting at the support, find shear areas on diagram equal to the resisting value of stirrup divided by $b=10$, or 627.2 lb. This may be done by scaling the ordinates above the line, af, and dividing 627.2 lb. by them. The space for the first stirrup would be about 9 inches, so by scaling the ordinate distant about 4 in. from a, we get the average ordinate equal to 73 lb. The first division, $\frac{627.2}{73} = 8.6$ inches may be laid off and the stirrup placed in the middle. Next scale an ordinate 4 inches from the end of first division and find the next spacing.

In our case, as the effect of the uniform load is small, we may simplify the matter by considering the ordinates in the portion aa_1b_2b and also in bb_1c_1c as constant and finding the spacings for these two constant values. Thus we find the spacing in portion ab to be 8.5 inches, and in portion bc, 10.5 inches. So we may make arbitrarily 3 spaces 8 inches and 6 spaces at 10 inches, giving the required number of stirrups.

The same method may be used in determining the spacing in the right end of the beam. The shear here is somewhat smaller, but it facilitates the erection to adopt the same spacing at both ends. Of course the stirrups must extend to the point d.

Bent Bars or Inclined Stirrups. When spacing bent bars or inclined stirrups by graphical method, we may proceed as in previous case except that the total amount of diagonal tension must be multiplied by seven-tenths. The spacing obtained gives the points of intersection of stirrups with the neutral axis.

If a combination of bent bars and stirrups is used, mark off on the dia-

gram the areas allotted to the bent bars and the stirrups and divide each of them separately into the required number of spaces.

STIRRUPS FOR MOVING LOADS

For beams carrying moving loads, as bridges and crane runways, it is necessary to determine for every section the maximum total shear, then draw the shear diagram and, bearing in mind that shear is the measure of diagonal tension, space the stirrups as suggested above. For heavy moving loads, it is advisable to add a certain percentage for impact, depending upon the character of the structure, the loading, and the

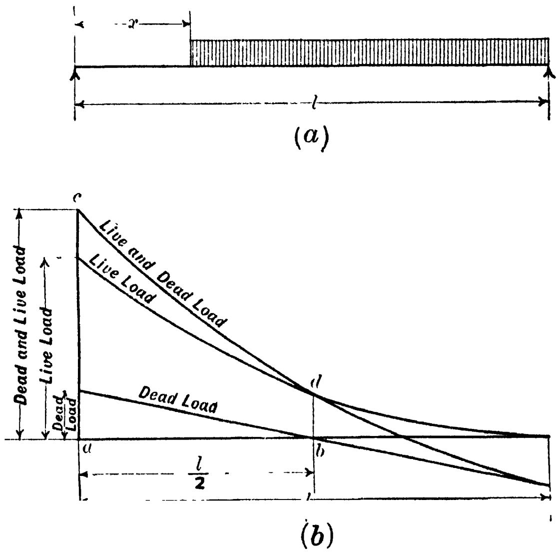


FIG. 162.—Shear Diagram for Uniformly Distributed Moving Load. (*See p.* 532.)

relation of the weight of the structure to the moving load. For railroad bridges and bridges carrying electric cars, the ordinary formulas may be used; for crane runways and highway bridges, from 25 to 50 per cent should be added. (See Chapter XXV.)

Shear Diagram for Uniformly Distributed Moving Load. Assume moving load w per lin. ft.; then the maximum positive shear at any section occurs when the load extends from the right support to the section under consideration and the portion between the left support and the section is unloaded. (See Fig. 162.)

The general equation of the maximum shear then is, $V = \frac{w(l-x)^2}{2l}$

This is an equation of a parabola.

For $x = 0$, $V = \frac{wl}{2}$; for $x = \frac{l}{2}$, $V = \frac{wl}{8}$; for $x = l$, $V = 0$.

To the shear due to moving loads, the shear due to stationary (dead) loads must be added. In Fig. 162 the diagrams for dead and live loads are drawn. Line (1) gives the diagram for dead load only; line (2) for moving load plus impact only; and line (3) for the sum of the two.

Stirrups must be provided for the sum of shears.

As the moving load can approach from either end, the spacings must be made the same for both ends.

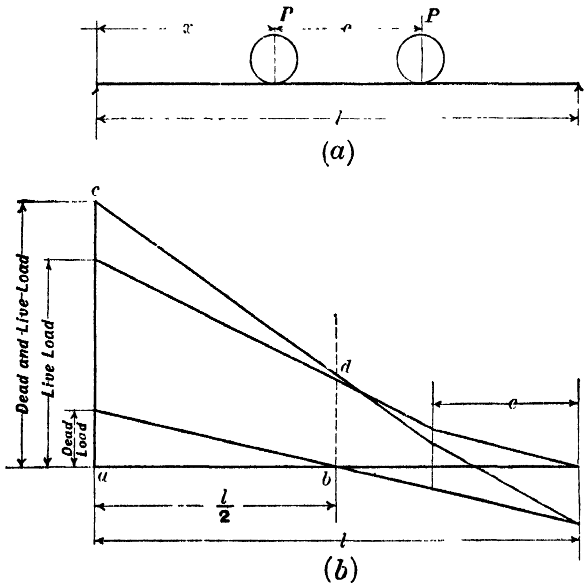


FIG. 163.—Shear Diagram for Two Moving Loads a Constant Distance Apart. (*See p.* 533.)

Shear Diagram for Two Equal Moving Loads a Constant Distance Apart. This case occurs in a beam carrying cranes and in highway bridges, (see p. 693.) The maximum shear is obtained by placing one load at the section considered. A general equation is:

Let

P = concentrated moving load.

e = constant distance between the loads P.

$$V = 2P\frac{l - \left(x + \frac{e}{2}\right)}{l} \quad \text{or} \quad P\frac{2(l - x) - e}{l} \text{ for } x < l - e, \text{ and}$$

$$V = P\frac{l - x}{l} \text{ for } x > l - e.$$

The variation in shear is a straight line for both equations. We need thus to determine two points. (See Fig. 163, page 532.)

Maximum shear where $x = 0$ is $V_{max} = P\frac{2l - e}{l}$ and

shear for $x = l - e$ is $V = P\frac{e}{l}$.

With these two values we may draw the shear diagram. To this must be added the shear due to the dead load. Having drawn the shear diagram, the spacing is determined as in previous cases.

For bridge design, the shear may be found as given in Chapter XXV on Bridge Design, the diagram plotted and the web reinforcement spaced as suggested above.

BOND OF STEEL TO CONCRETE IN A BEAM

The bonding of the steel to the concrete is discussed on page 429, the values being based on the resistance to slipping of a steel bar imbedded in concrete. **In a reinforced concrete beam the bond of the steel per unit of length must not exceed its safe working value.** The concrete surrounding the steel acts as a web between its tensile and compressive parts, and the pull in the rods as it becomes less and less, because of the reducing bending moment, passes into the beam, thus producing a bond stress between the steel and the concrete. If the bond is insufficient the rod will slip.

Care must be taken that the size of horizontal bars in a beam is not too large to give sufficient bond surface between the steel and the concrete.

Using the formula suggested by Prof. Talbot,* let

V = total shear.

v = shearing unit stress in lb. per sq. in.

u = bond unit stress in lb. per sq. in. of surface area of tension steel.

o = perimeter of bar in inches.

* Bulletin No. 4, University of Illinois, 1906, p. 19.

Σo = sum of perimeters of all horizontal tension bars at section considered.
jd = distance between centers of tension and compression.
d = depth from surface to center of tension steel.

Then*

$$u = \frac{V}{jd\,\Sigma o} \quad (36) \qquad \text{and} \quad \Sigma o = \frac{V}{jdu} \quad (36a)$$

These formulas apply to tension steel only.

The unit bond stress recommended by the Joint Committee for concrete whose strength is 2 000 pounds at 28 days is 80 pounds per square inch, and assuming also as a close approximation that $jd = \frac{7}{8}\,d$, the total perimeter of bars which is required at any point of a beam is

$$\Sigma o = \frac{V}{70\,d} \qquad (37)$$

The bond stresses being dependent upon the shear are, in a uniformly loaded beam, the maximum at the supports and decrease towards the middle. With concentrated loads, the maximum bond is at the support and is constant between the support and the nearest load.

In continuous beams at the support, this formula applies to the top steel which is in tension.

Special attention must be paid to bond in footings.

POINTS TO BEND HORIZONTAL REINFORCEMENT

The bending moment in a reinforced concrete beam decreases toward the ends, reducing in the same ratio the pull in the tension bars. Since these must be designed to take the maximum moment at the center of the beam, the steel at the ends, when the bars are carried horizontally through the whole length of the beam, is stressed away below its working strength. By bending up a part of the bars not required for tension, the inclined portion assists in providing for the diagonal tension, and by carrying the ends horizontally over the top of the supports the tension due to negative bending moment may be resisted there.

* The formula may be derived from the relation of the bond to the shear.

The tendency to slip, or the bond stress, is equal to the shearing stresses at the plane of bars because both are caused by the increment of the moment (see p. 368). Hence $u\Sigma o = vb$, from which, since $v = \frac{V}{bjd}$ then $u = \frac{V}{jd\,\Sigma o}$

The points where the bars may be bent may be obtained analytically.* It is easier, however, to obtain them graphically by means of the bending moment diagram. For uniformly distributed loads, one bending moment diagram may be used for a number of cases. For concentrated loads diagrams have to be made for each particular case, but they are quickly drawn.

Fig. 164 gives the bending moment diagram for simply supported beams, and also for the center span of continuous beams, and Fig. 165 for end spans of continuous beams. It may be noted that in the last two cases the curves for the negative bending moment are not a continuation of the positive bending moment curves. The reason for this is that the maximum positive bending moment is obtained for different positions of the loading than the maximum negative bending moment (see p. 504.)

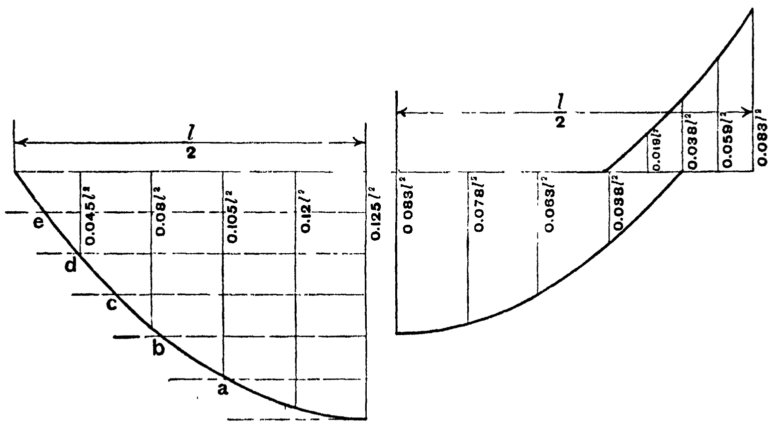


FIG. 164.—Bending Moment Diagram for Simply Supported Beams and for Center Span of Continuous Beams. (*See p.* 535.)

To use diagram proceed as follows: Suppose that a beam is reinforced in the center with six bars of the same diameter. The bending moment resisted by each bar is one-sixth of the total bending moment. Divide the ordinate of the maximum bending moment from the diagram into six equal parts and draw horizontal lines through the points of division (see Fig. 164); then, the distance between two successive horizontal lines will give the amount of the bending moment resisted by each bar. From the diagram we see that if the required number of bars at the center is

* Analytical treatment is given in editions of this book previous to 1916.

six, only five are needed at point *a*, four at point *b*, three at point *c*, and so on.

Points *a*, *b*, *c*, *d*, are theoretical points where one, two, and three bars respectively may be bent without overstressing the steel. To be well on the side of safety, it is advisable, however, to carry the bars beyond the theoretical points. A large proportion of the bars must not be bent at one place, neither must the angle of the bend be too steep. If the bars are bent at a steep angle and in one place, they suddenly stop being available as tensile reinforcement, and the stress in the remaining part will be increased suddenly with a consequent cracking of the beam at the point where the bars are bent up. An angle of 40° with the horizontal is satisfactory.

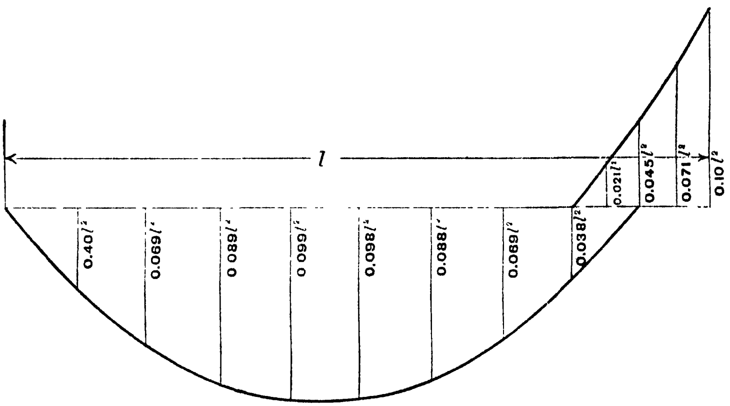


FIG. 165.—Bending Moment Diagram, End Span of Continuous Beams. (*See p.* 535.)

Having determined the points where bars may be bent see by the use of formula (39), page 539, that they are secure against slipping.

When bars are cut short at the places of reduced bending moment, (see p. 499) care must be taken that the number of bars to be cut short at one point be small (preferably one at a time) and the place where each bar is cut be a sufficient length distant from the theoretical point. The short bars also must be positively anchored to the remaining reinforcement, or else to the concrete above the bars by a curved hook, or by some other practical method. To illustrate the danger of cutting off the bars without anchorage, suppose that at a certain point we have four bars and the figured stress in each of them at this point is 8 000 pounds. So far as the allowable stresses in steel are concerned, two bars may be omitted at this point. If two bars are cut short here, we would have,

theoretically, in the long bars at the right of the point, a stress of 8 000 pounds, and at the left, 16 000 pounds. At the extreme ends of the short bars, there would exist an unbalanced stress of 8 000 pounds. As there is no provision for the large increase in stress of the longer bars and no means of transferring the stress from the short steel to the concrete, the short bars would tend to slip and form a crack. On account of the destroyed bond between the short bars and the concrete, they would then not be available as reinforcement for quite a distance and the stress would be thrown on the long bars.

The diagrams, Fig. 164 to 165, may be drawn to a large scale on tracing cloth, taking as a basis a span of say 10 feet. This diagram can then be used for spans of any length by multiplying the distances from the center to the point of bending bars by the ratio between the span and 10 feet. For dividing the bending moment into equal parts, the tracing may be placed over a cross-section paper on which the division lines may be marked.

LATERAL SPACING OF TENSION BARS IN A BEAM

The parallel bars in a beam must be a sufficient distance apart to properly transmit the stress to the concrete in the beam and prevent cleaving the concrete between the bars.

In practice it is advisable to make a rule that the rods shall not be spaced nearer together in the clear than 1½ times their diameter with 1 inch as a minimum. The minimum distance of the rods from the sides of the beam should be 1½ inches in the clear.

There is less danger of vertical than of horizontal splitting and where two layers of rods are used, the rods in a vertical plane may be placed directly over each other with sufficient space to permit the mortar to run between them. A limiting clear space of one-half inch is usually sufficient.

Prof. McKibben has suggested a mathematical method for determining the width of concrete required between the bars in order to make the resistance in shear equivalent to the adhesion of the concrete to the steel. A beam may fail in bond either by breaking the adhesion of the bars and the concrete around their whole circumference, or by shearing through the concrete between the bars on a plane with their centers and breaking the adhesion between the upper half of the bar and the concrete. To make the factor of safety equal for both cases, the shearing strength of the concrete between the bars must be equal to half the adhesion of the bars to the concrete. For a working bond unit stress,

$u=80$ lb. per sq. in., and a working direct shearing unit stress $v=120$ lb. per sq. in., and letting

s_b = distance in the clear between two bars in inches.

i = diameter of bar in inches.

Then*

$$s_b = 1.05\ i \tag{38}$$

Since the concrete is not easily placed between the bars, it may have a lower shearing strength there so that the lateral spacing of bars suggested above, is recommended.

DEPTH OF CONCRETE BELOW BARS

The selection of the thickness of the concrete below the bars is governed more by the proper fire and rust protection of the metal than by the stresses in the beam.

Prof. Charles L. Norton considers a thickness of 2 inches essential for efficient fire protection. (See p. 289.) Since an excessive thickness adds to the danger of cracking, because the tension in the concrete increases with the depth below the steel, this thickness, measured from the lower surface of the steel, and not from its center of gravity, may be taken as a maximum. For secondary members and floor slabs, ½ inch to 2 inches is enough.

The following thicknesses of concrete below the steel may be employed under ordinary conditions:

Thickness of Concrete below Steel.

Depth of slab or beam, inches	Thickness below lower surface of rods, inches
1½ to 2	½
2½ to 4	¾
4½ to 8½	1
9 to 12	1¼
13 to 18	1½
19 to 20	1¾
Greater than 20	2

The Joint Committee, 1916, recommends 2 inches for columns; 1½ inches for beams; and 1 inch for slabs.

* For a short length of bar l, equate the strength in shear of the concrete between the bars to the adhesion between the concrete and the upper half circumference of the bar.

Hence, if u = unit bond between steel and concrete. $s_b l v = \dfrac{\pi i l u}{2}$ $\quad s_b = 1.57 \dfrac{u}{v} i$

For $u = 80$ and $v = 129\ u$; $s_b = 1.05\ i$.

LENGTH OF BAR TO PREVENT SLIPPING

In cantilevers and restrained beams, also in the ends of columns, the full stress in the steel exists at the point of support. Therefore to transfer the stress from the bar to the support, the bars must be anchored to the support. The simplest means of anchoring is by imbedding the bar in the concrete of the support for a sufficient length **beyond the point of maximum stress** so that the bond (that is, the resistance to slipping) of the bar in the concrete is great enough to resist the direct tension or direct compression in the body of the bar. The support must be strong enough to withstand the stress transferred by the anchored member.

Whenever it is necessary to splice tension steel, the length of the lap is determined by the safe bond stress for the stress in bar at the splice.

Unless a bar is bent up or anchored by some mechanical means (see p. 438) the length of imbedment (also of lap) necessary to develop a required holding power through bond may be determined thus:

Let

f_s = actual tensile or compressive stress per square inch in the bar.
i = diameter of bar in inches.
u = bond in pounds per square inch of surface.
l_1 = necessary length of imbedment of bar in inches.

Then,* for both square and round bars.

$$l_1 = \frac{1}{4} \frac{f_s}{u} i \tag{39}$$

For 1 : 2 : 4 concrete of 2 000 lb. per sq. in. crushing strength, the necessary length of imbedment is

Plain Bars,		*Deformed Bars,*	
u = 80 lb. per sq. in.		u = 100 to 120 lb. per sq. in.	
f_s = 16 000	$l_1 = 50\ i$	f_s = 16 000	$l_1 = 40\ i$ to $33\ i$
f_s = 18 000	$l_1 = 56\ i$	f_s = 18 000	$l_1 = 45\ i$ to $37\ i$

For steel in compression, the stresses, f'_s, usually vary between 7 000 and 10 000 lb. per sq. in. The required length of imbedment, therefore, is smaller than given above for steel in tension and may be taken from table given on page 540.

* If the bar is round the total force to be developed in the body of the bar is $\frac{\pi i^2}{4} f_s$ while the holding power of the bar, or its resistance to slipping is $\pi i u l$. Equating these and solving for l, we obtain $l_1 = \frac{1}{4} \frac{f_s}{u} i$.

Length of Imbedment Required for Round or Square Bars.

STRESS IN STEEL f_s.	LENGTH OF BARS TO IMBED IN TERMS OF THE DIAMETER.				
	Allowable bond stress, pounds per square inch.				
Lb. per sq. in.	40	60	80	100	120
6 000	38	25	19	15	13
8 000	50	33	25	20	17
10 000	63	42	31	25	21
12 000	75	50	37	30	25
14 000	83	58	44	35	29
16 000	100	67	50	40	33
18 000	113	75	56	40	38
20 000	125	83	62	50	41

NOTE: The length of imbedment may be obtained by multiplying the value selected from this table by the diameter of the bar.

Recommendations as to the Size and Shape of Hook. Another expedient used is the bending of the end of the bar into a hook. The effectiveness of the hook, as shown by tests (see pages 438 to 439), depends to a great extent upon the strength of the concrete. In transferring the stress from the bar to the concrete, the hook exerts bearing stresses on concrete, which for a properly designed hook must not exceed the safe bearing stress on the confined concrete. The best anchorage is obtained by a combination of straight imbedment and a semi-circular hook, the diameter of which is at least four times the diameter of the bar. To insure against splitting of the concrete, it is advisable to place a cross-bar of proper length against the hook to distribute the bearing stresses on a large area of concrete. With a hook designed as suggested above, the elastic limit of the steel can be reached without causing excessive secondary stresses in concrete.

FLAT SLABS

The term "flat slab" is generally applied to the type of floor construction which consists of a flat plate, with no beams or girders, continuous over the whole floor and supported by columns only.

To reduce the thickness of the slab the column head is enlarged, thus shortening the span and increasing the shearing resistance at the support. The thickness of the slab at the center may be still further reduced and the rigidity of the structure increased by the use of a drop panel, or plinth resting on the column head. The sizes of column head and drop panel are discussed on page 551.

FLAT SLAB SYSTEMS

Several systems of reinforcement for flat slabs have been developed These may be classified, according to reinforcement, as the four-way, the two-way, and the circumferential system.

Four-way System. The four-way system, introduced by Mr. O. W. Norcross and Mr. C. A. P. Turner, consists of four bands of parallel

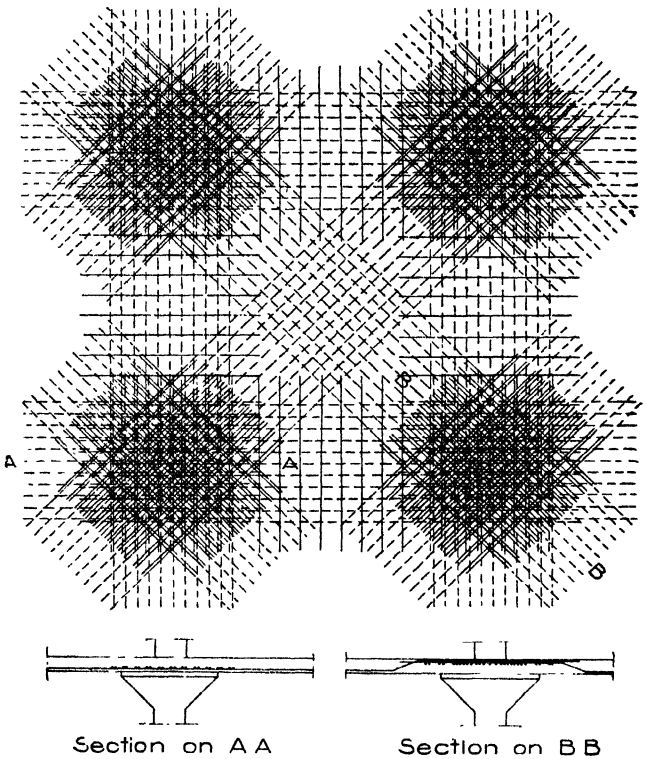


FIG. 166.—Four-Way Flat Slab System. (*See p.* 541.)

bars of small diameter placed lengthwise, crosswise, and in both diagonal directions, across the panels from column to column and across the columns, as shown in Fig. 166, page 541. All the bars are sometimes bent up over the column head, but the authors recommend that only two bands—usually the diagonal—be bent up at about the one-fifth point of the span to serve as tensile reinforcement, and that the other two bands continue across the column at the bottom of the slab to serve as compressive reinforcement. The compressive stresses in the slab at

the column head are sometimes high, but with only a single span loaded, the bottom of the slab is in tension. In either case the bottom of the slab should be reinforced.

Two-way System. The two-way system, developed by the Condron Company and the Corrugated Bar Company, consists of bars placed in two directions only, as shown in Fig. 167, page 542. The spacing of the bars right between the columns, where only one layer is used, is as

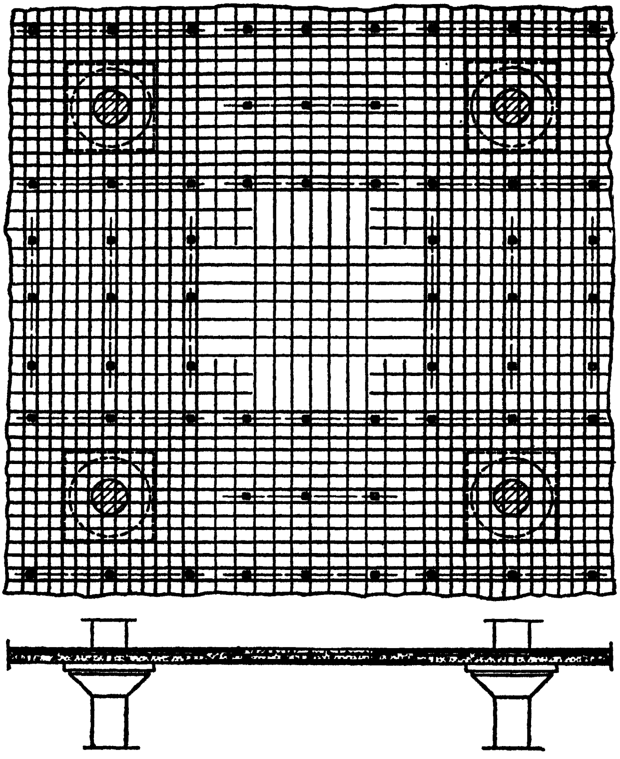


FIG. 167.—Two-Way Flat Slab System. *(See p. 542.)*

a rule closer than in the central portion of the slab where two layers of bars at right-angles are used. Part of the bars are carried through to serve as compressive reinforcement, and some of the bars are bent up at about one-fourth of the span and carried over the support to serve as negative bending moment reinforcement.

Circumferential System. The circumferential system, developed by Mr. Edward Smulski, (see Fig. 168, page 543,) consists of three types of units; one at the column head; a second between adjacent columns;

and a third in the center of the slab. Unit A at the column head consists of a combination of rings and hairpin-shaped bars placed radially as shown in Fig. 168, page 543. The upper prong, longer than the lower, runs from within the column head out beyond the point of inflection and serves as tensile reinforcement; the lower prong carries the compressive stresses; and the hook transfers the stress to the concrete through bond and bearing. The inner ring keeps the inner ends of the radials in place and reduces the bearing stresses. The rings on

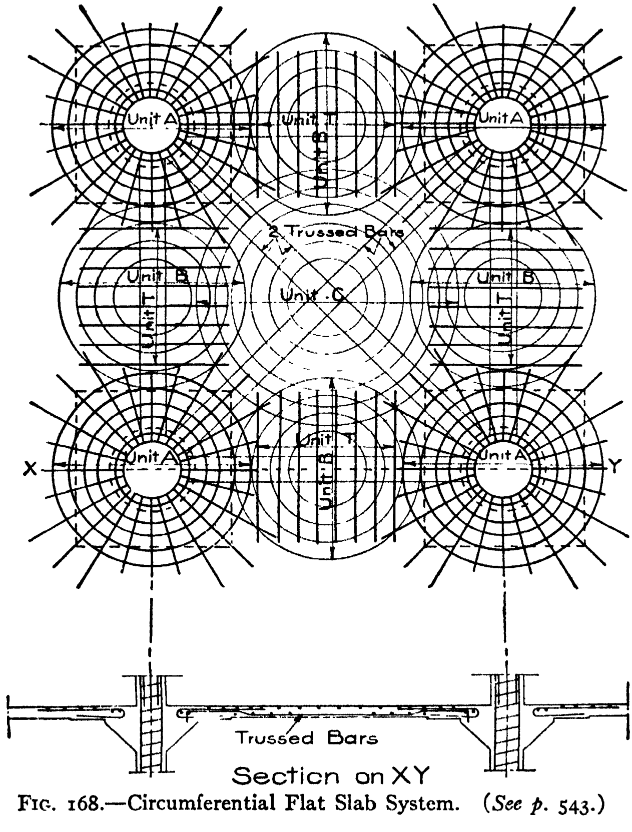


FIG. 168.—Circumferential Flat Slab System. (*See p.* 543.)

top of the radials resist circumferential and, with the radial bars, radial bending moment. The steel in Units A and C follows pretty closely the contour lines shown in Fig. 169, page 544.

Units B and C consist of two and four trussed bars, respectively, supporting rings, and extending from center ring to center ring and hooked around them. These trussed bars increase the steel at the column head.

The trussed bars are used to connect with the column head the parts of the slab under positive bending moment. The inclined part of the trussed bar, shown in section in Fig. 168, carries diagonal tension and the horizontal part in the top of the slab resists negative bending moment. The rings of the second and third units (those between columns) overlap and bind together all parts of the slab.

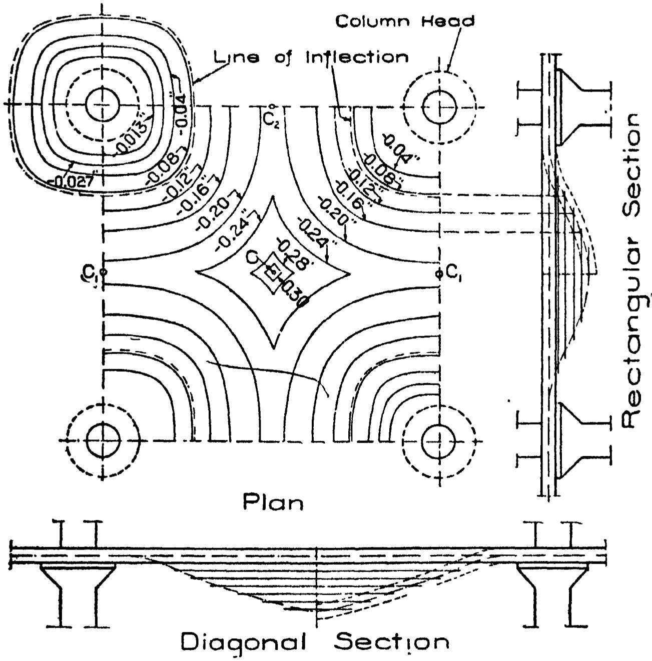


FIG. 169.—Contour Lines. (See p. 544.)

ACTION OF FLAT SLABS

To get an understanding of the action of a flat slab consider its deflection under load. Fig. 169 shows, by means of contour lines, the deflection of one panel taken from a continuous floor where it and the surrounding panels were uniformly loaded. The contour lines are curves connecting the points of equal deflection, *i.e.*, the points which, after loading, deflect an equal distance below the original level of the slab. The points were obtained by drawing cross-sections along the edge of the panel and along a diagonal section and plotting their deflection curves. This curve of deflection, just as for continuous beams, is determined by the requirements that the tangents at the support and at the center of the span shall be horizontal and that the points of in-

flexion shall be at about one-fifth of the span. Having drawn the deflection curves, the contour lines are plotted from the deflection curves. From the contour lines it is evident that a flat slab after deflection assumes the shape of an umbrella at the column head, of a trough between the columns, and of a saucer in the central portion.

Points of Inflexion. Since the bending moments change from negative at the column head to positive at the center of the slab, there must be a line of zero moment, or a line of points of inflection surrounding each column. This line of points of inflection divides the slab into circumferential cantilevers concentric with the columns and firmly clamped to them, and slabs extending between adjacent columns and supported on both ends, and square or rectangular panels supported on four edges.

Stresses at Column Head. It is evident from the contour lines and lines of deflection that the portion of the slab at the column head, which acts as a circumferential cantilever, is subjected to a negative bending moment causing tensile stresses at the top and compressive at the bottom. Assuming the umbrella shape, the slab undergoes deformation in two directions, namely, in the direction of the radius of any circle drawn on the slab around the column, and also along the circumference of the circle, because both the radius and the circumference are increased by the deflection. The particles, therefore, are subjected to two stresses, radial and circumferential, acting at right angles to each other. The reinforcement at the column head, therefore, must be placed in two directions, preferably radial and circumferential.

As explained in text books on mechanics, when a particle is subjected to forces acting at right angles to each other its actual deformation is smaller than if the stresses acted separately, because the deformation due to one force is decreased by the deformation of the force acting at right angles. The ratio of the decrease is equal to Poisson's ratio and is taken into account in fixing the constants on pages 547 and 548.

Stresses in Central Portion. The central portion of the slab between the points of inflection is subjected to positive bending moment, causing tension below and compression above. The portion of slab between adjacent columns develops stresses in one direction only since the contour lines are practically perpendicular to the center line through the columns. The portion in the middle of the panel, on the other hand, is stressed in two directions since the conditions there are somewhat similar to those at the column head, except reversed.

In the first case the steel must be placed in one direction only, while in the other case it must be circular or placed in two directions.

The bending moments recommended for design are given on page 547.

DESIGN OF FLAT SLABS

In designing flat slabs the following points must be considered:

(1) Stresses in concrete and steel at the column head due to negative bending moment.

(2) Stresses in concrete and steel in the central parts of the slab due to positive bending moment.

(3) Punching shear at the edge of the column capital.

(4) Diagonal tension at the capital, and edge of the dropped head.

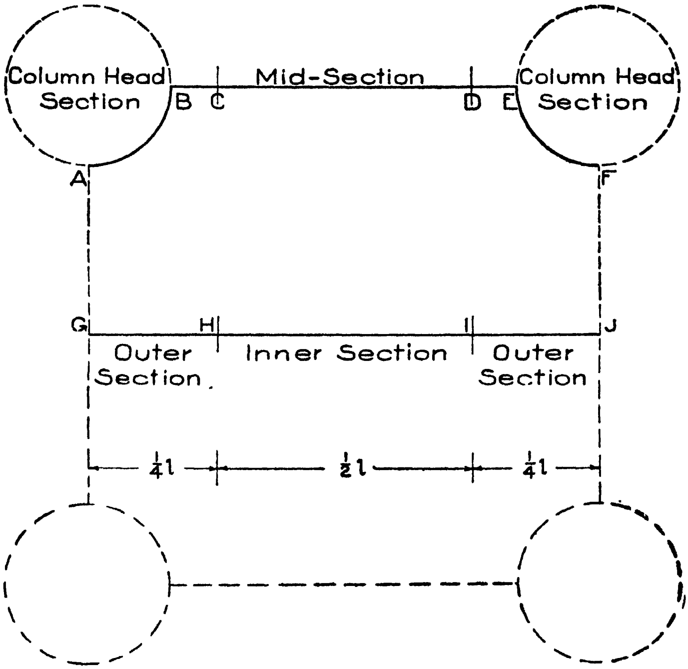


FIG. 170.—Division of Flat Slabs. (See p. 547.)

Bending Moments in Flat Slabs. The theoretical determination of bending moment is very complicated, but simple formulas based partly on theory and partly on tests have been evolved which give safe results.

For the purpose of determining stresses and bending moments the slab will be divided as shown in Fig. 170.

Negative Bending Moment. The negative bending moment, which acts in radial and circumferential directions, may be resolved into components acting along sections at right angles to each other. One section shown in the figure as *ABCDEF*, follows the circumference of the column head, and the panel edge from column head to column head.

The largest part of this negative moment is resisted by the portions *ABC* and *DEF*, called the column head sections, and whose projected width equals half the panel width. Section *CD*, half a panel wide, is called the mid-section. Similar bending acts at right angles to this section, and has the same relation to the column head and the panel.

Positive Bending Moment. The maximum positive bending moment acts along lines at right angles to each other drawn through the center of the panel, one of which is shown as *GHIJ* in Fig. 170. The sections *GH* and *IJ*, the combined width of which equals one-half of the panel width, is called the outer section, and *HI* the inner section.

FORMULAS FOR BENDING MOMENTS FOR FLAT SLABS

It is not necessary to use the full static moment, which would be obtained by following the method of analysis indicated above, because (1) The tensile resistance of the concrete appreciably reduces the stress in steel near support; (2) the stress in one direction is reduced by the resistance of the concrete in directions at right angles (see p. 476); (3) scarcely ever in practice are all panels of a floor fully loaded so as to produce maximum stresses at the column head; (4) experience with structures under load shows less stress and deflection than would be expected from the theoretical analysis.

In view of these considerations the authors recommend the use of coefficients lower than the purely statical analysis would require.* They correspond substantially to the requirements of the Chicago Building Code introduced in 1914.

Let w = total unit live and dead load.
l = length of panel.
c = diameter of column head.

Square Panels. Negative Bending Moment. For a square interior panel, the total negative bending moment along the line *ABCDEF* may be taken as $\frac{1}{17} wl\,(l-\frac{2}{3}c)^2$. Of this bending moment, 85 per cent should be provided for in the column head section *AB* and *EF*, and the remaining 15 per cent in the mid-section *CD*.

Square Panels. Positive Bending Moment. For a square interior panel the total positive bending moment along the line *GHIJ* may be taken as $\frac{1}{80} wl\,(l-\frac{2}{3}c)^2$. Of this moment, not more than 60 per cent should be placed in the outer sections *GH* and *IJ*.

Oblong Panels. The above formulas may be used for oblong panels in which the length of the panel does not exceed the width by more than

* The final report of the Joint Committee, 1916, uses a coefficient for negative moment, 12%, and for positive moment about 16% higher than those adopted by the authors.

5 per cent. In such a case the mean of the two sides should be taken as the length of span. For panels with larger variation in ratio of length to width, special formulas are required.

In addition to the notation on page 547,

Let l_1 = width of panel.
l_2 = length of panel.

Oblong Panels. Negative Bending Moment. For oblong panels on a section at the edge of the panel parallel to the width, l_1, as explained in connection with square panels, the negative bending moment may be taken as $\frac{1}{17} wl_2 (l_1 - \frac{2}{3} c)^2$, and along the length, l_2, the value is $\frac{1}{17} wl_1 (l_2 - \frac{2}{3}c)^2$. This bending moment should be distributed as recommended in connection with square panels.

Oblong Panels. Positive Bending Moment. For oblong panels the positive bending moment on a section through center of the panel parallel to the length, l_2, may be taken as $\frac{1}{30} wl_2 (l_1 - \frac{2}{3} c)^2$ and on a section parallel to the width, l_1, as $\frac{1}{30} wl_1 (l_2 - \frac{2}{3} c)^2$. The moments should be distributed among the inner and outer sections as recommended in connection with square panels.

Units. In all the above formulas, if w is in pounds per square foot and l, l_1, or l_2 in feet, the bending moment is in foot pounds. To get inch pounds, multiply the result by 12.

FLAT SLAB END PANELS

End panels must be subdivided into two classes, (1) where the wall columns are provided with brackets and concrete spandrel beams; and (2) where the wall panels rest on brick walls or on steel spandrel beams. In the first case, the slab is partially restrained at the wall column, and in the second case, it is simply supported.

End Panels with Spandrel Beams and Column Brackets. Since the slab is only partially restrained by the wall column, the bending moment at the first interior column head and in the center of slab is increased.

First Interior Column Head and Center of Span. The bending moment at the first interior column head and in the center of the span along sections parallel to the wall should be increased by 20 per cent. No increase is necessary at sections perpendicular to the wall.

Wall Column Head. The negative bending moment at the wall column should be 25 per cent smaller than given on page 547 for interior columns. Reinforcement should also be provided on the top of the slab at the juncture of spandrel and slab, the amount per foot computed for a bending moment, $\frac{wl^2}{80}$.

Spandrel Beams. The spandrel beams must be designed to carry, besides the brick wall, the live and dead load from the slab for one-fifth of the distance between the wall column and the adjacent interior column.

End Panels Resting on Brick Piers or Steel Spandrel Beams. Since the slab is completely unrestrained the bending moments must be increased above those where the slab is partly restrained.

Negative Bending Moment. For panels resting on brick piers, increase the negative bending moment at the nearest column along sections parallel to the wall by 30 per cent over that at interior columns.

Positive Bending Moment. The positive bending moment at sections parallel to the wall for interior panels should be increased 40 per cent over that given for interior panels on page 547.

Provision for Negative Bending Moment along the Wall. If the slab is freely supported on the wall and no masonry is placed above it, there can be no possibility of any negative bending moment, therefore, all the steel may be placed at the bottom. If the slab is restrained in any way there is a possibility of negative bending moment, and therefore of cracks and some reinforcement should be provided and hooked at the wall end. It may extend to about one-sixth of the span and need not exceed 0.25 per cent of the concrete section.

REINFORCEMENT FOR FLAT SLABS

After determining the bending moment the required area of steel is found in the ordinary way. (See formula (4a), p. 482). Only reinforcement crossing the section under consideration should be taken as effective in resisting the bending moment at that section. The effective area of bars crossing the section at an angle (as the diagonal bars in the four-way system or radial bars in the circumferential system) is found by multiplying their area by the sine of the angle between the bars and the perpendicular to the section. Since each ring in the circumferential system, is cut twice by any section, it may be considered as equivalent at that section to two straight bars, the sectional area of each being equal to the sectional area of the steel ring.

PUNCHING SHEAR AND DIAGONAL TENSION

Punching Shear. The punching shear at the edge of the column capital should not exceed the working value recommended on page 567.

Diagonal Tension. The critical sections so far as diagonal tension is concerned are (a) at the column head, and (b) at the edge of the drop

panel. At the column head the measure of diagonal tension is the unit shear, v, determined from formula (32), page 517, at a distance out from the column capital equal to the thickness of the slab plus the depth of the drop panel. At the edge of the drop panel the measure of diagonal tension is the unit shear, v, from formula (32) page 517, determined at a distance out from the drop panel equal to the thickness of the slab. The unit shear so determined must not exceed 60 pounds per square inch for 1 : 2 : 4 concrete. Larger values for v than in simple beams may be used, because failure by diagonal tension is retarded by the resistance of the slab adjoining the plane at which diagonal tension was figured.

In both of the above cases the total shear V in the formula is the vertical shear at the sections at which the unit shear is being figured.

DETAILS OF DESIGN OF FLAT SLABS.

Steel at the Column Head. If bands of steel are used, the reinforcement must be bent up and securely held at the proper distance above the form. The bands should be arranged in such a way that the negative bending of the steel is available where required. If the bars are extended beyond the column head and are assumed to take the bending moment on the opposite side of the column, they must be extended at least 6 inches beyond the point of inflection. The bars at the column head must be securely wired together so as to avoid the danger of misplacing during the pouring of the concrete. If practicable, the bars running over the columns shall be placed between those coming from the opposite direction so as to allow proper imbedment for all the bars. If the bars are not carried far enough across the column to serve as negative bending moment reinforcement, they must be extended a sufficient distance to develop their full strength. It is suggested that in such cases the bars should extend beyond the line through the center of columns 60 diameters with a minimum of 3 feet to allow for any discrepancy in the length of the long bars, and also to provide for any possibility of the bars extending farther over on one side than on the other, contingencies which are very apt to occur in the construction.

Bars should be actually bent and never allowed simply to sag to place, because the steel area will fall off more quickly than the bending moment so that the slab will be actually weaker away from the column than at the column head. This weakness does not show during the early stages of the loading because the concrete area is sufficient to take the stresses, but if the loading is continued, the slab eventually fails by

tension in the concrete instead of by tension in the steel. A good illustration of this case are the tests to destruction by Prof. Wm. H. Kavanaugh.* The failure of the slab occurred outside of the column by the breaking of the concrete.

In the circumferential system the rings at the column head should be properly lapped to develop the strength of the bars by bond as recommended on page 539. A still better plan is to hook the bars down when the hooked ends may be used as a chair to support the unit.

Column Heads. The column head may be considered as starting where the thickness below the slab is at least 2 inches and the shape of its cone must be such that the angle with the vertical must in no place be larger than 45°. The size of the column head is dependent upon the shear and compression in concrete. It is advisable to make the column head of a diameter equal to at least 0.225 of the span. The punching shear, as explained on page 520, must not exceed 120 pounds per square inch for 2 000 pound concrete, and the shear at a distance from the column head equal to the thickness of the slab (which may be considered as the measure of diagonal tension) must not exceed 60 pounds.

The cross section of the column head may be either octagonal or round. The forms are made of wood or of metal. In the latter case there is opportunity for ornamental treatment.

Drop Panel. The thickness of the slab at the column head may be increased by the introduction of a drop panel (sometimes called a plinth) either to decrease the shear or reduce compressive stresses. The width of the drop panel should be 0.4 of the span and its thickness should be limited to 0.6 of the depth of the slab.

The use of the drop panel reduces the amount of steel at the column head, but it complicates the form work. In many cases it is unnecessary.

Thickness of Slab. The thickness of slab is governed by the bending and shearing forces and sometimes, in the slabs, by deflection.

Let t = total thickness of slab in inches.
L = panel length in feet.
w = sum of live and dead load, pounds per square foot.

Then, $t = 0.023\ L\ \sqrt{w}$.

In no case should the slab thickness be less than 6 inches or less than $\frac{1}{32}L$. Roof slabs should be limited to $\frac{1}{40}L$.

* Flat Plate Theory of Reinforced Concrete Floor Slabs, by Henry T. Eddy, 1913, p. 71.

EXAMPLE OF BEAM AND SLAB DESIGN

The use of the formulas given in the preceding pages can be best illustrated by the design of a floor bay consisting of slabs, beams and girders. The design of reinforced concrete structures permits of so many variations by locating steel in different ways that more than one type of design for the same member is almost always possible. The dimensions and reinforcement shown illustrate common methods, and the arrangement of details in the different members is also given as typical. The principles of design follow the recommendations of the Joint Committee on Concrete and Reinforced Concrete, 1916.

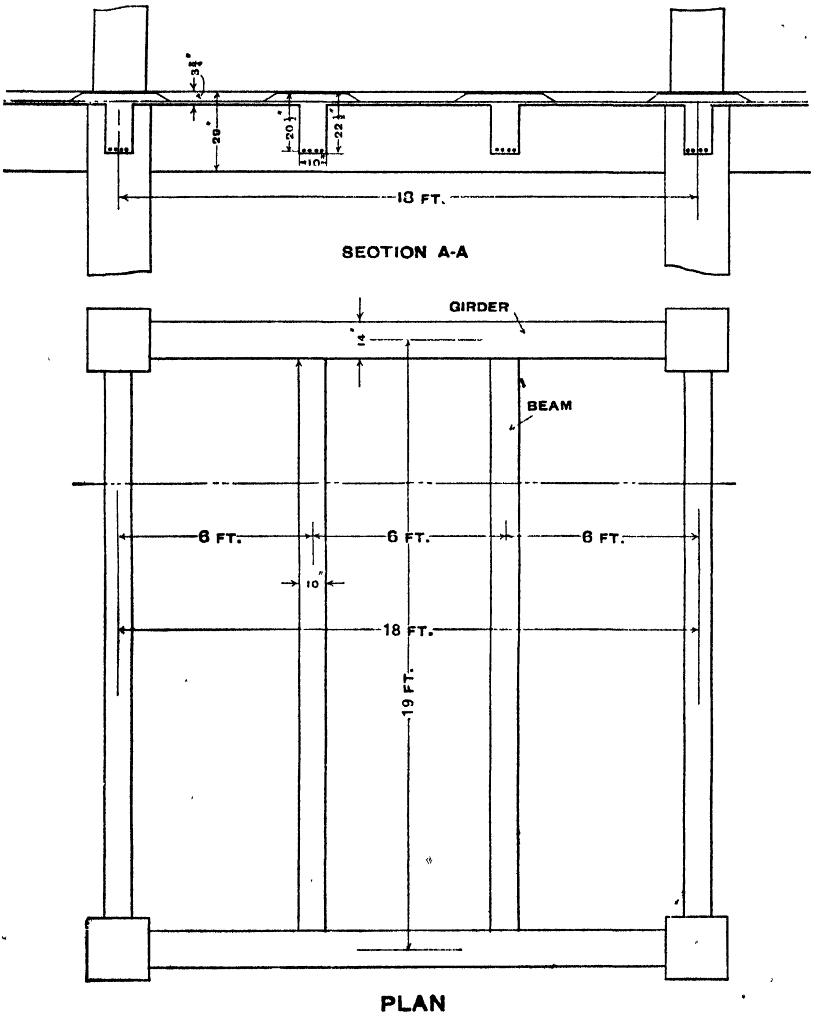


FIG. 171.—Design of Floor System. (See page 553.)

The computations are given with but few comments, but references are entered to the pages upon which each part of the calculation is based.

Example 8: Design a typical slab, beam and girder for a reinforced floor to support a live load of 250 pounds per square foot with columns spaced 18 by 19 feet on centers.

Solution: The girder will be made 18 feet long and the distance between centers of beams 6 feet. The beams are 19 feet long on centers.

	Refer to page
Take allowable fiber stress in concrete, 650 lb. per sq. in.	573
Take allowable tension in steel, 16 000 lb. per sq. in.	573
Take ratio of elasticity of steel to concrete, 15	477
Take direct shear in concrete, 120 lbs. per sq. in.	573
Take shear in concrete involving diagonal tension, 40 lb. per sq. in.	573
Take bond between concrete and plain bars, 80 lb. per sq. in.	573
Notation used in Example is Joint Committee standard	353

Slab. Span of slab is 6 ft.
Live load, 250 lb. per sq. ft.
Assumed dead load, 50 lb. per sq. ft.
Total loading, 300 lb. per sq. ft.

Use for moment, $M = \frac{wl^2}{12}$, then $M^* = \frac{300 \times 6^2 \times 12}{12} = 10\ 800$ in. lb. 512

Same value may be found directly from curves 606
Since $f_c = 650$, $f_s = 16\ 000$ and $n = 15$, then
$C_1 = 0.028$ and $p = 0.0077$, from table on page 483

Hence, depth to steel is, $d = 0.028 \sqrt{10\ 800} = 2.9$ in. 485

Taking $\frac{3}{4}$ in. concrete below steel, thickness of slab is $3\frac{3}{4}$ in. 538
Area steel, A_s, $= 2.9 \times 0.0077 = 0.0223$ sq. in. per inch of width 485
Round rods $\frac{3}{8}$ inch in diameter spaced 5 inches on centers will give required area. Table 1. To provide for bond stress use deformed bars 574
The same results may be obtained by using the Slab Table 5: 580

Since this table is based on $M = \frac{wl^2}{10}$ and we use here $M = \frac{wl^2}{12}$ the total unit load of 300 pounds per square foot may be reduced by $\frac{1}{6}$ or to 250 pounds and this value treated in the table, which gives a $3\frac{3}{4}$ inch slab.
Rods must be bent up to give same steel at top of slab over supports.

Beams. Span 19 feet.
Distance between beams, 6 feet.
Dead and live loads of the slab per foot of length of beam, $6 \times 300 = 1\ 800$ pounds.
Assumed dead load of stem of the beam, 200 pounds per foot of length.
Total unit loading, 2 000 pounds.

Use for moment $M = \frac{wl^2}{12}$, then $M = \frac{2000 \times 19^2 \times 12}{12} = 722\ 000$ inch pounds.

Reaction at support, which is the maximum shear, is

$$V = \frac{2000 \times 19}{2} = 19\ 000 \text{ pounds.}$$

Breadth of Flange. Taking 12 times the thickness of slab plus the breadth of stem of beam (assumed as 10 inches) $b = (12 \times 3\frac{3}{4}) + 10 = 55$ inches.

* Only one 12 is inserted in the numerator to change the 6 ft. to inches because the 300 is pound per foot.

Minimum Depth. The minimum depth, or depth at which steel and concrete stresses are the maximum permissible, must be found by trial. Assume a depth, d, and find from Table 11, page 586, the value of C_d and from Table 12, page 587, the corresponding value of j. Then from formula (17) page 489, compute the minimum depth. If it does not check the assumed depth closely another computation must be made. In this case assume $d = 11$ inches. Then $\frac{t}{d} = 0.34$ and, from Tables 11 and 12, $C_d = 45$ and $j = 0.876$. (See also example, p. 587.)

$$\text{minimum } d = \frac{722\ 000 \times 45}{0.876 \times 16\ 000 \times 55 \times 3.75} = 11.2 \text{ inches.}$$

A larger value of d will be used in order to reduce the steel.

Cross-section of Web as Determined by the Shear.

$$V = 19\ 000 \text{ pounds (see above) hence}$$

Refer to page

$$b'\left(d - \frac{t}{2}\right) \geqq \frac{19\ 000}{120} \text{ or } 158$$ formula (16), 489

Economical Depth. From formula (19), $d - \frac{t}{2} = \sqrt{\frac{rM}{f_s \times b'}}$ if the ratio of unit cost of steel to cost of concrete, $r = 70$ 490

$$\text{for } b' = 8,\ d - \frac{t}{2} = 19.85 \text{ inches or } d = 21.7 \text{ inch.}$$

$$b' = 9,\ d - \frac{t}{2} = 18.7 \quad \text{"} \quad \text{or } d = 20.6 \quad \text{"}$$

$$b' = 10,\ d - \frac{t}{2} = 17.8 \quad \text{"} \quad \text{or } d = 19.7 \quad \text{"}$$

For convenience in placing steel take

$$b' = 10 \text{ inches},\ d = 20\tfrac{1}{2} \text{ inches},\ h = 22\tfrac{1}{2} \text{ inches; hence } b'\left(d - \frac{t}{2}\right) = 186.$$ 537

With this value of d, $j = 0.918$. Table page 491, also Table 13, page 588

Sectional Area of Steel. From formula (20) 491

$$A_s = \frac{722\ 000}{0.918 \times 20.5 \times 16\ 000} = 2.4 \text{ square inches}$$

4 round bars ⅞ inches diameter will be sufficient. Two of these may be bent up and lap over the top of the support 496

Steel at Top and Bottom. Negative bending moment at support equals positive M at middle or $-M = 722\ 000$ inch pounds. 512

At support the flange of T-beam being in tension is negligible and since four ⅞-in. round bars are in tensile and two in compressive part of beam, the T-beam changes into a rectangular beam with steel in top and bottom.

The ratios of steel in tension and compression are respectively

$$p = \frac{2.4}{10 \times 20.5} = 0.0117 \text{ and } p' = \frac{p}{2} = 0.0058$$

With these values of p and p' and $a = 0.1$ we obtain, from Diagram 2 (p. 594), $\frac{f_s}{nf_c} = 1.5$. Assuming $j = 0.875$, the maximum tension in steel is

$$f_s = \frac{M}{jdA_s} = \frac{722\ 000}{0.875 \times 20.5 \times 2.4} = 16\ 750 \text{ lb. per sq. in.}$$ 496

and

$$f_c = \frac{f_s}{1.5\,n} = \frac{16\ 750}{1.5 \times 15} = 750 \text{ lb. per sq. in.}$$ 496

Refer to page

Allowable compression in concrete at the support may be 15% larger than that at middle, hence, no haunch necessary. 497

Girder. Span 18 feet, breadth to use for T-beam, 59 in. (assuming breadth of stem as 14 in.) 489
Concentrated loads at $\frac{1}{3}$ points.
Assumed dead load of the stem of the girder, 360 pounds per linear foot.
Load transmitted by the beams is considered as concentrated.
Reaction of concentrated loads, $V = 38\ 000$ pounds. 503
Maximum moment of concentrated loads with ends of beam simply supported would be, $M = 38\ 000 \times 6 \times 12 = 2\ 740\ 000$ inch pounds. 511
This corresponds to formula $M = \dfrac{wl^2}{8}$; to correspond to $M = \dfrac{wl^2}{12}$ it may be reduced by the ratio $\dfrac{8}{12}$ or 512

$M = 2\ 740\ 000 \times \dfrac{8}{12} = 1\ 827\ 000$ inch pounds.

Moment of dead load, $M = 116\ 600$ inch pounds, based on $\dfrac{wl^2}{12}$. 512
Total moment, $M = 1\ 943\ 600$ inch pounds.*

Minimum Depth. Assuming $d = 20$ and finding C_d and j from Tables 11 and 12 we have from formula (17), page 489,

$$\text{min. } d = \frac{1\ 943\ 600 \times 33}{0.917 \times 16\ 000 \times 59 \times 3.75} = 19.7 \text{ inches}$$

A somewhat greater depth is economical as shown below. 490

Cross-section Determined by Shear. $V = 38\ 000 + 3\ 240 = 41\ 240$ pounds 488
Using a limit of 120 pounds for total shear

$$b'\left(d - \frac{t}{2}\right) = \frac{41\ 240}{120} = 344 \text{ square inches}$$ 489

Select, by judgment
$b = 14$ inches, $d = 26.5$ inches, $h = 29$ in. (to allow for 2 layers of steel).

Steel Area. From Table 13, page 588, $j = 0.93$. Then from Formula (20) page 491, with $M = 1\ 943\ 000$, $A_s = 4.93$, for $\dfrac{d}{t} = 7.0$.

Check of Results by Exact Formulas (15) to (17) 356
(This check is unnecessary in practice for an experienced designer.)
$b' = 14$ inches, $b = 45 + 14 = 59$ inches, $t = 3\frac{3}{4}$ inches.
$A_s = 4.90$ square inches.

$$kd = \frac{4.90 \times 2 \times 15 \times 26.5 + 59 \times 3.75^2}{2 \times 4.90 \times 15 + 59 \times 3.75 \times 2} = \frac{3900 + 829}{147 + 442} = \frac{4729}{589} = 8.05 \text{ inches}$$

$$z = \frac{3 \times 8.05 - 7.5}{2 \times 8.05 - 3.75}\ \frac{3.75}{3} = 1.69 \text{ inches}$$

$$jd = 26.5 - 1.69 = 24.81$$

The value for jd would be a bit lower, when the compression in the stem is also considered. It is evident that the approximate value used in previous figuring, $jd = 0.936 \times 26.5 = 24.64$ in., is practically identical with the more exact moment arm.

Girder at Support.—$M = 1\ 943\ 600$ inch pounds. 511
Reinforcement at supports consists of $\frac{7}{8}$ inch round bars.
Eight bars are in tensile and four in compressive part of beam, hence

*By method suggested on page 502 the result would be 2 159 000 less 10 per cent. or 1 943 000 pounds, a result almost identical with the more exact one.

Refer to page

$$\text{ratio tension steel, } p = \frac{4.90}{14 \times 26.5} = 0.0132$$

$$\text{ratio compression steel, } p' = \frac{0.0132}{2} = 0.0066$$

With these values of p and p' and $a = 0.1$ we obtain, from Diagram 2 (p. 595), $\frac{f_s}{nf_c} = 1.4$

$$f_s = \frac{M}{jdA_s} = \frac{1\,943\,600}{0.875 \times 26.5 \times 4.9} = 17\,110 \text{ lb. per sq. in.}$$ 496

and

$$f_c = \frac{17\,110}{1.4 \times 15} = 815 \text{ lb. per sq. in.}$$ 406

which are excessive.

Depth and Length of a Haunch. For depth try $a = 0.1$, $d = 28$ inches 497
For this depth of beam the ratios of steel in tension and compression change to $p = 0.0132 \times \frac{26.5}{28} = 0.0125$, $p' = \frac{0.0125}{2} = 0.0063$.

From Diagram 2 (as before), $\frac{f_s}{nf_c} = 1.48$

$$f_s = \frac{1\,943\,600}{0.875 \times 28 \times 4.9} = 16\,200 \text{ lb. per sq. in.}$$ 406

and

$$f_c = \frac{16\,200}{1.48 \times 15} = 730 \text{ lb. per sq. in}$$ 406

This stress is allowable and the depth of haunch from top of beam of 28 inches will be accepted.

Length of haunch may be approximated Moment of resistance of beam without haunch. 496
$M_r = 0.875 \times 0.0132 \times 16\,000 \times (26.5)^2 \times 14 = 1\,820\,000$ inch pounds. Formula (18)
$M_b = 1\,943\,600$ inch pounds. 496
Hence from formula (28) length of haunch

$$x = \frac{123\,000}{1\,943\,600} \frac{18}{5} \times 12 = 2.6 \text{ inches}$$

Since maximum negative moment occurs in middle of column and necessary length of haunch is only 2.6 inches, no haunch will be introduced outside of the column.

Diagonal Tension Reinforcement of Beam. *Vertical stirrups* in beam on page 553

$V = 19\,000$ pounds $w = 2\,000$ pounds.
$b' = 10$ inches $jd = 18.625$ inches and the unit shear

$$v = \frac{19\,000}{10 \times 18.625} = 102 \text{ pounds per sq. in. (formula 32a),}$$ 517

Since this shearing unit stress is greater than 40 lb. per sq. in., stirrups are necessary.

Diameter of Stirrups. From Table on page 525, the maximum diameter for stirrups with straight ends for a beam 20.5 inches deep is ¼-inch. However, since U-shaped stirrups with hooked ends will be used ⅜-inch round bars will be selected.

Location of Stirrups. Stirrups are unnecessary [with $v = 102$ lb. and $v' = 40$ lb.] at a distance from the support $x_1 = \frac{19}{2}\left(1 - \frac{40}{102}\right) = 5.8$ feet.

Number and Spacing of Stirrups. Tensile value of one stirrup is $A_s f_s = 2 \times 0.11 \times 16\,000 = 3\,520$ pounds. Assume that stirrups take two-thirds of total diagonal tension. Total diagonal tension to be taken by stirrups may be represented by a trapezoid similar to Fig. 159. Area of the trapezoid times width of stem, b', is $\frac{2}{3}\,\frac{v+v'}{2} \times x_1 \times 12 \times b' = 32\,964$ lb. Divide this total diagonal tension by value of stirrup in pull gives 9.3 or 10 as the number of stirrups. The same results may be obtained more easily from Table 9, p. 585. Spacings of stirrups from this table, for $\frac{v}{v'} = \frac{102}{40} = 2.55$ are 4.9, 5.2, 5.3, 5.7, 6.1, 6.4, 7.0, 8.0, 8.9, 10.6 in. The same result may be obtained graphically. (See p. 526.)

MISCELLANEOUS EXAMPLES OF BEAM AND SLAB DESIGN.

Example 9. What is the value of C and the ratio of steel if pressure in concrete is limited to 500 pounds per square inch and pull in steel to 14 000 pounds per square inch, the ratio of moduli of elasticity being 15?

Solution: Approximate values, which are sufficiently exact, may be obtained from the Table 15, page 596, from which C equals 0.114, and ratio of steel, $p = 0.0062$.

Example 10: What is the value of C for a beam in which the pressure in the concrete is 650 pounds per square inch, the pull in the steel 16 000 pounds, and the area of steel 1.0%, the ratio of moduli of elasticity being 15?

Solution: The requirements in the example are impossible. With the pressure in the concrete limited to 650 pounds per square inch, the pull in the steel, if 1.0% is used, cannot be as high as 16 000 pounds. From Table 15, page 596, when $p = 0.010$ and $f_c = 650$, $C = 0.093$ and the pull in the steel is 14 000 pounds. Furthermore, comparing this item with the line for 0.008 steel in the same table, it is evident than an increase of 25% in the area of the steel, *i.e.*, from ratio 0.008 to ratio 0.010, decreases the value C, and therefore the depth of beam, only about 3%.

Example 11: What safe load per square foot can be supported by a slab 5 inches thick and 10-foot span reinforced with ½-inch round bars placed 8 inches apart?

Solution. From Slab Table, page 581, since the given reinforcement from page 574 is equivalent to $0.196 \times 1\frac{1}{2} = 0.294$ square inches for one foot of width, we find by inspection that for a 5-inch slab the nearest area of steel in column (18) is 0.288. Hence, the total safe load for a 10-foot span is slightly more than 136 pounds, say, 140 pounds per square foot; and deducting the weight per square foot of the slab, column (15), gives $140 - 64 = 76$ pounds per square foot safe live load. If slab is square, continuous and reinforced in two directions, the safe load of 140 pounds may be multiplied by 2. Deducting the dead load of 64 pounds, the live load will be $280 - 64 = 216$ pounds per square foot.

Example 12: What safe load per square foot can be placed upon an 8-inch slab 16 foot span, having steel reinforcement of 0.007?

Solution: Since by Rule 3, on page 580 total loads are inversely proportional to the squares of the span, the load for a 16-foot slab is ¼ the load for an 8-foot slab. For the total safe load of an 8-foot slab, we must interpolate between steel ratios of 0.006 and 0.008, thus obtaining

$$649 + 831$$

$= 740$ pounds per square foot. For the 16-foot slab the total safe load is therefore $\frac{740}{4} = 185$ pounds, and deducting the weight of the slab from column (15) gives a net live load of $185 - 103 = 82$ pounds per square foot.

Example 13: Using Table 4 of rectangular beams, page 578, what should be the dimensions and reinforcements of a beam 12 feet span, continuous and loaded uniformly with 1 000 pounds per foot of length?

Solution: The assumed stresses are the same as those adopted in the Beam Table. Assuming a width of beam 12 inches, a total load per inch of width of $\frac{1000}{12} = 84$ pounds per running foot. Referring directly to the Beam Table, we find that the total depth corresponding to a 12-foot beam with this load is about 12 inches. The reinforcement from column (25) is $0.083 \times 12 = 1.00$ square inch.

Example 14: What total load per foot of length can be carried by a 12-foot simply supported beam 12 inches wide and 25 inches deep?

Solution: There is no value in the Table 4, page 578, for a beam whose total depth is 25 inches, but since, from rule 4, loads are proportional to the square of the depth of the steel, we may calculate the load in this case from the load for a 26-inch beam 12 inches wide. Assuming in both cases that the depth to steel, d, is 2 inches less than the total depth, we have $364 \times \frac{23^2}{24^2} \times 12 = 4\ 000$ pounds per running foot of beam. Since the table is based on $M = \frac{wl^2}{10}$ for simply supported beams, deduct 20% from the above amount. Hence the safe load is $4\ 000 - 800 = 3\ 200$ pounds.

CONCRETE COLUMNS

Columns or piers of short length, not more than four times the least lateral dimension, may be built of plain concrete with no reinforcement provided the loading is central with no possibility of side thrust. The carrying capacity of such columns may be determined by multiplying the safe unit stress as given on page 573 by the effective cross-sectional area of the column.

The unit stress is determined by dividing the load to be sustained by the effective area.

Let

P = total load.
A = effective area of cross-section of column.
f_c = allowable compressive unit stress in concrete.

Then

$$P = Af_c \qquad (40) \qquad\qquad A = \frac{P}{f_c} \qquad (40a) \qquad\qquad f_c = \frac{P}{A} \qquad (40b)$$

Effective Area in Columns. The compression area used in computation should ordinarily be less than the total area to allow for surface damage in case of fire. An extra thickness where fireproofing is needed, varying from one to two inches in accordance with the inflammability of the contents of the building, should be allowed. The Joint Committee recommends that the protective covering shall be taken to a

depth of 1½ inches where fireproofing is required since in a severe fire the concrete to this depth may be affected by the heat.

The effective area of a hooped column must be taken as that within the hooping both for fireproofing and for strength.

Fireproofing. The steel in all cases should be imbedded at least 1½ to 2 inches. Where the fire risk is great, round columns should be used, as experience teaches that these suffer much less from extreme heat than square columns. Rounding or beveling the corners of square columns is always advisable.

DESIGN OF REINFORCED CONCRETE COLUMNS

Columns must be provided with reinforcement consisting either of vertical steel bars or of hooping, or a combination of the two.

Concrete is especially adapted for sustaining compression on account of its comparatively large compressive strength and its cheapness. Therefore, when conditions permit, the minimum allowable percentage of vertical steel should be used. For ordinary conditions, bars having a total cross-sectional area of 1% of the effective area of the column may be considered a minimum requirement. In building construction, it usually is difficult to keep the size of the columns, especially in the lower stories, within the limits required by the uses for which the building is constructed. To reduce the size of the columns, the following methods may be used separately, or in combination.

(1) Rich proportions of concrete.

(2) Increased amount of vertical reinforcement.

(3) Hooping with or without vertical steel.

(4) Structural steel shapes in combination with the concrete.

Rich Proportions of Concrete. The cheapest way of increasing the strength of a column is by using rich proportions of concrete, since the compressive strength of concrete is approximately proportional to the amount of cement which it contains. (See p. 454.) A rich concrete also works smoother in placing so that it is easier to produce a homogeneous column.

The strength of concrete for different mixtures is indicated on page 315, and recommended working stresses are given on page 573.

Vertical Steel Bar Reinforcement. The column may be strengthened by the introduction of vertical steel up to about 6% of the effective area of the concrete. Ordinarily it is not advisable, however, to use more

than 4% on account of the difficulty in accommodating any larger amount of steel, especially when the reinforcement is spliced by lapping.

Tests given on page 455 prove positively that the steel imbedded in concrete takes its proportion of compressive stresses as indicated in formulas (41) to (45), page 562.

Hooped Columns. As shown by the tests, hooping, if properly applied, increases the ultimate breaking strength of the column. The deformation, however, corresponding to the ultimate strength is excessive, so that the working stress in a hooped column must be based on the elastic limit of the column and not on ultimate strength (see tests, page 458).

Hooped columns without vertical steel are flexible and their use is not recommended.

Hooped columns with steel reinforcement are tough and more reliable than reinforced concrete columns with vertical steel only. If continuous spiral hooping is used, the danger of sudden failure, especially during construction, is lessened. An amount of spirals beyond 1% of the total volume of the column within the hooping does not seem to increase the elastic limit of the column so that it is economical to limit the amount of hooping to 1%. Not more than 6% of vertical steel should be used.

Recommendation for Column Design.* As a result of tests and practice, the authors recommend as follows:

(*a*) Piers, the length of which does not exceed four times the least lateral dimension and in which there is no danger of bending, may be built of plain concrete with allowable unit stresses given on page 573.

(*b*) Columns reinforced with not less than 1% of vertical steel and not more than 6%,† in which the unsupported length does not exceed fifteen times the least lateral dimension, may be designed by formulas on page 562 with unit stresses given on page 573. For longer columns, the working stresses should be reduced as stated on page 466.

(*c*) Columns reinforced with structural shapes, the area of which exceeds 6% of the cross-sectional area of concrete, may be designed as specified on page 563.

* These recommendations agree with those of the Joint Committee on Concrete and Reinforced Concrete except as indicated.

† Joint Committee limits the amount of vertical steel to 4%.

(*d*) Hoops and bands should not be figured as adding directly to the strength of the column.

(*e*) Columns should not be designed with hoops alone.

(*f*) Columns reinforced with not less than 1% and not more than 6%* of longitudinal bars and with not less than 1% in bands or hoops may be given a working stress in concrete 55% higher than columns with vertical steel only. The ratio of unsupported length to the diameter of the core in such columns must not exceed 10. If for 2 000-lb. concrete, the unit working stress in concrete for columns with vertical steel only is taken as 450 lb. per sq. in., the hooped and vertical reinforced columns may be thus given 700 lb. per sq. in. To the strength of the concrete in the column also must be added the strength of the steel according to the formula given below.

Design of Columns with Vertical Steel only and of Hooped Columns with Vertical Steel. The formulas given below apply to columns with vertical steel bar reinforcement and also to hooped columns. The difference between the two types is taken care of in the unit stress. The unit stresses, f_c, recommended, are given on page 573.

The derivation of the formulas is given on page 376.

Let

f = allowable average unit pressure upon the reinforced column, equal to the total load divided by the effective area.

f_c = allowable unit pressure upon the concrete of the column.

f'_s = allowable unit pressure upon the vertical steel in the column.

$n = \frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to modulus of elasticity of concrete.

P = load to be sustained by the column.

A = area of total effective cross-section of column (see p. 558.)

A_c = area of concrete in cross-section.

A_s = area of steel in cross-section.

$\frac{A_s}{A} = p$ = ratio of cross-section of steel to cross-section of column.

Formulas to be Used in Reviewing Columns Already Designed.

Find safe load, P. Given: unit stress, f_c; effective cross-sectional area of column, A; area of steel, A_s.

* Joint Committee limits the amount of vertical steel to 4%.

$$P = f_c\,[A + (n - 1)\,A_s] \qquad \text{or} \qquad (41)$$

$$P = f_c\,A\,[1 + (n - 1)\,p] \qquad (41b)$$

Find unit stress in concrete, f_c, and in steel, f'_s. Given: load, P; effective area of column, A; and area of steel, $A_s = pA$.

$$f_c = \frac{P}{A + (n - 1)\,A_s} \qquad \text{or} \qquad (42)$$

$$f_c = \frac{P}{A\,[1 + (n - 1)\,p]} \qquad (42a)$$

$$f'_s = nf_c \qquad (43)$$

Formulas to be Used in Designing Columns. *Find required area of steel, A_s, or required ratio of steel, p. Given: load, P; unit stress, f_c; and effective area of column, A.*

$$A = \frac{P - f_cA}{f_c(n - 1)} \qquad (44) \qquad\qquad p = \frac{P - f_cA}{f_c(n-1)A} \qquad (45)$$

Relation between the average unit stress, f, (which equals the load, P, divided by the effective area, A,) and the allowable unit stress in concrete, f_c;

$$f = \frac{P}{A} = f_c\,[1 + (n - 1)\,p] \qquad (46)$$

Values of f for different percentages of steel are given on page 599.

Data for Designing Spirals. The following formulas can be used to advantage in determining the pitch of the spirals, length of spirals, and weight of spirals.

Let

d = diameter of column in inches.
A_s = cross-sectional area of wire used for spiral.
s = pitch of spiral in inches.
L = length of spiral in feet per height of column.
h = height of column in feet.

Then

Pitch of spiral* for given percentage of spiral reinforcement and cross-sectional area of wire is

$$s = \frac{4A_s}{dp} \tag{47}$$

If A_s is in square inches and d in inches, the pitch is in inches. Where one per cent of spirals is used as recommended on page 561, the pitch of spiral equals

$$s = 400\frac{A_s}{d} \tag{48}$$

Total length of spiral in feet in a column of a given height, h, in feet, and a given pitch, s, in inches, equals

$$L = \pi\,\frac{dh}{s} \tag{49}$$

The weight of spiral can be obtained from the above formulas by multiplying the length of spiral by the weight per foot of bar used for spirals.

The above formulas are approximate because they consider the length of spiral as equal to the circumference of the column. The error is largest for small diameters of columns and large pitch, as then the difference between the actual length of spiral and the approximate length of spiral is largest. For columns ordinarily used in practice and small pitch, the error is only a fraction of 1 per cent. The maximum of error in spacing of spirals in any case does not exceed 2 per cent.

COLUMNS WITH STRUCTURAL STEEL

Structural Steel Reinforcement. Sometimes, when, in order to reduce the size of the column, larger percentage of steel is required than 4%, structural shapes are used for column reinforcement. If small struc-

* Above formulas were derived as follows:

Length of spiral per pitch equals πd inches (because of the pitch, the circumference of the spiral can be taken as equal to the circumference of the column). The total volume of spiral reinforcement per foot of height of column for a given ratio of steel to concrete, p, equals

$$\frac{\pi d^2}{4} \times 12 \times p = 3\pi d^2 p$$

Since the length of spiral per pitch equals πd inches and its volume equals $\pi d A_s$, the number of spirals per foot is obtained by dividing the volume of the required spiral reinforcement found above by the volume of one spiral, which is $\frac{3dp}{A_s}$. Finally the pitch of spiral is obtained by dividing 12 inches by the number of spirals required per foot.

Therefore, $s = \frac{4A_s}{dp}$.

tural members, such as angles, are used and the ratio of area of concrete to the area of steel does not exceed 6%, the formulas given for columns with vertical bars may be used. It is advisable, however, to tie the angles by occasional tie plates, or lacing, to keep them in place during erection.

In building construction, it is sometimes necessary, to reduce the size of column, to use structural steel columns of an area up to 15 or 20% of the concrete area. In this case, the column must be considered as a structural steel column strengthened by the concrete. The best structural steel shapes to be used are shown in Fig. 142, page 463. When properly laced, the strength of such columns may be considered as equal to the strength of the structural column, computed in the same fashion as for structural columns not imbedded in concrete, plus the strength of the concrete core,—that is, of the concrete enclosed by the steel, which may be figured on the basis of the unit stresses recommended for the columns with vertical steel only.

COLUMNS UNDER FLEXURE

Columns subject to an eccentric load require special formulas: these are given in Chapter XX on pages 377 to 389.

COLUMN TABLES

Table 18, page 599, gives values for f, for different values of p and f_c. Tables 19 to 21 give safe loads for columns of different diameters, with vertical steel and with spirals, for different mixes of concrete. For column details see Chapter XXIII.

COLUMN EXAMPLES

Example 15: What size of square column reinforced with 2 per cent. of longitudinal bars without spirals will be required for a load of 94 000 pounds?

Solution: By column (4), page 573, the allowable compression on 2 000 pounds concrete is limited to 450 pounds per square inch. For this allowable stress, using 2% of longitudinal reinforcement and a ratio of moduli of elasticity of 15, the area of column from formula (46), page 562, is

$$A = \frac{94\,000}{450\,(1 + 14 \times 0.02)}$$

= 163 square inches, corresponding to 12.8 inches square. Allowing 2 inches for protective covering gives 14.8 inches, or, say, 15 inches square.

Example 16: What sectional area of vertical steel will be required for a round spiraled column limited to 36 inches diameter, which has to bear 1 000 000 pounds with pressure in plain concrete limited to 450 pounds per square inch?

Solution: By column (4), page 573, in a column reinforced with vertical bars and 1% of spirals, the allowable pressure on the concrete may be increased 55% over that on plain concrete, hence $f_c = 450 + 55\% = 700$ pounds per square inch. Considering the area within hooping equal to $\frac{33^2 \times 3.14}{4} = 858$ square inches as effective, the unit pressure from page 558, will be

$$f = \frac{1\ 000\ 000}{858}$$

= 1160 pounds per square inch. Assume $n = 15$, then by transposing formula (46), page 562,

$$p = \frac{1160 - 700}{14 \times 700}$$

= 0.047, and area of steel $A_s = 858 \times 0.047 = 40.4$ square inches. From table on page 574, 22 square rods $1\frac{3}{8}$ inches thick are chosen.

Example 17: What should be the area of a column 10 feet high supporting 1 000 000 pounds, reinforced with 3.5% of longitudinal reinforcement and 1% of hooping for $n = 15$ and an allowable compression in plain concrete limited to 450 pounds?

Solution: Since the column is reinforced with longitudinal and hooping reinforcement, the unit compression on concrete may be taken as $f_c = 450 + 55\% = 700$ pounds per square inch (column (4), page 573.) Then from formula (46), page 562, the column area is

$$A = \frac{1\ 000\ 000}{700\ (1 + 14 \times 0.035)} = 960 \text{ square inches,}$$

requiring a 35-inch diameter inside the spiral.

REINFORCEMENT FOR TEMPERATURE AND SHRINKAGE STRESSES

All masonry is subject to temperature cracks, but when they are distributed in the many joints between bricks or stones they do not show so plainly as on the smooth surface of concrete.

Expansion from a rise in temperature rarely causes trouble except at angles where the lengthening of the surface may produce a buckling or a sliding of one portion of the wall past the end of the other. In a building, the walls and floors are generally so well bonded together and free to move

as a unit, that no provision need be made for expansion. In a structure like a square reservoir, the effect of expansion must be taken into account in the design to prevent failure at the corners.

Contraction is often more serious, although cracks are by no means necessarily dangerous. To prevent cracking due to the shrinkage of the concrete in hardening (see p. 261) or to the lowering of the temperature, reinforcement should be inserted or joints formed to localize the cracks. (See p. 259.)

Reinforcement properly placed distributes the contraction stresses so as to make the cracks very small, practically invisible, but it does not prevent them entirely.

The steel must be sufficient in quantity, and should be of small diameter and placed as close as practicable to the surfaces to distribute the cracks and thus make them very fine. Deformed bars, that is, bars with irregular surfaces which provide a mechanical bond with the concrete, are more effective than smooth bars, and steel of high elastic limit also is advantageous.

In practice, from $\frac{2}{10}$ of 1% to $\frac{4}{10}$ of 1% (a ratio of 0.002 to 0.004) of steel, based on the cross-section of the concrete, is commonly used as temperature or shrinkage reinforcement.

The tensile strength of concrete is so low that a small change in temperature will crack it. For example, the coefficient of expansion of concrete is 0.0000055 (see p. 261) and the modulus of elasticity is generally assumed as 2 000 000; therefore, the stress (see p. 400) per degree Fahrenheit is $0.0000055 \times 2\ 000\ 000 = 11$ pounds per square inch, and a fall in temperature of $\frac{300}{11} = 27°$ is sufficient to crack a concrete the tensile strength of which is 300 pounds per square inch.

It is evident, and it has been proved by experience, that there is less cracking in concrete laid in cold than in warm weather.

Longitudinal reinforcement is especially necessary in conduits which must be water-tight.

Shrinkage cracks due to the hardening of the concrete may be prevented by keeping the concrete wet. (See p. 261.)

It has been suggested by Mr. Charles M. Mills that the relation between the tensile strength of the concrete and the bond with the bars is an important factor in governing the size of the cracks, and the following analysis, based on his suggestions, gives a means of estimating the size and distance apart of the cracks so as to form a basis for judgment as to the sizes and percentages of steel to use.

The tensile stress in the steel at a crack tends to pull out the bars from

the concrete, and referring to Fig. 172, the bond stress of the bar in the length *ab* must equal the tensile stress in the whole cross-section of the concrete at *b* caused by the contraction of the concrete.

Let

x = distance apart of cracks.
D = diameter of round bar or side of square bar.
p = ratio of cross-section of steel to cross-section of concrete.

Then,* if, as is sufficiently accurate for practical purposes, the strength of concrete in tension is assumed to be equal to the bond between plain steel bars and concrete, the distance apart of cracks is

$$x = \frac{D}{2p} \text{ for square or round bars.}$$

The distance apart is inversely proportional to the unit bond*, so that a deformed bar having twice the bond strength would space the cracks one-half as far apart and allow them to be only one-half as wide.

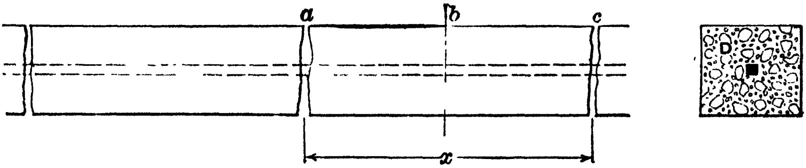


FIG. 172—Reinforcement for Temperature Stresses. *(See p. 567.)*

It is evident that the distance apart of the cracks is proportional to the diameter of the reinforcing bars, and inversely proportional to the percentage of steel.

From this formula is tabulated the estimated percentage of reinforcement for different spacing of cracks and different sizes of bars, assuming the bonding strength of the steel to the concrete to equal the tensile strength of the concrete.

* In addition to above notation, let

A_c = area of section of concrete.
A_s = area of section of steel.
o = perimeter of steel bar.
f'_c = tensile stress in concrete.
u = unit bond between plain steel and concrete.
f_s = unit tensile stress in steel.
D = diameter of bar.

Then $A_c f_c' = \frac{1}{2} uox$, or $x = \frac{2 A_c f_c}{u o}$. If $f'_c = u$, $x = \frac{2 A_c}{o}$, and since $p = \frac{A_s}{A_c}$, $x = \frac{2 A_s}{o p}$. Also, $\frac{A_s}{o} = \frac{D}{4}$ for both round and square bars, hence $x = \frac{1}{2} \frac{D}{p}$.

Estimated Percentage of Reinforcement for Different Spacing of Cracks

DISTANCE APART OF CRACKS WITH							
PLAIN BARS		12″	18″	24″	36″	48″	60″
DEFORMED BARS *		8″	12″	16″	24″	32″	40″
		%	%	%	%	%	%
Diameter of round or side of square bar	¼″	1.04	0.70	0.52	0.35	0.26	0.21
	⅜″	1.56	1.04	0.78	0.52	0.39	0.31
	½″	2.08	1.39	1.04	0.69	0.52	0.41
	⅝″	2.60	1.74	1.30	0.87	0.65	0.52
	¾″	3.12	2.08	1.56	1.04	0.78	0.62
	⅞″	3.65	2.44	1.82	1.22	0.91	0.73
	1″	4.17	2.78	2.08	1.39	1.04	0.83

NOTE: To express the steel as the ratio of area of cross-section of steel to cross-section of concrete, divide the percentages by 100; thus 1.04 becomes $p = 0.0104$.

* Assuming the bond of deformed bars to be 50% greater than plain.

The size of the crack is governed by the amount of shrinkage and for cracks due to temperature changes may be estimated as the product of the coefficient of contraction (0.0000055) by the number of degrees fall in temperature by the distance between cracks.

Estimated Width of Cracks for Different Distances Apart

WIDTH FOR DIFFERENT TEMPERATURE CHANGES	DISTANCE APART					
	12″	18″	24″	36″	48″	60″
30° Fahr*	0.0020	0.0030	0.0040	0.0059	0.0079	0.0099
50° "	0.0033	0.0050	0.0066	0.0099	0.0132	0.0165
70° "	0.0046	0.0069	0.0092	0.0139	0.0185	0.0232

From this, if it can be determined how large a crack will be allowable, the corresponding spacing can be obtained.

To avoid large cracks it may be necessary to use enough steel to prevent its passing its elastic limit. If the bars are continuous for such a length that the ends are practically immovable, as in a long retaining wall, a drop

* 30° corresponds to a shrinkage of 0.017%; 50° to 0.028%; 70° to 0.038%.

in temperature, tending to shorten them, produces a tensile stress which is independent of the distance between the restrained ends. Assuming the coefficient of expansion of steel the same as concrete and the modulus of elasticity of steel as 30 000 000, this stress is 30 000 000 × 0.0000055 = 165 pounds per square inch per degree of temperature, or for 50° Fahr. is 8250 pounds per square inch. This is well within the elastic limit of the steel and would not, of itself, cause the steel to take a permanent set. However, since the concrete surrounding the steel will be continuous except at certain cracks, the stretch in the steel may be unevenly distributed and largely confined to the immediate vicinity of the cracks. If cracks occur while steel is unstressed, through the concrete shrinking, the steel tends to resist the shrinkage by tension at the crack and compression at the center of the block of concrete, and the tensile stress will be equal to the compressive and each equal to one-half the tensile strength of the concrete. This may be expressed by the following formula, using the foregoing notation:*

$$f'_s = \frac{1}{2p} f'_c$$

Since the tensile stress in the concrete is liable to be low at the time shrinkage cracks are formed, it may be assumed, for illustration, as 200 pounds per square inch making

$$f'_s = \frac{100}{p}$$

This represents the stress due to local cracks which is additional to the temperature stresses above described. The total stress is, therefore, for 50° change of temperature $8250 + f'_s$ or $8250 + \frac{100}{p}$. If the elastic limit of the steel is 40 000 pounds per square inch, and we must keep below this

$$40\ 000 = 8250 + \frac{100}{p} \text{ and } p = 0.0031$$

For steel, the elastic limit of which is 50 000 pounds per square inch,

$$50\ 000 = 8250 + \frac{100}{p} \text{ and } p = 0.0024$$

These values of p represent the lowest theoretical ratio of area of cross-section of steel to area of cross-section of concrete which can be used without the steel passing its elastic limit at certain of the cracks when the ends are restrained or the length is so great that intermediate parts are practically restrained.

---

* $\frac{A_c f'_c}{2} = A_s f'_s$ or $f'_s = \frac{A_c}{2A_s} f'_c$ hence $f'_s = \frac{1}{2p} f'_c$

In view of the very slight stretch required to relieve the stress in the bars when the elastic limit is exceeded, and the probability of its distribution by the restraint to movement by the mass, it is not always essential to consider the elastic limit.

SYSTEMS OF REINFORCEMENT

One of the earliest recorded examples of the application of reinforced concrete is a boat of concrete and iron, built by Mr. L. J. Lambot in France, and shown at the Paris International Exhibition in 1855.* In 1861 Mr. Coignet began his investigations, and in 1866 Mr. Monier, to whom the invention of reinforced concrete is often attributed, applied the combination of concrete and iron to various structures, and laid the foundation for its future widespread applications.

As long ago as 1872, Mr. W. E. Ward,† at Port Chester, N. Y., built a house entirely of concrete, reinforced with iron I-beams and round rods.

The rapid development of reinforced concrete has resulted in the introduction of numerous systems, many of them covered by patents, for arranging the metal in the concrete, or for special forms of metal. These systems are fully described in the various French works on reinforced concrete.‡

A few of the systems, representing both the arrangement and the form of the metal, are described below, and forms of metal extensively used in the United States are illustrated in Fig. 173.

Systems of Reinforcement

Bonna. Metal of cruciform cross-section.

Berlini. Girder Frame. Horizontal tension members with vertical stirrups shrunk on to them.

Chaudy and Degon. Cross rods passing under bearing rods, but looped up between them.

Coignet. Round bars in top and bottom of beam connected by diagonal wire lacing.

Columbian. Vertical steel plates with horizontal ribs.

Corrugated. (See Fig. 173, page 571.)

Cottacin. Round rods interlaced in the same manner as in wire netting.

Cummings. Bars of different lengths having their ends bent up to an incline and formed into a loop to resist internal stresses.

* Christophe's Beton Armé, 1902, p. 1.

† Transactions American Society Mechanical Engineers, Vol. IV, p. 388.

‡ See among others Christophe's Beton Armé, 1902, pp. 10–71, and Morel's Ciment Armé, 1902, pp. 88 to 152.

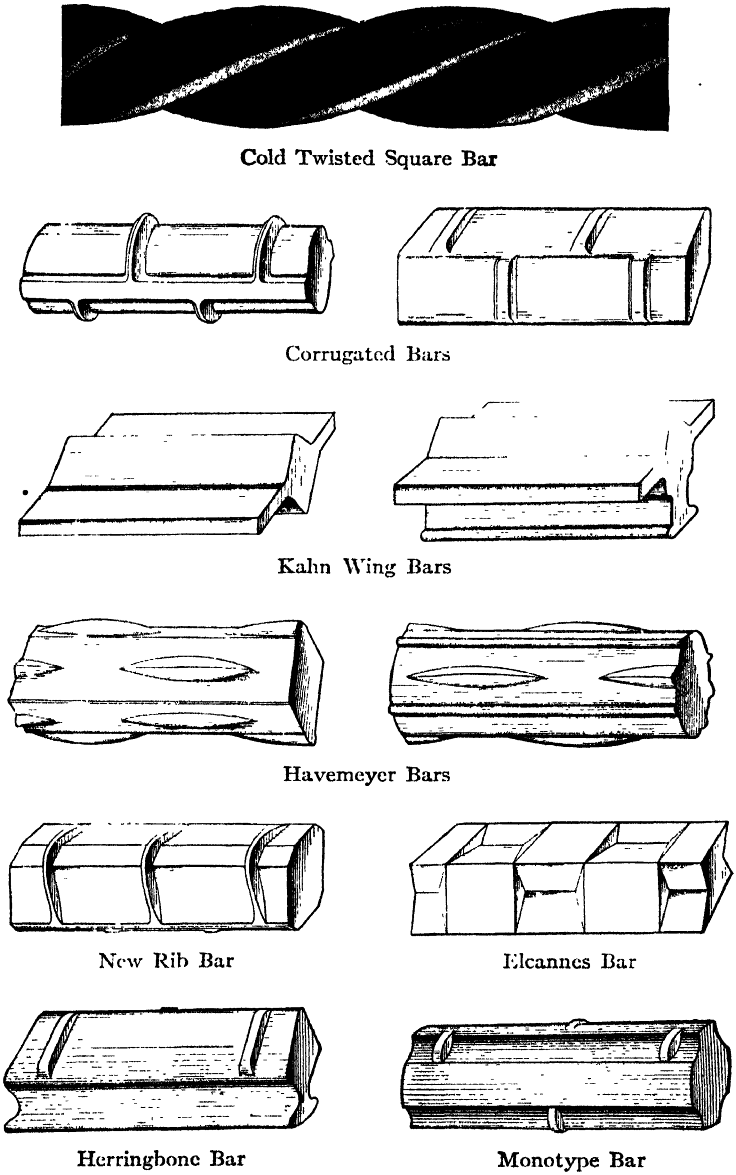


FIG. 173.—Types of Reinforcing Steel, (*See pp.* 570 *and* 572.)

Diamond Bar. Bars rolled round with parallel ribs passing along and around the bar forming diamond-shaped shoulders on its surface.

Donath. Inverted T-beams or I beams connected by horizontal diagonals of light, flat metal on edge.

Elcannes. (See Fig. 173, page 571.)

Expanded Metal. Sheet steel, slit and expanded to form a diamond mesh.

Ferroinclave. Sheet steel with inversely tapered corrugations to be covered on both sides with concrete.

Gabriel. Deformed tension members with trussing of hard drawn wire.

Habrich and Düsing. Flat metal twisted hot.

Havemeyer. (See Fig. 173, page 571.)

Hennebique. A combination of alternate straight bars and bars with ends bent up at an angle, with vertical U-bars, or stirrups, of flat iron passing around the straight bars and reaching nearly to the top of the beam.

Herringbone. (See Fig. 173, page 571.)

Holzer. Metal in form of I-beams.

Hyatt. Flat plates or bars set on edge and pierced with holes through which pass small round rods to form the cross reinforcements.

Johnson. Corrugated bars.

Kahn Wing. (See Fig 173, page 571.) The horizontal flanges are sheared up at intervals to serve as diagonal reinforcement.

Lock-Woven Steel Fabric. Steel wire mesh, locked at intersections.

Lug Bars. (See Fig. 173, page 571.)

Melan. Steel ribs, either I-beam or 4 angles latticed, imbedded in the concrete of the arch.

Monier. Two series of round parallel bars at right angles to each other.

Monotype. (See Fig. 173, page 571.)

Mushroom. Flat floor slabs supported by columns with enlarged heads.

New Rib. (See Fig. 173, page 571.)

Parmley. Bars with bent ends, to place in the sides of a conduit or the haunches of an arch to resist tension.

Rabitz. Various combinations employing galvanized wire.

Ransome. Square steel rods twisted cold. (See Fig. 173, p. 571.)

Roebling. Flat steel bars set on edge, clamped to supporting beams, and held in alignment by flat bar separators.

Schülter. Like Monier System except rods are placed diagonally.

Triangle Mesh. Wire mesh reinforcemeut with transverse metal placed diagonally.

Trussit. Expanded metal or herringbone lath bent to V-shaped section.

Visintini. Beams of concrete, cored out so as to form lattice girders.

Welded Wire Fabric. Wire mesh reinforcement with wires at right angles to each other and welded at intersections.

Working Unit Stresses

Kind of Stress.		Notation and No. of formula.	Allowable Working Stresses. Percentage of crushing strength at 28 days*	Allowable Working Stresses. For 2000 lb. Concrete. Lb. per sq. inch.	Remarks.
(1)		(2)	(3)	(4)	(5)
Bearing			32.5	650	
Axial compression			22.5	450	Length of pier not to exceed 4 diameters
Columns (as described on page 561)	Vertical steel 1 to 6%†	f_c (41) to (46)	22.5	450	Length not to exceed 15 diameters
	Vertical steel 1 to 4% and spirals 1%	f_c (41) to (46)	35	700	Length not to exceed 10 diameters of core
Compression in extreme fiber	Ordinary	f_c (4a) to (20)	32.5	650	Use in beam formulas
	In continuous beams adjacent to the support.	f_c (4a) to (20)	37.5	750	
Shear (punching shear)			6	120	
Shear (as measure of diagonal tension	Beams without web reinforcement	v (32)	2	40	
	Beams with web reinforcement	v (32)	6	120	Two-thirds of this stress must be provided for with web reinforcement
Bond	Plain bars	u (36)	4	80	
	Deformed bars	u (36)	5 to 6	100 to 120	
Steel in tension	Structural grade	f_s (4a) to (25)	16 000 lbs. per sq. in.		
	First class high carbon steel	f_s (4a) to (25)	18 000 lb. per sq. in.		

* Strengths at 28 days and other ages, and for different aggregates and different consistencies are given on pp. 310 to 329.

† Joint Committee limits the amount of vertical steel to 4%.

TABLE 1. AREAS, WEIGHTS AND CIRCUMFERENCES OF BARS.

Areas and Weights of Square and Round Rods and Circumferences of Round Rods.

One cubic foot weighs 490 lb.

Thickness or Diameter in inches	Area of Square Rod in square inches	Area of Round Rod in square inches.	Circumference of Round Rod in inches	Weight of Square Rod One Foot Long	Weight of Round Rod One Foot Long	Thickness or Diameter in inches	Area of Square Rod in square inches.	Area of Round Rod in square inches	Circumference of Round Rod in inches	Weight of Square Rod One Foot Long	Weight of Round Rod One Foot Long.
0						**2**	4.0000	3.1416	6.2832	13.60	10.68
1/16	0.0039	0.0031	0.1963	0.013	0.010	1/16	4.2539	3.3410	6.4795	14.46	11.36
1/8	0.0156	0.0123	0.3927	0.053	0.042	1/8	4.5156	3.5466	6.6759	15.35	12.06
3/16	0.0352	0.0276	0.5890	0.119	0.094	3/16	4.7852	3.7583	6.8722	16.27	12.78
1/4	0.0625	0.0491	0.7854	0.212	0.167	1/4	5.0625	3.9761	7.0686	17.22	13.52
5/16	0.0977	0.0767	0.9817	0.333	0.261	5/16	5.3477	4.2000	7.2649	18.19	14.28
3/8	0.1406	0.1104	1.1781	0.478	0.375	3/8	5.6406	4.4301	7.4613	19.18	15.07
7/16	0.1914	0.1503	1.3744	0.651	0.511	7/16	5.9414	4.6664	7.6576	20.20	15.86
1/2	0.2500	0.1963	1.5708	0.850	0.667	1/2	6.2500	4.9087	7.8540	21.25	16.69
9/16	0.3164	0.2485	1.7671	1.076	0.845	9/16	6.5664	5.1572	8.0503	22.33	17.53
5/8	0.3906	0.3068	1.9635	1.328	1.043	5/8	6.8906	5.4119	8.2467	23.43	18.40
11/16	0.4727	0.3712	2.1598	1.608	1.262	11/16	7.2227	5.6727	8.4430	24.56	19.29
3/4	0.5625	0.4418	2.3562	1.913	1.502	3/4	7.5625	5.9396	8.6394	25.00	20.20
13/16	0.6602	0.5185	2.5525	2.245	1.763	13/16	7.9102	6.2126	8.8357	26.90	21.12
7/8	0.7656	0.6013	2.7489	2.603	2.044	7/8	8.2656	6.4918	9.0321	28.10	22.07
15/16	0.8789	0.6903	2.9452	2.989	2.347	15/16	8.6289	6.7771	9.2284	29.34	23.04
1	1.0000	0.7854	3.1416	3.400	2.670	**3**	9.0000	7.0686	9.4245	30.60	24.03
1/16	1.1289	0.8866	3.3379	3.838	3.014	1/16	9.3789	7.3662	9.6211	31.89	25.04
1/8	1.2656	0.9940	3.5343	4.303	3.379	1/8	9.7656	7.6699	9.8175	33.20	26.08
3/16	1.4102	1.1075	3.7306	4.795	3.766	3/16	10.160	7.9798	10.014	34.55	27.13
1/4	1.5625	1.2272	3.9270	5.312	4.173	1/4	10.563	8.2958	10.210	35.92	28.20
5/16	1.7227	1.3530	4.1233	5.857	4.600	5/16	10.973	8.6179	10.407	37.31	29.30
3/8	1.8906	1.4849	4.3197	6.428	5.049	3/8	11.391	8.9462	10.603	38.73	30.42
7/16	2.0664	1.6230	4.5160	7.026	5.518	7/16	11.816	9.2806	10.799	40.18	31.56
1/2	2.2500	1.7671	4.7124	7.650	6.008	1/2	12.250	9.6211	10.996	41.65	32.71
9/16	2.4414	1.9175	4.9087	8.301	6.520	9/16	12.691	9.9678	11.192	43.14	33.90
5/8	2.6406	2.0739	5.1051	8.978	7.051	5/8	13.141	10.321	11.388	44.68	35.09
11/16	2.8477	2.2365	5.3014	9.682	7.604	11/16	13.598	10.680	11.585	46.24	36.31
3/4	3.0625	2.4053	5.4978	10.41	8.178	3/4	14.063	11.045	11.781	47.82	37.56
13/16	3.2852	2.5802	5.6941	11.17	8.773	13/16	14.535	11.416	11.977	49.42	38.81
7/8	3.5156	2.7612	5.8905	11.95	9.388	7/8	15.016	11.793	12.174	51.05	40.10
15/16	3.7539	2.9483	6.0868	12.76	10.02	15/16	15.504	12.177	12.370	52.71	41.40

BEAM AND SLAB TABLES

Beam Tables. Tables 2, 3, and 4, pages 576, 577 and 578, give the loading and reinforcement for beams, based on 1 inch of width under different conditions. For a beam 10 inches wide, for example, both the safe load per linear foot and the steel area will be ten times the values given in the tables.

The tables are for rectangular beams but may be used for T-beams which have a depth 3 or 4 times the thickness of slab by taking the width of flange as the breadth, b.

Table 2 is for a simply supported beam and is based on a working compressive stress in concrete of 500 pounds per square inch and in steel of 14,000 pounds per square inch—lower values than are customarily used in construction, but required in many building laws. If the compression in concrete is limited to 500 pounds, while 16,000 pounds is permitted in the steel, use the same loading but reduce the steel in the ratio of 16 to 14.

Tables 3 and 4 are for ordinary design, approved by the authors and corresponding to recommendations of the Joint Committee.

Slab Tables. Table 5 is for slab design with different working stresses in the steel and concrete. Ordinarily, the series at the top of the second page of the table is used.

Table 6 is more convenient for review of beams already designed. It is computed by using formulas (8) and (10) on page 355, and selecting the lower value of M. The most economical ratio of steel for the limiting stresses is $p = 0.0077$. For ratios lower than this the safe loads are governed by the tensile strength of the steel, while for larger ratios they are limited by the compressive strength of the concrete.

Table 7 is for designing fully continuous slabs.

Table 8 covers cinder concrete slabs.

Stirrup Tables. Tables 9 and 10 give, respectively, the number and spacing of stirrups in uniformly loaded beams.

T-Beam Tables. Tables 11 to 13 are for designing and reviewing T-beams.

Beams with Steel at Top and Bottom. Table 14 and Diagrams 1, 2, and 3 are for use in designing and reviewing beams with tensile and compressive steel.

Tables of Constants. Tables 15, 16, and 17 are useful in giving constants in convenient shape for use in beam and slab design.

Column Tables. Tables 18 to 21 are for designing and reviewing columns.

Bending Moment Diagrams. Diagrams 4, 5, and 6 give bending moments for different spans and loads.

TABLE 2. USE FOR SIMPLY SUPPORTED BEAMS FOR EXTRA CONSERVATIVE DESIGN

Safe Loading and Reinforcement for Rectangular Beams One Inch in Width. 1 : 2 : 4 *Concrete. Mild Steel.*

Based on $M = \frac{wl^2}{8}$ $n = 15$. $f_c = 500$. $f_s = 14000$. (*See p.* 575 *and* 596.)

For $M = \frac{wl^2}{10}$ add 25% to the safe loads using same steel area. For $M = \frac{wl^2}{12}$ add 50% to the safe loads using same steel area.

Depth of Beam. (h) in.	Total Safe Load (w) per Linear Foot for Beam One Inch Wide including Weight of Beam. *For safe live load deduct weight of beam in column* (22) (See important footnotes.) Span in Feet (l). 5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	25	30	35	Weight of Beam One Inch Wide per Linear Foot. lb.	Depth to Steel. (d) in.	Depth below Steel. (e) in.	Steel Area in a Beam One Inch Wide.* sq. in.	Safe Moment of Resistance. (See p. 355.) (M) in. lb.
																				(22)	(23)	(24)	(25)	(26)
5	33	23	17	13	10	8														5.4	4.0	1.0	0.024	1230
6	51	36	26	20	16	13	10	9												6.4	5.0	1.0	0.030	1925
7	74	51	38	29	23	18	15	13	11	9										7.5	6.0	1.0	0.037	2770
8	101	69	51	39	31	25	21	17	15	13	11									8.6	7.0	1.0	0.043	3770
9	123	86	63	48	38	31	25	21	18	16	14	10								9.6	7.75	1.25	0.047	4620
10	157	109	80	61	48	39	32	27	23	20	17	15								10.7	8.75	1.25	0.053	5890
11	195	135	99	76	60	49	40	34	29	25	22	19	17							11.8	9.75	1.25	0.059	7310
12	237	165	121	93	73	59	49	41	35	30	26	23	20							12.8	10.75	1.5	0.066	8900
13	274	188	138	106	84	68	56	47	40	35	30	26	23	21						13.9	11.50	1.5	0.070	10100
14	321	223	164	125	99	80	66	56	47	41	36	31	28	25						15.0	12.50	1.5	0.076	12030
15	374	260	191	146	115	93	77	65	55	48	41	36	32	29	26					16.0	13.50	1.5	0.082	14030
16	431	300	220	168	132	108	89	75	64	55	48	42	37	33	30					17.1	14.50	1.5	0.088	16170
17	493	342	251	193	152	123	102	86	73	63	55	48	43	38	34					18.1	15.50	1.5	0.094	18500
18	557	388	285	218	172	139	115	97	83	71	62	54	48	43	39					19.2	16.50	1.5	0.101	20960
19	593	411	303	232	183	148	122	103	88	77	66	58	51	46	41	37				20.3	17.00	2.0	0.104	22250
20		462	339	260	205	166	137	115	98	85	74	65	57	51	46	41				21.4	18.00	2.0	0.110	24940
22			419	321	253	205	169	142	121	104	91	80	71	63	57	51	33			23.6	20.00	2.0	0.122	30800
24			506	388	306	248	205	172	147	127	111	97	86	77	69	62	40			25.7	22.00	2.0	0.134	37260
26				462	365	295	244	205	175	151	131	115	102	91	82	74	47	33		27.9	24.00	2.0	0.146	44350
28				532	428	346	287	241	205	177	154	135	120	107	96	87	55	38		30.0	26.00	2.0	0.159	52020
30					496	402	332	279	238	205	179	157	139	124	111	100	64	45		32.1	28.00	2.0	0.171	60360
36						576	476	400	340	293	256	225	199	178	159	144	92	64	46	38.5	33.50	2.5	0.210	86400
42						800	661	556	473	408	355	312	277	247	222	200	128	89	64	41.9	39.50	2.5	0.240	120100
48							876	736	627	540	471	414	367	327	294	265	169	118	85	51.4	45.50	2.5	0.278	159400

TABLE 3. USE ONLY FOR CONTINUOUS BEAMS

Safe Loading and Reinforcement for Rectangular Beams One Inch in Width. 1 : 2 : 4 Concrete. Mild Steel.

Based on $M = \frac{wl^2}{12}$ $n = 15.$ $f_c = 650.$ $f_s = 16000.$ (*See p.* 575 *and* 596.)

Depth of Beam. (h) in.	Total Safe Load (w) per Linear Foot for Beam One Inch Wide including Weight of Beam. *For safe live load deduct weight of beam in column (22)* (See important foot-notes.) Span in Feet (l). 5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	25	30	35	Weight of Beam One Inch Wide per Linear Foot. lb.	Depth to Steel. (d) in.	Depth Below Steel. (e) in.	Steel Area in a Beam One Inch Wide.* sq. in.	Safe Moment of Resistance. (See p. 355.) (M) in.-lb.
																				(22)	(23)	(24)	(25)	(26)
5	70	48	36	28	22	17	14	12												5.4	4.0	1.0	0.031	1740
6	109	76	55	43	34	28	23	19	16											6.4	5.0	1.0	0.038	2720
7	155	109	80	61	48	40	32	28	23	20										7.5	6.0	1.0	0.046	3910
8	214	149	109	84	66	53	44	37	31	28	24									8.6	7.0	1.0	0.054	5320
9	262	182	133	102	80	66	54	46	38	34	29	25								9.6	7.75	1.25	0.060	6520
10	334	232	170	131	103	84	70	58	49	43	37	32	29							10.7	8.75	1.25	0.067	8310
11	415	288	211	162	128	103	85	72	61	53	46	41	36	32						11.8	9.75	1.25	0.075	10320
12	504	350	257	197	156	126	104	88	74	65	56	49	43	38	35					12.8	10.75	1.25	0.083	12540
13	577	401	294	226	178	144	119	101	85	73	64	56	50	44	40					13.9	11.50	1.5	0.089	14350
14	682	473	348	266	210	170	140	119	101	86	76	67	59	53	47	43				15.0	12.5	1.5	0.096	16950
15	796	552	406	311	245	199	164	138	118	101	89	78	68	61	55	49				16.0	13.5	1.5	0.104	19780
16	917	637	468	359	283	229	190	160	136	116	102	90	79	71	64	58				17.1	14.5	1.5	0.112	22820
17	1049	728	535	409	324	262	216	182	155	133	116	102	91	80	72	66	42			18.1	15.5	1.5	0.119	26070
18	1188	826	606	464	367	296	245	206	175	151	132	116	103	91	83	74	48			19.2	16.5	1.5	0.127	29540
19	1261	876	643	492	389	316	260	218	186	161	140	124	110	100	91	81	50			20.3	17.0	2.0	0.131	31360
20	1414	982	721	552	437	353	292	245	209	180	157	138	122	109	98	89	56			21.4	18.0	2.0	0.139	35160
22		1212	890	682	539	436	360	302	258	222	194	170	151	134	121	109	70	48		23.6	20.0	2.0	0.154	43400
24		1466	1078	824	652	528	436	367	312	269	235	206	182	163	146	132	84	59		25.7	22.0	2.0	0.169	52520
26			1282	982	775	628	518	437	372	320	280	245	217	194	174	157	101	70		27.9	24.0	2.0	0.185	62500
28				1152	910	737	610	512	442	376	328	288	256	228	204	185	118	82	60	30.0	26.0	2.0	0.200	73360
30				1336	1055	854	707	594	505	436	380	334	296	264	238	214	137	95	[illegible]	32.1	28.0	2.0	0.216	85080
36				1913	1511	1224	1012	850	725	624	544	478	424	378	340	306	196	136	100	38.5	33.5	2.5	0.258	121800
42				2660	2101	1702	1406	1182	1007	868	756	665	589	526	472	426	272	190	139	44.9	39.5	2.5	0.304	169300
48						2261	1869	1570	1338	1153	1005	883	782	697	626	565	361	251	185	51.4	45.5	2.5	0.350	224600

TABLE 4. USE FOR END SPANS OF CONTINUOUS BEAMS. FOR SUPPORTED BEAMS DEDUCT 20 PER CENT.

Safe Loading and Reinforcement for Rectangular Beams One Inch in Width. 1 : 2 : 4 Concrete. Mild Steel.

Based on $M = \frac{wl^2}{10}$, $n = 15$, $f_c = 650$, $f_s = 16\,000$ (*See p.* 575 *and* 596.)

For $M = \frac{wl^2}{8}$ deduct 20 per cent from safe loads, using same steel area.

Depth of Beam.	Total Safe Load (*w*) per Linear Foot for Beam One Inch Wide including Weight of Beam. *For safe live load deduct weight of beam in column* (22) (See important foot-notes.) Span in Feet (*l*).																			Weight of Beam One Inch Wide per Linear Foot	Depth to Steel.	Depth Below Steel.	Steel Area in a Beam One Inch Wide.*	Safe Moment of Resistance. (*See p.* 355.)
(*h*) in.	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	25	30	35	lb.	(*d*) in.	(*e*) in.	sq. in.	(*M*) in.-lb.
																				(22)	(23)	(24)	(25)	(26)
5	58	40	30	23	18	14	12	10												5.4	4.0	1.0	0.031	1740
6	91	63	46	36	28	23	19	16	13											6.4	5.0	1.0	0.038	2720
7	129	91	67	51	40	33	27	23	19	17										7.5	6.0	1.0	0.046	3910
8	178	124	91	70	55	44	37	31	26	23	20									8.6	7.0	1.0	0.054	5340
9	218	152	111	85	67	55	45	38	32	28	24	21								9.6	7.75	1.25	0.060	6520
10	278	193	142	109	86	70	58	48	41	36	31	27	24							10.7	8.75	1.25	0.067	8310
11	346	240	176	135	107	86	71	60	51	44	38	34	30	27						11.8	9.75	1.25	0.075	10320
12	420	292	214	164	130	105	87	73	62	54	47	41	36	32	29					12.8	10.75	1.25	0.083	12540
13	481	334	245	188	148	120	99	84	71	61	53	47	42	37	33					13.9	11.50	1.5	0.089	14350
14	518	394	290	222	175	142	117	99	84	72	63	56	49	44	39	36				15.0	12.5	1.5	0.096	16950
15	663	460	338	259	204	166	137	115	98	84	74	65	57	51	46	41				16.0	13.5	1.5	0.104	19780
16	764	531	390	299	236	191	158	133	113	97	85	75	66	59	53	48				17.1	14.5	1.5	0.112	22820
17	874	607	446	341	270	218	180	152	129	111	97	85	76	67	60	55	35			18.1	15.5	1.5	0.119	26070
18	990	688	505	387	306	247	204	172	146	126	110	97	86	76	69	62	40			19.2	16.5	1.5	0.127	29540
19	1051	730	536	410	324	263	217	182	155	134	117	103	91	83	75	66	42			20.3	17.0	2.0	0.131	31360
20	1178	818	601	460	364	294	243	204	174	150	131	115	102	91	82	74	47			21.4	18.0	2.0	0.139	35160
22		1010	742	568	449	363	300	252	215	185	162	142	126	112	101	91	58	40		23.6	20.0	2.0	0.154	43400
24		1222	898	687	543	440	363	306	260	224	196	172	152	136	122	110	70	49		25.7	22.0	2.0	0.169	52520
26			1068	818	646	523	432	364	310	267	233	204	181	162	145	131	84	58		27.9	24.0	2.0	0.185	62500
28				960	758	614	508	427	363	313	273	240	213	190	170	154	98	68	50	30.0	26.0	2.0	0.200	73360
30				1113	879	712	589	495	421	363	317	278	247	220	198	178	114	79	58	32.1	28.0	2.0	0.216	85080
36				1594	1259	1020	843	708	604	520	453	398	353	315	283	255	163	113	83	38.5	33.5	2.5	0.258	121800
42				2216	1751	1418	1172	985	839	723	630	554	491	438	393	355	227	158	116	44.9	39.5	2.5	0.304	169300
48						1884	1557	1308	1115	961	837	736	652	581	522	471	301	209	154	51.4	45.5	2.5	0.350	224600

RULES.
1. **For safe load of any width of beam** multiply by width in inches.
2. **For area of cross-section of steel** for any width of beam multiply column (25) by width in inches.
3. **Total loads of spans** (*l*) and same depth of steel are inversely proportional to the squares of the spans.
4. **Total loads for other depths of steel** (*d*) and same span are proportional to the squares of the depth of steel.
5. The values in this table may apply to a very carefully graded 1 : 2½ : 5 mixture.

* This is for a ratio of steel $p = 0.0077$ (0.77 per cent.) which is required for the given working stresses.

TABLE 5. USE FOR DESIGNING SLABS, IF FULLY CONTINUOUS, ADD 20% TO LOADS

Safe Loadings per Square Foot and Reinforcement for Slabs for Various Working Stresses in Steel (f_s) and Concrete (f_c). (See pp. 575 and 484.)

Based on $M = \frac{wl^2}{10}$. For supported ends, $\left(M = \frac{wl^2}{8}\right)$, deduct 20% from loads. For fully continuous, $\left(M = \frac{wl^2}{12}\right)$, add 20%. For square slabs multiply by 2. Use same steel area always.

	Total depth of slab. (h)	Total safe load (w) per square foot, including weight of slab. For safe live load deduct weight of slab in column (14). (See important foot-notes on opposite page.) Span in feet (l). 4	5	6	7	8	9	10	11	12	13	14	15	Weight of slab per square foot. lb. (14)	Depth to steel. (d) in. (15)	Depth below steel. (e) in. (16)	Steel area in section of slab one foot wide.* sq. in (17)	Safe Moment of resistance. See p. 355 M in.-lb. (18)
$f_s = 14000$, $f_c = 500$, $n = 15$, $p = 0.0062$	2½	147	94	66										32	1¾	¾	0.130	2830
	3	243	156	108	79									38	2¼	¾	0.167	4670
	3½	360	231	161	118	90								45	2¾	¾	0.205	6930
	4	507	326	226	166	127	100	81						51	3¼	¾	0.242	9750
	4½	589	378	262	192	147	116	94						58	3½	1	0.260	11320
	5	768	494	343	251	192	152	123	102					64	4	1	0.298	14780
	6	1201	772	536	393	300	238	192	159	134	113			77	5	1	0.372	23100
	7	1729	1111	771	565	432	342	276	229	193	163	143	123	90	6	1	0.446	33260
	8	2353	1512	1050	770	588	471	376	312	262	222	195	167	103	7	1	0.521	45270
	9	3081	1971	1369	1006	770	608	493	407	342	292	251	219	116	8	1	0.595	59140
	10	3898	2495	1733	1273	975	770	624	515	433	369	318	277	128	9	1	0.670	74840
$f_s = 14000$, $f_c = 600$, $n = 15$, $p = 0.0084$	2½	195	125	87	64									32	1¾	¾	0.176	3760
	3	322	207	143	105	80								38	2¼	¾	0.227	6190
	3½	480	308	214	157	120	95							45	2¾	¾	0.277	9230
	4	675	434	301	221	169	134	108						51	3¼	¾	0.328	12990
	4½	784	503	349	256	196	155	125	104					58	3½	1	0.353	15070
	5	1023	657	457	334	256	203	163	136	114	96			64	4	1	0.403	19680
	6	1599	1027	713	523	400	317	255	212	178	150	132	114	77	5	1	0.504	30750
	7	2303	1479	1027	753	575	456	367	305	257	217	190	163	90	6	1	0.605	44280
	8	3134	2013	1398	1024	783	621	500	417	349	295	259	223	103	7	1	0.706	60270
	9	4101	2625	1823	1339	1025	810	656	542	456	388	335	292	116	8	1	0.806	78720
	10	5191	3321	2307	1694	1298	1025	830	686	577	491	424	369	128	9	1	0.907	99630
$f_s = 14000$, $f_c = 700$, $n = 15$, $p = 0.0107$	2½	246	158	110	80									32	1¾	¾	0.225	4730
	3	407	261	181	133	102	81							38	2¼	¾	0.289	7830
	3½	604	388	269	197	151	120	96						45	2¾	¾	0.353	11620
	4	850	546	379	278	212	168	136	113	95				51	3¼	¾	0.417	16350
	4½	986	633	440	322	246	195	157	131	110	93			58	3½	1	0.449	18960
	5	1293	827	574	421	322	255	205	171	143	121	106		64	4	1	0.514	24770
	6	2012	1292	898	658	503	399	321	267	224	189	166	143	77	5	1	0.642	38700
	7	2898	1861	1293	947	724	574	462	384	323	273	239	206	90	6	1	0.770	55730
	8	3945	2534	1760	1289	986	781	629	523	440	372	326	281	103	7	1	0.899	75860
	9	5161	3303	2294	1685	1290	1019	826	682	573	489	421	367	116	8	1	1.027	99080
	10	6534	4181	2904	2133	1633	1290	1045	864	726	618	533	464	128	9	1	1.156	125400
$f_s = 16000$, $f_c = 500$, $n = 15$, $p = 0.0050$	2½	137	87											32	1¾	¾	0.105	2620
	3	226	145	101	74									38	2¼	¾	0.135	4340
	3½	335	215	149	109	84	66							45	2¾	¾	0.165	6440
	4	471	303	210	154	118	93	75						51	3¼	¾	0.195	9060
	4½	546	351	244	179	137	108	87	72	61				58	3½	1	0.210	10510
	5	713	458	318	233	178	141	114	95	80				64	4	1	0.240	13720
	6	1115	716	498	364	279	221	178	148	124	105	92		77	5	1	0.300	21450
	7	1605	1031	716	525	401	318	256	213	179	151	133	114	90	6	1	0.360	30880
	8	2186	1404	975	715	546	433	349	290	244	206	181	155	103	7	1	0.420	42040
	9	2860	1831	1271	934	715	565	457	378	318	271	233	203	116	8	1	0.480	54910
	10	3619	2317	1609	1182	905	715	579	479	402	343	295	257	128	9	1	0.540	69500
$f_s = 16000$, $f_c = 600$, 15, 0.0067	2½	183	117	82										32	1¾	¾	0.141	3520
	3	303	195	135	99									38	2¼	¾	0.181	5830
	3½	450	289	201	147	112	89	72						45	2¾	¾	0.221	8650
	4	632	406	282	207	158	125	101	84	70				51	3¼	¾	0.261	12160
	4½	734	471	327	240	183	145	117	97	82				58	3½	1	0.281	14110
	5	958	615	427	313	239	190	153	127	107	90	79		64	4	1	0.322	18430
	6	1497	962	668	489	374	297	239	199	167	141	124	106	77	5	1	0.402	[illegible]
	7	2156	1385	962	705	529	427	344	286	240	203	178	153	90	6	1	0.482	41470
	8	2935	1885	1309	959	734	581	468	389	327	277	243	209	103	7	1	0.563	56450

TABLE 5—Continued. Based on $\frac{wl^2}{10}$.

	Total depth of slab. (h) in.	Total safe load (w) per square foot including weight of slab. For safe live load deduct weight of slab, column (14) (See important footnotes.) Span in feet. (l) 4	5	6	7	8	9	10	11	12	13	14	15	Weight of slab per square foot. lb.	(d) Depth to steel. in.	(e) Depth below steel. in.	Steel area in a section of slab one foot wide. sq. in.	Safe moment of resistance. See p. 355 (M) in. lb.
$r = 15$, $f_s = 16000$, $p = 0.0077$, $f_c = 650$	2½	206	132	92	67	52								32	1¾	¾	0.162	3952
	3	340	218	151	111	85	61	54						38	2¼	¾	0.208	6536
	3½	509	326	226	166	127	101	81	67					45	2¾	¾	0.254	9770
	4	711	455	316	232	178	140	114	94	79	67			51	3¼	¾	0.300	13650
	4½	824	528	366	269	206	163	132	109	92	78	67		58	3½	1	0.323	15830
	5	1077	690	479	352	269	213	172	142	120	102	88	77	64	4	1	0.370	20680
	6	1683	1077	748	550	421	332	269	223	187	159	137	120	77	5	1	0.462	32310
	7	2423	1551	1077	791	606	479	388	320	269	229	198	172	90	6	1	0.554	46520
	8	3297	2111	1466	1077	824	651	528	436	366	312	269	234	103	7	1	0.647	63320
	9	4308	2758	1915	1407	1077	851	689	570	479	408	352	306	116	8	1	0.739	82720
	10	5454	3491	2424	1781	1366	1077	873	721	606	516	445	388	128	9	1	0.832	104700
$n = 15$, $f_s = 16000$, $p = 0.0087$, $f_c = 700$	2½	231	148	103	75									32	1¾	¾	0.183	4440
	3	382	245	170	125	95	76	61						38	2¼	¾	0.235	7340
	3½	567	304	253	185	142	112	90	75	63				45	2¾	¾	0.287	10900
	4	797	512	356	261	199	158	127	106	89	75			51	3¼	¾	0.339	15330
	4½	925	594	413	302	231	183	148	123	103	87			58	3½	1	0.365	17790
	5	1209	776	539	395	303	239	193	160	135	114	100		64	4	1	0.418	23230
	6	1887	1212	842	617	472	374	301	250	210	178	156	134	77	5	1	0.522	36300
	7	2719	1746	1213	889	679	538	434	361	303	256	225	193	90	6	1	0.626	52280
	8	3699	2390	1651	1209	925	732	590	491	413	349	306	263	103	7	1	0.731	71150
	9	4840	3098	2152	1581	1210	956	774	640	538	458	395	344	116	8	1	0.835	92940
	10	6126	3921	2723	2000	1531	1210	980	810	681	580	500	436	128	9	1	0.940	117600
$n = 15$, $f_s = 20000$, $p = 0.0034$, $f_c = 500$	2½	118	76	53										32	1¾	¾	0.071	2270
	3	195	126	87	64									38	2¼	¾	0.022	3760
	3½	290	186	129	95	72								45	2¾	¾	0.112	5580
	4	408	262	182	133	102	81							51	3¼	¾	0.133	7850
	4½	474	304	211	155	118	94	76						58	3½	1	0.143	9110
	5	619	398	276	202	155	122	99	82					64	4	1	0.163	11900
	6	967	621	431	316	242	192	154	128	108	91	80		77	5	1	0.204	18600
	7	1392	894	621	455	348	280	222	185	155	131	115	99	90	6	1	0.245	26780
	8	1895	1217	846	619	474	375	302	257	211	179	157	135	103	7	1	0.286	36450
	9	2480	1587	1102	810	620	490	397	[illegible]	276	235	202	176	116	8	1	0.326	47610
	10	3139	2009	1395	1025	785	620	502	[illegible]	349	297	256	223	128	9	1	0.367	60260
$n = 15$, $f_s = 20000$, $p = 0.0047$, $f_c = 600$	2½	161	103	72	53									32	1¾	¾	0.099	3100
	3	267	171	119	87	67								38	2¼	¾	0.127	5130
	3½	396	254	177	129	99	78							45	2¾	¾	0.155	7610
	4	556	357	248	182	139	110	89						51	3¼	¾	0.183	10700
	4½	646	415	288	211	161	128	103	86					58	3½	1	0.197	12420
	5	844	542	376	276	211	167	135	112	94	79			64	4	1	0.226	16220
	6	1318	846	588	431	329	261	210	175	147	124	109	94	77	5	1	0.282	25350
	7	1898	1219	847	620	474	376	303	252	212	179	157	135	90	6	1	0.338	36500
	8	2584	1659	1152	844	646	572	412	343	288	243	214	184	103	7	1	0.395	49680
	9	3381	2164	1502	1104	845	668	541	447	376	320	276	240	116	8	1	0.451	64900
	10	4279	2738	1901	1397	1070	845	684	566	475	405	349	304	128	9	1	0.508	82140
$n = 15$, $f_s = 20000$, $p = 0.0060$, $f_c = 700$	2½	202	130	90	66									32	1¾	¾	0.126	3890
	3	334	215	149	109	83								38	2¼	¾	0.162	6430
	3½	497	319	221	162	124	98							45	2¾	¾	0.198	9550
	4	698	449	311	228	175	138	111	93					51	3¼	¾	0.234	13430
	4½	810	520	361	265	202	160	129	107	90				58	3½	1	0.252	15580
	5	1058	679	472	346	264	210	169	140	118	100	87		64	4	1	0.288	20350
	6	1653	1062	737	540	413	327	264	219	184	156	137	117	77	5	1	0.360	31800
	7	2381	1530	1062	778	595	472	380	316	266	224	197	169	90	6	1	0.432	45790
	8	3240	2082	1445	1059	810	642	517	430	361	305	268	231	103	7	1	0.504	62330
	9	4241	2715	1885	1385	1060	838	678	561	471	402	346	302	116	8	1	0.576	81420
	10	5368	3435	2386	1753	1342	1060	859	710	596	508	438	382	128	9	1	0.648	103030

TABLE 6. USE FOR REVIEWING DESIGNS. IF FULLY CONTINUOUS ADD 20% TO LOADS.

Safe Loads per Square Foot and Reinforcement for Slabs. Proportions 1 : 2 : 4.
(See p. 575).

Based on $M = \frac{wl^2}{10}$ For supported ends, $\left(M = \frac{wl^2}{8}\right)$, deduct 20% from loads

f_c = or < 650 $n = 15$ For fully continuous, $\left(M = \frac{wl^2}{12}\right)$, add 20% to loads

f = or < 16000   For square slabs, $\left(M = \frac{wl^2}{20}\right)$, multiply loads by 2.

Ratio of cross-section steel to beam above steel. (p)*	Total depth of slab. (h) in.	Total safe load (w) per square foot including weight of slab. For safe live load deduct weight of slab in column (15). (See important footnotes.) Span in feet (l.) 4	5	6	7	8	9	10	11	12	13	14	15	Weight of slab per square foot. lb. (15)	Depth to steel. (d) in. (16)	Depth below steel. (e) in. (17)	Steel area in a section of slab one foot wide. sq. in. (18)	Safe moment of resistance. See p. 355 (M) in. lb. (19)
0.002	3	95	60	42										38	2¼	¾	0.054	1800
	4	198	125	88	64	49	39							51	3¼	¾	0.078	3760
	5	300	190	133	97	74	59	48						64	4	1	0.096	5702
	6	469	297	207	151	116	92	74						77	5	1	0.120	8910
	7	675	428	298	218	167	132	107						90	6	1	0.144	12830
	8	919	582	406	296	227	180	146	120					103	7	1	0.168	17460
	9	1201	760	531	387	296	235	190	157					116	8	1	0.192	22810
	10	1519	962	671	388	375	298	241	199	167				128	9	1	0.216	28870
0.004	3	185	117	82	60	46	36	29						38	2¼	¾	0.108	3510
	4	385	244	170	124	95	76	61	50					51	3¼	¾	0.156	7324
	5	584	370	258	188	144	114	92	77	64				64	4	1	0.192	11100
	6	913	578	403	294	225	179	144	120	100				77	5	1	0.240	17340
	7	1314	832	581	423	324	257	208	172	144				90	6	1	0.288	24970
	8	1788	1133	790	576	441	350	283	234	196	167	145	126	103	7	1	0.336	33980
	9	2336	1479	1032	752	576	458	370	306	257	219	189	164	116	8	1	0.384	44380
	10	2957	1873	1307	952	730	579	468	387	325	277	239	208	128	9	1	0.432	56180
0.006	3	272	172	120	87	67	52	43	36					38	2¼	¾	0.162	5160
	4	567	359	250	183	140	111	90	74	62				51	3¼	¾	0.234	10770
	5	858	544	379	276	212	168	136	112	94				64	4	1	0.288	16310
	6	1342	850	593	432	331	263	212	176	147				77	5	1	0.360	25490
	7	1932	1223	854	622	477	378	306	253	212				90	6	1	0.432	36700
	8	2630	1665	1162	847	649	515	416	345	289	246	213	185	103	7	1	0.504	49960
	9	3435	2175	1518	1106	848	673	544	450	377	321	278	242	116	8	1	0.576	65260
	10	4348	2753	1921	1400	1073	852	688	570	478	407	351	306	128	9	1	0.648	82600
0.008	3	348	220	153	112	86	68	55	46					38	2¼	¾	0.216	6610
	4	726	460	321	234	179	142	115	95	80				51	3¼	¾	0.312	13790
	5	1100	697	486	354	271	215	174	144	121				64	4	1	0.384	20900
	6	1719	1088	760	553	424	337	272	225	189				77	5	1	0.480	32650
	7	2475	1567	1094	797	611	485	392	324	272	231	200		90	6	1	0.576	47020
	8	3369	2134	1489	1085	831	660	533	441	370	315	272	237	103	7	1	0.672	64000
	9	4401	2787	1915	1417	1086	862	697	577	483	412	356	310	116	8	1	0.768	*83600
	10	5570	3527	2401	1793	1374	1091	882	730	612	521	450	392	128	9	1	0.864	105800
0.010	3	374	237	165	120	92	73	59	49					38	2¼	¾	0.270	7100
	4	781	494	345	251	103	153	124	102	86				51	3¼	¾	0.390	14820
	5	1182	749	522	381	292	232	187	155	130				64	4	1	0.480	22400
	6	1847	1170	816	595	456	362	292	242	203	173	149		77	5	1	0.600	35090
	7	2660	1684	1175	856	656	521	421	348	292	249	215	187	90	6	1	0.720	50520
	8	3618	2292	1599	1165	893	709	573	474	397	339	292	255	103	7	1	0.840	68750
	9	4727	2993	2089	1522	1166	926	748	619	519	442	382	333	116	8	1	0.960	89800
	10	5986	3790	2645	1927	1477	1172	948	784	657	560	484	421	128	9	1	1.080	113700

TABLE 7. USE FOR DESIGNING FULLY CONTINUOUS SLABS.

Safe Loadings per Square Foot, with Thickness of Slab and Steel Required for Given Live Load.

Based on $M = \frac{wl^2}{12}$, $f_c = 650$, $f_s = 16\,000$, $n = 15$, 1:2:4 concrete. For $M = \frac{wl^2}{10}$ (wall spans) add 9% to depth of slab and area of steel. For $M = \frac{wl^2}{8}$ (supported spans) add 20% to depth of slab and area of steel.
h = total thickness of slab. A_s = Area of cross section of steel per foot of width.

Span	Live Load Pounds per Square Foot*																							
	40*		50*		60*		70*		75*		80*		90*		100*		115*		125*		140*		150*	
	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s
ft.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in.	sq. in.
4	3	0.04	3	0.04	3	0.05	3	0.06	3	0.06	3	0.06	3	0.06	3	0.07	3	0.07	3	0.08	3	0.09	3	0.10
5	3	0.06	3	0.07	3	0.08	3	0.09	3	0.09	3	0.09	3	0.10	3	0.11	3	0.12	3	0.13	3	0.14	3	0.15
6	3	0.09	3	0.10	3	0.11	3	0.13	3	0.13	3	0.13	3	0.15	3	0.16	3	0.17	3	0.19	3	0.20	3½	0.18
7	3	0.12	3	0.14	3	0.15	3	0.17	3	0.18	3	0.18	3	0.20	3½	0.18	3½	0.20	3½	0.21	3½	0.23	4	0.21
8	3	0.16	3	0.18	3	0.21	3½	0.19	3½	0.20	3½	0.21	3½	0.22	4	0.21	4	0.23	4	0.24	4	0.27	4½	0.27
9	3	0.20	3½	0.20	3½	0.22	3½	0.24	4	0.22	4	0.23	4	0.25	4	0.26	4½	0.28	4½	0.30	4½	0.32	5	0.31
10	3½	0.22	4	0.22	4	0.24	4	0.26	4	0.28	4	0.28	4½	0.30	4½	0.32	4½	0.35	5	0.33	5	0.36	5	0.38
11	4	0.24	4	0.26	4	0.29	4½	0.31	4½	0.32	4½	0.33	4½	0.36	5	0.35	5	0.38	5½	0.37	5½	0.40	5½	0.42
12	4	0.28	4½	0.31	4½	0.34	5	0.34	5	0.35	5	0.37	5	0.39	5½	0.38	5½	0.42	6	0.41	6	0.44	6	0.46
13	4½	0.33	5	0.34	5	0.37	5½	0.37	5½	0.39	5½	0.40	5½	0.43	6	0.42	6	0.46	6½	0.45	6½	0.48	6½	0.50
14	5	0.36	5½	0.37	5½	0.40	6	0.40	6	0.42	6	0.43	6	0.46	6½	0.46	6½	0.50	7	0.50	7	0.53	7	0.55
15	5	0.41	5½	0.42	6	0.43	6½	0.44	6½	0.45	6½	0.47	6½	0.50	6½	0.53	7	0.54	7½	0.54	7½	0.57	7½	0.60
16	5½	0.44	6	0.45	6½	0.47	6½	0.50	6½	0.50	7	0.51	7	0.54	7	0.57	7½	0.58	7½	0.61	8	0.62	8	0.65
17	6	0.46	6½	0.49	7	0.51	7	0.54	7	0.56	7½	0.55	7½	0.58	7½	0.62	8	0.63	8	0.66	8½	0.67	8½	0.70
18	6½	0.51	7	0.53	7	0.57	7½	0.58	7½	0.60	7½	0.62	8	0.63	8	0.66	8½	0.68	9	0.69	9	0.73	9	0.76
19	7	0.55	7½	0.57	7½	0.61	8	0.63	8	0.65	8	0.66	8½	0.67	8½	0.71	9	0.73	9½	0.74	9½	0.78	10	0.78
20	7½	0.59	8	0.61	8	0.65	8½	0.67	8½	0.69	8½	0.71	9	0.72	9	0.76	9½	0.78	10	0.79	10	0.84	10½	0.84

Live Load Pounds per Square Foot*

Span	165*		175*		190*		200*		215*		225*	
	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s
ft.	in.	sq. in.	in.	sq. in.	in.	sq. in.	in	sq in	in.	sq in	in.	sq in
4	3	0.10	3	0.11	3	0.12	3	0.12	3	0.13	3	0.13
5	3	0.16	3	0.17	3	0.18	3½	0.16	3	0.20	3	0.21
6	3½	0.19	3½	0.20	3½	0.22	4	0.19	3½	0.24	3½	0.25
7	3½	0.26	4	0.24	4	0.26	4	0.27	4	0.28	4	0.20
8	4	0.30	4½	0.30	4½	0.32	4½	0.33	5	0.31	5	0.33
9	5	0.33	5	0.35	5	0.36	5	0.38	5½	0.36	5½	0.38
10	5½	0.37	5½	0.38	5½	0.41	5½	0.43	6	0.41	6	0.43
11	6	0.41	6	0.43	6	0.46	6	0.47	6½	0.46	6½	0.48
12	6½	0.46	6½	0.48	6½	0.50	6½	0.52	7	0.52	7	0.53
13	7	0.49	7	0.53	7	0.56	7½	0.55	7½	0.57	7½	0.59
14	7½	0.55	7½	0.58	8	0.58	8	0.60	8	0.63	8	0.65
15	8	0.60	8	0.63	8½	0.63	8½	0.65	8½	0.68	8½	0.70
16	8½	0.66	8½	0.68	9	0.69	9	0.71	9	0.74	9½	0.73
17	9	0.71	9	0.74	9½	0.75	9½	0.77	10	0.78	10	0.80
18	9½	0.77	9½	0.80	10	0.80	10	0.83	10½	0.84	10½	0.86
19	10	0.82	10½	0.83	10½	0.87	11	0.87	11	0.90	11	0.93
20	10½	0.88	11	0.89	11½	0.90	11½	0.93	11½	0.97	12	0.97

Span	240*		250*		265*		275*		285*		300*	
	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s	h	A_s
ft.	in.	sq. in.	in.	sq in.	in.	sq in.	in.	sq. in.	in.	sq. in.	in.	sq. in.
4	3	0.14	3	0.14	3	0.15	3	0.16	3	0.16	3	0.17
5	3	0.22	3½	0.19	3½	0.20	3½	0.21	3½	0.21	3½	0.22
6	3½	0.27	4	0.23	4	0.25	4	0.26	4	0.26	4½	0.26
7	4	0.31	4½	0.30	4½	0.32	4½	0.33	5	0.30	5	0.31
8	5	0.34	5	0.35	5	0.37	5½	0.35	5½	0.36	5½	0.38
9	5½	0.39	5½	0.41	6	0.39	6	0.40	6	0.41	6	0.43
10	6	0.45	6	0.46	6½	0.45	6½	0.46	6½	0.47	6½	0.50
11	6½	0.50	6½	0.52	7	0.50	7	0.52	7	0.53	7	0.56
12	7	0.56	7½	0.54	7½	0.56	7½	0.58	7½	0.60	8	0.59
13	8	0.58	8	0.60	8	0.63	8	0.64	8	0.65	8½	0.65
14	8½	0.64	8½	0.66	8½	0.69	8½	0.71	9	0.69	9	0.72
15	9	0.70	9	0.73	9	0.75	9½	0.74	9½	0.76	9½	0.79
16	9½	0.77	9½	0.79	10	0.79	10	0.81	10	0.83	10½	0.83
17	10	0.83	10½	0.83	10½	0.85	10½	0.88	10½	0.88	11	0.90
18	11	0.87	11	0.90	11	0.93	11½	0.92	11½	0.94	11½	0.98
19	11½	0.93	11½	0.96	12	0.97	12	0.99	12	1.01	12½	1.01
20	12	1.12	12	1.03	12½	1.04	12½	1.06	13	1.06	13	1.10

* Allowance is made for weight of concrete slab but not for superimposed granolithic or wood flooring. To allow for granolithic (if not considered as part of structural slab) or wood flooring with cinder fill use the column in table corresponding to 15 pounds larger load than the live load.

TABLE 8. CINDER CONCRETE SLABS

A ratio of elasticity of $n = 35$ is used in the table below, although it is permissible to design with a ratio of 15 in very conservative practice.

The loads for slabs with a ratio of steel of 0.002 are limited by the working strength of the steel, and the values with the higher ratios by the working strength of the cinder concrete.

It is noticeable that less steel can be used economically for a given thickness of slab than with broken stone or gravel concrete, because the strength of the slab is more apt to be limited by the strength of the cinder concrete than by the strength of the steel.

Safe Loading and Reinforcement for CINDER CONCRETE SLABS One Foot in Width.
Proportions 1 : 2½ : 5. *Mild Steel.* (See p. 584).

Based on $M = \frac{wl^2}{10}$, f_c = or < 225, f_s = or < 14 000, $n = 35$

Ratio cross-section steel to beam above steel. (p)*	Total depth of slab. in.	Total safe load (w^1) per square foot including weight of slab. For safe live load deduct weight of slab in column (12). (See important foot-notes.) Span in Feet (l) 4	5	6	7	8	9	10	Weight of slab per square foot. lb.	Depth to steel. (d) in.	Depth below steel. (e) in.	Steel area in a section of slab one foot wide. sq. in.	Safe moment of resistance. (See p. 355.) (M_R) in. lb.
									(10)	(11)	(12)	(13)	(14)
0.002	2½	48	31						24	1¾	¾	0.042	920
	3	70	51	35	26				29	2¼	¾	0.054	1520
	3½	119	76	53	39				34	2¾	¾	0.066	2280
	4	166	106	74	54	41			39	3¼	¾	0.078	3180
	4½	192	123	85	63	48			43	3½	1	0.084	3690
	5	251	161	112	82	63	50		48	4	1	0.096	4820
	6	392	251	174	128	98	78	63	58	5	1	0.120	7530
	7	565	361	251	184	141	112	90	68	6	1	0.144	10840
	8	768	492	341	351	192	152	123	77	7	1	0.168	14750
0.004	2½	76	48	34	25				24	1¾	¾	0.084	1460
	3	125	80	56	41	31			29	2¼	¾	0.108	2400
	3½	187	120	83	61	47	37		34	2¾	¾	0.132	3590
	4	261	167	116	85	65	52	42	39	3¼	¾	0.156	5020
	4½	303	194	135	99	76	60	48	43	3½	1	0.168	5820
	5	396	253	176	129	99	78	63	48	4	1	0.192	7600
	6	619	396	275	202	155	122	99	58	5	1	0.240	11880
	7	891	570	396	291	223	176	143	68	6	1	0.288	17110
	8	1213	776	539	396	303	240	194	77	7	1	0.336	23290
0.006	2½	86	55	38	28				24	1¾	¾	0.126	1640
	3	141	90	63	46	35			29	2¼	¾	0.162	2710
	3½	211	135	94	60	53	42	34	34	2¾	¾	0.198	4050
	4	295	189	131	96	74	58	47	39	3¼	¾	0.234	5660
	4½	342	219	152	112	85	68	55	43	3½	1	0.252	6570
	5	447	286	199	146	112	88	72	48	4	1	0.288	8580
	6	698	447	310	228	175	138	112	58	5	1	0.360	13400
	7	1005	643	447	328	251	199	161	68	6	1	0.432	19300
	8	1368	876	608	447	342	270	219	77	7	1	0.504	26270

* Percentages of steel are values in this column multiplied by 100.

RULES. 1. **For load for any width of slab** multiply by width in feet.

2. **For area of cross-section of steel** for any width of slab multiply column (13) by width in feet.

3. **Total loads for other spans** (e) and same depth of steel are inversely proportional to the squares of the spans.

4. **Total loads for other depths of steel** (d) and same span are proportional to the squares of the depths of steel.

Table 9. NUMBER OF U-STIRRUPS IN UNIFORMLY LOADED BEAM

Number of stirrups per beam is $2N_s = C_n bl$.

$2N_s$ = number of stirrups per entire beam.
l = span of beam in feet.
b = breadth of beam in inches (in T-beam, breadth of stem).
v = maximum shearing unit stress in beam in lb. per sq. in.
v' = allowable shearing unit stress (or diagonal tension) in concrete alone in lb. per sq. in.
C_n = constant.

Note: Table is based on general formula $2N_s = \frac{4(v^2 - v'^2)}{vA_s f_s} bl$ in which stress f_s = 16 000 lb. per sq. in. has been accepted as a unit stress steel and A_s the corresponding area of two legs of the U stirrup.

Values of Constant C_n for Finding Number of Stirrups in Beam.

	$\frac{5}{16}$-in. round U-stirrup, A_s = .15		$\frac{3}{8}$-in. round U-stirrup, A_s = .22		$\frac{7}{16}$-in. round U-stirrup, A_s = .30		$\frac{1}{2}$-in. round U-stirrup, A_s = .39		$\frac{5}{8}$-in. round U-stirrup, A_s = .61	
v	v' = 40	60	v' = 40	60	v' = 40	60	v' = 40	60	v' = 40	60
70	0.079	0.031	0.054	0.021	0.039	0.015	0.030	0.012	0.019	0.008
75	0.090	0.045	0.061	0.031	0.044	0.022	0.034	0.017	0.022	0.011
80	0.100	0.058	0.068	0.040	0.049	0.029	0.038	0.022	0.024	0.014
85	0.110	0.071	0.075	0.049	0.054	0.035	0.042	0.027	0.027	0.017
90	0.120	0.083	0.082	0.057	0.059	0.041	0.046	0.032	0.029	0.020
95	0.130	0.095	0.089	0.065	0.064	0.047	0.050	0.036	0.032	0.023
100	0.140	0.107	0.096	0.073	0.069	0.052	0.054	0.041	0.034	0.026
105	0.150	0.118	0.102	0.080	0.073	0.058	0.057	0.045	0.037	0.029
110	0.159	0.129	0.108	0.088	0.078	0.063	0.061	0.049	0.039	0.032
115	0.169	0.140	0.115	0.095	0.083	0.068	0.065	0.053	0.041	0.034
120	0.178	0.150	0.121	0.102	0.087	0.074	0.068	0.057	0.044	0.037
125	0.187	0.160	0.128	0.109	0.092	0.079	0.072	0.061	0.046	0.039
130	0.196	0.170	0.134	0.116	0.096	0.084	0.075	0.065	0.048	0.042
140	0.214	0.190	0.146	0.130	0.105	0.093	0.082	0.073	0.052	0.047
150	0.232	0.210	0.158	0.143	0.114	0.103	0.089	0.080	0.057	0.051
160	0.250	0.229	0.171	0.156	0.123	0.112	0.096	0.088	0.061	0.056

Table 10. SPACING OF STIRRUPS IN BEAMS WITH UNIFORMLY DISTRIBUTED LOADING

Spacing in inches, $s = C_l\, l$

l = span of beam in feet.
N_s = number of stirrups in each end of beam.
v = maximum shearing unit stress in beam in lb. per sq. in.
v' = allowable shearing unit stress (or diagonal tension) in concrete alone in lb. per sq. n.

Values of Constant C_l for Finding Spacing of Stirrups.

$\frac{v}{v'} = 2.0$

N_s	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
2	1.26	1.74								
3	0.80	0.95	1.24							
4	0.59	0.67	0.78	0.97						
5	0.47	0.51	0.57	0.65	0.80					
6	0.39	0.42	0.45	0.50	0.57	0.67				
7	0.33	0.35	0.38	0.41	0.45	0.50	0.59			
8	0.29	0.30	0.32	0.34	0.37	0.41	0.45	0.52		
9	0.26	0.27	0.28	0.30	0.32	0.34	0.37	0.41	0.46	
10	0.23	0.24	0.25	0.26	0.28	0.29	0.31	0.34	0.37	0.42

$\frac{v}{v'} = 2.5$

N_s	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
2	1.43	2.17								
3	0.91	1.11	1.58							
4	0.67	0.76	0.92	1.25						
5	0.53	0.58	0.66	0.79	1.04					
6	0.44	0.47	0.52	0.59	0.69	0.89				
7	0.37	0.40	0.43	0.47	0.53	0.62	0.77			
8	0.32	0.34	0.37	0.40	0.44	0.49	0.56	0.69		
9	0.29	0.30	0.32	0.34	0.37	0.40	0.45	0.51	0.62	
10	0.26	0.27	0.28	0.30	0.32	0.34	0.37	0.42	0.47	0.56

$\frac{v}{v'} = 3.0$

N_s	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
2	1.53	2.47								
3	0.97	1.20	1.83							
4	0.71	0.82	1.01	1.46						
5	0.56	0.62	0.72	0.87	1.23					
6	0.46	0.50	0.56	0.64	0.77	1.05				
7	0.39	0.42	0.46	0.51	0.58	0.70	0.93			
8	0.34	0.37	0.39	0.43	0.47	0.54	0.64	0.83		
9	0.30	0.32	0.34	0.37	0.40	0.44	0.50	0.59	0.75	
10	0.27	0.29	0.30	0.32	0.34	0.37	0.41	0.46	0.54	0.69

$\frac{v}{v'} = 3.5$

N_s	1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
2	1.59	2.70								
3	1.00	1.26	2.02							
4	0.73	0.86	1.07	1.64						
5	0.58	0.65	0.75	0.93	1.38					
6	0.48	0.52	0.58	0.68	0.83	1.19				
7	0.41	0.44	0.48	0.54	0.62	0.75	1.05			
8	0.36	0.38	0.41	0.45	0.50	0.57	0.69	0.95		
9	0.31	0.33	0.36	0.38	0.42	0.46	0.53	0.64	0.86	
10	0.28	0.30	0.31	0.33	0.36	0.39	0.43	0.50	0.59	0.78

If larger number of stirrups are used divide the number by 2, find the spacing for this number from the table, and place intermediate stirrups between.

TABLE 11. T-BEAM TABLE.

Constants C_d in Formula for Minimum Depth, min. $d = \dfrac{MC_d}{jf_s bt}$ *for Different Stresses in Steel and Concrete and for Different Ratios of* $\dfrac{t}{d}$. *(See page 489, and example, page 587.)*

Unit Stress in Steel f_s	Unit Stress in Concrete f_c	Values of C_d																		
		Ratio of Thickness of Flange to Depth of Beam $\frac{t}{d}$																		
lb. per sq. in.	lb. per sq. in.	0	.10	.12	.14	.16	.18	.20	.22	.24	.26	.28	.30	.32	.34	.36	.38	.40	.42	.44
14 000	300	47	59	62	66	70	74	79	85	92										
	400	35	42	44	46	48	50	53	55	58	62	66	70							
	500	28	33	34	35	36	38	39	41	43	45	47	49	52	55					
	600	23	27	28	28	29	30	31	32	34	35	36	38	39	41	43	45			
	650	22	25	25	26	27	28	28	29	30	32	32	34	35	37	38	40	42		
	700	20	23	23	24	25	23	26	27	28	29	30	31	32	33	35	36	38	39	
	800	18	20	20	21	21	22	22	23	24	24	25	26	27	28	29	30	31	32	33
	900	16	17	18	18	19	19	20	20	21	21	22	22	23	24	25	25	26	27	28
	1 000	14	16	16	16	17	17	17	18	18	19	19	20	20	21	21	22	23	24	24
16 000	300	53	69	73	79	84	91	98												
	400	[illegible]	49	51	54	57	60	63	67	71	76									
	500	[illegible]	38	39	41	43	45	47	49	51	54	57	60							
	600	27	31	32	33	34	36	37	38	40	42	44	46	48	51					
	650	25	28	29	30	31	32	33	35	36	38	39	41	43	45	47				
	700	23	26	27	28	29	30	31	32	33	34	35	37	38	40	42	44			
	800	20	23	23	24	25	25	26	27	28	29	30	31	32	33	34	36	37	39	
	900	18	20	20	21	22	22	23	23	24	25	26	26	27	28	29	30	32	33	34
	1 000	16	18	18	19	19	20	20	21	21	22	23	23	24	25	25				
18 000	300	60	80	86	92	100	109	120												
	400	45	56	59	63	66	70	75	80	87										
	500	36	43	45	47	49	52	55	58	61	65	69								
	600	30	35	37	38	39	41	43	45	47	49	52	55	58						
	650	28	32	33	35	36	37	39	40	42	44	46	48	51	54					
	700	26	30	31	32	33	34	35	37	38	40	42	43	45	48	50				
	800	23	26	26	27	28	29	30	31	32	33	35	36	38	39	41	43	45		
	900	20	23	23	24	25	25	26	27	28	29	30	31	32	33	34	36	37	39	
	1 000	18	20	21	21	22	22	23	24	24	25	26	27	28	29	30	31	32	33	35

Values of Ratio of Moment Arm to Depth of Beam, j, for Different Values of $\frac{t}{d}$ *and* C_d.

Based on $n = 15$ and Formula (16a) p. 357.

$\frac{t}{d}$	Values of j.											
	Values of C_d from Table 11.											
	20	30	40	50	60	70	80	90	100	110	120	130
0.10	0.952	0.952	0.953	0.953	0.954	0.955	0.955	0.956	0.956	0.957	0.958	0.958
0.12	0.943	0.944	0.945	0.045	0.946	0.947	0.948	0.940	0.950	0.950	0.951	0.952
0.14	0.934	0.936	0.937	0.938	0.939	0.940	0.941	0.943	0.944	0.945	0.946	0.947
0.16	0.925	0.927	0.928	0.930	0.932	0.933	0.935	0.936	0.938	0.930	0.941	0.942
0.18	0.917	0.919	0.921	0.923	0.925	0.927	0.929	0.931	0.933	0.935	0.937	0.939
0.20	0.909	0.911	0.914	0.916	0.919	0.921	0.924	0.926	0.928	0.931	0.933	0.936
0.22	0.900	0.903	0.906	0.909	0.912	0.915	0.918	0.921	0.924	0.927	0.930	0.933
0.24	0.893	0.897	0.900	0.904	0.908	0.911	0.915	0.919	0.922	0.926	0.929	0.933
0.26	0.886	0.890	0.894	0.899	0.903	0.907	0.911	0.916	0.920	0.924	0.929	0.933
0.28	0.878	0.883	0.888	0.893	0.898	0.903	0.908	0.913	0.918	0.923	0.928	0.933
0.30	0.870	0.876	0.882	0.888	0.894	0.899	0.905	0.911	0.917	0.923	0.929	0.935
0.32	0.863	0.870	0.877	0.883	0.890	0.897	0.904	0.911	0.917	0.924	0.931	0.938
0.34	0.857	0.865	0.872	0.880	0.888	0.896	0.903	0.911	0.919	0.927	0.934	0.942
0.36	0.851	0.860	0.869	0.878	0.886	0.895	0.904	0.913	0.922	0.930	0.939	0.948

Use Tables 11 and 12 to determine minimum depth of T-Beam. Minimum depth of T-beam as determined by compression, is min. $d = \frac{MC_d}{jf_s bt}$ where M, f_s, b, and t are known, and C_d and j are constants from Tables 11 and 12.

The value of C_d is taken from Table 11, corresponding to the working stresses f_s and f_c, and to an assumed ratio of depth of slab to depth of beam, value $\frac{t}{d}$.* The value of j should be taken from Table 12, corresponding to the assumed value $\frac{t}{d}$ and the value of C_d from Table 11. After determining min. d compare actual d with assumed d. If the difference is large, repeat computation with a new value of $\frac{t}{d}$.

Example. Find minimum depth of T-beam for compression when $M = 1\,500\,000$ inch pounds, $f_c = 650$, $f_s = 16\,000$, $n = 15$, $t = 4$ inches, and $b = 58$ inches.

Solution. Assume $\frac{t}{d} = 0.24$ (This value may correspond to minimum depth required for shear.) From Table 11, for $f_s = 16\,000$, $f_c = 650$, and $\frac{t}{d} = 0.24$, we find $C_d = 36$. From Table 12 for $\frac{t}{d} = 0.24$ and $C_d = 36$, we find by interpolation $j = 0.899$, therefore

$$\text{min. } d = \frac{1\,500\,000 \times 36}{0.899 \times 16\,000 \times 58 \times 4} = 16.2 \text{ inches}$$

A smaller value of d would give compressive stresses exceeding $f_c = 650$. A larger depth may be more economical.

Example for Table 13. Rule for use of Table 13: Find k, use k to determine j and thus f_s in Formula (20), page 491. Use k also to find C_T and hence f_c.

Example. Given $M = 1\,900\,000$ inch pounds resisted by a T-beam with the following dimensions: $b = 58$ inches, $t = 4$ inches, $d = 22$ inches. $A_s = 6.2$ square inches. Find stresses f_c and f_s, assuming $n = 15$.

Solution. Compute the ratios $\frac{nA_s}{bt} = \frac{15 \times 6.2}{58 \times 4} = 0.4$ and $\frac{t}{d} = \frac{4}{22} = 0.18$. Find in Table 13, corresponding to the above ratios, $k = 0.35$. With this value of k and $\frac{t}{d} = 0.18$, find from the lower part of the table $j = 0.920$, and, by interpolation, $C_T = 0.036$. Stresses in steel therefore $f_s = \frac{1\,900\,000}{0.920 \times 22 \times 6.2} = 15\,200$ pounds per square inch, and $f_c = C_T f_s = 0.036 \times 15\,200 = 547$ pounds per square inch.

*The minimum value of d, determined by shear, may be taken in computing the assumed ratio $\frac{t}{d}$.

TABLE 13. USE FOR REVIEW OF T-BEAMS; MOMENT ARM AND STRESSES

Values of k, j, and C_T for Different Values of $\frac{nA_s}{bt}$ (See pages 356 and 357.)*

Values of k. Use to find j and C_T below.

Ratio $\frac{nA_s}{bt}$	Ratios of Thickness of Flange to Depth of Beam, $\frac{t}{d}$											
	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32
0.1	0.136	0.145	0.154	0.164								
0.2	0.208	0.217	0.225	0.233	0.242	0.250	0.258	0.267	0.275	0.283		
0.3	0.269	0.277	0.285	0.292	0.300	0.308	0.315	0.323	0.331	0.338	0.346	0.354
0.4	0.321	0.329	0.336	0.343	0.350	0.357	0.364	0.371	0.379	0.386	0.393	0.400
0.45	0.345	0.352	0.359	0.365	0.372	0.379	0.386	0.393	0.400	0.407	0.414	0.421
0.5	0.367	0.373	0.380	0.387	0.393	0.400	0.407	0.413	0.420	0.427	0.433	0.440
0.55	0.387	0.393	0.400	0.406	0.413	0.419	0.426	0.432	0.439	0.445	0.452	0.458
0.6	0.406	0.412	0.419	0.425	0.431	0.437	0.444	0.450	0.456	0.462	0.469	0.475
0.65	0.424	0.430	0.436	0.442	0.448	0.455	0.461	0.467	0.473	0.479	0.485	0.491
0.7	0.441	0.447	0.453	0.459	0.465	0.471	0.476	0.482	0.488	0.494	0.500	0.506
0.75	0.457	0.463	0.469	0.474	0.480	0.486	0.491	0.497	0.503	0.509	0.514	0.520
0.8	0.472	0.478	0.483	0.489	0.494	0.500	0.506	0.511	0.517	0.522	0.528	0.533

Values of j Use to find f_s in Formula $f_s = \frac{M}{A_s j d}$.

Values of k	Ratios of Thickness of Flange to Depth of Beam, $\frac{t}{d}$											
	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24	0.26	0.28	0.30	0.32
0.21	0.955	0.948	0.940	0.937	0.933	0.930						
0.22	0.955	0.948	0.940	0.935	0.931	0.927						
0.23	0.955	0.947	0.939	0.934	0.929	0.925	0.924					
0.24	0.954	0.947	0.938	0.933	0.928	0.923	0.921					
0.25	0.954	0.946	0.938	0.933	0.927	0.922	0.919	0.917				
0.26	0.954	0.946	0.937	0.932	0.926	0.920	0.917	0.914	0.913			
0.27	0.954	0.946	0.937	0.931	0.925	0.919	0.916	0.912	0.909			
0.28	0.954	0.946	0.936	0.931	0.924	0.918	0.914	0.910	0.906	0.907		
0.29	0.953	0.945	0.936	0.930	0.924	0.917	0.913	0.908	0.904	0.904		
0.30	0.953	0.945	0.936	0.930	0.923	0.916	0.912	0.907	0.902	0.901	0.900	
0.31	0.953	0.945	0.935	0.929	0.922	0.915	0.911	0.905	0.900	0.899	0.897	
0.32	0.953	0.945	0.935	0.929	0.922	0.915	0.910	0.904	0.898	0.896	0.894	0.893
0.33	0.953	0.945	0.935	0.929	0.921	0.914	0.909	0.903	0.897	0.895	0.892	0.890
0.34	0.953	0.944	0.935	0.928	0.921	0.913	0.908	0.902	0.896	0.893	0.890	0.887
0.35	0.953	0.944	0.934	0.928	0.920	0.913	0.907	0.901	0.894	0.891	0.888	0.885
0.36	0.953	0.944	0.934	0.928	0.920	0.912	0.907	0.900	0.893	0.890	0.886	0.883
0.37	0.953	0.944	0.934	0.927	0.920	0.912	0.906	0.899	0.892	0.889	0.884	0.880
0.38	0.953	0.944	0.934	0.927	0.919	0.911	0.905	0.899	0.891	0.887	0.883	0.879
0.40	0.952	0.944	0.933	0.927	0.919	0.911	0.904	0.897	0.890	0.885	0.880	0.875
0.42	0.952	0.943	0.933	0.926	0.918	0.910	0.903	0.897	0.889	0.883	0.880	0.872
0.44	0.952	0.943	0.932	0.926	0.918	0.910	0.903	0.896	0.888	0.881	0.879	0.870
0.48	0.952	0.943	0.932	0.925	0.917	0.909	0.902	0.896	0.887	0.879	0.878	0.868
0.52	0.951	0.943	0.931	0.925	0.916	0.909	0.901	0.895	0.886	0.877	0.870	0.866

Values of C_T in Formula $f_c = C_T f_s$.*

	Values of k										
	0.22	0.24	0.26	0.28	0.30	0.32	0.34	0.36	0.38	0.40	0.42
15	0.019	0.021	0.023	0.026	0.029	0.031	0.034	0.038	0.041	0.044	0.048
12	0.024	0.026	0.029	0.032	0.036	0.039	0.043	0.047	0.051	0.055	0.060
10	0.028	0.032	0.035	0.039	0.043	0.047	0.052	0.056	0.061	0.067	0.073

*$f_c = C_T f_s$ is another form of Formula (19) page 357.

TABLE 14. USE FOR DESIGN OF BEAMS WITH COMPRESSION STEEL.

Values of Ratio of Compression Steel p' for Different Values of a, f_s, f_c, and p_1. Based on Formula (21) p. 492.

f_s	f_c	p_1	Ratios of Compression Steel, p' — Ratios of Depth of Compression to Tension Steel, a											
			0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24
16 000	500 ($n=15$)	0.006	0.002	0.002	0.003	0.003	0.003	0.003	0.004	0.004	0.005	0.006	0.007	0.009
		0.008	0.007	0.007	0.008	0.009	0.009	0.010	0.011	0.013	0.015	0.017	0.021	0.026
		0.010	0.011	0.012	0.013	0.014	0.016	0.017	0.019	0.021	0.025	0.029	0.034	0.043
		0.012	0.016	0.017	0.018	0.020	0.022	0.024	0.027	0.030	0.034	0.040	0.048	0.060
		0.014	0.021	0.022	0.024	0.026	0.028	0.031	0.034	0.039	0.044	0.052	0.062	0.078
		0.016	0.025	0.027	0.029	0.031	0.034	0.038	0.042	0.047	0.054	0.063	0.076	0.095
		0.018	0.030	0.032	0.034	0.037	0.040	0.045	0.050	0.056	0.064	0.074	0.089	0.112
		0.020	0.034	0.037	0.039	0.043	0.047	0.051	0.057	0.064	0.074	0.086	0.103	0.129
		0.022	0.039	0.042	0.045	0.048	0.053	0.058	0.065	0.073	0.083	0.097	0.117	0.147
		0.024	0.043	0.046	0.050	0.054	0.059	0.065	0.072	0.081	0.093	0.109	0.131	0.164
		0.026	0.048	0.051	0.055	0.060	0.065	0.072	0.080	0.090	0.103	0.120	0.145	0.181
		0.028	0.052	0.056	0.061	0.066	0.072	0.079	0.088	0.099	0.113	0.132	0.158	0.198
		0.030	0.057	0.061	0.066	0.071	0.078	0.086	0.095	0.107	0.123	0.143	0.172	0.216
16 000	600 ($n=15$)	0.008	0.003	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.006	0.007
		0.010	0.006	0.007	0.007	0.008	0.008	0.009	0.010	0.011	0.012	0.013	0.015	0.018
		0.012	0.010	0.011	0.011	0.012	0.013	0.014	0.016	0.017	0.019	0.022	0.025	0.029
		0.014	0.014	0.015	0.016	0.017	0.018	0.020	0.022	0.024	0.026	0.030	0.034	0.040
		0.016	0.018	0.019	0.020	0.022	0.023	0.025	0.027	0.030	0.034	0.038	0.043	0.051
		0.018	0.022	0.023	0.024	0.026	0.028	0.031	0.033	0.037	0.041	0.046	0.052	0.062
		0.020	0.025	0.027	0.029	0.031	0.033	0.036	0.039	0.043	0.048	0.054	0.062	0.072
		0.022	0.029	0.031	0.033	0.035	0.038	0.041	0.045	0.050	0.055	0.062	0.071	0.083
		0.024	0.033	0.035	0.037	0.040	0.043	0.047	0.051	0.056	0.062	0.070	0.081	0.094
		0.026	0.037	0.039	0.042	0.045	0.048	0.052	0.057	0.063	0.070	0.078	0.090	0.105
		0.028	0.040	0.043	0.046	0.049	0.053	0.057	0.063	0.069	0.077	0.087	0.099	0.116
		0.030	0.044	0.047	0.050	0.054	0.058	0.063	0.069	0.076	0.084	0.095	0.108	0.127
16 000	650 ($n=15$)	0.008	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
		0.010	0.004	0.004	0.005	0.005	0.005	0.006	0.006	0.007	0.007	0.008	0.009	0.010
		0.012	0.008	0.008	0.008	0.009	0.010	0.010	0.011	0.012	0.014	0.015	0.017	0.019
		0.014	0.011	0.012	0.012	0.013	0.014	0.015	0.017	0.018	0.020	0.022	0.025	0.028
		0.016	0.014	0.015	0.016	0.017	0.019	0.020	0.022	0.024	0.026	0.029	0.033	0.037
		0.018	0.018	0.019	0.020	0.022	0.023	0.025	0.027	0.029	0.032	0.036	0.041	0.046
		0.020	0.021	0.023	0.024	0.026	0.028	0.030	0.032	0.035	0.039	0.043	0.048	0.055
		0.022	0.025	0.026	0.028	0.030	0.032	0.035	0.037	0.041	0.045	0.050	0.056	0.065
		0.024	0.028	0.030	0.032	0.034	0.037	0.039	0.043	0.047	0.051	0.057	0.064	0.074
		0.026	0.032	0.034	0.036	0.038	0.041	0.044	0.048	0.052	0.058	0.064	0.072	0.083
		0.028	0.035	0.037	0.040	0.042	0.045	0.049	0.053	0.058	0.064	0.071	0.080	0.092
		0.030	0.039	0.041	0.044	0.047	0.050	0.054	0.058	0.064	0.070	0.078	0.088	0.101
16 000	750 ($n=15$)	0.010	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
		0.012	0.003	0.004	0.004	0.004	0.004	0.005	0.005	0.005	0.006	0.006	0.007	0.008
		0.014	0.006	0.007	0.007	0.008	0.008	0.009	0.009	0.010	0.011	0.012	0.013	0.015
		0.016	0.009	0.010	0.010	0.011	0.012	0.013	0.014	0.015	0.016	0.017	0.019	0.021
		0.018	0.012	0.013	0.014	0.015	0.016	0.017	0.018	0.019	0.021	0.023	0.025	0.028
		0.020	0.015	0.016	0.017	0.018	0.019	0.021	0.022	0.024	0.026	0.028	0.031	0.035
		0.022	0.018	0.019	0.020	0.022	0.023	0.025	0.026	0.028	0.031	0.034	0.037	0.041
		0.024	0.021	0.022	0.024	0.025	0.027	0.029	0.031	0.033	0.036	0.039	0.043	0.048
		0.026	0.024	0.026	0.027	0.029	0.030	0.033	0.035	0.038	0.041	0.045	0.049	0.055
		0.028	0.027	0.029	0.030	0.032	0.034	0.037	0.039	0.042	0.046	0.050	0.055	0.062
		0.030	0.030	0.032	0.034	0.036	0.038	0.041	0.043	0.047	0.051	0.056	0.061	0.068
16 000	800 ($n=12$)	0.010	0.000	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.00[illegible]
		0.012	0.005	0.005	0.005	0.006	0.006	0.006	0.007	0.008	0.008	0.009	0.011	0.01[illegible]
		0.014	0.008	0.009	0.009	0.010	0.010	0.011	0.012	0.013	0.015	0.016	0.019	0.021
		0.016	0.012	0.012	0.013	0.014	0.015	0.016	0.018	0.019	0.021	0.024	0.027	0.031
		0.018	0.015	0.016	0.017	0.018	0.020	0.021	0.023	0.025	0.028	0.031	0.035	0.04[illegible]
		0.020	0.019	0.020	0.021	0.023	0.024	0.026	0.028	0.031	0.034	0.038	0.043	0.04[illegible]
		0.022	0.022	0.024	0.025	0.027	0.029	0.031	0.034	0.037	0.040	0.045	0.051	0.05[illegible]
		0.024	0.026	0.027	0.029	0.031	0.033	0.036	0.039	0.042	0.047	0.052	0.059	0.06[illegible]
		0.026	0.029	0.031	0.033	0.035	0.038	0.041	0.044	0.048	0.053	0.059	0.067	0.07[illegible]
		0.028	0.033	0.035	0.037	0.039	0.042	0.046	0.050	0.054	0.060	0.066	0.075	0.08[illegible]
		0.030	0.036	0.038	0.041	0.044	0.047	0.051	0.055	0.060	0.066	0.071	0.08[illegible]	0.09[illegible]

f_s	f_c	p_1	Ratios of Compression Steel, p'											
			Ratios of Depth of Compression to Tension Steel, a											
			0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24
16 000	850	0.012	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.006	0.006	0.007
	($n=12$)	0.014	0.006	0.007	0.007	0.007	0.008	0.008	0.009	0.010	0.011	0.012	0.013	0.01[illegible]
		0.016	0.009	0.010	0.011	0.011	0.012	0.013	0.014	0.015	0.017	0.018	0.021	0.023
		0.018	0.013	0.014	0.014	0.015	0.016	0.018	0.019	0.021	0.023	0.025	0.028	0.032
		0.020	0.016	0.017	0.018	0.019	0.021	0.022	0.024	0.026	0.028	0.031	0.035	0.040
		0.022	0.019	0.021	0.022	0.023	0.025	0.027	0.029	0.031	0.034	0.038	0.042	0.048
		0.024	0.023	0.024	0.025	0.027	0.029	0.031	0.034	0.037	0.040	0.044	0.050	0.056
		0.026	0.026	0.028	0.029	0.031	0.033	0.036	0.039	0.042	0.046	0.051	0.057	0.064
		0.028	0.029	0.031	0.033	0.035	0.037	0.040	0.043	0.047	0.052	0.057	0.064	0.073
		0.030	0.033	0.035	0.037	0.039	0.042	0.045	0.048	0.053	0.058	0.064	0.071	0.081
16 000	900	0.012	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.003
	($n=12$)	0.014	0.004	0.004	0.005	0.005	0.005	0.006	0.006	0.007	0.007	0.008	0.009	0.010
		0.016	0.007	0.008	0.008	0.009	0.009	0.010	0.011	0.012	0.013	0.014	0.015	0.017
		0.018	0.010	0.011	0.012	0.012	0.013	0.014	0.015	0.017	0.018	0.020	0.022	0.025
		0.020	0.014	0.014	0.015	0.016	0.017	0.018	0.020	0.021	0.023	0.026	0.028	0.032
		0.022	0.017	0.018	0.019	0.020	0.021	0.023	0.024	0.026	0.029	0.032	0.035	0.039
		0.024	0.020	0.021	0.022	0.024	0.025	0.027	0.029	0.031	0.034	0.037	0.041	0.047
		0.026	0.023	0.024	0.026	0.027	0.029	0.031	0.033	0.036	0.039	0.043	0.048	0.054
		0.028	0.026	0.028	0.029	0.031	0.033	0.035	0.038	0.041	0.045	0.049	0.055	0.061
		0.030	0.029	0.031	0.033	0.035	0.037	0.040	0.042	0.046	0.050	0.055	0.061	0.069
16 000	1000	0.014	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002
	($n=12$)	0.016	0.004	0.004	0.004	0.004	0.005	0.005	0.005	0.006	0.006	0.007	0.007	0.008
		0.018	0.007	0.007	0.007	0.008	0.008	0.009	0.009	0.010	0.011	0.012	0.013	0.014
		0.020	0.009	0.010	0.010	0.011	0.012	0.012	0.013	0.014	0.016	0.017	0.018	0.020
		0.022	0.012	0.013	0.014	0.014	0.015	0.016	0.017	0.019	0.021	0.022	0.024	0.026
		0.024	0.015	0.016	0.017	0.018	0.019	0.020	0.021	0.023	0.026	0.027	0.029	0.032
		0.026	0.018	0.019	0.020	0.021	0.022	0.024	0.025	0.027	0.030	0.032	0.035	0.038
		0.028	0.021	0.022	0.023	0.024	0.026	0.027	0.029	0.031	0.035	0.037	0.040	0.044
		0.030	0.023	0.025	0.026	0.027	0.029	0.031	0.033	0.036	0.040	0.042	0.046	0.051
18 000	500	0.005	0.002	0.003	0.003	0.003	0.003	0.004	0.004	0.005	0.006	0.007	0.009	0.012
	($n=15$)	0.006	0.005	0.005	0.006	0.006	0.007	0.008	0.009	0.010	0.012	0.014	0.018	0.025
		0.008	0.010	0.011	0.012	0.013	0.014	0.016	0.018	0.021	0.024	0.029	0.037	0.051
		0.010	0.015	0.016	0.018	0.020	0.022	0.024	0.027	0.031	0.037	0.044	0.056	0.077
		0.012	0.020	0.022	0.024	0.026	0.029	0.032	0.036	0.042	0.049	0.059	0.075	0.103
		0.014	0.026	0.028	0.030	0.033	0.036	0.040	0.045	0.052	0.061	0.074	0.095	0.120
		0.016	0.031	0.033	0.036	0.039	0.043	0.048	0.055	0.063	0.074	0.089	0.114	0.156
		0.018	0.036	0.039	0.042	0.046	0.051	0.056	0.064	0.073	0.086	0.104	0.133	0.182
		0.020	0.041	0.044	0.048	0.053	0.058	0.065	0.073	0.084	0.099	0.119	0.152	0.208
		0.022	0.046	0.050	0.054	0.059	0.065	0.073	0.082	0.094	0.111	0.134	0.171	0.234
		0.024	0.051	0.055	0.060	0.066	0.072	0.081	0.091	0.105	0.123	0.150	0.190	0.260
		0.026	0.056	0.061	0.066	0.072	0.080	0.089	0.100	0.115	0.136	0.165	0.209	0.286
		0.028	0.062	0.066	0.072	0.079	0.087	0.097	0.110	0.126	0.148	0.180	0.228	0.312
		0.030	0.067	0.072	0.078	0.086	0.094	0.105	0.119	0.137	0.160	0.195	0.247	0.339
18 000	600	0.006	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.003	0.003	0.004
	($n=15$)	0.008	0.005	0.006	0.006	0.007	0.007	0.008	0.009	0.010	0.011	0.013	0.015	0.018
		0.010	0.010	0.010	0.011	0.012	0.013	0.014	0.016	0.017	0.020	0.023	0.027	0.032
		0.012	0.014	0.015	0.016	0.017	0.019	0.020	0.023	0.025	0.028	0.033	0.038	0.047
		0.014	0.018	0.019	0.021	0.022	0.024	0.027	0.029	0.033	0.037	0.043	0.050	0.061
		0.016	0.022	0.024	0.026	0.028	0.030	0.033	0.036	0.041	0.046	0.053	0.062	0.075
		0.018	0.027	0.029	0.031	0.033	0.036	0.039	0.043	0.048	0.055	0.063	0.074	0.090
		0.020	0.031	0.033	0.035	0.038	0.042	0.045	0.050	0.056	0.063	0.073	0.086	0.104
		0.022	0.035	0.038	0.040	0.044	0.047	0.052	0.057	0.064	0.072	0.083	0.097	0.118
		0.024	0.039	0.042	0.045	0.049	0.053	0.058	0.064	0.071	0.081	0.093	0.109	0.133
		0.026	0.044	0.047	0.050	0.054	0.059	0.064	0.071	0.079	0.089	0.103	0.121	0.147
		0.028	0.048	0.051	0.055	0.059	0.064	0.071	0.078	0.087	0.098	0.113	0.133	0.161
		0.030	0.052	0.056	0.060	0.065	0.070	0.077	0.085	0.095	0.107	0.123	0.145	0.176
18 000	650	0.008	0.003	0.004	0.004	0.004	0.004	0.005	0.005	0.006	0.006	0.007	0.008	0.010
	($n=15$)	0.010	0.007	0.008	0.008	0.009	0.010	0.010	0.011	0.013	0.014	0.016	0.018	0.022
		0.012	0.011	0.012	0.013	0.014	0.015	0.016	0.018	0.019	0.021	0.025	0.028	0.033
		0.014	0.015	0.016	0.017	0.018	0.020	0.022	0.024	0.026	0.029	0.033	0.038	0.045
		0.016	0.019	0.020	0.022	0.023	0.025	0.027	0.030	0.033	0.037	0.042	0.048	0.057
		0.018	0.023	0.024	0.026	0.028	0.030	0.033	0.036	0.040	0.044	0.050	0.058	0.068
		0.020	0.027	0.029	0.031	0.033	0.035	0.039	0.042	0.047	0.052	0.059	0.068	0.080
		0.022	0.031	0.033	0.035	0.038	0.041	0.044	0.048	0.053	0.059	0.068	0.078	0.092
		0.024	0.035	0.037	0.040	0.042	0.046	0.050	0.054	0.060	0.067	0.076	0.088	0.104
		0.026	0.039	0.041	0.044	0.047	0.051	0.055	0.061	0.067	0.074	0.085	0.098	0.115
		0.028	0.043	0.045	0.048	0.052	0.056	0.061	0.067	0.074	0.082	0.093	0.108	0.127
		0.030	0.047	0.050	0.053	0.057	0.061	0.067	0.073	0.081	0.089	0.102	0.117	0.130

TABLE 14. BEAMS WITH COMPRESSION STEEL—Continued.

f_s	f_c	p_1	Ratios of Compression Steel, p' — Ratios of Depth of Compression to Tension Steel, a											
			0.02	0.04	0.06	0.08	0.10	0.12	0.14	0.16	0.18	0.20	0.22	0.24
18 000	750 (n=15)	0.010	0.003	0.004	0.004	0.004	0.004	0.005	0.005	0.006	0.006	0.007	0.008	0.009
		0.012	0.007	0.007	0.008	0.008	0.009	0.009	0.010	0.011	0.012	0.013	0.015	0.017
		0.014	0.010	0.011	0.011	0.012	0.013	0.014	0.015	0.016	0.018	0.020	0.022	0.025
		0.016	0.014	0.014	0.015	0.016	0.017	0.019	0.020	0.022	0.024	0.027	0.030	0.034
		0.018	0.017	0.018	0.019	0.020	0.022	0.023	0.025	0.027	0.030	0.033	0.037	0.042
		0.020	0.020	0.021	0.023	0.024	0.026	0.028	0.030	0.033	0.036	0.040	0.045	0.051
		0.022	0.024	0.025	0.027	0.028	0.030	0.033	0.035	0.038	0.042	0.047	0.052	0.059
		0.024	0.027	0.029	0.030	0.032	0.035	0.037	0.040	0.044	0.048	0.053	0.060	0.068
		0.026	0.030	0.032	0.034	0.036	0.039	0.042	0.045	0.049	0.054	0.060	0.067	0.076
		0.028	0.034	0.036	0.038	0.040	0.043	0.046	0.050	0.055	0.060	0.067	0.075	0.085
		0.030	0.037	0.039	0.042	0.044	0.048	0.051	0.055	0.060	0.066	0.073	0.082	0.093
18 000	800 (n=12)	0.008	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.002
		0.010	0.005	0.005	0.005	0.006	0.006	0.007	0.008	0.008	0.009	0.011	0.012	0.015
		0.012	0.009	0.009	0.010	0.011	0.012	0.013	0.014	0.015	0.017	0.019	0.022	0.027
		0.014	0.013	0.014	0.015	0.016	0.017	0.018	0.020	0.022	0.025	0.028	0.033	0.039
		0.016	0.017	0.018	0.019	0.020	0.022	0.024	0.026	0.029	0.033	0.037	0.043	0.051
		0.018	0.021	0.022	0.024	0.025	0.027	0.030	0.033	0.036	0.040	0.046	0.053	0.063
		0.020	0.025	0.026	0.028	0.030	0.033	0.036	0.039	0.043	0.048	0.055	0.063	0.075
		0.022	0.029	0.031	0.033	0.035	0.038	0.041	0.045	0.050	0.056	0.063	0.073	0.087
		0.024	0.033	0.035	0.037	0.040	0.043	0.047	0.051	0.057	0.064	0.072	0.084	0.099
		0.026	0.037	0.039	0.042	0.045	0.048	0.053	0.058	0.064	0.071	0.081	0.094	0.111
		0.028	0.041	0.043	0.046	0.050	0.054	0.058	0.064	0.071	0.079	0.090	0.104	0.123
		0.030	0.045	0.047	0.051	0.055	0.059	0.064	0.070	0.078	0.087	0.099	0.114	0.135
18 000	850 (n=12)	0.010	0.003	0.003	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.006	0.007	0.008
		0.012	0.007	0.007	0.007	0.008	0.009	0.009	0.010	0.011	0.012	0.014	0.016	0.018
		0.014	0.010	0.011	0.012	0.012	0.013	0.015	0.016	0.017	0.019	0.022	0.025	0.029
		0.016	0.014	0.015	0.016	0.017	0.018	0.020	0.022	0.024	0.026	0.030	0.034	0.039
		0.018	0.018	0.019	0.020	0.022	0.023	0.025	0.027	0.030	0.033	0.037	0.043	0.050
		0.020	0.021	0.023	0.024	0.026	0.028	0.030	0.033	0.036	0.040	0.045	0.052	0.060
		0.022	0.025	0.027	0.029	0.031	0.033	0.036	0.039	0.043	0.047	0.053	0.061	0.071
		0.024	0.029	0.031	0.033	0.035	0.038	0.041	0.045	0.049	0.054	0.061	0.070	0.081
		0.026	0.033	0.035	0.037	0.040	0.043	0.046	0.050	0.055	0.061	0.069	0.079	0.092
		0.028	0.036	0.039	0.041	0.044	0.048	0.051	0.056	0.062	0.068	0.077	0.088	0.102
		0.030	0.040	0.043	0.045	0.049	0.052	0.057	0.062	0.068	0.075	0.085	0.097	0.112
18 000	900 (n=12)	0.010	0.001	0.001	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.003
		0.012	0.005	0.005	0.005	0.006	0.006	0.006	0.007	0.008	0.008	0.009	0.011	0.012
		0.014	0.008	0.009	0.009	0.010	0.010	0.011	0.012	0.013	0.015	0.016	0.019	0.021
		0.016	0.012	0.012	0.013	0.014	0.015	0.016	0.018	0.019	0.021	0.024	0.027	0.031
		0.018	0.015	0.016	0.017	0.018	0.020	0.021	0.023	0.025	0.028	0.031	0.035	0.040
		0.020	0.019	0.020	0.021	0.023	0.024	0.026	0.028	0.031	0.034	0.038	0.043	0.049
		0.022	0.022	0.024	0.025	0.027	0.029	0.031	0.034	0.037	0.040	0.045	0.051	0.058
		0.024	0.026	0.027	0.029	0.031	0.033	0.036	0.039	0.042	0.047	0.052	0.059	0.068
		0.026	0.029	0.031	0.033	0.035	0.038	0.041	0.044	0.048	0.053	0.059	0.067	0.077
		0.028	0.033	0.035	0.037	0.039	0.042	0.046	0.050	0.054	0.060	0.066	0.075	0.086
		0.030	0.036	0.038	0.041	0.044	0.047	0.051	0.055	0.060	0.066	0.074	0.083	0.095
18 000	1 000 (n=12)	0.012	0.001	0.001	0.001	0.002	0.002	0.002	0.002	0.002	0.002	0.002	0.003	0.003
		0.014	0.004	0.005	0.005	0.005	0.006	0.006	0.007	0.007	0.008	0.008	0.009	0.011
		0.016	0.008	0.008	0.009	0.009	0.010	0.010	0.011	0.012	0.013	0.014	0.016	0.018
		0.018	0.011	0.011	0.012	0.013	0.014	0.015	0.016	0.017	0.019	0.020	0.023	0.026
		0.020	0.014	0.015	0.016	0.017	0.018	0.019	0.020	0.022	0.024	0.026	0.029	0.033
		0.022	0.017	0.018	0.019	0.020	0.022	0.023	0.025	0.027	0.029	0.032	0.036	0.041
		0.024	0.020	0.021	0.023	0.024	0.026	0.027	0.030	0.032	0.035	0.038	0.043	0.048
		0.026	0.023	0.025	0.026	0.028	0.030	0.032	0.034	0.037	0.040	0.044	0.049	0.056
		0.028	0.027	0.028	0.030	0.032	0.034	0.036	0.039	0.042	0.046	0.050	0.056	0.063
		0.030	0.030	0.031	0.033	0.035	0.038	0.040	0.043	0.047	0.051	0.056	0.063	0.071

DESIGN OF BEAMS WITH COMPRESSION STEEL.

Table 14. For the design of beams with steel in top and bottom Table 14, pages 589 to 591, may be used. Diagrams 2 and 3, pages 594 and 595, may be used also but are less convenient except for stresses not covered in Table 14. The use of Table 14 is illustrated as follows:

Example: Given, bending moment, $M = 2\ 000\ 000$; available depth and breadth of beam, 32 inches and 14 inches; allowable stresses $f_s = 18000$ and $f_c = 750$; and $n = 15$. The depth of the compression steel is $ad = 2$ inches.

Determine the amount of tensile and compressive steel.

Solution: Since $h = 32$ inches, $d = 29$ inches and $a = \frac{2}{29} = 0.069$. From the formula $A_s = \frac{M}{jdf_s}$, where we may assume $j = 0.89$, (see page 496) we have $A_s = \frac{2\ 000\ 000}{0.89 \times 29 \times 18\ 000} = 4.30$ square inches. Compute $p = \frac{A_s}{bd} = \frac{4.30}{14 \times 29} = 0.0106$. Refer to Table 14 in the section for $f_s = 18\ 000$ and $f_c = 750$, and find the value of p' corresponding to $a = 0.06$ and $p_1 = 0.0106$. $p' = 0.0052$.

Check the value of j by referring to Diagram 1, Page 593, and recompute if necessary.

REVIEW OF BEAMS WITH COMPRESSION STEEL.

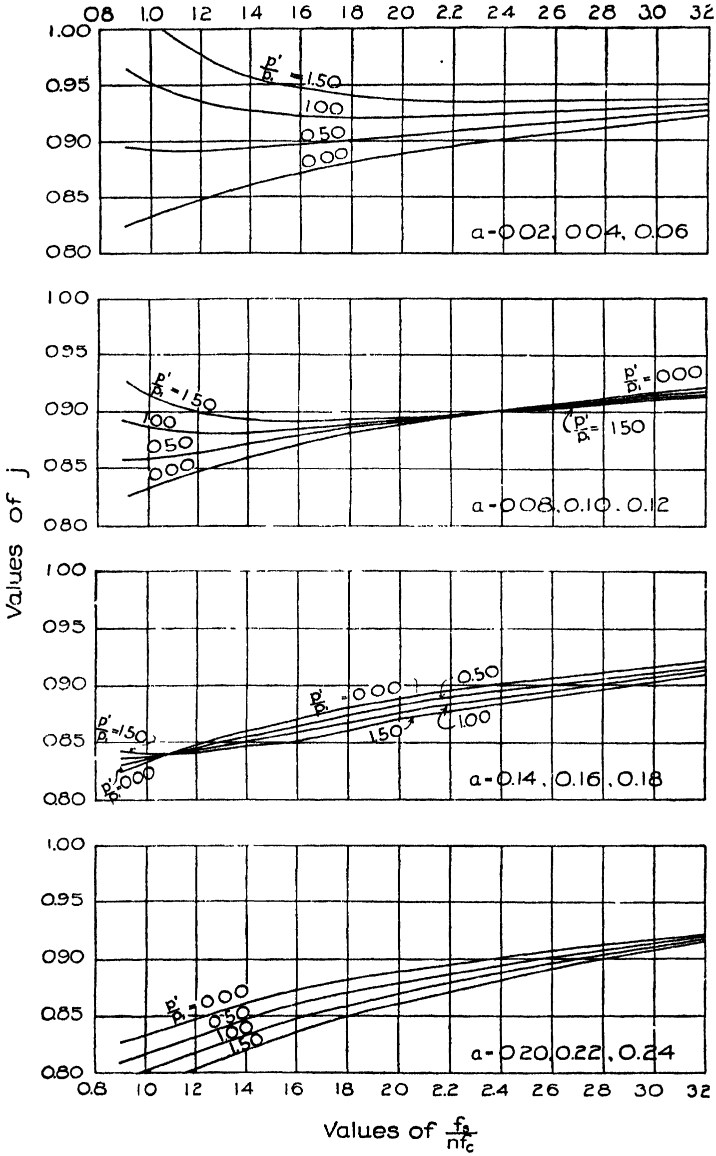
Diagrams 2 and 3. For the review of beams with steel in top and bottom, where it is required to determine the stresses when the dimensions of the beam and steel area are given, Diagrams 2 and 3, pages 594 and 595, are to be used, as illustrated below.

Example: Given $A_{s_2} = 3\ 5$ square inches; $M = 1\ 230\ 000$ inch pounds; $A'_s = 2.0$ square inches; and ratio $\frac{p_1}{p'} = 1.75$. The depth of compressive steel is $ad = 1.5$ inches. Find f_s and f_c.

Solution: Since $h = 24.5$ inches, $d = 23$ inches and $a = \frac{1.5}{23} = 0.065$. Compute $p_1 = \frac{3.5}{23 \times 10} = 0.0152$. and $p' = \frac{2\ 0}{23 \times 10} = 0.0087$. From Diagram 2, (page 594) for $a = 0.06$ and $p_1 = 0.0152$ and $p' = 0.0087$ we have, by interpolation, a value of $\frac{f_s}{nf_c} = 1.3$. From the formula $f_s = \frac{M}{jdA_{s_1}}$ where $j = 0.89$, (see p. 496) we have $f_s = \frac{1\ 230\ 000}{0.89 \times 23 \times 3.5} = 17\ 200$ pounds per square inch. Since $\frac{f_s}{nf_c} = 1.3$ and $f_s = 17\ 200$ we have, for $n = 15$, $f_c = \frac{17\ 200}{1.3 \times 15} = 880$ pounds per square inch.

Check the value of j by referring to Diagram 1, page 593, and recompute if necessary.

DIAGRAM 1.—VALUES OF j FOR BEAMS WITH STEEL IN TOP AND BOTTOM. (*See p.* 496.)



a = ratio of depth of steel in compression to depth of steel in tension.

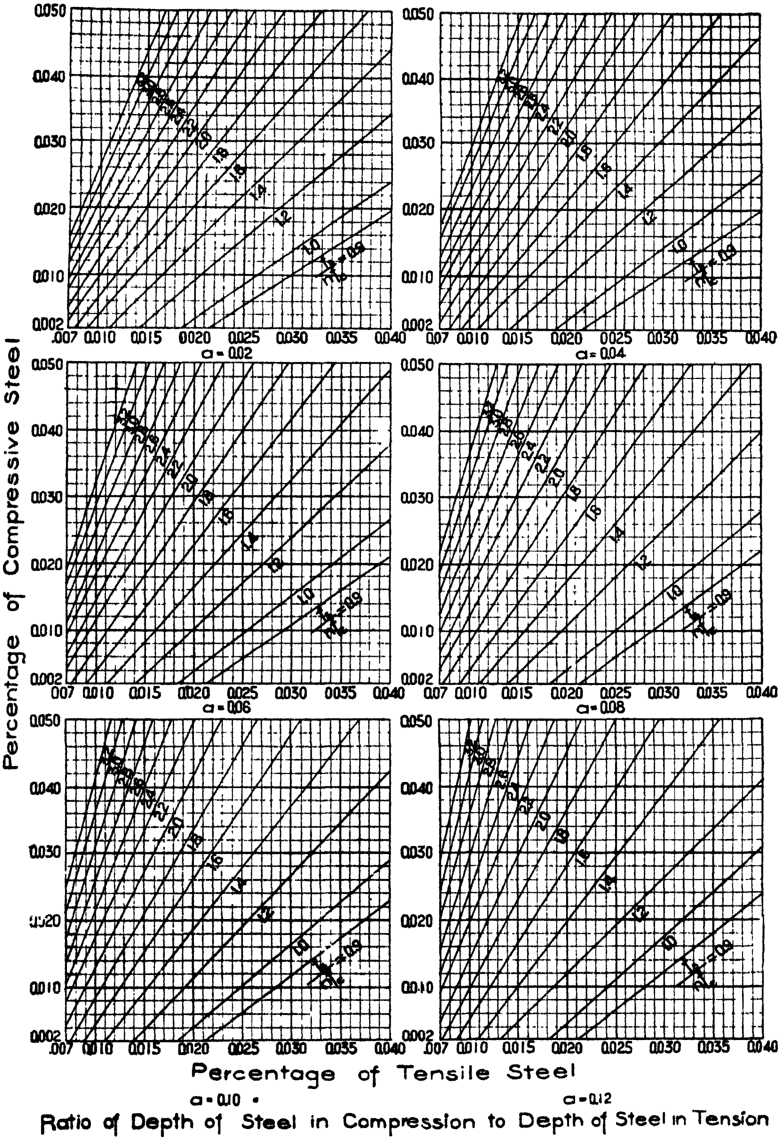


DIAGRAM 2.—Relation between Tensile and Compressive Steel in Beams with Steel in Top and Bottom. (*See p.* 492.)

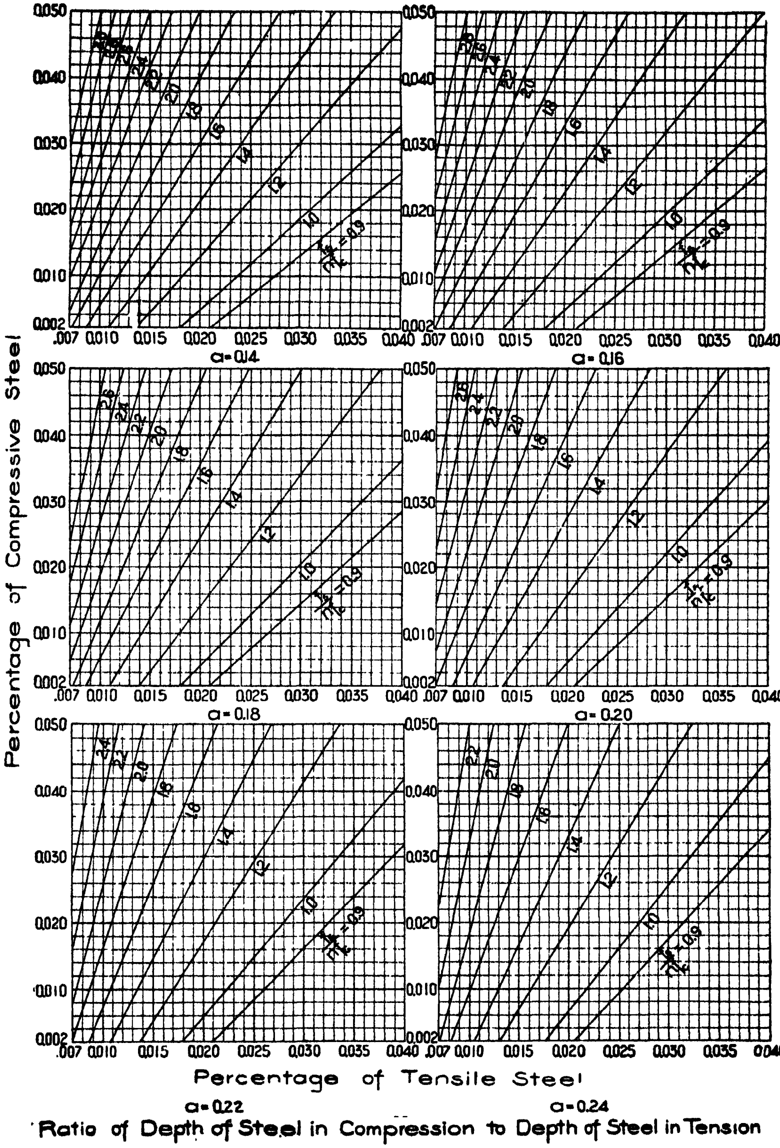


DIAGRAM 3.—Relation between Tensile and Compressive Steel in Beams with Steel in Top and Bottom. (*See p.* 492.)

TABLE 15. TABLE FOR CONSTANT C FOR BEAMS

Data for Determining Depth of Beam, Moment of Resistance and Reinforcement

To be used in formula for Depth of rectangular beams or slabs, $d = C\sqrt{\frac{M}{b}}$ and in formula for Moment of Resistance $M = \frac{bd^2}{C^2}$.

(See. pp. 483 and 355) Based on dimensions in inches and moments in inch-pounds.

		Ratio of Moduli of Steel to Concrete $n=12$				**Ratio of Moduli of Steel to Concrete $n = 15$**			
Working Strength of Steel f_s lb per sq. in.	Working Strength of Concrete f_c lb. per sq. in.	Ratio Depth of Neutral Axis to Depth of Steel k	Ratio of Moment Arm to Depth of Steel $\left(1 - \frac{k}{3}\right)$ j	Ratio Area of Steel to Beam Above Steel p	Safe Working Value of Constant C C	Ratio Depth of Neutral Axis to Depth of Steel k	Ratio of Moment Arm to Depth of Steel $\left(1 - \frac{k}{3}\right)$ j	Ratio Area of Steel to Beam Above Steel p	Safe Working Value of Constant C C
12 000	850	0.460	0.847	0.0163	0.077	0.515	0.828	0.0183	0.074
14 000	500	0.300	0.900	0.0054	0.122	0.348	0.884	0.0062	0.114
	550	0.320	0.893	0.0063	0.113	0.372	0.876	0.0073	0.106
	600	0.340	0.887	0.0073	0.105	0.392	0.869	0.0084	0.099
	650	0.358	0.881	0.0083	0.099	0.409	0.861	0.0095	0.093
	700	0.375	0.875	0.0094	0.093	0.428	0.857	0.0107	0.088
	750	0.391	0.870	0.0105	0.088	0.446	0.851	0.0120	0.083
	800	0.407	0.864	0.0116	0.084	0.462	0.846	0.0132	0.080
	850	0.422	0.860	0.0128	0.081	0.477	0.841	0.0145	0.077
16 000	500	0.273	0.909	0.0043	0.127	0.319	0.894	0.0050	0.118
	550	0.292	0.903	0.0050	0.117	0.339	0.887	0.0058	0.110
	600	0.310	0.897	0.0058	0.109	0.358	0.881	0.0067	0.103
	650	0.328	0.891	0.0067	0.102	0.378	0.874	0.0077	0.096
	700	0.344	0.885	0.0075	0.097	0.397	0.868	0.0087	0.091
	750	0.360	0.880	0.0085	0.092	0.414	0.862	0.0097	0.086
	800	0.375	0.875	0.0094	0.087	0.429	0.857	0.0107	0.083
	850	0.389	0.870	0.0103	0.083	0.444	0.852	0.0118	0.079
	900	0.403	0.866	0.0113	0.080	0.458	0.847	0.0129	0.075
18 000	500	0.250	0.917	0.0035	0.132	0.294	0.902	0.0041	0.123
	550	0.268	0.911	0.0040	0.123	0.314	0.895	0.0048	0.113
	600	0.286	0.905	0.0048	0.113	0.333	0.889	0.0056	0.106
	650	0.302	0.899	0.0055	0.106	0.351	0.883	0.0063	0.099
	700	0.318	0.894	0.0062	0.100	0.360	0.877	0.0072	0.094
	750	0.333	0.889	0.0070	0.094	0.385	0.872	0.0080	0.089
	800	0.348	0.884	0.0077	0.090	0.400	0.867	0.0089	0.085
	850	0.362	0.879	0.0085	0.086	0.415	0.862	0.0098	0.081
	900	0.375	0.875	0.0094	0.082	0.429	0.857	0.0107	0.077
20 000	500	0.231	0.923	0.0029	0.137	0.272	0.909	0.0034	0.127
	550	0.248	0.917	0.0034	0.126	0.292	0.903	0.0040	0.118
	600	0.265	0.912	0.0040	0.117	0.311	0.896	0.0047	0.109
	650	0.281	0.907	0.0046	0.110	0.328	0.891	0.0053	0.103
	700	0.296	0.901	0.0052	0.103	0.344	0.885	0.0060	0.097
	750	0.310	0.897	0.0058	0.098	0.359	0.880	0.0067	0.092
	800	0.324	0.892	0.0065	0.093	0.374	0.875	0.0075	0.087
	850	0.338	0.887	0.0072	0.088	0.389	0.870	0.0081	0.083
	900	0.351	0.883	0.0079	0.085	0.403	0.866	0.0091	0.080
24 000	850	0.298	0.901	0.0053	0.093	0.347	0.884	0.0061	0.088

TABLE 16. DATA FOR DETERMINING DEPTH OF RECTANGULAR BEAM OR SLAB OR MOMENT OF RESISTANCE FOR DIFFERENT PERCENTAGES OF STEEL.

Ratio of elasticity, $n = 15$.

Rule 1. To find depth of beam or slab for a given percentage of steel:

On line with the given percentage, select the higher value of C. This, substituted in formula

$$d = C\sqrt{\frac{M}{b}}$$

(see p. 481), gives the smallest permissible depth. Thus for 0.004 steel ratio the value of C from column (9) must be used instead of from column (6) because the latter would stress the steel to 23 700 pounds, which would not be allowable. It is evident also that the ratio of steel is too low for economy, because concrete is stressed only to 440 pounds.

Rule 2. To find amount of steel for a given beam or slab and given loading with stress in concrete limited to 650 pounds per square inch and stress in steel to 16 000 pounds per square inch:

Compute value of C from formula $M = \frac{bd^2}{C^2}$ (see p. 355). Locate this value either in column (6) or (9), whichever satisfies the allowed stresses, and find the corresponding value of p in the first column. Thus, if $C = 0.097$, it must be located in column (9) instead of column (6), because the latter would give a higher stress in steel than is allowable. The desired ratio of steel is therefore 0.0077. If $C = 0.088$, it must be located in column (6) because column (9) would give too high a stress in concrete.

Ratio area of steel to beam above steel.	Ratio depth of neutral axis to depth of steel.	Ratio moment arm to depth of steel.	Working compressive strength of concrete Lb. per sq. in.	Maximum fibre stress in steel corresponding to $f_c = 650$	Constant in formula $d = C\sqrt{\frac{M}{b}}$ see page 483	Working tensile strength of steel Lb. per sq. in.	Maximum fiber stress in concrete corresponding to $f_s = 16\,000$	Constant in formula $d = C\sqrt{\frac{M}{b}}$ see page 483
p (1)	k (2)	j (3)	f_c (4)	f_s (5)	C (6)	f_s (7)	f_c (8)	C' (9)
0.002	0.217	0.928	650	32900	0.124	16000	290	0.183
0.003	0.258	0.914	650	28000	0.114	16000	370	0.151
0.004	0.292	0.903	650	23700	0.108	16000	440	0.132
0.005	0.320	0.893	650	20800	0.104	16000	500	0.118
0.006	0.344	0.885	650	18600	0.100	16000	560	0.108
0.007	0.365	0.878	650	16900	0.098	16000	610	0.101
0.008	0.384	0.872	650	15600	0.096	16000	670	0.095
0.009	0.402	0.866	650	14500	0.094	16000	720	0.089
0.010	0.418	0.861	650	13600	0.092	16000	760	0.085
0.012	0.446	0.851	650	12100	0.090	16000	860	0.078
0.014	0.471	0.843	650	11000	0.088	16000	950	0.072
0.016	0.493	0.836	650	10000	0.086	16000	1040	0.068
0.018	0.513	0.829	650	9300	0.085	16000	1120	0.065
0.020	0.531	0.823	650	8600	0.084	16000	1210	0.061

TABLE 17. PROPORTIONAL DEPTHS OF NEUTRAL AXIS

Proportional Depth of Neutral Axis Below Top of Beam, k, and Ratio of Stress in Steel to Stress in Concrete, $\frac{f_s}{f_c}$, *for Different Percentages of Steel and Various Ratios of Moduli of Elasticity.* (*See p.* 400.)

The table below gives the proportional depths of the neutral axis, *k*, calculated from formula 6, page 484, for various percentages of steel and moduli of elasticity and the corresponding ratio of stress in steel to stress in concrete, $\frac{f_s}{f_c}$. Its principal use is for determining the moment of resistance, and consequently the safe loading for beams already built. Its use is *not* advised for ordinary calculations of moments of resistance and dimensions of beams or slabs, because it presents no means of determining, without further calculation, the stress in the steel or the concrete, and therefore is liable to lead to uneconomical design.

p Ratio of Area of Steel to Cross-Section of Beam Above Steel	$n = 10$		$n = 12$		$n = 15$		$n = 20$		$n = 30$		$n = 35$	
	k	$\frac{f_s}{f_c}$	k	$\frac{f_s}{f_c}$	k	$\frac{f_s}{f_c}$	k	$\frac{f_s}{f_c}$	k	$\frac{f_s}{f_c}$	k	$\frac{f_s}{f_c}$
0.001	0.132	66	0.143	72	0.158	79	0.181	91	0.217	109	0.232	116
0.002	0.181	45	0.196	49	0.217	54	0.246	62	0.292	73	0.311	78
0.003	0.217	36	0.235	39	0.258	43	0.292	49	0.344	57	0.365	61
0.004	0.246	31	0.266	33	0.292	37	0.328	41	0.384	48	0.420	53
0.005	0.270	27	0.292	29	0.320	32	0.358	36	0.418	42	0.442	44
0.006	0.292	24	0.314	26	0.344	29	0.384	32	0.446	37	0.471	39
0.007	0.311	22	0.334	24	0.365	26	0.407	29	0.471	34	0.497	36
0.008	0.328	21	0.353	22	0.384	24	0.428	27	0.493	31	0.519	32
0.009	0.344	19	0.369	21	0.402	22	0.446	25	0.513	29	0.539	30
0.010	0.358	18	0.384	19	0.418	21	0.463	23	0.531	27	0.557	28
0.012	0.384	16	0.402	17	0.446	19	0.493	21	0.562	23	0.588	25
0.014	0.407	15	0.436	16	0.471	17	0.519	19	0.588	21	0.614	22
0.016	0.428	13	0.457	14	0.493	15	0.542	17	0.611	19	0.637	20
0.018	0.446	12	0.476	13	0.513	14	0.562	16	0.631	18	0.657	18
0.020	0.463	12	0.493	12	0.531	13	0.580	15	0.649	16	0.675	17
0.030	0.531	9	0.562	9	0.599	10	0.649	11				
0.040	0.580	7	0.611	8	0.649	8	0.697	9				
0.050	0.618	6	0.649	6	0.686	7	0.732	7				

TABLE 18. AVERAGE WORKING UNIT STRESS, f, ON CONCRETE COLUMNS

Reinforced with Longitudinal Bars, for Different Unit Stresses in Concrete and Different Percentages of Steel. (*See p.* 564.)

Based on $f = f_c\,[1 + (n - 1)\,p]$. (*See p.* 562.)

Ratio of Moduli of Elasticity	Ratio of Steel p	Allowable Average Unit Stress, f, on Columns in Lb. per Sq. In.									
		f_c = 400	f_c = 450	f_c = 500	f_c = 550	f_c = 600	f_c = 650	f_c = 700	f_c = 750	f_c = 800	
n	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
15	0.0075	442	497	553	608	663	718	774	829	884	
	0.0100	456	513	570	627	684	741	798	855	912	
	0.0125	470	529	588	646	705	764	823	881	940	
	0.0150	484	545	605	666	726	787	847	908	968	
	0.0175	498	560	623	685	747	809	872	934	996	
	0.0200	512	576	640	704	768	832	896	960	1024	
	0.0225	526	592	658	723	789	855	921	986	1052	
	0.0250	540	608	675	743	810	878	945	1013	1080	
	0.0275	554	623	693	762	831	900	970	1039	1108	
	0.0300	568	639	710	781	852	923	994	1065	1136	
	0.0325	582	655	728	800	873	946	1019	1091	1164	
	0.0350	596	671	745	820	894	969	1043	1118	1192	
	0.0375	610	686	763	839	915	991	1068	1144	1220	
	0.0400	624	702	780	858	936	1014	1092	1170	1248	
	0.0425	638	718	798	877	957	1037	1117	1196	1276	
	0.0450	652	734	815	896	978	1059	1141	1223	1304	
	0.0475	666	749	833	916	999	1082	1165	1249	1332	
	0.0500	680	765	850	935	1020	1105	1190	1275	1360	
	0.0550	708	797	885	973	1062	1150	1239	1328	1416	
	0.0600	736	828	920	1012	1104	1196	1288	1380	1472	
		f_c = 550	f_c = 600	f_c = 650	f_c = 700	f_c = 750	f_c = 800	f_c = 850	f_c = 900	f_c = 950	f_c = 1000
12	0.0075	595	650	704	758	812	866	920	974	1028	1083
	0.0100	611	666	722	777	833	888	943	999	1055	1110
	0.0125	626	683	739	796	853	910	967	1024	1081	1138
	0.0150	641	699	757	816	874	932	990	1048	1107	1165
	0.0175	656	716	775	835	894	954	1014	1073	1133	1193
	0.0200	671	732	793	854	915	976	1037	1098	1159	1220
	0.0225	686	749	811	873	936	998	1060	1123	1185	1248
	0.0250	701	765	829	893	957	1020	1084	1147	1211	1275
	0.0275	716	782	847	912	977	1042	1107	1172	1237	1303
	0.0300	732	798	865	931	998	1064	1130	1197	1263	1330
	0.0325	747	815	882	950	1018	1086	1154	1222	1290	1358
	0.0350	762	831	900	970	1039	1108	1177	1247	1316	1385
	0.0375	777	848	918	989	1059	1130	1201	1271	1342	1413
	0.0400	792	864	936	1008	1080	1152	1224	1296	1368	1440
	0.0425	807	881	954	1027	1101	1174	1247	1321	1394	1468
	0.0450	822	897	972	1047	1121	1196	1270	1345	1420	1495
	0.0475	837	914	990	1066	1142	1218	1294	1370	1446	1523
	0.0500	853	930	1007	1085	1163	1240	1317	1395	1472	1550
	0.0550	883	963	1043	1124	1204	1284	1364	1444	1525	1605
	0.0600	913	996	1079	1162	1245	1328	1411	1494	1577	1660
		f_c = 600	f_c = 650	f_c = 700	f_c = 750	f_c = 800	f_c = 850	f_c = 900	f_c = 950	f_c = 1000	f_c = 1100
10	0.0075	641	694	747	801	854	907	961	1014	1068	1174
	0.0100	654	709	763	818	872	926	981	1035	1090	1199
	0.0125	668	723	779	834	890	946	1001	1057	1113	1224
	0.0150	681	738	795	851	908	965	1022	1078	1135	1249
	0.0175	695	752	810	868	926	984	1042	1100	1158	1273
	0.0200	708	767	826	885	944	1003	1062	1121	1180	1298
	0.0225	722	782	842	902	962	1022	1082	1142	1203	1323
	0.0250	735	796	858	919	980	1041	1103	1164	1225	1348
	0.0275	749	811	873	936	998	1060	1123	1185	1248	1372
	0.0300	762	826	889	953	1016	1079	1143	1206	1270	1397
	0.0325	776	840	905	969	1034	1099	1163	1228	1293	1422
	0.0350	789	855	921	986	1052	1118	1183	1249	1315	1447
	0.0375	803	869	936	1003	1070	1137	1204	1271	1338	1471
	0.0400	816	884	952	1020	1088	1156	1224	1202	1360	1496
	0.0425	830	899	968	1037	1106	1175	1244	1313	2383	1522
	0.0450	843	913	984	1054	1124	1194	1264	1335	1405	1546
	0.0475	857	928	999	1071	1142	1213	1285	1356	1428	1570
	0.0500	870	942	1015	1087	1160	1232	1305	1377	1450	1595
	0.0550	897	972	1047	1121	1196	1271	1345	1420	1495	1645
	0.0600	924	1000	1078	1155	1232	1309	1386	1463	1540	1694

 TABLE 19. USE FOR DESIGNING SQUARE COLUMNS WITH VERTICAL REINFORCEMENT.

Safe Loadings for Column of Various Sizes and Steel Required for Given Load. (*See p.* 564.)

Based on $P = Af_c\,[1 + (n-1)\,p]$ (*See p.* 562.)

Of the total width of column, 1½ inches on all faces is considered as protective covering and is not included in the area (A) carrying load. If total area of column is to be used, select sizes 3 inches smaller than those given.

Width of column.	Effective Width of Column.	Ratio of Area of Steel to Effective Area of Concrete.													
		$p = 0.010$		$p = 0.015$		$p = 0.020$		$p = 0.025$		$p = 0.030$		$p = 0.035$		$p = 0.040$	
		Safe Load.	Area of Steel.	Safe Load.	Area of Steel.	Safe Load.	Area of Steel.	Safe Load.	Area of Steel.	Safe Load.	Area of Steel.	Safe Load.	Area of Steel.	Safe Load.	Area of Steel.
		P	A_s	P	A_s	P	A_s	P	A_s	P	A_s	P	A_s	P	A_s
in.	in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.
		1: 2: 4 Concrete						$f_c = 450$		$n = 15$					
10	7	25 100	0.5	26 700	0.7	28 200	1.0	29 800	1.2	31 300	1.5	32 900	1.7	34 400	2.0
11	8	32 800	0.6	34 900	1.0	36 900	1.3	38 900	1.6	40 900	1.9	42 900	2.2	44 900	2.6
12	9	41 500	0.8	43 100	1.2	46 700	1.6	49 200	2.0	51 800	2.4	54 300	2.8	56 900	3.2
13	10	51 300	1.0	54 500	1.5	57 600	2.0	60 800	2.5	63 900	3.0	67 100	3.5	70 200	4.0
14	11	62 100	1.2	65 900	1.8	69 700	2.4	73 500	3.0	77 300	3.6	81 100	4.2	84 900	4.8
15	12	73 800	1.4	78 400	2.2	83 000	2.9	87 500	3.6	92 000	4.3	96 600	5.0	101 000	5.8
16	13	86 700	1.7	92 000	2.5	97 400	3.4	103 000	4.2	108 000	5.1	113 000	5.9	119 000	6.8
17	14	101 000	2.0	107 000	2.9	113 000	3.9	119 000	4.9	125 000	5.9	131 000	6.9	138 000	7.8
18	15	115 000	2.3	123 000	3.4	130 000	4.5	137 000	5.6	144 000	6.8	151 000	7.9	158 000	9.0
19	16	131 000	2.6	139 000	3.8	147 000	5.1	156 000	6.4	164 000	7.7	172 000	9.0	180 000	10.2
20	17	148 000	2.9	157 000	4.3	166 000	5.8	176 000	7.2	185 000	8.7	194 000	10.1	203 000	11.6
22	19	185 000	3.6	197 000	5.4	208 000	7.2	219 000	9.0	231 000	10.8	242 000	12.6	253 000	14.4
24	21	226 000	4.4	240 000	6.6	254 000	8.8	268 000	11.0	282 000	13.2	296 000	15.4	310 000	17.6
26	23	271 000	5.3	288 000	7.9	305 000	10.6	321 000	13.2	338 000	15.9	355 000	18.5	371 000	21.2
28	25	321 000	6.3	340 000	9.4	360 000	12.5	380 000	15.6	399 000	18.8	419 000	21.9	439 000	25.0
30	27	374 000	7.3	397 000	10.9	420 000	14.6	443 000	18.2	466 000	21.9	489 000	25.5	512 000	29.2
32	29	431 000	8.4	458 000	12.6	484 000	16.8	511 000	21.0	537 000	25.2	564 000	29.4	590 000	33.6
34	31	493 000	9.6	523 000	14.4	554 000	19.2	584 000	24.0	614 000	28.8	644 000	33.6	675 000	38.4
36	33	559 000	10.9	593 000	16.3	627 000	21.8	662 000	27.2	696 000	32.7	730 000	38.1	765 000	43.6
38	35	628 000	12.3	667 000	18.4	706 000	24.5	744 000	30.6	783 000	36.8	821 000	42.9	860 000	40.0
		1: 1½: 3 Concrete						$f_c = 570$		$n = 12$					
10	7	31 000	0.5	32 500	0.7	34 100	1.0	35 600	1.2	37 100	1.5	38 700	1.7	40 200	2.0
11	8	40 500	0.6	42 500	1.0	44 500	1.3	46 500	1.6	48 500	1.9	50 500	2.2	52 500	2.6
12	9	51 300	0.8	53 800	1.2	56 300	1.6	58 800	2.0	61 400	2.4	64 000	2.8	66 500	3.2
13	10	63 300	1.0	66 400	1.5	69 500	2.0	72 700	2.5	75 800	3.0	79 000	3.5	82 100	4.0
14	11	76 600	1.2	80 400	1.8	84 100	2.4	87 900	3.0	91 700	3.6	96 000	4.2	99 300	4.8
15	12	91 100	1.4	95 600	2.2	100 000	2.9	105 000	3.6	109 000	4.3	114 000	5.0	118 000	5.8
16	13	107 000	1.7	112 000	2.5	118 000	3.4	123 000	4.2	128 000	5.1	133 000	5.9	139 000	6.8
17	14	124 000	2.0	130 000	2.9	136 000	3.9	142 000	4.9	149 000	5.9	155 000	6.9	161 000	7.8
18	15	142 000	2.3	149 000	3.4	156 000	4.5	164 000	5.6	171 000	6.8	178 000	7.9	185 000	9.0
19	16	162 000	2.6	170 000	3.8	178 000	5.1	186 000	6.4	194 000	7.7	202 000	9.0	210 000	10.2
20	17	183 000	2.9	192 000	4.3	201 000	5.8	210 000	7.2	219 000	8.7	228 000	10.1	237 000	11.6
22	19	228 000	3.6	240 000	5.4	251 000	7.2	262 000	9.0	274 000	10.8	285 000	12.6	296 000	14.4
24	21	279 000	4.4	293 000	6.6	307 000	8.8	321 000	11.0	334 000	13.2	348 000	15.4	362 000	17.6
26	23	335 000	5.3	351 000	7.9	368 000	10.6	385 000	13.2	401 000	15.9	418 000	18.5	434 000	21.2
28	25	395 000	6.3	415 000	9.4	435 000	12.5	454 000	15.6	474 000	18.8	493 000	21.9	513 000	25.0
30	27	461 000	7.3	484 000	10.9	507 000	14.6	530 000	18.2	553 000	21.9	576 000	25.5	598 000	29.2
32	29	532 000	8.4	559 000	12.6	585 000	16.8	611 000	21.0	638 000	25.2	664 000	29.4	690 000	33.6
34	31	608 000	9.6	638 000	14.4	668 000	19.2	698 000	24.0	720 000	28.8	759 000	33.6	789 000	38.4
36	33	689 000	10.9	723 000	16.3	757 000	21.8	791 000	27.2	826 000	32.7	860 000	38.1	894 000	43.6
38	35	775 000	12.3	814 000	18.4	852 000	24.5	890 000	30.6	929 000	36.8	967 000	42.9	1 005 000	49.0
		1: 1: 2 Concrete						$f_c = 680$		$n = 10$					
10	7	36 300	0.5	37 800	0.7	39 300	1.0	40 800	1.2	42 300	1.5	43 800	1.7	45 300	2.0
11	8	47 400	0.6	49 400	1.0	51 400	1.3	53 300	1.6	55 300	1.9	57 200	2.2	59 200	2.6
12	9	60 000	0.8	62 500	1.2	65 000	1.6	67 500	2.0	70 000	2.4	72 400	2.8	74 900	3.2
13	10	74 100	1.0	77 200	1.5	80 200	2.0	83 300	2.5	86 400	3.0	89 400	3.5	92 500	4.0
14	11	89 600	1.2	93 400	1.8	97 100	2.4	101 000	3.0	104 000	3.6	108 000	4.2	112 000	4.8
15	12	107 000	1.4	111 000	2.2	116 000	2.9	120 000	3.6	124 000	4.3	129 000	5.0	133 000	5.8
16	13	125 000	1.7	130 000	2.5	136 000	3.4	141 000	4.2	146 000	5.1	151 000	5.9	156 000	6.8
17	14	145 000	2.0	151 000	2.9	157 000	3.9	163 000	4.9	169 000	5.9	175 000	6.9	181 000	7.8
18	15	167 000	2.3	174 000	3.4	181 000	4.5	187 000	5.6	194 000	6.8	201 000	7.9	208 000	9.0
19	16	190 000	2.6	198 000	3.8	205 000	5.1	213 000	6.4	221 000	7.7	229 000	9.0	237 000	10.2
20	17	214 000	2.9	223 000	4.3	232 000	5.8	241 000	7.2	250 000	8.7	258 000	10.1	267 000	11.6
22	19	268 000	3.6	279 000	5.4	290 000	7.2	301 000	9.0	312 000	10.8	323 000	12.6	334 000	14.4
24	21	327 000	4.4	340 000	6.6	354 000	8.8	367 000	11.0	381 000	13.2	394 000	15.4	408 000	17.6
26	23	392 000	5.3	408 000	7.9	425 000	10.6	441 000	13.2	457 000	15.9	473 000	18.5	489 000	21.2
28	25	463 000	6.3	482 000	9.4	502 000	12.5	521 000	15.6	540 000	18.8	559 000	21.9	578 000	25.0
30	27	540 000	7.3	563 000	10.9	585 000	14.6	607 000	18.2	630 000	21.9	652 000	25.5	674 000	29.2
32	29	623 000	8.4	649 000	12.6	675 000	16.8	701 000	21.0	726 000	25.2	752 000	29.4	779 000	33.6
34	31	712 000	9.6	742 000	14.4	771 000	19.2	801 000	24.0	830 000	28.8	859 000	33.6	889 000	38.4
36	33	807 000	10.9	841 000	16.3	874 000	21.8	907 000	27.2	941 000	32.7	974 000	38.1	1 007 000	43.6
38	35	908 000	12.3	946 000	18.4	983 000	24.5	1 020 000	30.6	1 058 000	36.8	1 095 000	42.9	1 133 000	49.0

TABLE 20. USE FOR DESIGNING ROUND COLUMNS WITH VERTICAL REINFORCEMENT.

Safe Loadings for Columns of Various Sizes and Steel Required for Given Load. *(See p. 564.)*

Based on $P = Afc\,[1 + (n - 1)\,p]$. *(See p. 562.)*

Of the total diameter of the column, 1½ inches on all sides is considered as protective coating and is not included in the area (A) carrying load. If total area of column is to be used, select sizes 3 inches smaller than those given

Diameter of Column	Effective Diameter of Column.	Ratio of Area of Steel to Effective Area of Concrete. $p = 0.010$ Safe Load.	$p = 0.010$ Area of Steel.	$p = 0.015$ Safe Load.	$p = 0.015$ Area of Steel.	$p = 0.020$ Safe Load.	$p = 0.020$ Area of Steel.	$p = 0.025$ Safe Load.	$p = 0.025$ Area of Steel.	$p = 0.030$ Safe Load.	$p = 0.030$ Area of Steel.	$p = 0.035$ Safe Load.	$p = 0.035$ Area of Steel.	$p = 0.040$ Safe Load.	$p = 0.040$ Area of Steel.
		P	A_s	P	A_s	P	A_s	P	A_s	P	A_s	P	A_s	P	A_s
in.	in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.
1:2:4 Concrete						$f_c = 450$				$n = 15$					
10	7	19 700	0.4	20 900	0.6	22 200	0.8	23 400	1.0	24 600	1.2	25 800	1.3	27 000	1.5
11	8	25 800	0.5	27 400	0.8	29 000	1.0	30 500	1.3	32 100	1.5	33 700	1.8	35 300	2.0
12	9	32 600	0.6	34 600	1.0	36 600	1.3	38 700	1.6	40 700	1.9	42 700	2.2	44 700	2.5
13	10	40 300	0.8	42 800	1.2	45 200	1.6	47 700	2.0	50 200	2.4	52 700	2.7	55 100	3.1
14	11	48 800	1.0	51 700	1.4	54 700	1.9	57 700	2.4	60 700	2.9	63 700	3.3	66 700	3.8
15	12	58 000	1.1	61 600	1.7	65 100	2.3	68 700	2.8	72 300	3.4	75 800	4.0	79 400	4.5
16	13	68 100	1.3	72 300	2.0	76 500	2.7	80 600	3.3	84 800	4.0	89 000	4.6	93 200	5.3
17	14	79 000	1.5	83 800	2.3	88 700	3.1	93 500	3.8	98 400	4.6	103 000	5.4	108 000	6.2
18	15	90 700	1.8	96 200	2.7	102 000	3.5	107 000	4.4	113 000	5.3	118 000	6.2	124 000	7.1
19	16	103 000	2.0	109 000	3.0	116 000	4.0	122 000	5.0	128 000	6.0	135 000	7.0	141 000	8.0
20	17	116 000	2.3	124 000	3.4	131 000	4.5	138 000	5.7	145 000	6.8	152 000	7.9	159 000	9.1
22	19	145 000	2.8	154 000	4.3	163 000	5.7	172 000	7.1	181 000	8.5	190 000	9.9	199 000	11.3
24	21	178 000	3.5	189 000	5.2	200 000	6.9	210 000	8.7	221 000	10.4	232 000	12.1	243 000	13.8
26	23	213 000	4.2	226 000	6.2	239 000	8.3	252 000	10.4	265 000	12.5	279 000	14.5	292 000	16.6
28	25	252 000	4.9	267 000	7.4	283 000	9.8	298 000	12.3	314 000	14.7	329 000	17.2	345 000	19.6
30	27	294 000	5.7	312 000	8.6	330 000	11.5	348 000	14.3	366 000	17.2	384 000	20.0	402 000	22.9
32	29	339 000	6.6	360 000	9.9	381 000	13.2	401 000	16.5	422 000	19.8	443 000	23.1	464 000	26.4
34	31	387 000	7.5	411 000	11.3	435 000	15.1	459 000	18.9	482 000	22.6	506 000	26.4	530 000	30.2
36	33	439 000	8.6	466 000	12.8	493 000	17.1	520 000	21.4	547 000	25.7	574 000	29.9	600 000	34.2
38	35	494 000	9.6	524 000	14.4	554 000	19.2	585 000	24.1	615 000	28.9	645 000	33.7	675 000	38.5
1:1½:3 Concrete						$f_c = 570$				$n = 12$					
10	7	24 300	0.4	25 500	0.6	26 800	0.8	28 000	1.0	29 200	1.2	30 400	1.3	31 600	1.5
11	8	31 800	0.5	33 400	0.8	35 000	1.0	36 500	1.3	38 100	1.5	39 700	1.8	41 300	2.0
12	9	40 300	0.6	42 300	1.0	44 200	1.3	46 200	1.6	48 200	1.9	50 200	2.2	52 200	2.5
13	10	49 700	0.8	52 200	1.2	54 600	1.6	57 100	2.0	59 500	2.4	62 000	2.7	64 500	3.1
14	11	60 100	1.0	63 100	1.4	66 100	1.9	69 100	2.4	72 100	2.9	75 000	3.3	78 000	3.8
15	12	71 600	1.1	75 100	1.7	78 700	2.3	82 200	2.8	85 800	3.4	89 300	4.0	92 800	4.5
16	13	84 000	1.3	88 100	2.0	92 300	2.7	96 500	3.3	101 000	4.0	105 000	4.6	109 000	5.3
17	14	97 000	1.5	102 000	2.3	107 000	3.1	112 000	3.8	117 000	4.6	122 000	5.4	126 000	6.2
18	15	112 000	1.8	117 000	2.7	123 000	3.5	128 000	4.4	134 000	5.3	140 000	6.2	145 000	7.1
19	16	127 000	2.0	134 000	3.0	140 000	4.0	146 000	5.0	152 000	6.0	159 000	7.0	165 000	8.0
20	17	144 000	2.3	151 000	3.4	158 000	4.5	165 000	5.7	172 000	6.8	179 000	7.9	186 000	9.1
22	19	179 000	2.8	188 000	4.3	197 000	5.7	206 000	7.1	215 000	8.5	224 000	9.9	233 000	11.3
24	21	219 000	3.5	230 000	5.2	241 000	6.9	252 000	8.7	263 000	10.4	273 000	12.1	284 000	13.8
26	23	263 000	4.2	276 000	6.2	289 000	8.3	302 000	10.4	315 000	12.5	328 000	14.5	341 000	16.6
28	25	311 000	4.9	326 000	7.4	341 000	9.8	357 000	12.3	372 000	14.7	388 000	17.2	403 000	19.6
30	27	362 000	5.7	380 000	8.6	398 000	11.5	416 000	14.3	434 000	17.2	452 000	20.0	470 000	22.9
32	29	418 000	6.6	439 000	9.9	459 000	13.2	480 000	16.5	501 000	19.8	521 000	23.1	542 000	26.4
34	31	478 000	7.8	501 000	11.3	525 000	15.1	549 000	18.9	572 000	22.6	596 000	26.4	620 000	30.2
36	33	541 000	8.6	568 000	12.8	595 000	17.1	622 000	21.4	649 000	25.7	675 000	29.9	702 000	34.2
38	35	609 000	9.6	639 000	14.4	669 000	19.2	699 000	24.1	729 000	28.9	760 000	33.7	790 000	38.5
1:1:2 Concrete						$f_c = 680$				$n = 10$					
10	7	28 500	0.4	29 700	0.6	30 900	0.8	32 000	1.0	33 200	1.2	34 400	1.3	35 600	1.5
11	8	37 300	0.5	38 800	0.8	40 300	1.0	41 900	1.3	43 400	1.5	44 900	1.8	46 500	2.0
12	9	47 200	0.6	49 100	1.0	51 000	1.3	53 000	1.6	54 900	1.9	56 900	2.2	58 800	2.5
13	10	58 200	0.8	60 600	1.2	63 000	1.6	65 400	2.0	67 800	2.4	70 200	2.7	72 600	3.1
14	11	70 400	1.0	73 400	1.4	76 300	1.9	79 200	2.4	82 100	2.9	85 000	3.3	87 900	3.8
15	12	83 800	1.1	87 300	1.7	90 900	2.3	94 200	2.8	97 700	3.4	101 000	4.0	105 000	4.5
16	13	98 400	1.3	102 000	2.0	107 000	2.7	111 000	3.3	115 000	4.0	119 000	4.6	123 000	5.3
17	14	114 000	1.5	119 000	2.3	124 000	3.1	128 000	3.8	133 000	4.6	138 000	5.4	142 000	6.2
18	15	131 000	1.8	136 000	2.7	142 000	3.5	147 000	4.4	153 000	5.3	158 000	6.2	163 000	7.1
19	16	149 000	2.0	155 000	3.0	161 000	4.0	167 000	5.0	174 000	6.0	180 000	7.0	186 000	8.0
20	17	168 000	2.3	175 000	3.4	182 000	4.5	189 000	5.7	196 000	6.8	203 000	7.9	210 000	9.1
22	19	210 000	2.8	219 000	4.3	228 000	5.7	236 000	7.1	245 000	8.5	254 000	9.9	262 000	11.3
24	21	257 000	3.5	267 000	5.2	278 000	6.9	289 000	8.7	299 000	10.4	310 000	12.1	320 000	13.8
26	23	308 000	4.2	321 000	6.2	333 000	8.3	346 000	10.4	359 000	12.5	372 000	14.5	384 000	16.6
28	25	364 000	4.9	379 000	7.4	394 000	9.8	409 000	12.3	424 000	14.7	439 000	17.2	454 000	19.6
30	27	424 000	5.7	442 000	8.6	459 000	11.5	477 000	14.3	494 000	17.2	512 000	20.0	530 000	22.9
32	29	490 000	6.6	510 000	9.9	530 000	13.2	550 000	16.5	571 000	19.8	591 000	23.2	611 000	26.4
34	31	560 000	7.5	583 000	11.3	606 000	15.1	629 000	18.9	652 000	22.6	675 000	26.4	698 000	30.2
36	33	634 000	8.6	660 000	12.8	686 000	17.1	713 000	21.4	739 000	25.7	765 000	29.9	791 000	34.2
38	35	713 000	9.6	743 000	14.4	772 000	19.2	802 000	24.1	831 000	28.9	860 000	33.7	890 000	38.5

TABLE 21. USE FOR DESIGNING ROUND COLUMNS WITH VERTICAL REINFORCEMENT AND 1% SPIRAL HOOPING.

Safe Loadings for Columns of Various Sizes and Steel Required for Given Load (*See p.* 564).

Based on $P = Af_c[1 + (n - 1)p]$ (See page 562.)

Of the total diameter of column, $1\frac{1}{2}$ inches on all sides is considered as fireproofing and is not included in the area (A) carrying load. If no fireproofing is required, use columns 3 inches smaller than those given.

Diameter of Columns	Effective Diameter of Columns	Ratio of Area of Steel to Effective Area of Concrete																			
		$p = 0.010$		$p = 0.015$		$p = 0.020$		$p = 0.025$		$p = 0.030$		$p = 0.035$		$p = 0.040$		$p = 0.045$		$p = 0.050$		$p = 0.060$	
		Safe Load	Area of Steel	Safe Load	Area of Steel	Safe Load	Area of Steel	Safe Load	Area of Steel	Safe Load	Area of Steel	Safe Load	Area of Steel	Safe Load	Area of Steel	Safe Load	Area of Steel	Safe Load	Area of Steel	Safe Load	Area of Steel
		P	A_s	P	A_s	P	A_s	P	A_s	P	A_s	P	A_s	P	A_s	P	A_s	P	A_s	P	A_s
in.	in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb.	sq. in.	lb	sq in.	lb.	sq in.	lb.	sq. in.	lb.	sq. in.	lb.	sq in.	lb.	sq. in.
		1:2:4 Concrete. $f_c = 700$. $n = 15$																			
10	7	30 700	0.4	32 500	0.6	34 400	0.8	36 300	1.0	38 400	1.2	40 100	1.3	42 000	1.5	43 800	1.7	45 800	1.9	49 500	2.3
11	8	40 100	0.5	42 500	0.8	45 600	1.0	47 500	1.3	49 900	1.5	52 400	1.8	55 000	2.0	57 200	2.3	59 800	2.5	64 800	3.0
12	9	50 800	0.6	53 900	1.0	57 000	1.3	60 000	1.6	63 200	1.9	66 400	2.2	69 600	2.5	71 800	2.9	75 700	3.2	82 000	3.8
13	10	62 600	0.8	66 400	1.2	70 400	1.6	74 200	2.0	78 000	2.4	81 600	2.7	85 800	3.1	89 400	3.5	93 400	3.9	101 000	4.7
14	11	75 900	1.0	80 500	1.4	85 200	1.9	89 800	2.4	94 400	2.9	98 800	3.3	103 800	3.8	108 000	4.3	113 000	4.8	122 400	5.7
15	12	90 200	1.1	95 700	1.7	101 000	2.3	106 000	2.8	112 400	3.4	117 500	4.0	123 500	4.5	128 800	5.1	134 500	5.7	145 500	6.8
16	13	105 000	1.3	112 000	2.0	118 000	2.7	124 000	3.3	131 200	4.0	137 300	4.6	144 000	5.3	150 500	6.0	157 000	6.6	170 000	8.0
17	14	122 000	1.5	130 000	2.3	137 000	3.1	145 000	3.8	153 000	4.6	159 000	5.4	168 000	6.2	174 400	6.9	182 000	7.7	197 000	9.2
18	15	140 000	1.8	149 000	2.7	158 000	3.5	167 000	4.4	175 000	5.3	183 500	6.2	192 000	7.1	200 500	8.0	209 200	8.8	227 000	10.6
19	16	160 000	2.0	170 000	3.0	180 000	4.0	190 000	5.0	200 000	6.0	209 000	7.0	220 000	8.0	229 000	9.0	239 000	10.1	259 000	12.1
20	17	181 000	2.3	191 000	3.4	203 000	4.5	213 000	5.7	225 000	6.8	235 000	7.9	247 000	9.1	257 500	10.2	269 000	11.3	291 000	13.6
22	19	225 000	2.8	231 000	4.3	253 000	5.7	267 000	7.1	281 000	8.5	295 000	9.9	309 000	11.3	322 000	12.8	337 000	14.2	365 000	17.0
24	21	276 000	3.5	293 000	5.2	310 000	6.9	327 000	8.7	344 000	10.4	360 000	12.1	378 000	13.8	394 000	15.6	412 000	17.3	445 000	20.8
26	23	331 000	4.2	351 000	6.2	372 000	8.3	392 000	10.4	412 000	12.5	432 000	14.5	453 000	16.6	473 000	18.7	494 000	20.8	535 000	24.9
28	25	391 000	4.9	415 000	7.4	440 000	9.8	463 000	12.3	487 000	14.7	510 000	17.2	536 000	19.6	558 000	22.1	583 000	24.5	632 000	29.5
30	27	456 000	5.7	484 000	8.6	513 000	11.5	541 000	14.3	569 000	17.2	595 000	20.0	625 000	22.9	652 000	25.8	680 000	28.6	737 000	34.[illegible]
32	29	527 000	6.6	559 000	9.9	593 000	13.2	623 000	16.5	656 000	19.8	686 000	23.1	720 000	26.4	752 000	29.7	786 000	33.0	850 000	39.6
34	31	600 000	7.5	638 000	11.3	676 000	15.1	713 000	18.9	750 000	22.6	784 000	26.4	824 000	30.[illegible]	860 000	34.0	896 000	37.7	970 000	45.3
36	33	681 000	8.6	724 000	12.8	766 000	17.1	808 000	21.4	850 000	25.7	890 000	29.9	934 000	34.2	974 000	38.5	1 015 000	42.8	1 100 000	51.3
38	35	766 000	9.6	814 000	14.4	863 000	19.2	909 000	24.1	956 000	28.9	1 000 000	33.7	1 050 000	38.5	1 095 000	43.3	1 145 000	48.1	1 280 000	57.7

1:1½:3 Concrete. $f_c = 890$. $n = 12$

		$p = 0.010$		$p = 0.015$		$p = 0.020$		$p = 0.025$		$p = 0.030$		$p = 0.035$		$p = 0.040$		$p = 0.045$		$p = 0.050$		$p = 0.060$	
10	7	38 000	0.4	39 900	0.6	41 700	0.8	43 700	1.0	45 500	1.2	47 400	1.3	49 400	1.5	51 200	1.7	53 000	1.9	56 800	2.3
11	8	49 600	0.5	52 000	0.8	54 500	1.0	57 000	1.3	59 400	1.5	62 000	1.8	64 500	2.0	66 800	2.3	69 400	2.5	74 300	3.0
12	9	62 700	0.6	66 000	1.0	69 000	1.3	72 300	1.6	75 300	1.9	78 400	2.2	81 600	2.5	84 600	2.9	87 800	3.2	94 000	3.8
13	10	77 700	0.8	81 500	1.2	85 000	1.6	89 000	2.0	92 900	2.4	96 800	2.7	101 000	3.1	105 000	3.5	108 400	3.9	116 000	4.7
14	11	93 800	1.0	98 500	1.4	100 000	1.9	109 000	2.4	113 000	2.9	117 000	3.3	122 000	3.8	127 000	4.3	131 000	4.8	141 000	5.7
15	12	111 500	1.1	117 000	1.7	123 000	2.3	128 000	2.8	134 000	3.4	140 000	4.0	145 000	4.5	151 000	5.1	156 000	5.7	167 000	6.8
16	13	130 000	1.3	137 000	2.0	143 000	2.7	150 000	3.3	157 000	4.0	164 000	4.6	170 000	5.3	177 000	6.0	183 000	6.6	196 000	8.0
17	14	151 000	1.5	159 000	2.3	166 000	3.1	174 000	3.8	182 000	4.6	190 000	5.4	198 000	6.2	205 000	6.9	212 000	7.7	227 000	9.2
18	15	173 000	1.8	182 000	2.7	191 000	3.5	200 000	4.4	209 000	5.3	218 000	6.2	227 000	7.1	235 000	8.0	244 000	8.8	261 000	10.6
19	16	198 000	2.0	208 000	3.0	218 000	4.0	228 000	5.0	238 000	6.0	248 000	7.0	258 000	8.0	268 000	9.0	277 000	10.1	297 000	12.1
20	17	223 000	2.3	234 000	3.4	245 000	4.5	256 000	5.7	269 000	6.8	280 000	7.9	291 000	9.1	302 000	10.2	313 000	11.3	335 000	13.6
22	19	279 000	2.8	293 000	4.3	307 000	5.7	322 000	7.1	336 000	8.5	350 000	9.9	364 000	11.3	377 000	12.8	391 000	14.2	419 000	17.0
24	21	342 000	3.5	359 000	5.2	376 000	6.9	392 000	8.7	410 000	10.4	427 000	12.1	444 000	13.8	461 000	15.6	478 000	17.3	512 000	20.8
26	23	410 000	4.2	430 000	6.2	450 000	8.3	471 000	10.4	492 000	12.5	512 000	14.5	533 000	16.6	552 000	18.7	574 000	20.8	614 000	24.9
28	25	484 000	4.9	508 000	7.4	531 000	9.8	557 000	12.3	580 000	14.7	605 000	17.2	623 000	19.6	652 000	22.1	676 000	24.5	725 000	29.5
30	27	564 000	5.7	593 000	8.6	620 000	11.5	650 000	14.3	678 000	17.2	706 000	20.0	735 000	22.9	762 000	25.8	790 000	28.6	846 000	34.4
32	29	650 000	6.6	685 000	9.9	715 000	13.2	750 000	16.5	782 000	19.8	814 000	23.1	848 000	26.4	878 000	29.7	912 000	33.0	976 000	39.6
34	31	745 000	7.5	782 000	11.3	820 000	15.1	850 000	18.9	893 000	22.6	930 000	26.4	968 000	30.2	1 004 000	34.0	1 041 000	37.7	1 115 000	45.3
36	33	845 000	8.6	885 000	12.8	927 000	17.1	970 000	21.4	1 011 000	25.7	1 054 000	29.9	1 096 000	34.2	1 137 000	38.5	1 180 000	42.8	1 262 000	51.3
38	35	950 000	9.6	998 000	14.4	1 060 000	19.2	1 090 000	24.1	1 138 000	28.9	1 185 000	33.7	1 235 000	38.5	1 280 000	43.3	1 328 000	48.1	1 420 000	57.7

1:1:2 Concrete. $f_c = 1050$. $n = 10$

10	7	44 400	0.4	46 300	0.6	48 100	0.8	50 000	1.0	51 800	1.2	53 600	1.3	55 500	1.5	57 200	1.7	59 100	1.9	62 800	2.3
11	8	58 100	0.5	60 400	0.8	62 800	1.0	65 200	1.3	67 600	1.5	70 000	1.8	72 500	2.0	74 800	2.3	77 200	2.5	82 100	3.0
12	9	73 600	0.6	76 600	1.0	79 600	1.3	82 700	1.6	85 600	1.9	83 600	2.2	91 800	2.5	94 800	2.9	97 800	3.2	104 000	3.8
13	10	90 800	0.8	94 500	1.2	98 200	1.6	102 000	2.0	106 000	2.4	110 000	2.7	113 000	3.1	117 000	3.5	121 000	3.9	128 200	4.7
14	11	110 000	1.0	114 000	1.4	119 000	1.9	123 000	2.4	128 000	2.9	132 000	3.3	137 000	3.8	142 000	4.3	146 000	4.8	155 000	5.7
15	12	131 000	1.1	136 100	1.7	142 000	2.3	147 000	2.8	152 200	3.4	158 000	4.0	163 000	4.5	169 000	5.1	174 000	5.7	184 800	6.8
16	13	153 000	1.3	160 000	2.0	166 000	2.7	172 000	3.3	179 000	4.0	185 000	4.6	192 000	5.3	198 000	6.0	204 000	6.6	217 000	8.0
17	14	178 000	1.5	185 000	2.3	192 000	3.1	200 000	3.8	207 000	4.6	214 000	5.4	222 000	6.2	229 000	6.9	237 000	7.7	251 000	9.2
18	15	204 000	1.8	213 000	2.7	221 000	3.5	230 000	4.4	238 000	5.3	246 000	6.2	255 000	7.1	263 000	8.0	272 000	8.8	289 000	10.6
19	16	232 000	2.0	242 000	3.0	251 000	4.0	261 000	5.0	271 000	6.0	280 000	7.0	290 000	8.0	299 000	9.0	309 000	10.1	328 000	12.1
20	17	252 000	2.3	273 000	3.4	284 000	4.5	295 000	5.7	305 000	6.8	316 000	7.9	327 000	9.1	338 000	10.2	349 000	11.3	370 000	13.6
22	19	328 000	2.8	341 000	4.3	354 000	5.7	368 000	7.1	382 000	8.5	395 000	9.9	409 000	11.3	422 000	12.8	436 000	14.2	463 000	17.0
24	21	400 000	3.5	417 000	5.2	433 000	6.9	450 000	8.7	466 000	10.4	483 000	12.1	500 000	13.8	516 000	15.6	532 000	17.3	566 000	20.8
26	23	480 000	4.2	500 000	6.2	520 000	8.3	540 000	10.4	559 000	12.5	579 000	14.5	600 000	16.6	618 000	18.7	638 000	20.8	678 000	24.9
28	25	567 000	4.9	590 000	7.4	613 000	9.8	637 000	12.3	660 000	14.7	684 000	17.2	708 000	19.6	740 000	22.1	754 000	24.5	801 000	29.5
30	27	662 000	5.7	689 000	8.6	716 000	11.5	744 000	14.3	771 000	17.2	798 000	20.0	826 000	22.9	852 000	25.8	880 000	28.6	936 000	34.4
32	29	764 000	6.6	795 000	9.9	826 000	13.2	858 000	16.5	890 000	19.8	920 000	23.1	939 000	26.4	990 000	29.7	1 021 000	33.0	1 078 000	39.6
34	31	872 000	7.5	908 000	11.3	944 000	15.1	980 000	18.9	1 015 000	22.6	1 051 000	26.4	1 089 000	30.2	1 123 000	34.0	1 160 000	37.7	1 231 000	45.3
36	33	989 000	8.6	1 028 000	12.8	1 068 000	17.1	1 110 000	21.4	1 150 000	25.7	1 191 000	29.9	1 235 000	34.2	1 272 000	38.5	1 315 000	42.8	1 395 000	51.3
38	35	1 112 000	9.6	1 156 000	14.4	1 202 000	19.2	1 249 000	24.1	1 295 000	28.9	1 340 000	33.7	1 389 000	38.5	1 433 000	43.3	1 480 000	48.1	1 570 000	57.7

DIAGRAM 4. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{8}$$

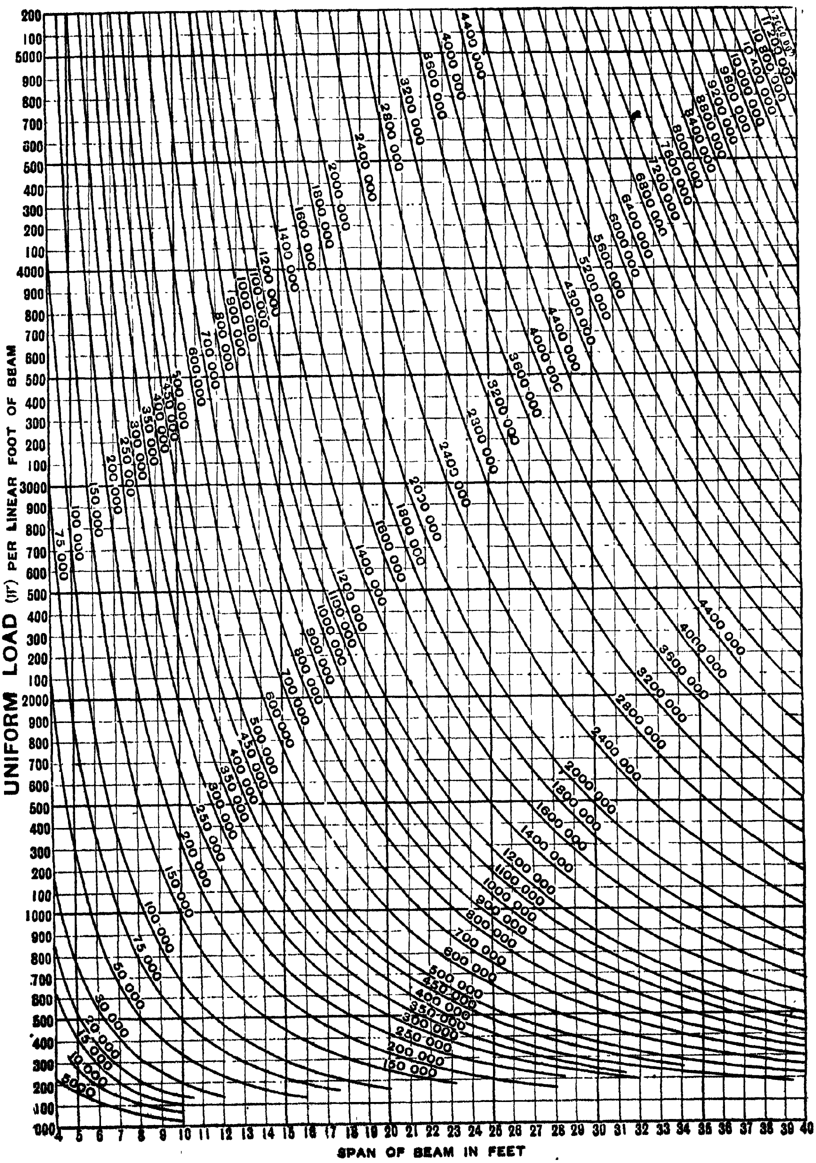


DIAGRAM 5. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{10}$$

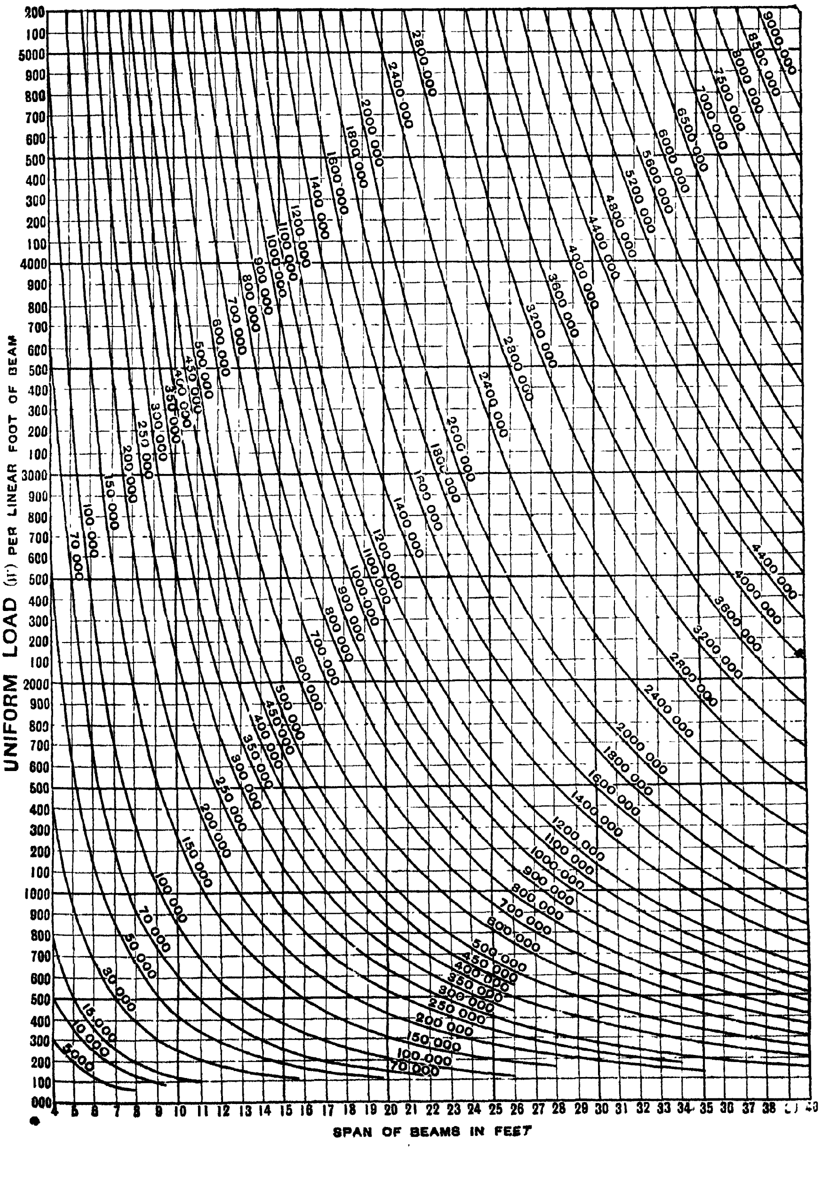
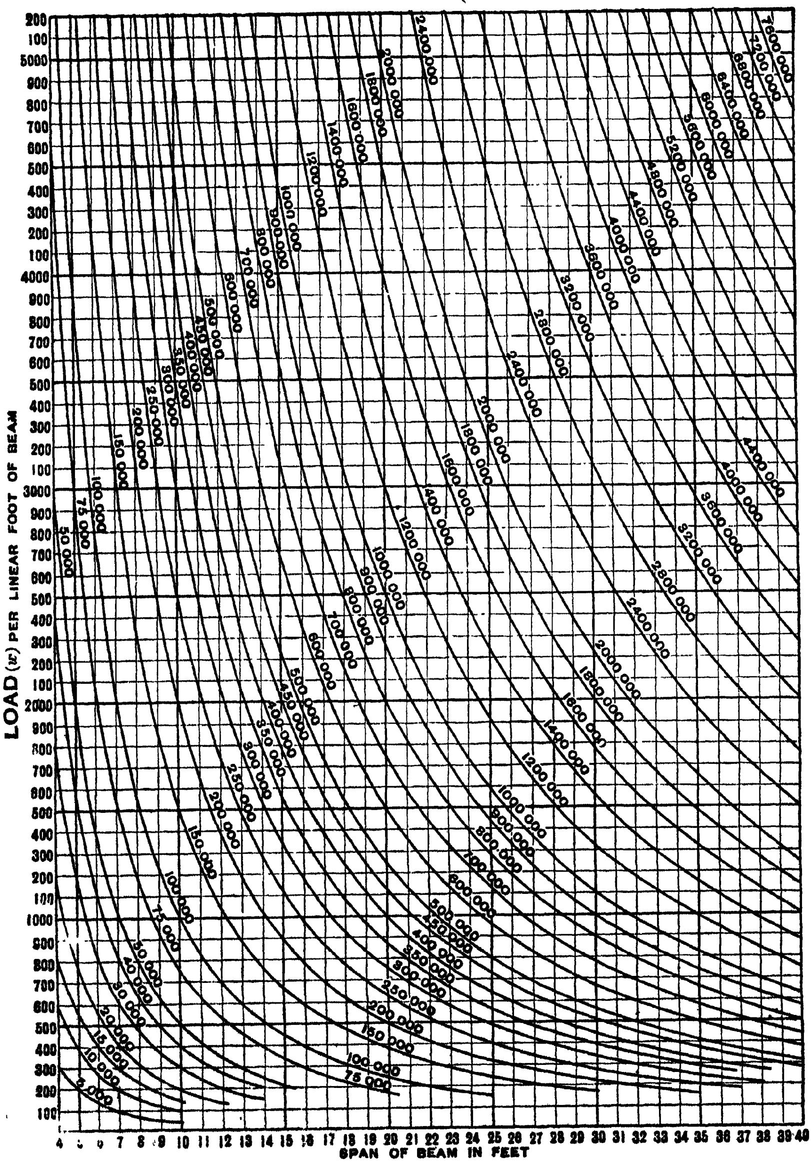


DIAGRAM 6. BENDING MOMENTS FOR DIFFERENT SPANS AND LOADS.

$$M = \frac{wl^2}{12}$$



CHAPTER XXIII

BUILDING CONSTRUCTION

Reinforced concrete has taken its place as an established material for building construction. Durable, fireproof, and economical in first cost; adaptable to various types of design; capable of carrying heavy loads; and at the same time susceptible of pleasing architectural treatment, its position as a building material is unique.

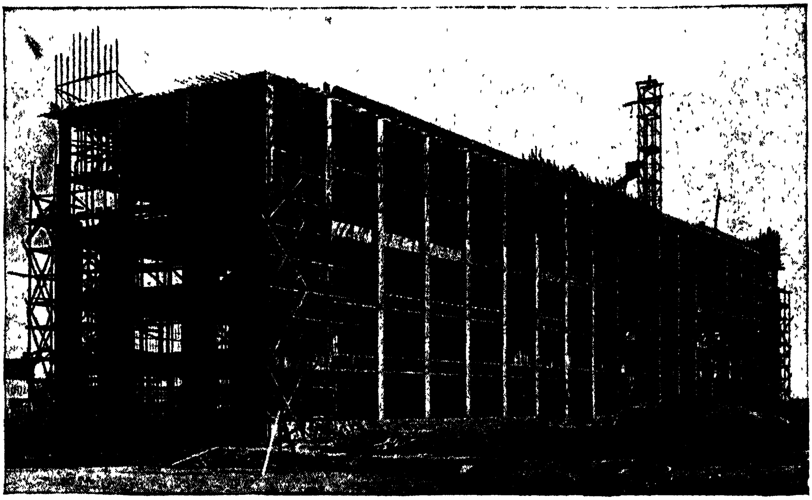


FIG. 174.—Placing Brick Veneer on Concrete Building. (*See p.* 612.)
Gray and Davis Building, Cambridge, Mass.

Used as the structural frame of factory and office buildings, for foundations and floors of steel frame structures; or as artificial stone for facing or trimming, its adaptability is recognized. For small buildings such as dwellings, its use is not so general because of larger unit costs on small jobs, but in certain cases where, on the one hand, expense is not the criterion, and, on the other hand, where duplication of design reduces the costs, it is being adopted to advantage.

For first-class construction there are three requisites: (1) thoroughly tested materials; (2) design by an engineer familiar with reinforced

concrete design; and (3) construction by an experienced builder working under careful supervision.

RELATIVE COSTS OF BUILDINGS OF DIFFERENT MATERIALS

For industrial and office buildings reinforced concrete naturally competes with the steel frame, plain or fireproofed, and with mill construction. Cost is usually the important factor, but sometimes speed after breaking ground is the main consideration, as is the case, for example, in high buildings in the business sections of large cities, and structural steel may be selected on this account. If the time of rolling of the structural steel must be included, however, the concrete building can be put up in a shorter time.

In selecting the type of building, the first cost should not be considered alone, but only in connection with the average annual expense and depreciation over a term of years. In other words, it is economical to increase the first cost for the sake of an annual saving in expense that ultimately, in the course of the useful life of the building, makes up for the higher initial expenditure.

Fireproofed steel frame construction almost invariably is more expensive in first cost than reinforced concrete. This is due chiefly to the fact that in the reinforced concrete structure the concrete itself is not simply for fireproofing, but at the same time, by its strength in compression, forms a load-carrying part of the members. Moreover, placing the fireproofing on the steel frame is a separate and expensive operation that is practically incidental in a reinforced concrete building.

The first cost of the reinforced concrete structure, in turn, may be greater than that of a steel frame, not fireproofed, or of mill construction. This depends, however, to a considerable extent on the type of building. Thus, with very heavy loads, especially on long spans, concrete is cheaper than steel or mill construction. The dividing line varies with relative costs of material. Frequently it occurs at loads of 200 pounds per square foot on spans in the neighborhood of 20 feet.*

To get any real comparison between buildings of different materials a detailed estimate should be made of the first cost and the annual expenditures. The following table† issued by the Universal Portland Cement Company‡ illustrates the method.

* See paper by L. C. Wason in Proceedings National Association of Cement Users, Vol. VII, 1911, p. 448.

† Similar computations are given by J. P. H. Perry in Proceedings National Association of Cement Users, Vol. VII, 1911, p. 443.

‡ See also *Concrete Cement Age* July 1916, p. 25.

Comparative First and Maintenance Costs of Reinforced Concrete and Mill Constructed Buildings

(From standpoint of Owner)

Building 100 ft. x 175 ft. 7 stories and basement. Total Floor Area 140 000 sq. ft.

	Reinforced Concrete (Fireproof)	Mill Construction (Not Fireproof)
First cost of building, ...	$189 000 00	$168 000 00
First cost of sprinkler system	14 000 00	14 000 00
Total first investment	$203 000 00	$182 000 00

First cost fireproof more than mill construction $21 000 00

Maintenance

Interest on first investment. . . .	6%	$12 180 00	$10 920 00	6%
Tax on first investment	1%	2 030 00	1 820 00	1%
Depreciation on building	0.5%	945 00	3 360 00	2%
Obsolescence	1.0%	1 890 00	3 360 00	2%
Depreciation on sprinkler	10%	1 400 00	1 400 00	10%
Repairs to building	0.25%	472 50	1 680 00	1%
Damage to building by vermin	None		200 00	Est. low
Auxiliary fire equipment ...	..Estimated	200 00	300 00	Estimated
Fire insurance on building,	None required		235 20	14 cts. on $100
		$19 117 50	$23 275 20	

Yearly expense fireproof less than non-fireproof . . . $4 157 70

The yearly saving of $4 157 70 capitalized at 6% represents $69 295. Therefore, actual cost of concrete building is $119 705, in comparison with one of mill construction costing $168 000.

Furthermore there is a lower rate for fire insurance on the contents of the concrete building which still further reduces the cost.

Reinforced concrete has been used economically for dwelling houses, but only where cheap cottages can be built in groups of similar pattern. With this exception wood is cheaper and, in fact, the cost of forms alone exceeds in many cases that of the material and concrete labor. Precast blocks, requiring no forms, can best be used for this class of work, but unless the surfaces are tooled the appearance is apt to be monotonous. In estimating the labor where forms are used allowance must be made for time lost waiting for the concrete to harden so that the forms can be raised. For this reason a small gang of men should be used,—only enough to lay concrete to the height of one section of the forms per day.

Average Costs of Concrete Buildings per Square Foot of Floor Area (See p. 611.)

Costs include all items except interior finish

COST IN DOLLARS PER SQUARE FOOT OF FLOOR AREA

Width in Feet	Length of Building in Feet						Length of Building in Feet					
	50 $	100 $	200 $	300 $	400 $	600 $	$	100 $	200 $	300 $	400 $	600 $
	1-Story						**2-Story**					
25	2.34	1.83	1.60	1.46	1.40	1.38	2.29	1.77	1.55	1.43	1.37	1.30
50	1.67	1.43	1.26	1.14	1.08	1.05	1.64	1.30	1.15	1.05	1.01	0.98
75	1.52	1.32	1.15	1.03	0.98	0.95	1.44	1.19	1.03	0.96	0.91	0.87
100	1.44	1.24	1.08	0.98	0.91	0.89	1.35	1.10	0.97	0.89	0.84	0.81
150	1.39	1.18	1.03	0.93	0.86	0.84	1.27	1.04	0.91	0.83	0.79	0.76
	4-Story						**6 to 10-Story**					
25	2.22	1.68	1.46	1.37	1.31	1.25	2.22	1.66	1.45	1.35	1.32	1.25
50	1.54	1.20	1.07	1.00	0.97	0.93	1.53	1.18	1.06	1.00	0.97	0.93
75	1.35	1.08	0.96	0.90	0.87	0.84	1.33	1.08	0.96	0.89	0.85	0.83
100	1.25	1.01	0.89	0.83	0.80	0.78	1.24	0.99	0.88	0.82	0.79	0.77
150	1.18	0.95	0.84	0.78	0.75	0.72	1.16	0.93	0.82	0.77	0.75	0.72

Average Costs of Concrete Buildings per Cubic Foot of Volume (See p. 611.)

Costs include all items except interior finish

COST IN DOLLARS PER CUBIC FOOT OF VOLUME

Width in Feet	Length of Building in Feet						Length of Building in Feet					
	50 $	100 $	200 $	300 $	400 $	600 $	50 $	100 $	200 $	300 $	400 $	600 $
	1-Story						**2-Story**					
25	0.195	0.153	0.133	0.122	0.117	0.115	0.191	0.147	0.129	0.119	0.114	0.108
50	0.139	0.119	0.105	0.095	0.090	0.087	0.137	0.108	0.096	0.088	0.084	0.082
75	0.126	0.110	0.096	0.086	0.082	0.079	0.120	0.099	0.087	0.080	0.076	0.072
100	0.120	0.104	0.090	0.082	0.076	0.074	0.113	0.092	0.081	0.074	0.070	0.067
150	0.116	0.098	0.086	0.077	0.072	0.070	0.106	0.087	0.076	0.069	0.066	0.063
	4-Story						**6 to 10-Story**					
25	0.185	0.140	0.122	0.114	0.109	0.104	0.185	0.138	0.121	0.112	0.110	0.104
50	0.128	0.100	0.089	0.083	0.081	0.077	0.128	0.098	0.088	0.083	0.081	0.077
75	0.112	0.090	0.080	0.075	0.072	0.070	0.111	0.090	0.080	0.074	0.071	0.069
100	0.104	0.084	0.074	0.069	0.067	0.065	0.103	0.082	0.073	0.068	0.066	0.064
150	0.098	0.079	0.070	0.065	0.063	0.060	0.097	0.077	0.068	0.064	0.062	0.060

Values are based on conditions outlined on page 611. The tables are taken from "Concrete Costs" by the same authors, and the values are made up from tables of unit times and costs given in the same book carefully checked by contractors' estimates. For more complete details and for the unit values which are adapted to all conditions, see other tables and examples in "Concrete Costs."

Values are for symmetrical buildings.

Values must be corrected for the high prices obtaining during the war,

For cellar and foundation walls of all classes of buildings, including brick and frame (see p. 643), concrete is superseding rubble masonry except where rubble stone is taken from the excavation so as to be very cheap.

Cement mortar plastered on to metal or wood lathing is used not only for outside walls but in some cases for fire resisting partitions in large buildings. (See p. 645).

ACTUAL COST OF REINFORCED CONCRETE BUILDINGS

The tables presented on page 610 present the approximate average cost per square foot of floor area and per cubic foot of volume of plain rectangular reinforced concrete buildings of various sizes and heights.

The costs include all details of construction, not only the concrete forms and reinforcement, but also windows, stairs, roof covering, and plumbing. Interior finish, which varies widely with the type of construction, is not included. The basis of the tables is as follows:

(1) Floor loads, 150 pounds per square foot.
(2) Story heights: first floor on a 3-foot fill;
other floors 12 feet from slab surface to slab surface.
(3) Column spacing, 18 feet on centers.
(4) Floor design: girders between columns in one direction; beams between columns in other direction with two intermediate beams.
(5) Excavation and foundations.*

Story Height	Outside Walls per Linear Foot	Inside Walls per Linear Foot
1	$2.00	$1.75
2	2.90	2.25
3	3.80	2.80
4	4.70	3.40
5	5 60	3.90
6	6.50	4.50

(6) Filling under first floor: 3-foot fill at 50¢ per cubic yard in place.
(7) Stairs: material and labor, $100 per flight per story.
(8) Stairways and elevator towers:
2 stairways and 1 elevator tower for buildings up to 150 feet long.
2 stairways and 2 elevator towers for buildings up to 300 feet long.
3 stairways and 3 elevator towers for buildings over 300 feet long.

* Taken from paper presented before the New England Cotton Manufacturers' Association, April 1904, by Mr. Charles T. Main. Prices revised by Mr. Main to conform to prices prevailing about January, 1910. Values must be corrected for the high prices obtaining during the war.

(9) Floor finish: all floors of concrete with granolithic finish.

(10) Walls:

(a) Curtain walls between pilasters, 3 feet high and 8 inches thick;

(b) Concrete walls for penthouses, 6 inches thick. Dimensions of penthouse are 10 feet by 10 feet;

(c) Concrete walls around the elevator and stairway openings are taken 6 inches thick, the elevator opening being 10 by 20 feet and the stairways 10 by 10 feet, these two being adjacent so that the one intermediate 10-foot wall serves for both openings;

(d) For toilets, concrete walls 6 inches thick and 20 feet long, one wall for each 5 000 square feet of floor space.

Walls 8 inches thick, including reinforcement and forms, \$0.35 per square foot.

Walls 6 inches thick, including reinforcement and forms, \$0.30 per square foot.

(11) Windows and doors: all openings for windows and doors, \$0.40 per square foot.

(12) Roof and flashing: five-ply tar and gravel roofing, \$0.60 per square foot.

(13) Plumbing: two fixtures on each floor up to 5 000 square feet of floor surface, and one additional fixture for each additional 5 000 square feet, \$75.00 per fixture.

(14) Labor rates: carpenter labor, \$0.50 per hour; steel labor, \$0.30 per hour; and common labor, \$0.25 per hour.

(15) Concrete in place (including labor and materials): \$7.00 per cubic yard, or \$0.26 per cubic foot.

(16) Form lumber: \$30.00 per 1 000 feet B. M., delivered.

(17) Steel for reinforcement: \$37.00 per ton, delivered.

For lighter loads than specified, the costs are slightly decreased, this decrease running up to 12½ cents per square foot for a 75-pound load in a 10-story building. For a 300-pound load, the prices are increased from 6 cents for a 2-story building up to 12½ cents per square foot for 10 stories. **It must be remembered that the tables are based on rectangular, symmetrical buildings. Allowance must be made for irregular layouts, which increase materially the cost of form construction.**

The variation in cost due to variation in spacing of columns is small. If columns are spaced fifteen feet apart the cost is 6 per cent greater than where columns are spaced twenty-five feet apart.

BUILDING DESIGN AND CONSTRUCTION

The factory* of Gray & Davis, Inc., Cambridge, Mass., built by the Aberthaw Construction Company, is shown in Fig. 174, page 607. The photograph shows the brick veneer being laid from a scaffold, but the same builders now omit the scaffold on their work and use a patented platform swung from the roof.

In the frontispiece is shown the new buildings of the Massachusetts

* Monks & Johnson, Engineers.

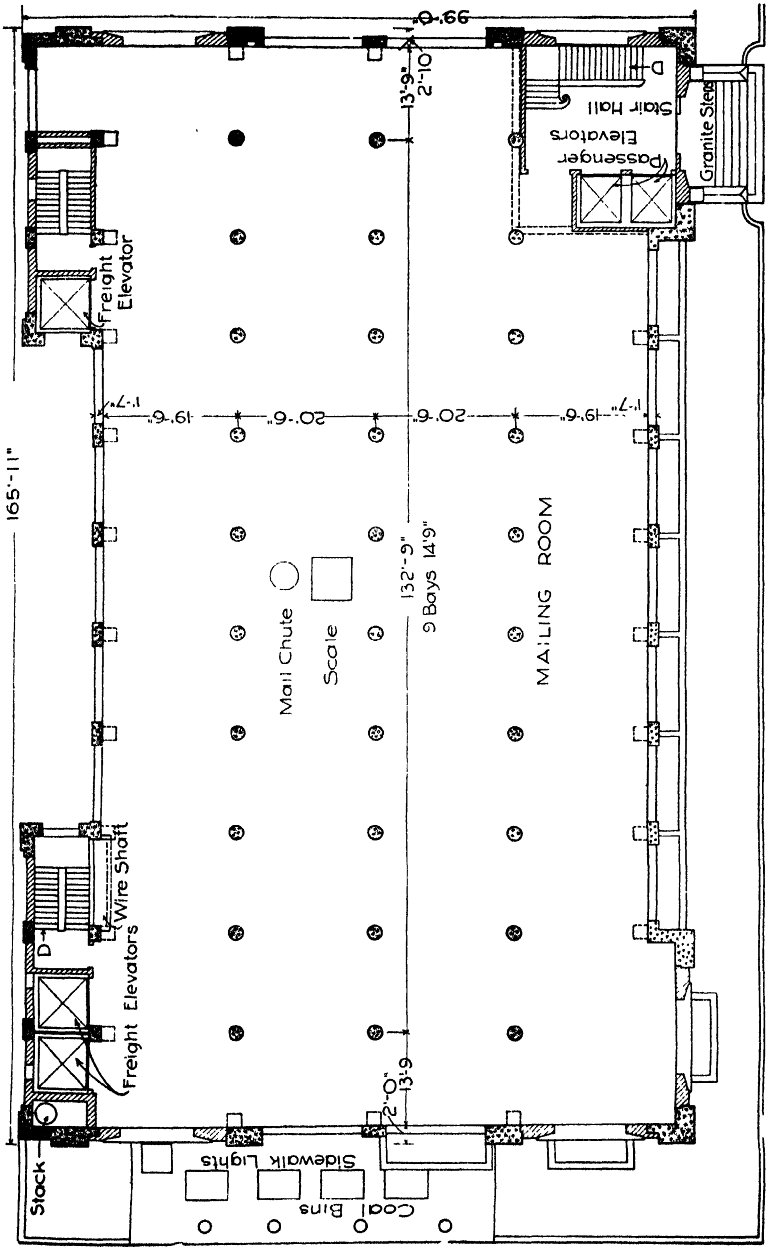


FIG. 175.—Typical Floor Plan. Youth's Companion Building, Boston, Mass. (See p. 616.)

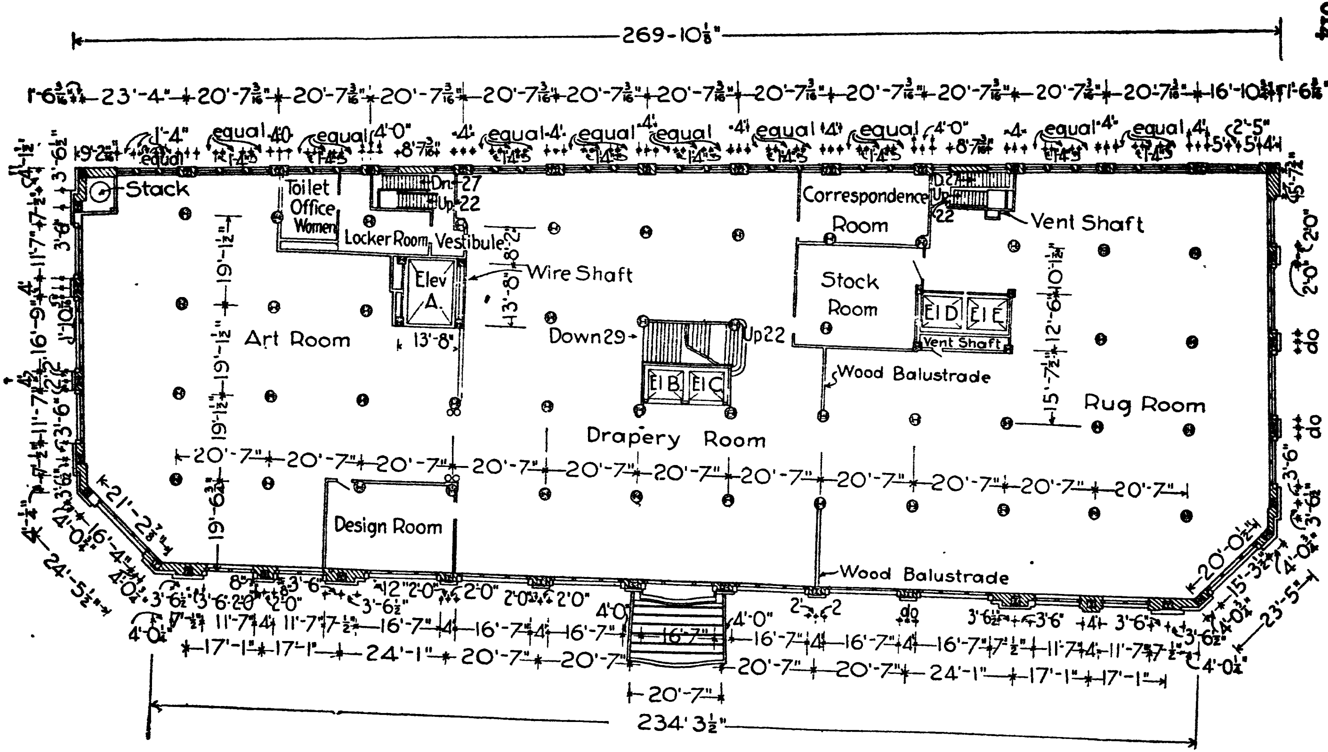
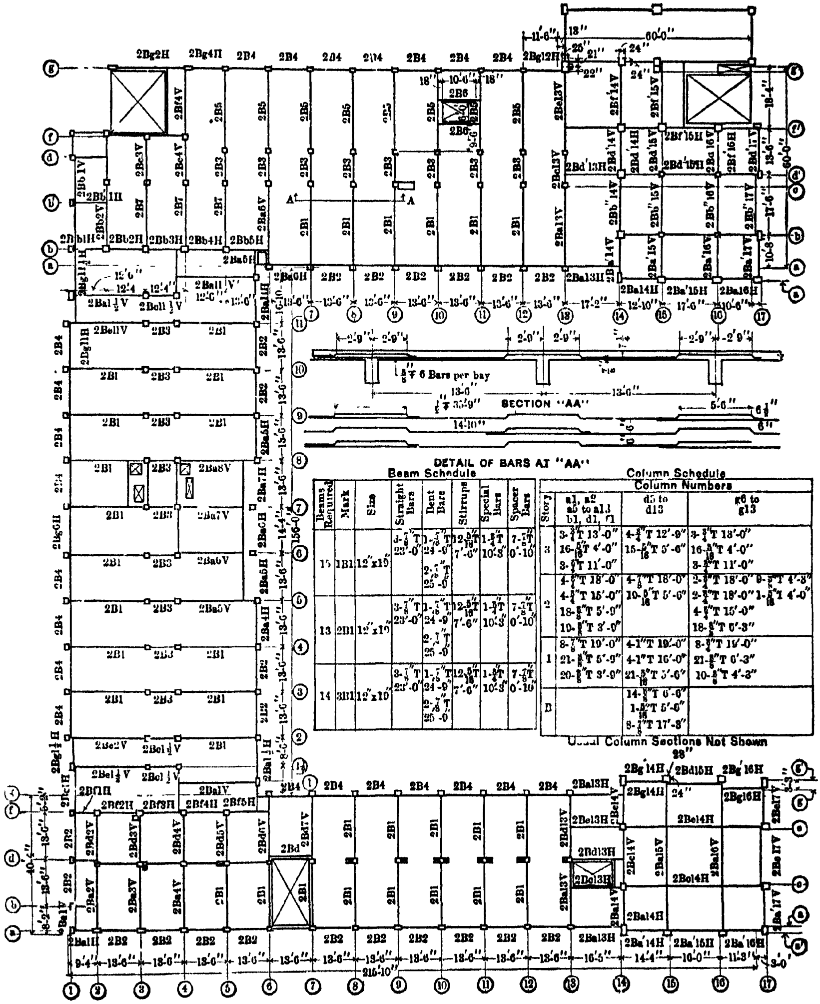


Fig. 176.—Typical Floor Plan. Paine Furniture Building, Boston, Mass. (*See p.* 616.)

Institute of Technology, Cambridge, Mass., designed and built by the Stone & Webster Engineering Corporation; William W. Bosworth, Architect. The concrete design and construction was under the super-



Beam Schedule

Beams Required	Mark	Size	Straight Bars	Bent Bars	Stirrups	Special Bars	Spacer Bars
15	1B1	12"x19"	3-7/8"T 23'-0"	1-7/8"T 24'-9" 2-7/8"T 25'-0"	12-5/16"T 7'-6"	1-3/4"T 10'-3"	7-7/8"T 0'-10"
13	2B1	12"x19"	3-7/8"T 23'-0"	1-7/8"T 24'-9" 2-7/8"T 25'-0"	12-5/16"T 7'-6"	1-3/4"T 10'-3"	7-7/8"T 0'-10"
14	3B1	12"x10"	3-7/8"T 23'-0"	1-7/8"T 24'-9" 2-7/8"T 25'-0"	12-5/16"T 7'-6"	1-3/4"T 10'-3"	7-7/8"T 0'-10"

Column Schedule

Story	Column Numbers: a1, a2, a5 to a13, b1, d1, f1	d5 to d13	g6 to g13
3	3-3/4"T 13'-0" 16-5/16"T 4'-0" 3-3/4"T 11'-0"	4-3/4"T 12'-9" 15-5/16"T 5'-6"	3-3/4"T 18'-0" 16-5/16"T 4'-0" 3-3/4"T 11'-0"
2	4-3/4"T 18'-0" 4-3/4"T 15'-0" 18-5/8"T 5'-9" 10-5/8"T 3'-0"	4-7/8"T 18'-0" 10-5/16"T 5'-6"	2-3/4"T 18'-0" 9-5/8"T 4'-3" 2-3/4"T 18'-0" 1-5/16"T 4'-0" 4-3/4"T 15'-0" 18-5/8"T 6'-3"
1	8-7/8"T 19'-0" 21-5/8"T 5'-9" 20-5/8"T 3'-9"	4-1"T 19'-0" 4-1"T 16'-0" 21-5/16"T 5'-6"	8-3/4"T 19'-0" 21-5/8"T 6'-3" 10-5/8"T 4'-3"
B		14-5/8"T 6'-6" 1-5/16"T 5'-0" 8-7/8"T 17'-8"	

FIG. 177.—Typical Framing Plan and Steel Schedules. (See p. 617.)
Buildings 1, 3, and 5, Massachusetts Institute of Technology, Cambridge, Mass.

vision of Mr. Sanford E. Thompson, Consulting Engineer. This work is one of the most comprehensive schemes for an educational institution that has been developed in this country. The buildings cover an area of 3½ acres and if placed end for end would extend some 2 500 feet in length. Except for the structural steel stairways and exterior finish (see Fig. 180, page 619) reinforced concrete was used throughout. About 40 000 cubic yards of concrete and 3 600 tons of steel reinforcement were used.

Typical Layouts. A number of figures are given to show practical solutions of problems encountered in ordinary building design. Fig. 175, page 613, shows the first floor plan of the Youth's Companion

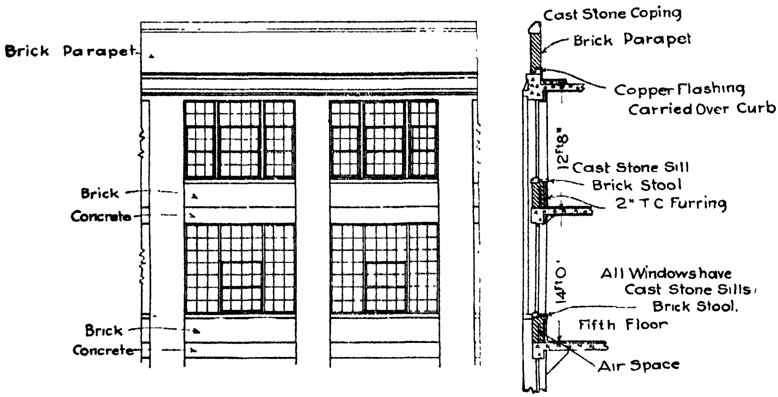


FIG. 178.—Elevation Showing Brick Veneer and Concrete Columns and Spandrel Beams; also Wall Section. (*See p.* 617.)
Youth's Companion Building, Boston, Mass.

Building,* Boston, Mass., Densmore and LeClear, Architects and Engineers.* The size of the panels was determined by the arrangement of the printing presses. The type of reinforcement is illustrated in Fig. 168, page 543. The construction is simplified by the location, outside of the building proper, of the stairs and elevator wells.

The first floor plan of the Paine Furniture Building, Boston, Densmore and LeClear, Architects and Engineers,† is shown in Fig. 176, page 614. In this case the layout is governed by the allowable loads on the soil and by the requirements of retail salesrooms and shops. The floor is a 4-way flat slab system. (See Fig. 166, p. 541.)

The framing plan of one floor of Buildings 1, 3, and 5 of the New

* W. F. Kearns Co., Builders; Sanford E. Thompson, Consulting Engineer.
† James Stewart & Co., Builders: Sanford E. Thompson, Consulting Engineer.

Technology, Cambridge, Mass., is shown in Fig. 177, page 615. This is the key plan which ties in the numerous detail sheets covering the individual slabs, beams, and columns. The figure shows also the typical slab reinforcement and beam and column steel schedule. Fig. 181, page 624, shows a steel schedule for a beam.

A portion of the elevation and wall section of the Youth's Companion Building is shown in Fig. 178, page 616. The building is situated on a boulevard in Boston, and is a combination of a factory and a commercial building. For such cases the arrangement of brick veneer and concrete columns and spandrel beams shown in the drawing is specially suitable. (See also Fig. 174, p. 607.) For the fine architectural treatment of the new Technology buildings limestone and granite were used except on unimportant parts which were faced with brick. (See Fig. 180, p. 619.)

A cross-section of one of the new Technology buildings is shown in Fig. 179, page 618. In this instance a long corridor, with class rooms opening out on either side, ran the entire length of the building. By using long span beams in the class room and short spans in the corridors, all columns were located in the walls and partitions except in large drawing rooms and shops. (See Fig. 183, p. 626). In some large class or lecture rooms columns were avoided by using heavy long span beams as shown in Fig. 182, p. 625. (See also page 627.)

FLOOR LOADS

In designing any structure the local building laws must be consulted. To illustrate good practice the following provisions for floor loads are taken from the 1916 New York City Building Code, Bureau of Manhattan.

Floor loads. Every floor, roof, yard, court or sidewalk shall be of sufficient strength in all parts to bear safely any imposed loads, whether permanent or temporary, in addition to the dead loads depending thereon, provided, however, that no floor in any building or extension to an existing building hereafter erected, shall be designed to carry less than the following live loads per square foot of area, uniformly distributed, according as the floor may be intended or used for the purposes indicated.

40 pounds for residence purposes,

100 pounds for places of assembly or public purpose, except that for classrooms of schools or other places of instruction the floor need not be designed for more than 75 pounds, and

120 pounds for any other purpose,* except that the floors of offices need not be designed for more than 60 pounds. The live load for which any and every floor may be designed shall be clearly shown in the application and on the plans before any permit to erect is issued.

*Loads on warehouse floors run from 200 to 500 pounds with an average of 250 to 300.—Authors.

Concentrated loads. Every steel floor beam in any building hereafter erected used for any business purpose shall be capable of sustaining a live load concentrated at its centre of at least 4 000 pounds.

Moving loads. Running machinery or other moving loads shall be considered as increasing the live loads in proportion to the degree of vibratory impulse transmitted to the floor.

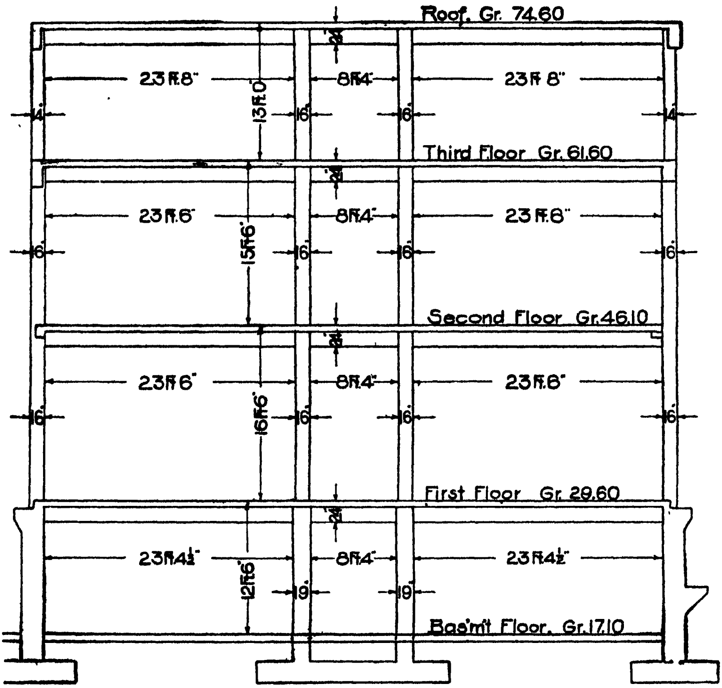


FIG. 179.—Typical Cross-Section Showing Classrooms Opening on Corridor.
(*See p.* 617.)
Massachusetts Institute of Technology, Cambridge, Mass.

Roof loads. Every roof hereafter erected, shall be proportioned to bear safely a live load of 40 pounds per square foot of surface when the pitch of such roof is twenty degrees or less with the horizontal, and thirty pounds per square foot measured on a horizontal plane, when the pitch is more than twenty degrees.

Loads on vertical supports. Every column, post or other vertical support shall be of sufficient strength to bear safely the combined live and dead loads of such portions of each and every floor as depend upon it for support, except that in buildings more than five stories in height the live load on the floor next below the top floor may be assumed at ninety-five per cent. of the allowable live load, on the next lower floor at ninety per cent., and on each succeeding lower floor at correspondingly decreasing percentages, provided that in no case shall less than fifty per cent. of the allowable live load be assumed.

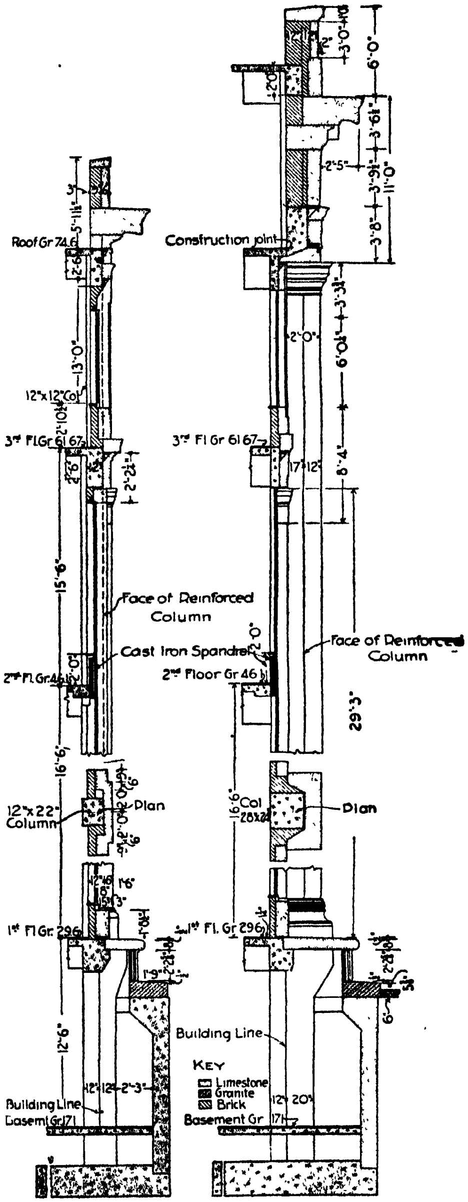


FIG. 180.—Ornamental Façades on Major and Minor Courts. Massachusetts Institute of Technology, Cambridge. (*See p.* 617.)

Sidewalk loads. For sidewalks between the curb and building lines, the live load shall be taken at 300 pounds per square foot.

Yard and court loads. For yards and courts inside the building line, the live loads shall be taken at not less than 120 pounds per square foot.

Weight of Concrete. The weight of concrete varies with the weight of the aggregate and with the proportions. For ordinary stone and gravel concrete, a weight of 150 pounds per cubic foot can be used in computation. An approximate value of 144 pounds per cubic foot is sometimes used because of convenience in cancellation when figuring bending moments. For certain conditions, more exact values are necessary, in which case the weights on page 9 should be employed, and the excess weight of the steel reinforcement, which sometimes amounts to several pounds per cubic foot of the structure, may be allowed for.

Tables of volume of concrete in slabs, beams, and columns of

different dimensions, are given in the authors' book on "Concrete Costs," pp. 526–533. These are convenient to use for computations for both quantities and weights.

MATERIALS FOR BUILDING CONSTRUCTION

A first-class Portland cement must be used which fulfills the standard specifications given on page 62. The rules for the selection of the aggregates are the same as for other classes of concrete. Since the concrete may be loaded way up to its working stresses, it is particularly necessary to see that the sand is satisfactory. (See page 115.) The size of the coarse aggregate is often limited to 1 inch. If well graded, however, so that the larger pieces will not collect and prevent the flow of the mortar around the steel, the limit of size where the floor thickness is not less than 4 inches may be as high as $1\frac{1}{2}$ inches.

The usual proportions are 1: 2: 4, that is, one barrel, or four bags, of Portland cement, 8 cubic feet of sand, and 16 cubic feet of broken stone or gravel, the relative proportions of the sand and stone being varied slightly to suit the particular aggregates. In certain cases, especially where the cost of cement is relatively low, it is economical to use richer proportions, such as 1: $1\frac{1}{2}$: 3, and in columns, particularly, where size is an important factor, the proportions may be even richer. Although higher unit stresses may be used, it must be remembered that the modulus, and therefore the ratio of elasticity (see page 483), as well as the strength, changes with the proportions so that there is less reduction in the size of the member on account of a richer mix than might be expected. It is necessary to make allowance for this difference in modulus both in figuring columns (see p. 376) and also in the formulas for beams and slabs (see pp. 353 to 362).

The quantities of cement and aggregates required per cubic yard of concrete in different proportions are tabulated on page 214. Convenient tables for figuring material costs are presented in "Concrete Costs," pp. 165 to 173.

Steel requirements are discussed on page 478.

Cinder Concrete. For short-span floors of steel frame buildings, cinder concrete sometimes may be used economically for light loads because of its light weight. The span for cinder concrete slabs is generally limited to 6 or 8 feet. Cinders are not suitable for the structural members of reinforced concrete structures. For fireproofing (see p. 289) cinder concrete may be employed providing a first-class cinder is available.

Cinders for concrete should contain but little unburned coal and be free from soot. A clean cinder will not discolor the palm when held in it and rubbed with the fingers. Usually a better mixture can be obtained by screening the fine stuff from the cinders, and then, if gritty, mixing it with sand, than by using unscreened material, although if the fine stuff is found by tests to be uniformly distributed through the mass, it may be used without screening and a smaller proportion of sand added.

Concrete Blocks. Concrete blocks are specially adapted to dwellings, farm buildings and similar structures. None should be used, however, unless they are known by test and experience to be satisfactory and durable. The National Board of Fire Underwriters specifies that the compressive strength calculated on the gross area, including cellular spaces, shall be not less than 800 pounds per square inch with the cells vertical; and not less than 300 pounds per square inch, with no block testing at less than 200 pounds per square inch, with the cells horizontal. The average absorption of three blocks shall not exceed 10 per cent. in 48 hours nor 15 per cent. in any case.

Special care is needed in selecting and proportioning aggregates. One of the chief difficulties has been the selection of too lean proportions.

Ornamental Stone. For balustrades, cornices, interior work, and the like, artificial stone made with special aggregates is cast in carefully shaped molds. Such products have been found satisfactory architecturally and from the point of view of durability.

To prevent hair cracking or checking, density is the most essential requirement. The aggregates must be accurately graded with as large a maximum size as possible, half-inch or even three-quarters inch, preferably. The consistency must not be too wet, a sluggish consistency about like very thick pea soup or even stiffer is best. A very wet mix is almost sure to check, while too dry a mix is apt to be porous.

Molding in sand produces blocks of a rough, rather pleasing surface, and is also suitable for blocks which are to be tooled. A wooden core is made and fine damp white sand is packed around it, then the core is removed and the mortar is poured in. When this concrete is hard the sand is removed and the blocks stored where they may be kept moist for at least two weeks. If flasks are used, they may be stored in the flasks. The requirements mentioned as to grading and consistency must be followed.

Tooling the surface of concrete blocks is an effective treatment, exposing the aggregate and producing a pleasing surface resembling

cut stone. If sand molded blocks are tooled so as to produce sharp arrises or corners it is possible to use a stiff consistency and coarse aggregate.

CONCRETE FLOOR SYSTEMS

In reinforced concrete construction, the panel in a floor system, that is, the space between four columns, may be of three general types:

(1) Reinforced concrete slabs, beams, and girders.

(2) Reinforced concrete slabs supported on steel beams.

(3) Reinforced concrete flat or girderless slabs supported directly on columns.

Design of Members. In designing the members of a reinforced concrete floor system, the formulas and allowable unit stresses given in Chapter XXII, on Reinforced Concrete Design should be used.

The economy of a design depends in a large measure upon the proper selection of the type of panel to use and the proper spacing of the columns.

In warehouses and factory buildings, the spacing of columns, and therefore the size of the panels, often is governed by economical considerations, while in other structures, the architectural requirements control to a large extent. With free choice, economical spans range between 18 feet and 25 feet. In determining the economical spacing of columns for a building of given size, it frequently is well worth while to make comparative sketches for panels of different sizes and shapes and then figure the amount of concrete, steel, and form work required for each size. In comparative estimates, it is useless to figure the quantities of concrete and steel unless the cost of form work is taken accurately. Such computations may be made accurately and very quickly by reference to "Concrete Costs." In that book are given tables of volumes and costs of concrete and costs of forms for concrete members of different sizes. To illustrate the accuracy with which the cost of different designs may be compared, the table on page 623 is quoted from a paper by Mr. Thompson presented before the American Concrete Institute.

In many cases the comparison must be made between flat slab and beam and girder construction, the same method, however, being used for this as in the case given and the data required being taken from the various tables in "Concrete Costs."

Frequently when the spans of the panels in two alternate designs are nearly alike, the difference in cost of the columns and footings themselves is small and need not be considered. In other cases where the building is high with heavy floor loads, that may be the controlling

Item	Unit Cost	Concrete Slab Span 15 ft. 6 in. Amount	Concrete Slab Span 15 ft. 6 in. Cost	Concrete Slab Span 7 ft. 9 in. 1 intermediate Beam. Amount	Concrete Slab Span 7 ft. 9 in. 1 intermediate Beam. Cost	Hollow Tile 15 ft. 6 in. Amount	Hollow Tile 15 ft. 6 in. Cost	Page Reference. "Concrete Costs."
Concrete per panel:								
Slab....cu. yd.	....	10.7	....	6.1	....	6.6	....	....
Beam....cu. yd.	....	2.8	....	2.6	....	2.2	....	....
Girder....cu. yd.	....	....	....	0.65	....	....	....	....
Total	$5.82	13.5	$78.50	9.35	$55.00	8.8	$51.22	....
Steel per panel:								
Slab....lb.	....	1 080	....	435	....	607	....	....
Beam ....lb.	....	944	....	1 620	....	944	....	....
Girder....lb.	....	....	....	690	....	....	....	....
Total....	0.027	2 024	55.60	2 745	75.00	1 551	41.88	....
Tile per panel:								
Slab ....piece	0 200	....	....	....	....	286	57.20	....
Total cost of concrete..	.. .	....	**$134 10**		**$130 00**		**$150.30**	....
Form Lumber:								
Slab ....1¼in. stock ft. B.M.	....	1 772	....	435	....	1 772	....	617
Beam....1¼in. stock ft. B M.	....	496	....	680	....	429	....	619
Girder. ....1¼in. stock ft. B.M	....	.. .	....	267	....	....	....	619
Total for 4 floors	...	2 268	....	2 382	....	2 201	....	....
Total per floor per panel ..M	$30.00	567	$17.00	596	$17 86	550	$16.50	....
Form Labor, Slab:	Carpenter labor, 50c. per hour. Ordinary labor, 25c. per hour.	....	....	....	....	....	....	644
Make		$4.39	....	$3 78	....	$4.39	....	....
Place and remove first floor		12.88	....	11.48	....	12.88	....	....
Place and remove second floor .		9.88	....	8.57	....	9.88	....	....
Place and remove third floor ..		11 02	....	9 31	....	11.02	....	....
Place and remove fourth floor...		11 02	....	9 31	....	11 02	....	....
Total cost 4 floors		$49 19	....	$42.45	....	$49.19	....	....
Cost per panel per floor		....	$12.30	....	$10 61	....	$12.30	....
Form Labor, Beams:								639
Make		$3.00	....	$2.10	....	$2.70	....	....
Place and remove first floor . ..		5.79	....	4.40	....	5.10	....	....
Place and remove second floor ..		5.20	....	3.90	....	4.60	....	....
Place and remove third floor . ..		6 92	....	5 10	....	6.80	....	....
Place and remove fourth floor		6.92	....	5.10	....	6.80	....	....
Total cost per beam 4 floors		$27.83	....	$20.60	....	$26.00	....	....
Cost per panel per floor		....	$6.95	....	$10 30	....	$6.40	....
Form Labor, Girders:								641
Make		....	....	$2.30	....	....	....	....
Place and remove first floor.. ..		....	....	4.80	....	....	....	....
Place and remove second floor .		....	....	4.10	....	....	....	....
Place and remove third floor....		....	....	6.20	....	....	....	....
Place and remove fourth floor...		....	....	6.20	....	....	....	....
Total cost 4 floors....		....	....	$23.60	....	....	....	....
Cost per panel per floor		....	....	....	$9.50†	....	....	....
Total cost forms....	....	....	**$36.25**	....	**$48.27**	....	**$35.20**	....
Total cost....	....	....	**$170.35**	....	**$178.27**	....	**$185.50**	....

Cost of forms, not cost of materials used in beams and slab, determines relative cost of alternate designs. Forms assumed to be used four times and remade once for a change in size of columns. Costs are from "Concrete Costs" by Taylor and Thompson according to page numbers given. Size of beam below slab, 12 in. by 19 in. Size of girder below slab, 14 in. by 19 in.

*** From Design and Construction of the Massachusetts Institute of Technology Buildings by Sanford E. Thompson, Journal American Concrete Institute, July 1915, page 377.**

† Multiply cost per panel per floor by 1.6 instead of 2.0 to allow for spandrel beam.

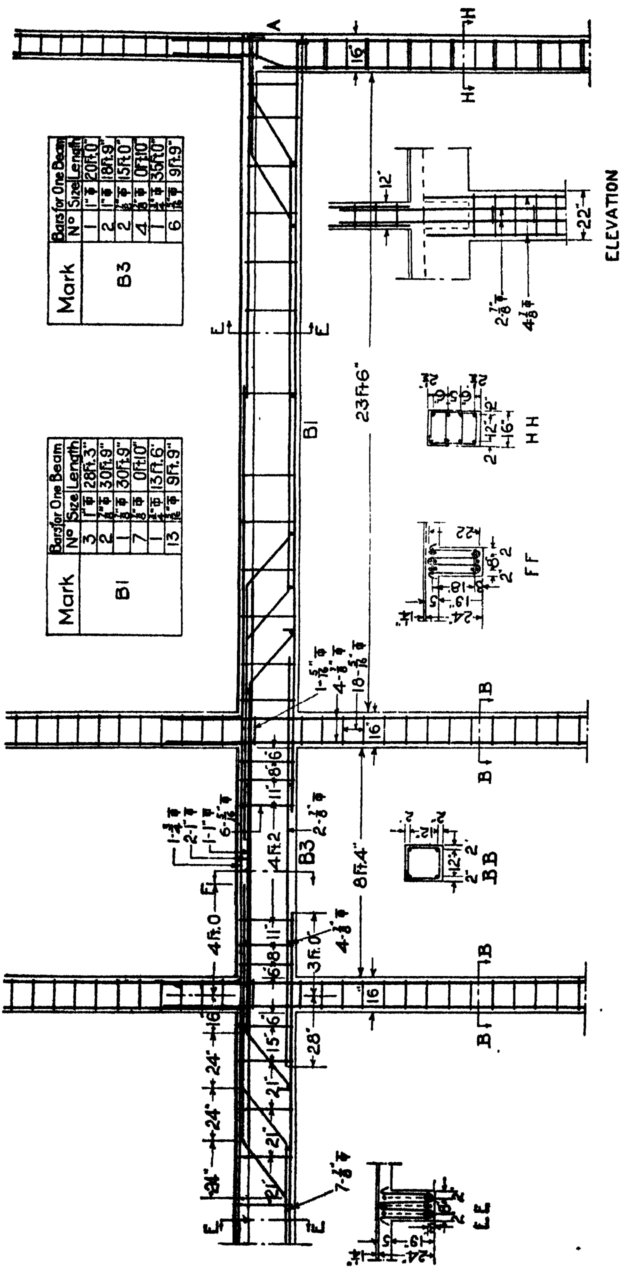


FIG. 181.—Typical Beam and Column Reinforcement. (See p. 626.)
Massachusetts Institute of Technology, Cambridge, Mass.

factor. For large panels it may be necessary to use structural steel reinforcement instead of bars. As this is more expensive and complicates, to some extent, the erection, smaller panels may be desirable although by themselves they may show no advantage.

The thickness of the structural slab should not be smaller than 3 inches or better still 4 inches because of the difficulty of placing the concrete properly, and preferably not greater than 8 inches, except in occasional cases of flat slab construction, because of the weight of the structure.

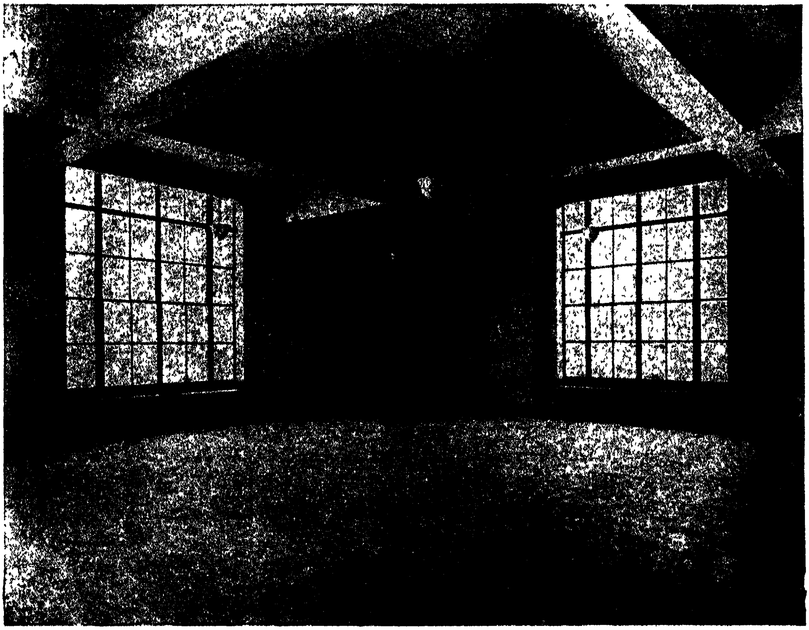


Fig. 182.—Arrangement of Long Span Beams in Large Lecture Room.
(*See p.* 627.)
Massachusetts Institute of Technology, Cambridge, Mass.

Where possible, the layouts should provide for a typical design all through the building and do away as far as possible with odd panels, which not only complicate the design but also make the construction more difficult and costly. In rectangular buildings, the dimensions of the panels should be preferably multiples of the width and the length of the building.

Panels of Reinforced Concrete Beam and Slab Design. The design of a complete floor system with reinforced concrete beams, girders, and slabs, is illustrated on page 552, and on pages 553 to 557 are given the complete computations for determining sizes of members and reinforcement. Other arrangements of beams are shown in Figure 181, page 624.

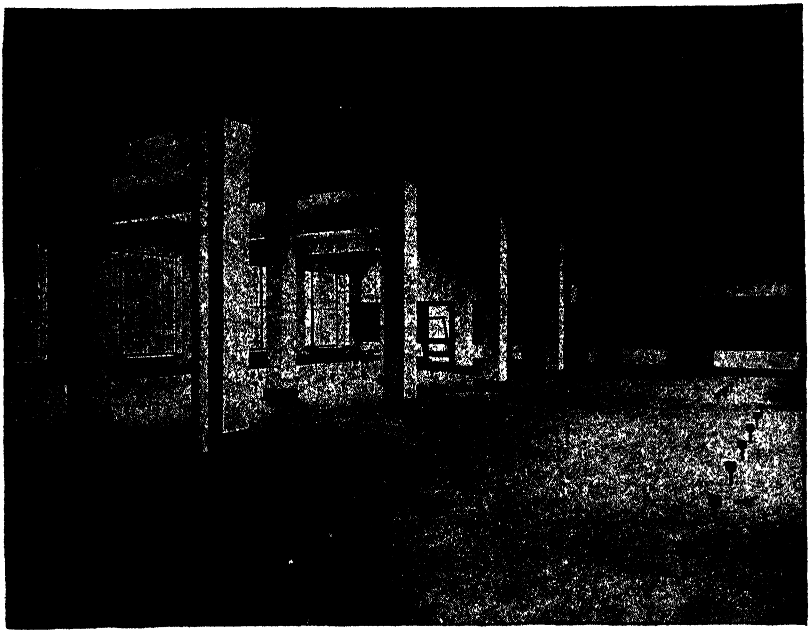


FIG. 183.—Typical Arrangement of Beams and Columns for Drawing Rooms and Classrooms Opening on a Corridor. (*See p.* 617.)
Massachusetts Institute of Technology, Cambridge, Mass.

The method of comparing the cost with different spacings of beams is illustrated on page 623. The slab span between beams gives the minimum quantity of steel and concrete, but on account of the excess cost of forms, and in some cases the possibility of omitting girders, longer spans may be more economical, as in the example just referred to.

As different floors in the building are designed for different loadings, it is sometimes economical to keep the stems of the T-beams alike to permit the repeated use of the same beam forms and change only the reinforcement and the design of the floor slabs in accordance with the loads.

The common type of beam and girder floor is made up of girders

running from column to column with beams running into the same columns and one or two intermediate beams between columns. The principal slab reinforcement runs at right angles to the beams. By using longer spanned slabs, the girders and intermediate beams frequently may be omitted entirely, as shown in Figure 187, page 630.

In girders carrying concentrated loads from beams fewer stirrups are required to carry the shear than is the case in beams carrying a distributed load. Except for the dead load and possibly some live load the shear is constant between the support and the first beams. This permits a uniform and a wider spacing than in beams.

Square Panels. To obtain smooth ceilings, the panels are sometimes made square, or nearly so, with beams on all four sides. The reinforcement then consists of bars running in two directions, or arranged as shown in Figure 168, page 543. The latter arrangement, in which certain features are patented, requires less steel because the bars run more nearly in the direction of the actual stresses (see p. 544). For a

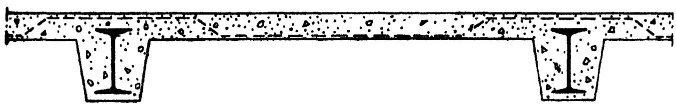


FIG. 184.—Steel Beams, Fireproofed, Supporting Concrete Slab.* (*See p.* 628.)

large square room, like a lecture room, where it is advisable to have no obstacles, the girders may be placed in the walls with two intermediate beams running in each direction and intersecting at the third points in the span. (See Fig. 182, page 625.) By assuming a distribution of the load to all of these beams, spans as long as 40 feet may be readily attained without excessive depth of beam.

Panels with Slab Supported on Steel Beams. A structural steel frame for beams, girders, and columns may be preferable to reinforced concrete for certain structures, such as city office buildings, simply for the reason that if the steel is fabricated in advance the buildings can be erected more rapidly although at higher cost. For structural steel buildings concrete is commonly used for the slabs. Concrete slabs supported by beams framed into girders are reinforced in one direction only. Steel may be run over the top of the beams so as to make the slab continuous, or the surface of the slab may be flush with the top of the I-beam. The former plan is usually preferable, requiring thinner slabs, because of the continuous action with less danger of cracks over

* Redrawn from a cut prepared by the author for Marks' Mechanical Engineers' Handbook.

the beams. If the panel is square, or nearly so, the slab between the steel beams is reinforced by bars running in two directions, or by radials and circles as shown in Figure 168, page 543.

For fireproof construction, the steel beams, girders, and columns must be encased in concrete, tile, or other fireproof material. This fireproofing increases the cost so as to make the steel frame building always more costly than reinforced concrete construction. (See page 608. The fireproofing of steel girders is shown in Figure 184, page 627.

When floors are built of a combination of steel girders and reinforced concrete beams and slabs, care must be taken to see that proper seats in the steel frame are provided for the concrete beams.

The slab between steel beams sometimes is constructed in the form of a concrete arch. If unreinforced, the beams should be connected with tie rods spaced to resist any possible unbalanced thrust. For arches with curved upper surfaces, a fill of cinders or a very lean concrete is used for leveling.

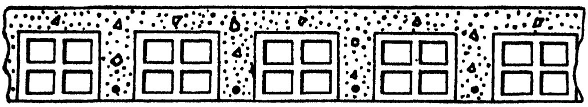


FIG. 185.—Details of One-Way Hollow Tile Floor Slab.* (*See p.* 628.)

Hollow Tile and Reinforced Concrete Floors. For floors of comparatively long span and light load a combination of concrete and hollow tile is suitable in certain cases. The substitution of light weight hollow tile for a part of the concrete below the neutral axis reduces the dead load of the building, and the amount of steel is also reduced because the depth of the tile may be greater than is customary with concrete.

The one-way system shown in Figure 185, page 628, consists of a series of reinforced concrete ribs from one to two feet on centers with hollow tile between, and a slab of concrete 2 inches or more in thickness covering it and extending from rib to rib. In designing, each rib may be treated as a T-beam and the formulas for bending moments and shears used as recommended in Chapter XXII, page 487. Diagonal tension and bond stresses must receive special attention (see pages 516 and 539). Along the beams the tile are omitted and the slab is made solid to provide a flange for the beam and to reduce the compressive and shearing stresses in the joists at the support.

* Redrawn from a cut prepared by the author for Marks' Mechanical Engineers' Handbook.

In the two-way system shown in Figure 186, page 629, the ribs run in two directions, as shown, and the load may be considered as distributed equally in the two directions.

In constructing this combination of tile and concrete slab, flat form work is built and the tiles are placed on this centering in proper position. The steel is placed between and the concrete of the ribs and slabs is poured as in any other monolithic construction.

The advantages of this construction are that the joists may be made of the same thickness as the supporting beams so as to give a smooth ceiling and the weight of the construction is somewhat reduced. The

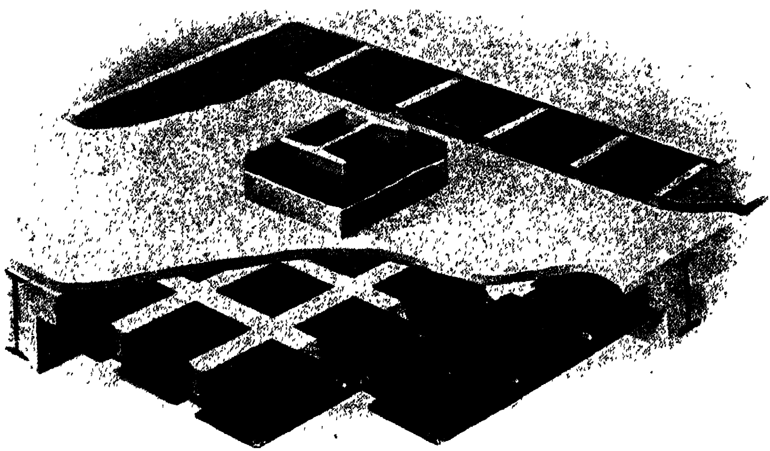


FIG. 186.—Details of Two-Way Hollow Tile Floor Slab and Structural Steel Column Fireproofing. (*See p.* 629.)

disadvantages are the high cost of tiles and of the labor of placing and keeping them in position during construction. The ceilings also have to be plastered or else the appearance is not neat. The flat ceiling surface in most cases can be obtained more readily by the use of flat slab construction, except in the case of narrow buildings.

Flat Slab Floors. In recent years, girderless floors, or flat slabs, supported directly on columns, provided usually with a flaring head, have come into very common use. This type of construction has the following advantages:

(1) Reduced story height because of elimination of beams and girders.
(2) Better distribution of light.
(3) Economy in construction.
(4) Reduced cost of form work because of omission of beams.

Figures 166, 167 and 168, pages 541, 542 and 543, illustrate the methods of reinforcing girderless floors, and Fig. 187, page 630, shows the interior of a flat slab building.

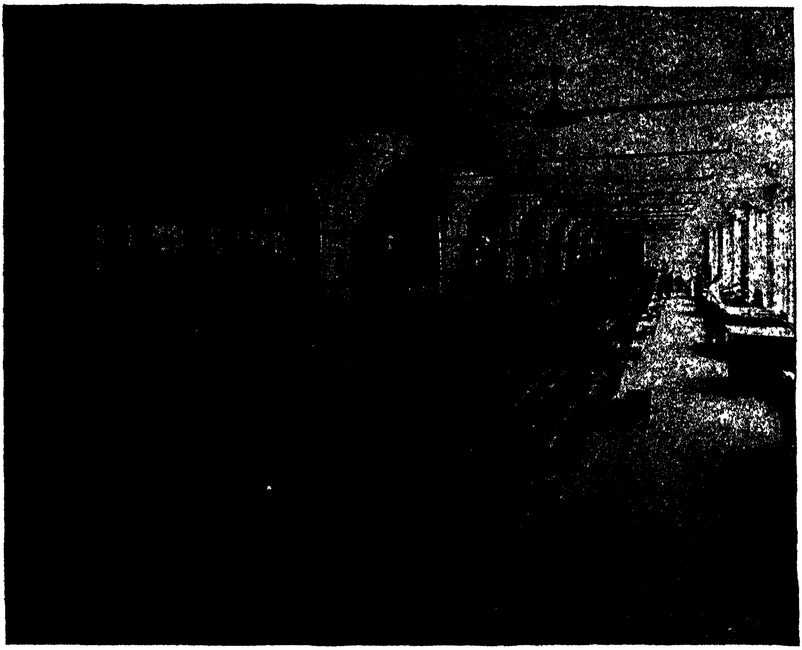


FIG. 187.—Typical Flat Slab Interior Showing Flat Ceiling, Columns, and Wall Column Brackets. (*See p.* 630.)
Youth's Companion Building, Boston, Mass.

Shafting Hangers and Inserts.—Shafting, sprinkler systems, and the like may be suspended from concrete beams and slabs by various means. Many types of sockets are on the market intended to be placed on the forms and imbedded in the concrete when it is laid. Expansion bolts set into the concrete are widely used, especially for heavy loads. Frequently several types are used in the same building. At the New Technology buildings, for example, two styles of patented sockets were concreted into the slab (see Fig. 188, p. 631) but were not considered

strong enough for the heavy shafting. The shafting hangers were fastened, by lag screws,* to hard pine stringers which were in turn fastened to the slab by expansion bolts.

Concrete Columns. In designing concrete columns it is necessary, first, to determine the size of the column, the proportion of the concrete, and, from formula (44), page 562, the required amount of reinforcement. The methods of designing are given on pages 558 to 565 and the allowable working stresses on page 573. Steel in compression is more expensive than concrete, therefore the most economical column is obtained where the minimum amount of steel is used, which, as explained on page 559, is one per cent of the net area. Ordinarily, however, in building construction the size of the columns is limited; therefore the strength of the concrete column must be increased by either of four methods: (1) a richer mix; (2) a larger amount of vertical steel; (3) spiral steel; (4) structural steel with or without spiral steel.

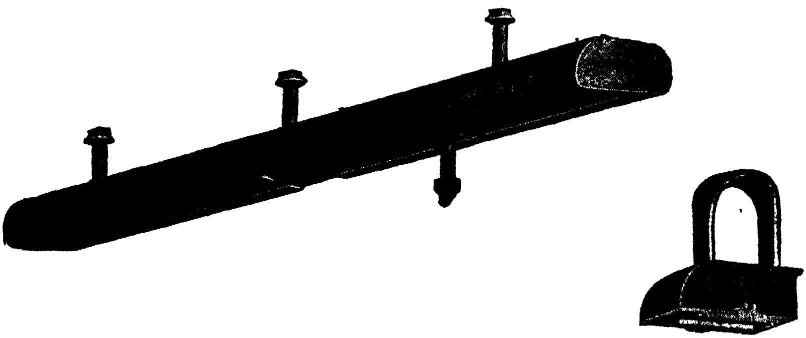


FIG. 188.—Inserts Used in Slabs at Massachusetts Institute of Technology, Cambridge, Mass. (*See p.* 630.)

The cheapest column usually is that in which the required strength is obtained by a rich mix using only a minimum amount of vertical steel, say about 1%. Proportions 1 : 1½ : 3 or 1 : 1 : 2 are customary. When the size of the column is limited so that a large percentage of vertical steel is required, it is likely to be cheaper to use spirals with vertical bars, and thus obtain the advantage of the larger unit stresses allowable. With a structural steel core the size of the column may be still further reduced although at additional cost. Each case should be studied to determine the relative advantages from the cost and

* Through bolts are sometimes considered preferable to lag screws.

structural standpoints. The following table illustrates a practical comparison of the cost per lineal foot of a 22-inch column carrying 230 000 pounds with very heavy vertical steel (5.8%) and a similar column reinforced with spirals and vertical bars. With $1\frac{1}{2}$ inches fireproofing, the effective diameter is 19 inches and the effective area is 282 square inches. The average stress, therefore, is

$$f = \frac{250\ 000}{282} = 815 \text{ lb. per sq. in.}$$

and the required amount of reinforcement for the proper working stresses, f_c, can be taken from Table 18, page 599.

Relative Economy of Reinforced Concrete Columns in a Particular Case. (*See p.* 632.)

Cost of steel per pound in vertical bars, 3.5c.; in spirals, 3.9c;
Cost of concrete per cubic foot; proportions 1:2:4, 20c; proportions 1:$1\frac{1}{2}$:3, 25c.
Volume of concrete per foot of length: 22 inch round columns, 2.64 cubic feet; 21 inch round columns, 2.4 cubic feet.

Item.		1:2:4 Concrete		1:$1\frac{1}{2}$:3 Concrete	
		Vertical bars only	Spirals and vertical bars	Vertical bars only	Spirals and vertical bars
Diameter of column		22 in.	22 in.	22 in.	21 in.
Effective area		282 sq. in.	282 sq. in.	282 sq. in.	252 sq. in.
Percentage, area, and weight of steel per foot of column	Vertical bars	5.8%, 16.4 sq. in. 55.8 lb.	1.48%, 4.1 sq. in. 14 lb.	4.0%, 11.2 sq. in. 38.2 lb.	1%, 2.52 sq. in., 8.6 lb.
	Spirals	....	1%, 2.8 sq. in. 9.6 lb	....	....
Cost of steel per foot of column	Vertical bars	\$1.95	\$0.49	\$1.34	\$0.30
	Spirals	....	0.38	....	0.34
Cost of concrete per foot of column		0.53	0.53	0.66	0.60
Total cost per foot of column		\$2.48	\$1.40	\$2.00	\$1.24

It is evident from the table that a 1:$1\frac{1}{2}$:3 mix with spirals is the most economical column in this particular case. With a large diameter of column permissible the relative results would be different. However, if the job is small and the inspection not efficient, it may be advisable to use spiralled columns and 1:2:4 concrete, even if the expense is larger, because of the possibility of failure on the part of the workmen to use the richer mix in the columns.

Details of Design. In buildings of several stories it is advisable to design the columns so as to make the number of changes in size from story to story as small as possible. The reduction of the size of columns in the upper story requires not only the remaking of column forms used in the floor below, but also remaking of beam forms, because of the increase in length of the net span. Ordinarily it is possible, as the building goes up, to reduce the strength of the column by gradually reducing the amount of steel and possibly omitting the spiral reinforcement. In that way the same size of columns can be kept through several stories. Sometimes it may be advisable to waste some concrete in the columns to avoid the remaking of column and beam forms. Methods of design are treated on pages 558 to 565.

In flat slab construction metal forms are generally used for columns and column caps. As the metal forms are very easily adjustable, little attention needs to be paid to the changes in sizes of the columns.

Round columns and columns with rounded corners are less affected by fire than columns with sharp corners. (See p. 289.)

Concrete Columns Reinforced with Vertical Bars Only. The reinforcement consists as a rule of bars up to $1\frac{1}{2}$ inches in diameter, placed around the circumference of the column about 2 inches from the outside face. The bars should be evenly distributed except in columns subject to

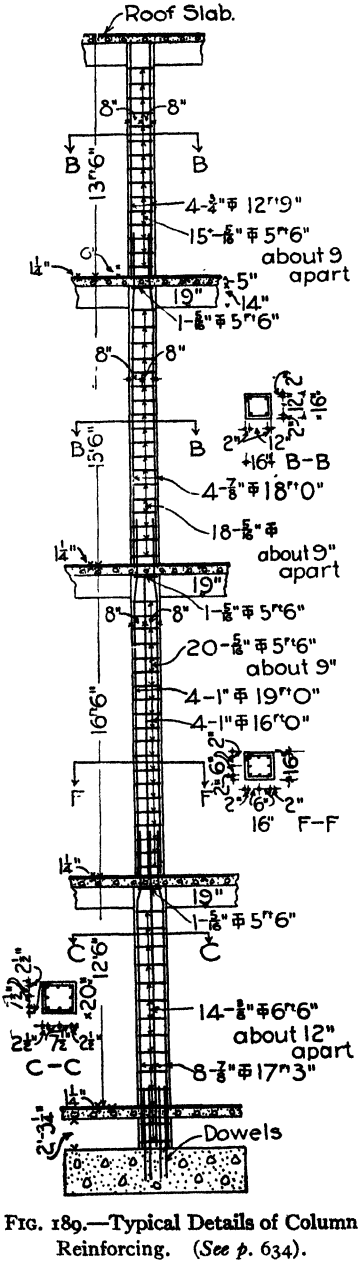


FIG. 189.—Typical Details of Column Reinforcing. (*See p.* 634).

eccentric loads, or bending moments, in which case the largest number of bars should be placed near the surface under the largest stress. If tension occurs in the column, it must be provided for.

The bars as a rule are carried through only one story and are spliced on the floor level, as shown in Fig. 189, page 633. For bars up to one inch diameter the splicing is effected by extending the bars a sufficient length above the top of the floor to develop their strength by bond (see p. 533). Bars over one inch diameter should be faced true and butted. To keep the bars in position, their ends should be enclosed in a tight fitting sleeve. If there is a possibility of tension in such columns, shorter bars extending below and above the floor should be used.

The bars should be held in place at regular intervals by ties of small bars $\frac{1}{4}$-inch diameter in columns up to 20 inches diameter, and $\frac{3}{8}$-inch diameter for larger columns. The spacing of these ties should not exceed 18 inches, nor the smallest diameter of the column.

When erecting columns, it is ordinarily advisable to assemble the bars and the ties in the yard and place them as a complete unit. If the amount of steel in the column is too large, making the assembled unit too heavy to handle, it may be advisable to make a unit using a portion of the bars only and place the remaining bars after the skeleton is in place.

If the sizes of the columns in two successive stories are different, it is necessary to bend the bars of the lower column so as to enable them to fit into the smaller column form without interfering with the steel of the column above. If the size of the bars and the difference in sizes of the columns is large, it is advisable to bend the bars beforehand. Small differences can be adjusted after the bars are in place.

In column footings the stress in the steel must be distributed on the concrete by bearing plates or by dowels long enough to transfer the stress by bond.

Spiral Columns. Spiral columns consist of vertical bars and circular spirals placed outside of the vertical bars. The pitch of the spiral preferably should not be greater than $\frac{1}{10}$ of the diameter of column. In no case, however, should it exceed $2\frac{1}{2}$ inches or $\frac{1}{6}$ of the diameter of column. Formulas for pitch and weight of spirals are given on page 563.

The requirements for spirals are: (1) The pitch should be uniform; (2) The spiral should be continuous, or else properly welded or spliced; (3) The spiral should be even, as any irregularities are harmful. After erection the core should be straight and well centered.

In best practice, spirals are built in the shop and transported in collapsed form to the job. Guide angles, notched to insure accurate spacing, are used. The spiral must extend from the bottom of the column practically to the top of the slab.

Structural Steel Columns Imbedded in Concrete. If the area of the structural steel column does not exceed 6 per cent of the section of concrete and the shapes consist of small angles, the column should be treated as reinforced concrete. In such cases it is sufficient to imbed the angles in concrete and provide $\frac{3}{8}$-inch round ties, spaced 12 inches on centers. At the floor level the angles should be faced and butted and provided with bolted splice-angles.

If the structural shapes are designed to resist the larger part of the load they must be built rigidly in conformity with standard practice for structural steel columns. The shapes should be latticed or provided with tie bars.

The structural steel columns may consist of four angles placed in corners of a rectangle with legs turned in. Another type sometimes used is the Gray column, consisting of eight angles arranged in four groups of two angles each placed face to face. (See Fig. 142, p. 463.) Bethlehem H sections and built-up columns are sometimes used.

The splices of structural shapes must be made according to standard practice. Seats must be provided for beams and girder. In flat slab design the load is transferred to the column by means of clip angles, placed near the bottom of the column head.

To prevent concrete from separating from the structural column, it is advisable to use ties spaced not more than 18 inches on centers.

FLOOR SURFACES*

The type of floor surface to select for reinforced concrete or fireproofed steel frame buildings must be governed by the use to which the building is to be put and the relative costs of different materials. Granolithic made with the right materials, properly proportioned and laid makes a most durable and satisfactory floor. Special conditions may lead to the adoption of terrazzo, mosaic, magnesium composition, hard wood, or a covering of battleship linoleum upon the concrete.

Granolithic Floors. A cement or granolithic surface is in keeping with the type of structure of a reinforced concrete building, and not-

* For more complete discussion, including a treatment of methods and costs of different types of floors and specifications for laying granolithic see paper on "Floor Surfaces in Fireproof Buildings" by Sanford E. Thompson, Transactions American Society of Mechanical Engineers, Vol. 36, 1914, p. 387.

withstanding the numerous instances of floors which dust and ravel under service, it is possible to lay satisfactory and durable floors which will resist severe wear, and even trucking, with inappreciable dusting.

The objection occasionally heard of coldness is not borne out by the facts except where the floor is laid directly upon the ground or is over an unheated room. If the building is warm the floors will be warm. The color of cement, a dead gray, is not particularly pleasing but can be improved upon by adding coloring matter, or by coating the surface with linseed oil or similar material. The darkening of the surface also produces a "warmer" color, in fact it changes the appearance so that it gives the sensation of greater warmth to the occupants of the room. The hardness of the surface from a practical standpoint is more apparent than real as it is found that operatives in the plant readily become accustomed to the slight difference and do not notice it. Machinery can be readily held in place and shafting can be hung by bolts imbedded as described on pages 269 and 630.

The essentials for a surface which will resist wear and prevent appreciable dusting are: the selection of aggregates which contain no dust and consist chiefly of particles ranging from $\frac{1}{16}$ to $\frac{1}{2}$ inch in size; proportions about one part cement to two parts mixed aggregate; a consistency that will not flow but that will hold its shape in a pile without settling; a perfect bond with the concrete base; the avoidance of temperatures below fifty degrees Fahr.; trowelling so that there is no excess water brought to or remaining at any time on the surface; and maintaining a wet surface at all times for at least ten days or two weeks after laying.

The soft dusty surface so often found on granolithic floors is usually due to one or a combination of 3 causes: (1) excess water in mixing, giving a weak, white concrete; (2) the use of too fine sand and screenings; (3) water remaining on the surface, especially serious in cold weather, which prevents the proper crystallization of the cement. Improper curing, that is, too rapid drying out through lack of moist covering or because of excessive heat in the building may produce checking of the surface.

Compounds of various kinds have been brought out and many of them patented for use on granolithic floors. With the proper construction, however, no treatment is necessary. In case of a poor surface with a good body of granolithic, that is, where the soft material is only a small fraction of an inch in thickness, a hardening compound may be of value to aid in resisting abrasion. If the poor surface, however, goes to any depth there is no material which will penetrate satis-

factorily so as to give permanent results. In such cases probably the best treatment, although a very radical one, is to grind down the surface to hard substance, somewhat as described below for a new floor.

Brief Specifications for Laying Granolithic Finish on a Set Concrete Base. The following specifications are quoted in substance from the paper by Mr. Thompson referred to on page 635.

1. Roughen surface of base concrete at the age of about 24 hours, so as to remove most of surface scum.

2. If surfaces have not been thus roughened, pick with a bushhammer to remove a part but not all of the surface skin.

3. Spread dilute muriatic acid about one part acid to four parts water over the surface, allow it to stand for a few minutes, then soak thoroughly with water, and wash off the surface.

4. Sweep off the excess water on the surface of the concrete and spread on a coating about $\frac{1}{8}$ in. thick of stiff neat cement paste, and broom it well into the concrete. (Do not use dry cement for this.)

5. Mix the granolithic in proportions 1 part cement to $\frac{3}{4}$ parts coarse sand, like Plum Island, to $1\frac{1}{4}$ part crushed granite or trap screened through a $\frac{3}{4}$-in. screen and caught on $\frac{3}{16}$-in. dust jacket.

An alternate plan* is to use a single aggregate consisting of fine stone retained on a No. 30 sieve, with no sand.

6. Make the consistency of granolithic rather stiff so that the mortar will just flush to the surface.

7. Have the screeds laid parallel and level so that the granolithic can be spread even with straight-edge. Run over the screeds. See that plenty of material is being pushed ahead of the straight-edge at all times so as to avoid pockets in the surface.

8. Ram granolithic with light square-faced tamper.

9. Trowel granolithic surface as soon as it begins to stiffen.

10. Trowel granolithic surface hard as soon as the proper stage has been reached. (If surface is to be ground do not give surface this final trowelling.)

11. Cover the surfaces of the granolithic about 24 hours after laying with wet burlap or similar material which will hold water. Wet material each day, and oftener if necessary, for a period of 14 days.

Bond of Granolithic to Base Concrete. One of the most important essentials in laying granolithic is to see that it is properly bonded to the concrete base. The best and in fact the cheapest plan is to lay the granolithic immediately after placing the base concrete, say within a

* L. C. Wason in Transactions American Society of Mechanical Engineers, 1914, p. 400.

half hour of placing, so that they will bond together and form a monolithic mass. This avoids special treatment of the concrete base. In many cases this is inconvenient, because of danger of injury by workmen and the possibility of sudden showers which will roughen the surface.

If the granolithic is to be laid after the concrete is hard, the base concrete after it has stiffened, but before it is thoroughly set, can be roughened with a wire brush so as to remove any scum and leave an irregular surface. In this case, however, the surface must be gone over in places before or when ready to place granolithic to remove any soft spots. A still further and positive precaution against separation of the granolithic is to go over the entire surface of the base concrete with a hammer and chisel or a pneumatic tool and cut down deep enough to get into the body of the concrete. In any case it is essential when laying the granolithic to thoroughly wet the base and spread on neat cement, as described in the brief specifications.

Preparing Base for Other Surface Materials. If hardwood finish, composition or other surfacing is to be used, the base can be brought sufficiently level by careful screeding and troweling of rough places and filling of holes. An allowance of one cent per square foot may be made in cost estimates for this treatment. For linoleum only a thin mortar surface is required.

Concrete Surface Without Granolithic. It is possible where an especially smooth surface is not required and where the wear will not be very severe to trowel the concrete of the base without laying granolithic. It is especially necessary in such cases that the sand used in the concrete be coarse, and that an excess of water be avoided in mixing.

Grinding Granolithic Surface. A method which has been followed satisfactorily in practical construction and which prevents any tendency to dust and produces a pleasing appearance, is the grinding of the surface of the granolithic, when it is a few days old—the time being usually from four to seven days—with a machine similar to that which is used in grinding terrazzo floors (see Fig. 190, p. 639). This plan was followed in the floors of the New Technology Buildings, laid in 1916. In this case the aggregates were specially selected, using for the coarser material a crushed granite which contained numerous black particles, in some cases mixed with crushed marble. The grinding removes the scum and the top film of the surface and cuts into the sand and stone grains so as to expose them and leave the surface smooth, but not shiny. If any small pinholes remain in the surface they may be filled by rubbing in neat cement paste. Care must be taken in spreading the granolithic

to see that the surface is level without excessive trowelling,—in fact the final troweling may be omitted.

CONCRETE STAIRS

The design of concrete stairs is a simple problem in reinforced concrete construction. A stairway may consist (1) of an inclined slab of reinforced concrete with the steps molded upon its upper surface, or (2) of two or, for a wide stairway, three inclined girders to form the stringers, with the stairs between them. The first method is suitable for short flights not over 8 or 10 feet in length measured on the slope, and the

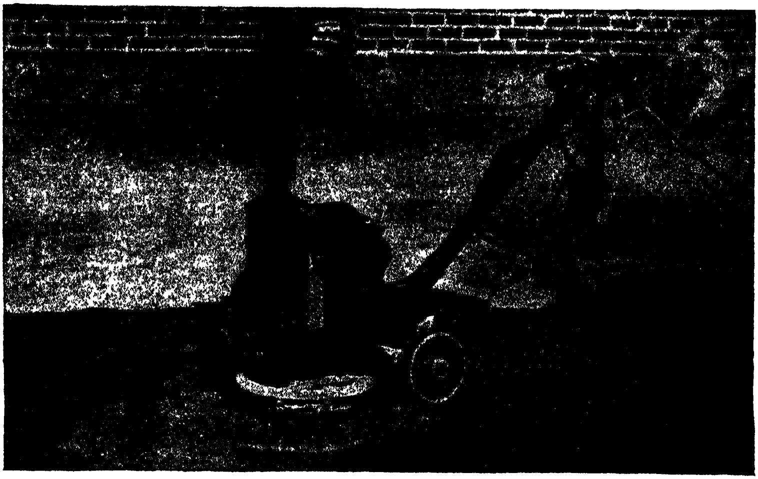


FIG. 190.—Machine Used for Grinding Granolithic Floors. *(See p. 638.)*

thickness and reinforcement are calculated as for a slab supported at the ends. (See pp. 484 to 487.) The principal reinforcement is of course in the direction of the length with occasional cross metal for stiffening. To prevent cracking provision must be made at top and bottom of the flight for negative bending moment. This necessitates steel in the upper part of the slab at these points. A slab 5 inches thick measured at the foot of the risers is suitable for a stairway half a story high.

When built with side girders, the dimensions of each of the latter may be calculated as a concrete beam with reinforcement near the lower surface. A small bar also runs across from girder to girder at the foot of

each riser so that the risers are practically reinforced beams. It is usually cheaper to construct the under side of the stairs as a slab than to build forms for each stair. The forms for the stringers may consist of planks notched for treads and risers, with boards nailed across as molds for the faces. If a fine finish is desired, the method of surfacing described for curbing may be followed. (See p. 806.)

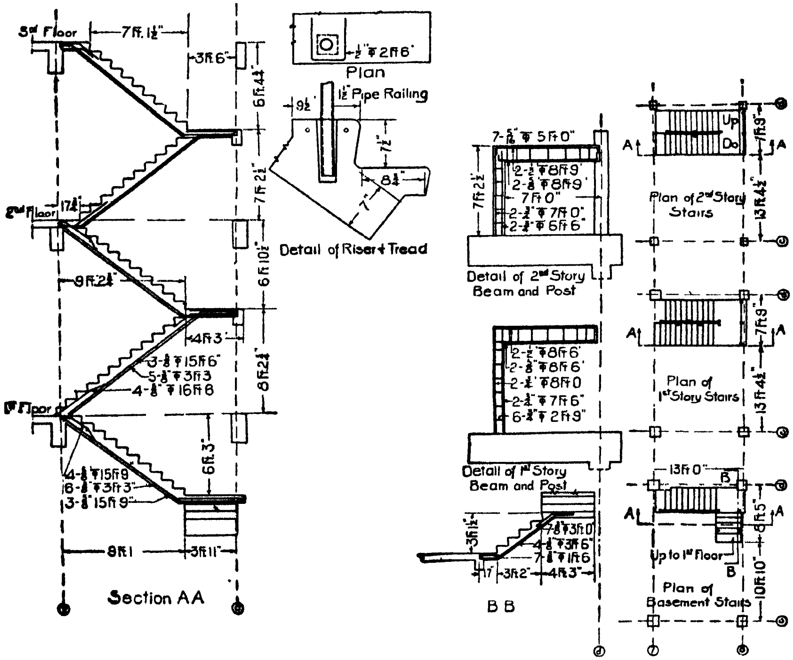


FIG. 191.—Details of Concrete Stairs Designed as Inclined Slabs. (*See p.* 639.) Massachusetts Institute of Technology, Cambridge, Mass.

Representative details of concrete stairs built by the first method (as an inclined slab) are shown in Fig. 191, page 640.

On stairs of factories, office buildings, and similar structures a live load of 70 pounds per square foot is customary. The span of a flight of stairs is the horizontal distance between supports.

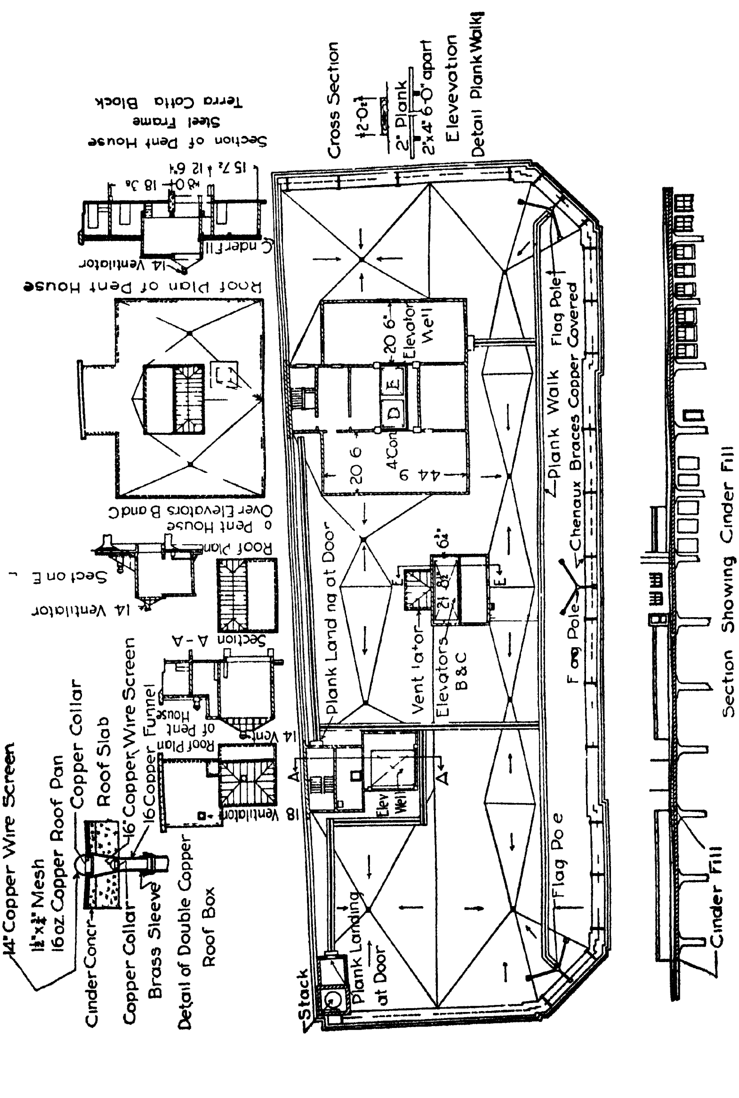


FIG. 192.—Typical Plan and Sections of Roof on a Large Building Showing Penthouses, Drainage Slopes and Double Roof. Paine Furniture Building, Boston, Mass. (*See p. 642*)

CONCRETE ROOFS

Roof design and construction is similar to that of floor slabs. As the live load is usually relatively small, cinder concrete is more suitable than for floors. The thinness is frequently governed by construction limit instead of strength.

There may be in cold weather some annoyance from condensation of vapor on the under side of concrete roofs. This can be obviated by a suspended ceiling, a double roof, or some form of insulation. Ordinary cinder covering (see Fig. 192, p. 641) also acts as a non-conductor.

The plan and sections of the roof of the Paine Furniture building are shown in Fig. 192, page 641. This is a typical layout for penthouses for elevators, sprinkler tanks, and any mechanical apparatus that may be necessary. The cinder fill is used partly to give the proper drainage pitch and partly to provide a double roof to prevent condensation on the ceiling below. A four inch conductor box was used for every 20 000 square feet of surface.

Saw Tooth Roof. In manufacturing plants, to obtain more light, a saw-tooth roof is often built. The cross-section of this type of roof is a series of triangles, similar to the teeth of a saw. The longer side is a concrete slab supported, at the top, by inclined posts between which are placed the windows. The slab and posts may be supported on longitudinal beams across the tops of the columns or the longitudinal beams may run across transverse beams supported, in turn, by the columns. By the second method no horizontal thrust is transmitted to the columns. At the junction of the post and the slab, at the peak of the roof, reinforcement must be used to take the bending moment.

Rigid Frames. Sometimes the roof girders are built monolithic with the columns and may be designed according to the formulas for the rigid frame method so as to permit longer spans and lighter framing.* The rigid frame construction is very popular in Europe, but as yet has not come into use to any great extent in America. It is well adapted to manufacturing plants and in a good many cases besides being fireproof may easily compete with structural steel trusses.

Roofs of Special Design. Reinforced concrete is admirably adapted to the construction of roofs of special design, such as domes and roof arches. In domes concrete can take all the compressive stresses and the steel the tensile stresses developed in the lower curves of the dome and in the arch ring. Roof trusses have been built but are not generally recommended.

* "Rigid Frames in Concrete Construction", by Sanford E. Thompson and Edward Smulski, *Engineering and Contracting*, January 15, 1913, p. 75.

Roof Loads. A roof load is made up of the weights of the roof itself, the roof covering, the snow load, and the wind load.

The weight of the concrete may be obtained from the tables mentioned.

Prof. Mansfield Merriman* gives the following estimates for the weight of roof covering:

Tin, 1 lb. per square foot of roof surface.
Iron, 1 to 3 lb. per square foot of roof surface.
Slate, 10 lb. per square foot of roof surface.
Tiles, 12 to 25 lb. per square foot of roof surface.

Average may be taken at 12 lb. per square foot.

The snow load varies with the slope of the roof and the locality. Prof. Merriman allows for an approximate average 15 lb. per square foot of horizontal area.

The wind load, which acts horizontally, varies with the velocity of the wind, a usual pressure being assumed as 40 lb. per square foot of vertical surface. This pressure multiplied by the sine of the angle of slope of the roof gives the pressure normal to the surface.

In practice it is common to specify a minimum value for the roof load to include the weight of the roof covering, snow, wind and any moving loads which may come upon it. A usual value for this total is 30 pounds per square foot.

CONCRETE WALLS

Concrete building walls above ground are built of single or double thickness. For cellar walls or foundations they are built solid. Interior partitions may be built of concrete also but various forms of tile or fireproof compounds are lighter and, requiring no form work, are cheaper to erect. A 6-inch wall of concrete will cost no more than a 12-inch wall of brick and will be stronger and more durable.

Cellar Walls. Cellar or basement walls adapted to withstand earth pressure may be thinner when of concrete than when built of stone, because laid as a continuous vertical slab supported at top and bottom.

For a wall of $1:2\frac{1}{2}:5$ Portland cement concrete with a spreading base imbedded in the earth, a thickness of 10 inches will withstand, without reinforcing metal, a pressure of 6 feet of earth. If the top of the wall is strengthened by a wooden sill imbedded in or dogged to the concrete, and the sill is stiffened by floor joists, the wall becomes a slab

* Merriman's "Roofs and Bridges," p. 4.

supported at its bottom by the earth and at its top by the sill. A 6-inch wall 8 feet high will thus withstand the pressure against it of 6 feet of earth. However, $\frac{3}{8}$-inch bars, spaced about 2 feet apart in both directions, will greatly stiffen so thin a wall, and prevent cracks before the concrete is thoroughly hard. If desired, a coping of concrete wider than the wall itself may be formed at the top and a $\frac{1}{2}$-inch rod placed

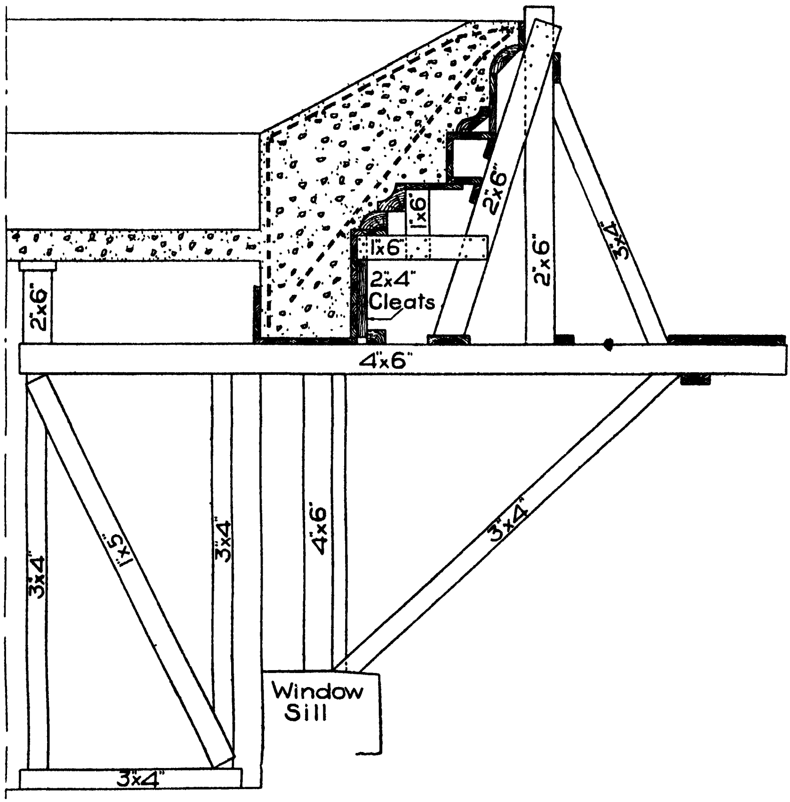


FIG. 193.—Design of Overhanging Cornice Cast in Place. (*See p.* 645.)

horizontally in its inner face. The earth must not be filled in against the back of the wall until three or four weeks after placing, unless portions of the interior forms are left in place and carefully braced. A cellar wall failed* completely by overturning, in Chicago, because allowed to stand unsupported during construction.

* Henry Blood in *Engineering News*, October 29, 1914, p. 894.

Building Walls. The double building wall is advantageous because it is more completely moisture proof. Moisture is likely to collect on the inside of a wall. especially on a north wall.

A single concrete wall 4 inches thick with its base spread to provide a footing is at least equivalent to an 8-inch brick wall, and 6 inches of concrete is at least equivalent to 12 inches of brick. It is advisable to place small reinforcing bars about $\frac{3}{8}$-inch in diameter, 12 inches to 2 feet apart in walls 6 inches thick or under, not only to increase their permanent strength, but to guard against accidents during or immediately after construction. Occasional projections or pilasters improve the appearance and add to the strength of a single wall.

Each face of a hollow wall is usually 3 to 4 inches thick, 3 or $3\frac{1}{2}$ inches being the minimum thickness at which concrete can be conveniently placed.

Cornice.—In reinforced concrete buildings the cornice may be built either entirely of reinforced concrete or the frame may be built of concrete and the facing of stone or terra cotta. To support the facing, anchors, angles, or other structural shapes, are sometimes imbedded in concrete.

Fig. 180, page 619, illustrates two of the cornices used on the New Technology and Fig. 193, page 644, a reinforced concrete cornice molded in place on a 12-story warehouse by the Aberthaw Construction Company. In the latter figure the forms also are shown.

WALLS OF MORTAR PLASTERED UPON METAL LATH

Partitions of plaster from metal lathing are used extensively for fireproof office buildings and hotels, and are also adapted, when made with Portland cement mortar, to certain classes of outside walls.

For a one-story building, timber or steel posts may be set upon concrete foundations, and the walls constructed by using $\frac{3}{4}$-inch or 1-inch channel irons for studding, to which the metal lathing is attached, and then covered (on both sides) with Portland cement mortar about 2 inches in total thickness, the studding being generally set from 12 to 16 inches on centers, depending on the height of wall. Such walls are also adapted for high buildings where steel frames are used, as the studding can be securely bolted to the steel work, and the metal lathing and cement applied in the same manner as for one-story buildings.

For curtain walls the first coat of mortar is usually mixed with one barrel of first-class Portland cement to three barrels of coarse sand, and

one cask of lime putty, or paste, into which is mixed a small quantity of long cattle hair. The second coat, which is applied before the first coat is thoroughly dry, consists of one barrel of Portland cement to three barrels of sand with about a bucketful of lime putty, without hair. The finish coat is generally mixed in the proportions of one part Portland cement to two parts sand. This finish coat may be trowelled or floated to a smooth or rough surface, as may be desired, or it may be given what is known as a "slap-dash" finish by throwing the mortar on with a brush or twig broom.

UNIT BUILDING CONSTRUCTION

Buildings and bridges of precast separately molded units have been used to advantage on structures where a large number of members of the same dimensions are required. This scheme permits of considerable saving in form lumber and labor and by using a central plant or factory much of the work can be done under more advantageous conditions than on the job itself, and better concrete results. For buildings the members are usually made on the job; for bridges, especially on railroad work, a central plant is used.

More material, especially steel, is required for this type of construction because beams and slabs are not continuous. In spite of this, however, it is an economical method under certain conditions.

FORMS FOR BUILDING CONSTRUCTION

Forms for building construction are important because they constitute so large a proportion of the total cost. Standardized designs and methods of construction are therefore essential. A few designs for columns, beams, walls, and slabs, are shown on pages 649 to 657. These and alternate designs are given, and more fully discussed than is possible here in "Concrete Costs," chapter XVI.

A designer familiar with structural layout and also practical building construction can save money by making detail sketches of all forms so as to use the minimum amount of lumber and of labor in making and placing. Tables giving the spacings of column clamps, and joists, studs, and stringers for various conditions, are given in Chapter XX of "Concrete Costs." It is important to know the order of removal of forms, this being usually column sides, joists, girder sides, beam sides, slab bottoms, girder and beam bottoms. Walls usually are built independently and the forms can be removed without disturbing the

rest. Wall and slab forms should be built in sections to prevent binding when removing. For the same reason beam forms, if removed as a unit, should be built with slightly tapering sides.

For making forms most easily the carpenter's bench* should be designed so that cleat holders may be set in place for each type of section and piece after piece of forms made up with no further measuring.

Lumber for Forms. The best lumber for forms or molds for concrete is white pine because it is easily worked and retains its shape after exposure to the weather. Except, however, where a very fine face is required, motives of economy usually prompt the use of cheaper material, such as spruce or fir, or, for very rough work, even hemlock. Green lumber is preferable to dry because it is less affected by the water in the concrete.

If the planks or boards are thoroughly oiled and are not exposed too long a time to the hot sun and dry air, which tend to warp them, they may be used over and over again. Long exposure, however, will throw the surface out of true, and open up the joints. In some instances the same lumber can be employed in different places. For example, in the construction of a one-story factory building, Mr. Thompson specified 2-inch tongued-and-grooved roof plank of green spruce for the forms, and after using at least four times, no difficulty was found in laying it on the roof. The planks were merely slightly gritty and discolored by the oil employed to prevent adhesion of cement.

Lumber which is planed on one side is essential to a smooth face, and where the forms must be removed within 24 or 48 hours it is sometimes advantageously employed for rough work because the concrete adheres less to planed lumber and that which does stick is easily scraped off, thus effecting a saving of labor which more than balances the cost of planing. Many concrete experts advise the use of beveled edge stuff in preference to tongue-and-grooved. The edges crush as the board or plank swells, and this prevents buckling.

Square corners and thin projections should be avoided when possible; beveled strips in the corners of forms will eliminate the former.

Steel vs. Lumber. The use of steel forms on building work is limited; the first cost is high, and it is difficult to adapt them to changes in dimensions of the structure. They are useful where they can be used repeatedly without changes, as is sometimes the case for slab and wall panels. For circular columns steel forms are especially satisfactory.

* See drawing of carpenter's bench in "Concrete Costs," page 487.

Removal of Forms. The time that forms have to remain in place depends upon the character of the members, weather conditions, the span, if a beam or slab, and the relation of the dead to the live load.

Vertical members, such as walls thicker than 4 inches, or columns, will bear their own weight when quite green, while horizontal members, such as floors, must harden until the concrete can sustain the dead weight and the load during construction.

The weather conditions greatly affect the setting and hardening of concrete. Heat causes it to harden quickly while cold retards the hardening and therefore prevents early removal of forms. If, through accident, the concrete should be frozen, it will not begin to harden until it has thawed and then it may require several months to attain the strength usually reached in two or three weeks.

A long span beam or slab must be supported, in general, a longer time than a short one, chiefly because of the larger dead load. If the dead load, *i.e.*, the weight of the concrete, is heavy in comparison with the live load, *i.e.*, the load which the floor must bear later on, forms must be left a longer time because the compression in the concrete is large even before the live load comes upon it.

Experienced builders have definite rules for the minimum time which the forms must be left in ordinary weather and then these times are lengthened for poor weather conditions and special members according to judgment.

As a guide to practice the following rules are suggested:*

Walls in mass work: One to 3 days or until the concrete will bear pressure of the thumb without indentation.

Thin walls: In summer, 2 days; in cold weather, 5 days.

Columns: In summer, 2 days; in cold weather, 4 days, provided the girders are shored to prevent an appreciable weight reaching the columns.

Slabs up to 7-foot spans: In summer, 6 days; in cold weather, 2 weeks.

Beam and girder sides: In summer, 6 days; in cold weather, 2 weeks.

Beam and girder bottoms and long span slabs: In summer, 10 days or 2 weeks; in cold weather, 3 weeks to 1 month. Time to vary with the conditions.

Conduits: 2 or 3 days provided there is not a heavy fill upon them.

Arches: If of small size, 1 week; large arches with heavy dead load, 1 month.

* See also paper on "Form Construction" by Sanford E. Thompson, in Bulletin No 13, Association of American Portland Cement Manufacturers, and Proceedings National Association of Cement Users, Vol. 3, p. 64, 1907.

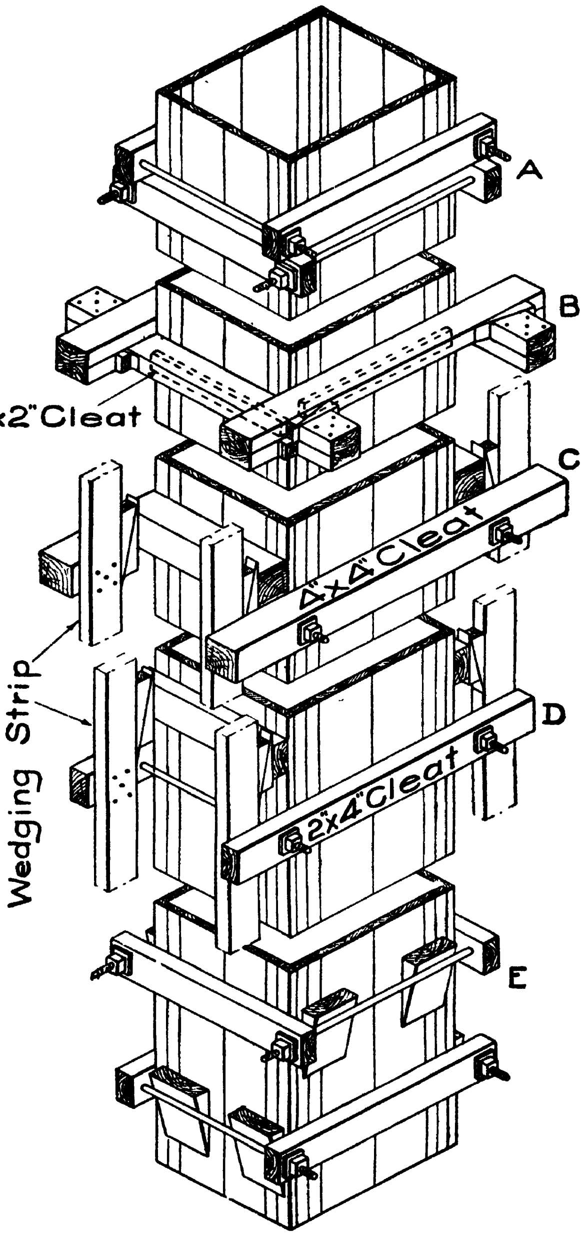


FIG. 194.—Column Form Showing Various Designs. (*See p.* 651.)

All these times are of course simply approximate, the exact time varying with the temperature and moisture of the air and the character of the construction. Even in summer, during a damp, cloudy period, wall forms sometimes cannot be removed inside of 5 days, and other members are delayed proportionally. Occasionally, too, batches of concrete will set abnormally slow, either because of slow

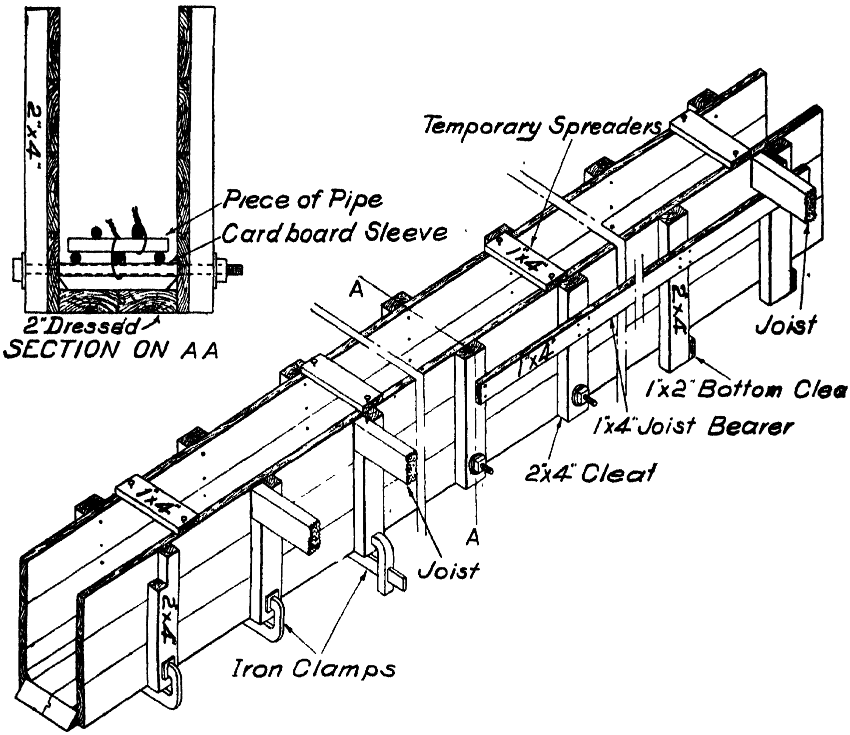


FIG. 195.—Beam Form Showing Three Methods of Construction. (*See p.* 651.)

setting cement or impurities in the sand, and the foreman and inspector must watch very carefully to see that the forms are not removed too soon. Trial with a pick may help to determine the right time.

One large builder* requires that a 20-penny spike driven into the concrete must double up before it has penetrated one inch.

A plan which is being introduced on some of the best construction work is to take a sample of concrete from the mixer once or twice a

* Mr. C. A. P. Turner.

day and allow it to set out-of-doors, under the same conditions as the construction work, until the date when the forms should be removed, then, before beginning to remove, find the actual strength of the concrete by crushing the blocks in a testing machine to see whether it is strong enough to carry the dead and the construction load.

Column Forms. Column forms for square or rectangular columns differ principally in the type of clamp. (See Fig. 194, p. 649.)

For round columns metal forms are commonly used. The most economical method of erecting wood column forms is to nail three sides together before erecting, and the fourth side afterward. If the column sizes are to be reduced in the upper stories it is convenient to use in the largest sizes, in making, narrow strips of sheathing that can be removed without splitting the boards when the column is made smaller. Every column form should be made with a clean-out opening at the lower end.

Beam and Girder Forms. The principal methods of beam construction are shown in Fig. 195, page 650.

Girder forms are similar to beam forms except for the beam openings, which are framed by inch or inch and a half stock, as is the case with beam openings in column forms. If the beams are of the same or nearly the same depth as the girder, the girder sides should be made in sections between beam openings; if the beams are shallower than the girders the forms may be made in one piece.

Beam and girder forms should be erected as a unit after assembling the sides and bottom on the floor below. Beam sides may be made of 1-inch ($\frac{7}{8}$-inch) or 2-inch ($1\frac{7}{8}$-inch) stock—the former is more economical. Beam bottoms, unless very narrow, should not be thinner than 2-inch stock. Cleats should be spaced symmetrically about the center line.

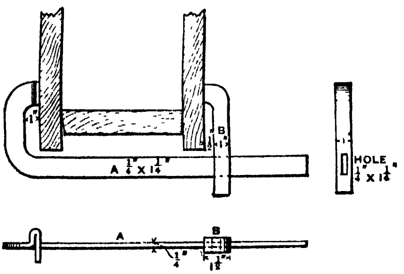


FIG. 196.—Clamp for Beam or Small Column Form. (*See p.* 651.)

A simple form of clamp for beam or small column forms, used originally in Europe, is shown in Fig. 196. The hook, A, is a plain piece of flat iron $\frac{1}{4}$ inch by $1\frac{1}{4}$ inches, with one end bent and curved as shown. The dog, B, is a square piece of iron, with the end slightly turned and a hole slightly larger than the flat iron, A, punched through it. This is tightened by hammering on its

lower end. The outward pressure of the form boards upon its upper end causes it to bind, and prevents it from slipping back. If it fails to hold, in any case, a wooden wedge is readily driven in to assist in tightening.

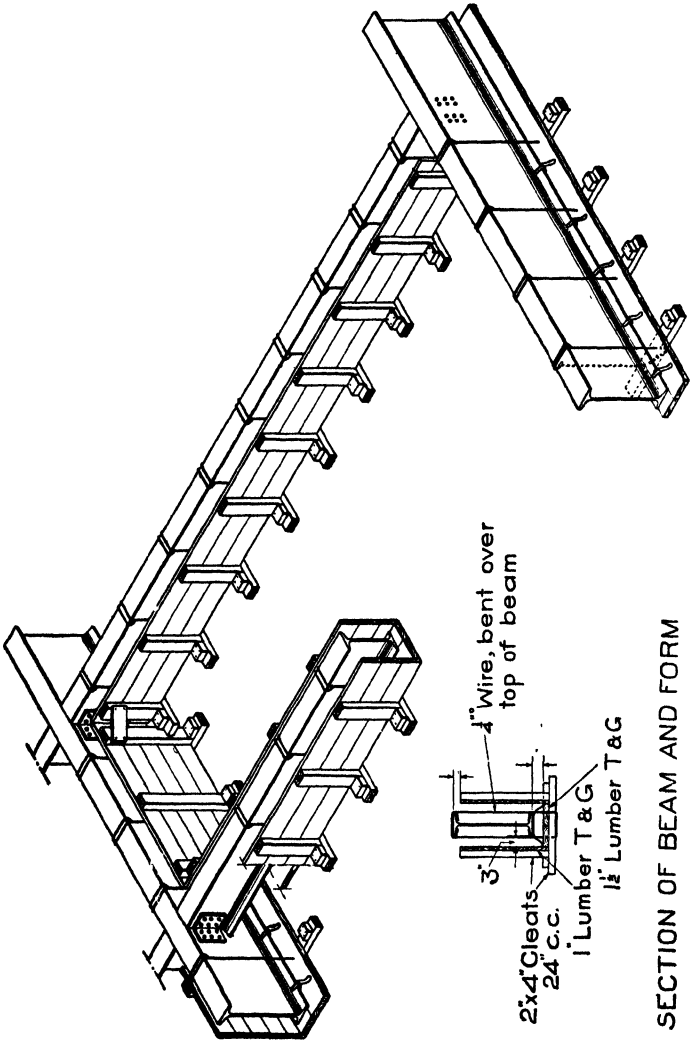


FIG. 197.—Forms for Fireproofing Steel Beams. (See p. 653.)

Beam Forms for Fireproofing Steel Beams. If the building is of steel frame construction with concrete slabs, the beams and columns

may be covered with concrete for fire protection. In such a case, the concrete should be carried around under the bottom of the flange to protect it from fire.

One type of beam form in this class of construction is shown in Fig. 197, page 652.

The section shows in detail one method of construction. Wire is passed through the bottom form around the cleat and then bent over

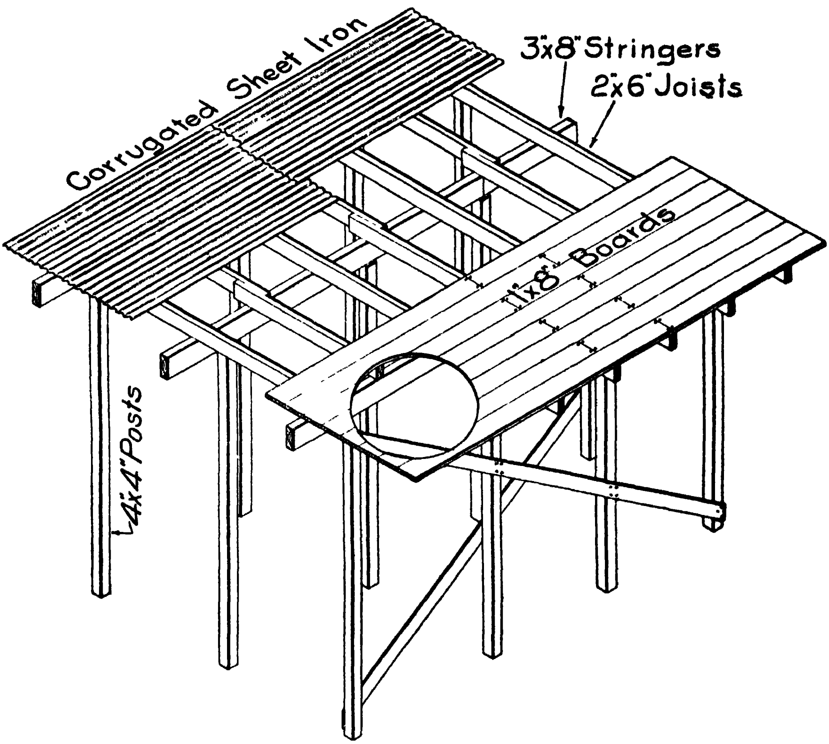
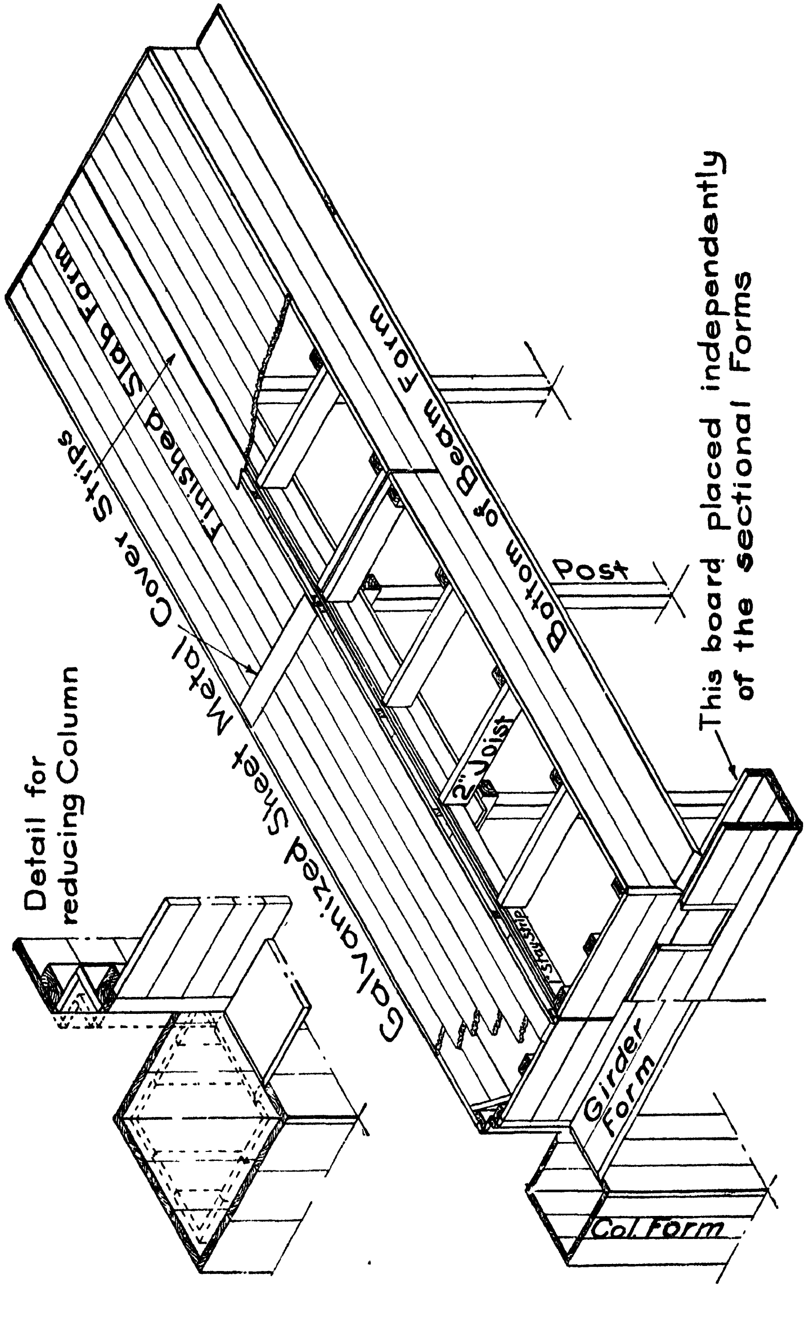


FIG. 198.—Forms for Flat Slab Construction Using Joists and Stringers. (*See p.* 655.)

the top of the I-beam to hold the form in place. To be really fire-proof, the concrete must surround not only the sides but also the bottom of the I-beam. To reinforce the strip of concrete under the lower flange of the I-beam, clips may be attached to the flange as shown in the figure. There are several patented designs of clips in the market.

Slab Forms. Slab forms for a panel should usually be made in two, three, or four sections, to facilitate removal. The sheathing may be made up into panels with thin cleats or battens, and the panels supported


Finished Slab Form
Galvanized Sheet Metal Cover Strips
Bottom of Beam Form
Post
2" Joist
Stay Strip
This board placed independently of the sectional Forms
Girder Form
Col. Form
Detail for reducing Column

by joists or joists and stringers. The latter method is most economical and is illustrated in Fig. 198, page 653.

Another type of form, designed by William O. Lichtner, where the panels, including the joists and the sides of the beam forms, are made up in advance, is shown in Fig. 199, p. 654.

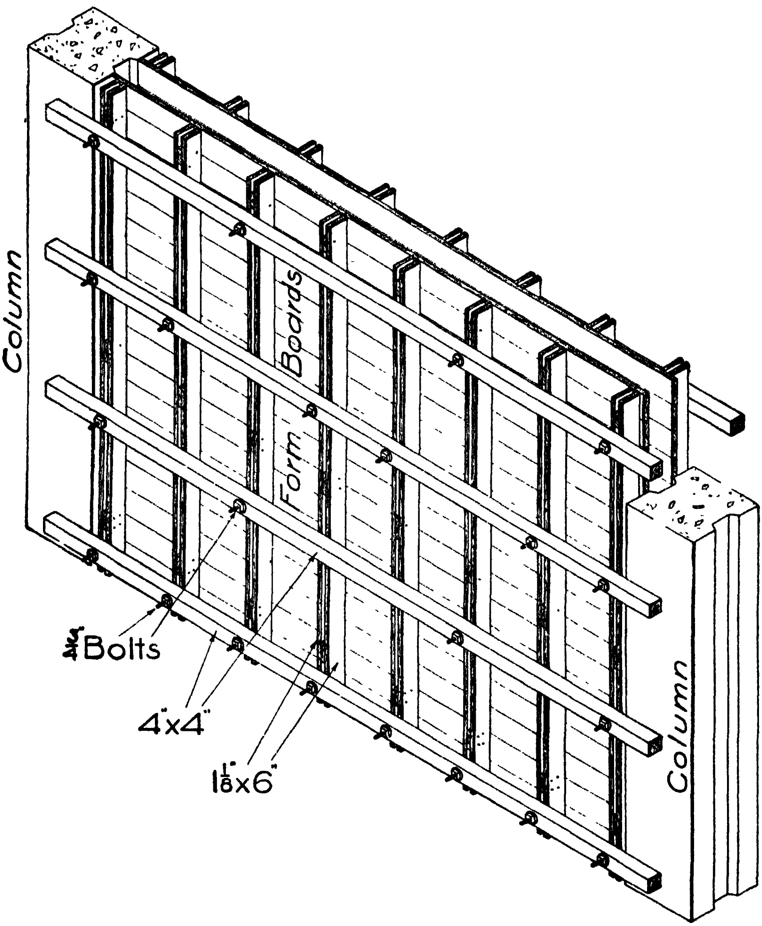


FIG. 200.—Forms for Curtain Walls Between Columns. (*See p.* 650.)

Flat slab forms must be entirely supported by posts, but in beam and girder construction the slab joists may be supported on a horizontal ledger or joist bearer nailed to the beam cleats, or these beam cleats may be notched to receive the joists.

In remaking slab forms when the beams are made narrower the joists are lengthened on alternate ends by nailing on short lengths. To avoid increasing the length or width of the sheathing, a strip of zinc may be placed over the crack. Whenever columns are reduced in size, the panels must be cut back to beyond the first cleat and patched out to fit the new size.

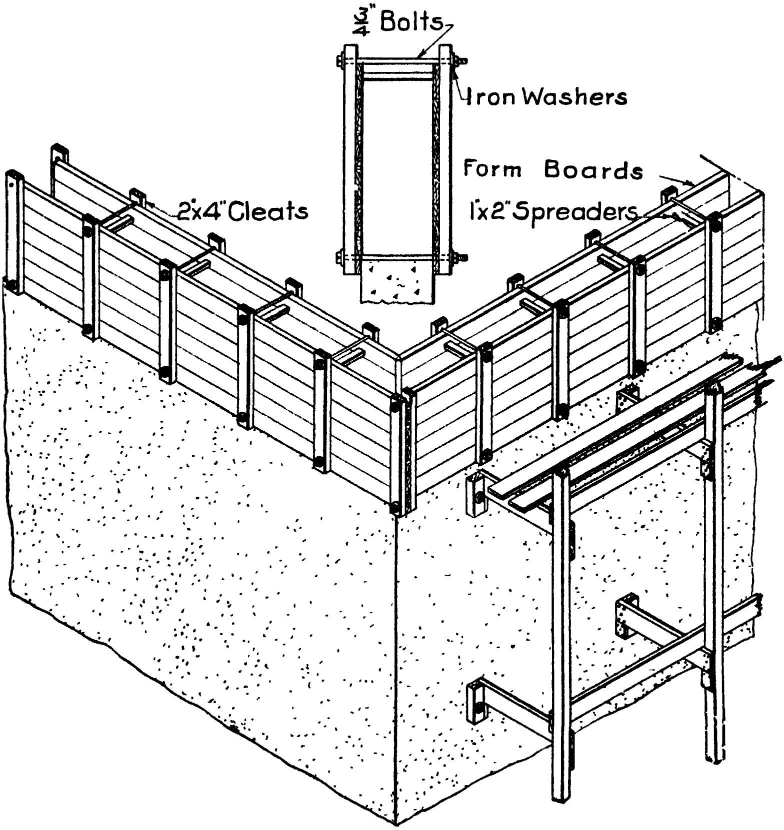


FIG. 201.—Sectional Wall Form. (*See p.* 657.)

Wall Forms. A form for a cellar wall is illustrated in Fig. 6, page 19. Occasionally the face of the excavation can be trimmed so that only one side of the form is necessary.

A form for a curtain wall is shown in Fig. 200, page 655. This type of wall is usually built after the structural part of the building is complete.

For building solid walls a sectional form is convenient. A very economical and much used type of wall form is shown in Fig. 201, page 656. This wall form is made in sections 3 feet high by 12 feet long and is bolted as shown. A form of this size is very easily handled by two carpenters. The bolt holes left in the wall can be utilized for attaching an outside scaffolding, as shown in Fig. 201, after which they can be very easily plugged up in the usual manner.

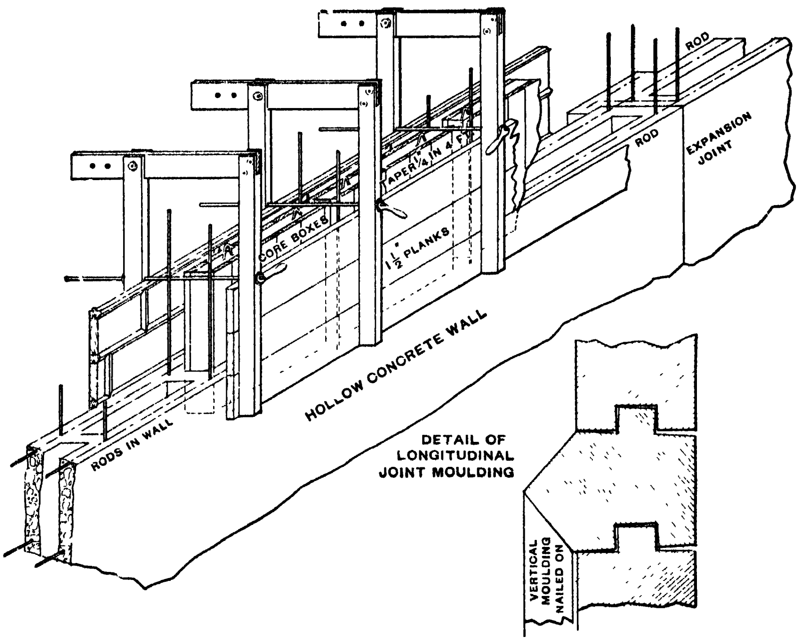


FIG. 202.—Forms for Hollow Walls. (See p. 657.)

A design for a form for a hollow wall is shown in Fig. 202. The ribs and bolts are so arranged that the latter do not pass through the concrete, the form being raised when the concrete reaches their level. In the same figure is shown a style of tongued and grooved molding with edges slightly beveled, which may be used to form the horizontal joint instead of nailing a triangular strip upon the planks. If the surface is finished as a monolith of course no moldings are required. The forms must be nearly watertight, to prevent the mortar running away from the stones.

STRENGTH OF FORMS

Forms that are sufficiently strong and rigid and at the same time economical in the use of lumber must have the dimensions and spacing of all joists, stringers, studs, and posts determined by computation. The loads to consider are: (1) on beam, girder, and slab forms, the vertical pressure of concrete (assumed—for convenience in figuring—at 144 pounds per cubic foot), and the construction load of—in average cases—75 pounds per square foot; and (2) on column and wall forms the hydraulic pressure of a liquid weighing 144 pounds per cubic foot. The head to be used in figuring the hydraulic pressure should be the depth poured in the time that the concrete takes to begin to set; in summer this is about half-an-hour.*

The allowable stress in 3 by 4-inch posts is 350 and in 4 by 4-inch, 450 pounds per square inch; in joists and stringers subject to a vertical load, 1 200 pounds per square inch; and in studs and column clamps, where the load is somewhat relieved as the concrete sets, 2 400 to 3 000 pounds per square inch. The bending moment formula is $M = \frac{1}{10}Wl$, and the deflection formula is $d = \frac{3}{384} \cdot \frac{Wl^3}{EI}$.

The stress in column clamps is governed to a certain extent by the type of clamp; the closer to the sheathing the bolts or ties are placed the shorter the span and the smaller the bending moment. In wide columns an extra bolt can be run through the middle of the column.

If weak or poorly braced forms are forced out of place by the concrete they can be realigned only at the risk of cracking and seriously injuring the green concrete.

Forms should be designed so that the pressure of the concrete forces the boards against their cleats; nails are then required only to hold the parts in place before concreting, and very few are needed.

CONSTRUCTION METHODS

Construction methods are covered fully in the preceding chapters. The most important references are to be found in Chapter XIII, Mixing; Chapter XIV, Depositing; and Chapter XVI, Laying Concrete in Freezing Weather.

It is important to keep the bottom steel above the forms in order to imbed the bars enough to develop their strength and to get the proper thickness of fireproofing. Near columns, especially in flat slabs, the location of the top steel is essential. Either concrete blocks or wire

* For other conditions see Concrete Costs, Table 127, page 610.

chairs are satisfactory for keeping steel in position. Although done in many cases, it is bad practice to place the steel directly on the forms and pry it up as the concrete is placed.

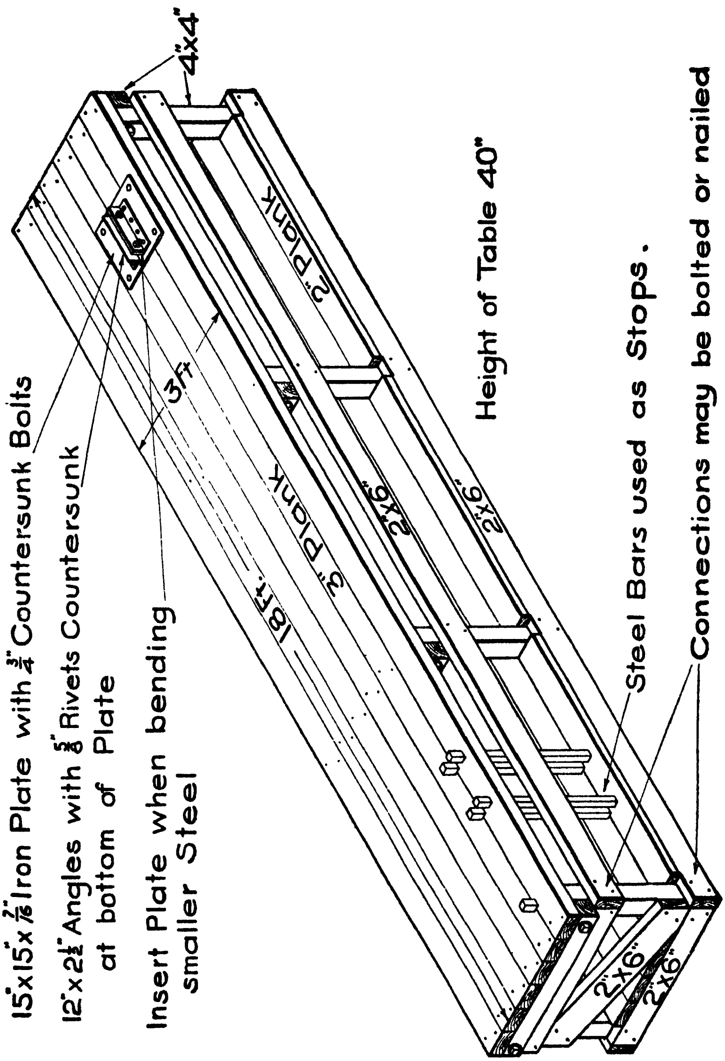


FIG. 203.—Table for Bending Reinforced Bars. (See p. 659.)

BENDING STEEL

Numerous patented machines for bending steel are on the market. A table with two devices for bending bars is shown in Fig. 203, page 659

REINFORCED CONCRETE CHIMNEYS

High factory chimneys of reinforced concrete are being built in this country and abroad. The cost, especially of those over 100 feet high, is usually much less than brick. If designed and built upon the same principles and by the same methods which have proved essential in other types of reinforced concrete construction, they can be depended upon to give permanent satisfaction.

Reports* from a large number of chimneys have shown that concrete is unaffected by the heat from an ordinary steam boiler plant. The temperature in such chimneys seldom exceeds 700° Fahr. while 400° to 500° Fahr. is more usual. Experimental tests also indicate that concrete is not appreciably injured at temperatures of 600° to 700° Fahr.†

To provide for extremes, it is advisable, however, to build an independent inner shell of concrete or firebrick for at least a portion of the height. Concrete should not be used for a chimney in connection with special high temperature furnaces.

Since concrete and steel have substantially the same coefficient of expansion‡ there is no danger of heat causing a separation of the reinforcement from the concrete.

The expansive effect of heat is a more serious question. Stresses are set up in the shell of any masonry chimney because of the hot interior and cold exterior surfaces. A concrete chimney, however, has thinner walls so that the stress is less than in one of brick or tile and it is also better reinforced. Provision for temperature stresses are discussed in paragraphs on design which follow.

Construction. A reinforced concrete chimney is more difficult to construct than many other kinds of concrete construction because of its height and shape, and it therefore should be handled by experienced builders.

It is essential in chimney construction that the materials be very carefully selected. The sand as well as the cement should be tested by determining the actual tensile strength of mortar made from it. The stone preferably should be of the nature of a hard trap rock ½ inch maximum size. Proportions 1 : 2 : 3 have been found to give good results. A dry mix should not be used, since insufficient water will produce a porous concrete which does not adhere to the steel. The consistency must be wet enough to quake and form jelly-like mass when lightly rammed, so as to properly imbed and

* A special investigation of reinforced concrete chimneys was made by Sanford E. Thompson in 1907 for the Association of American Portland Cement Manufacturers. Many of the points here discussed are summarized from the report, which is printed as Bulletin No. 18 of the Association

† Tests of Metals, U. S. A.

‡ See page 261.

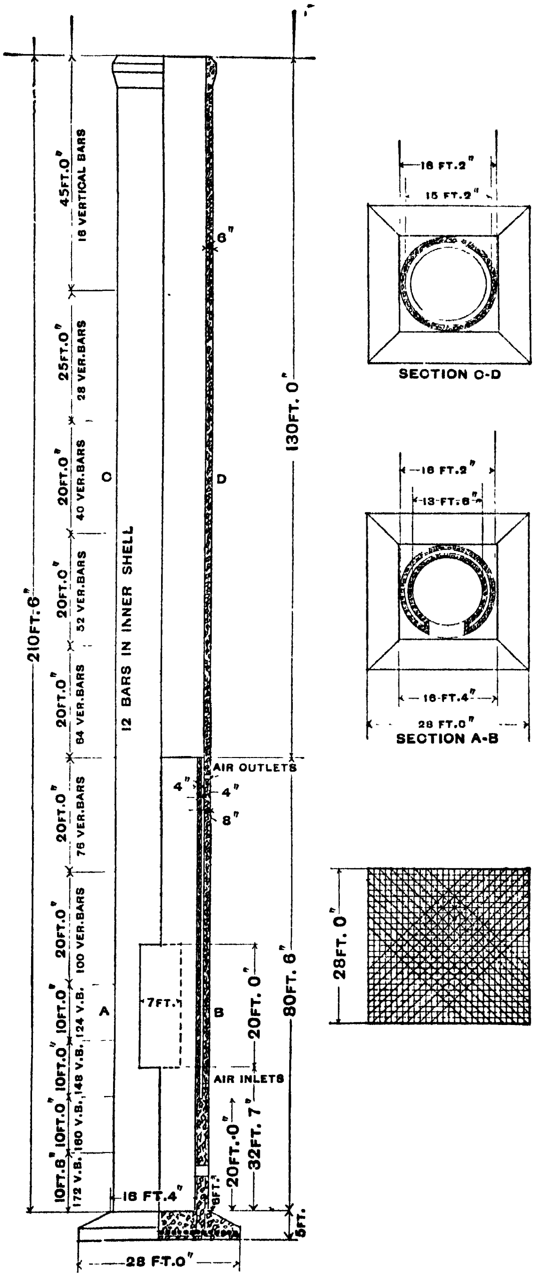


FIG. 204.—Design of Chimney of the Edison Electric Illuminating Co., Brooklyn, N. Y. (*See p.* 662.)

bond the reinforcement. No exterior plastering should be permitted because it is liable to check and scale. The steel should be good quality round or deformed bars. Bars with flat surfaces like T-bars are inferior because the flat surfaces give a poor bond and the angles make the placing of the concrete difficult. Deformed bars of small size quite closely spaced are specially good for the horizontal steel to distribute the temperature stresses and high carbon steel of first-class quality also has advantages for the horizontal reinforcement.

Design. The design of a chimney built in Brooklyn, N. Y., in 1907 is illustrated in Fig. 204.

Design of Reinforced Concrete Chimneys. A reinforced concrete chimney consists primarily of a concrete shell with vertical steel bars imbedded in it all around the chimney. The shell must be of proper thickness and the steel bars sufficient in size and number to withstand the stresses due to the weight of the chimney and to the action of the wind. A chimney of this type differs essentially from one of brick in that the diameter at the base is so small as compared to the height that it would overturn under a heavy wind were it not for the vertical bars of steel which serve as anchors and hold it on the windward side.

Wind, in blowing against a chimney, causes compression on the side opposite to the wind and tension on the side against which the wind is acting. This compression is resisted by the concrete and steel on the leeward side, while the tension or pull is taken by the steel on the windward side.

In addition to the vertical reinforcement, a reinforced concrete chimney should be provided with horizontal hoops of steel, the object of which is to stiffen the vertical steel, to distribute cracks in the concrete due to a difference in temperature between the interior and exterior and to resist the diagonal tension.

In designing a reinforced concrete chimney the problem then is primarily to determine at various horizontal sections the necessary thickness of the concrete shell and the required amount of vertical reinforcement, so that the allowable working stresses in the concrete and in the steel shall not be exceeded under the action of the forces to which the structure may be subjected. The problem is one in mechanics, involving the equilibrium of a system of forces, and, with certain reasonable assumptions, the laws of mechanics may therefore be applied to these forces, producing thereby certain rational formulas from which the necessary proportions of the chimney may be determined. The complete analysis and development of the most useful formulas are given in Chapter XX, page 390, of this treatise, the formulas themselves being reproduced below.

The problem of the determination of stresses due to the difference in temperature between the interior and the exterior of the shell involves many uncertainties. The heat tends to expand the inner surfaces, producing tension in the outside surface of the shell and compression in the interior surface. Although the distribution of the stress is not clearly known, the variation of the heat through the shell not being uniform, tentative computations indicate high stresses so that it is a question whether vertical temperature cracks can be entirely prevented any more than they can be prevented in brick or tile chimneys. The function of the horizontal steel may therefore be to distribute these cracks and to resist the vertical shear or diagonal tension. This horizontal steel should be distributed therefore by using small diameter bars closely spaced rather than large bars spaced further apart. Because of the possibility of vertical temperature cracks, the concrete should never be relied upon to carry tension or vertical shear, and the amount of horizontal reinforcement to resist this may be obtained in a similar fashion to the det^rmination of vertical stirrups in a beam. In Chapter XX, page 397, the analysis for the shearing stresses is indicated, and the final formula is presented below together with suggestions for adapting the horizontal reinforcement to temperature stresses.

The amount of vertical reinforcement, the thickness of the shell, and the percentage of horizontal reinforcement may be obtained from the following formulas, the derivation of which is given in Chapter XX, page 390.

Let

W = weight in pounds of the chimney above the section under consideration.

M = moment in inch-pounds of the wind about that section.

f_s = maximum tension in the steel in pounds per square inch.

f_c = maximum compression in the concrete in pounds per square inch (measured at the mean circumference).

$n = \frac{E_s}{E_c}$ = ratio of modulus of elasticity of steel to that of concrete.

D = mean diameter of shell in inches (i. e., diameter of center of ring).

r = mean radius of shell in inches.

t = total thickness of shell in inches.

A_s = total cross-sectional area, in square inches, of reinforcing bars in the section under consideration.

k = ratio of distance of neutral axis, from mean circumference on compression side, to the mean diameter D.

z, C_P, C_T = constants for any given value of k, Tables 1 and 2, pages 665, 666.

p_0 = ratio of area of steel hoop to area of concrete.
h_1 = height in feet of chimney above section under consideration.
F = effective wind pressure against chimney in pounds per square foot.

Then

$$A_s = \frac{8\ (M - W\ z\ D)}{C_T f_s D} \tag{1}$$

$$t = \frac{2\ W + (C_T f_s - C_P f_c n)\ \frac{A_s}{\pi}}{C_P f_c D} + \frac{A_s}{\pi D} \tag{2}$$

$$p_0 = \frac{h_1\ F}{18.8\ f_s\ t} + 0.0025 \tag{3}$$

Formulas (1), (2), *and* (3) *correspond to formulas* (96), (97), *and* (98) *in Chapter XX.*

In the formula for p_0, the first term gives the ratio of steel to resist vertical shear or diagonal tension, and the second term is an arbitrary ratio designed to distribute the temperature strains. To best distribute the temperature strains, a maximum spacing of the horizontal bars is recommended as 6 inches to 10 inches.

In the formulas the terms z, C_P and C_T are constants, the values of which are fixed for any given position of the neutral axis. By means of tables 1 and 2 (pp. 665-6) these constants may be easily and quickly determined so that the solution of formulas (1) and (2) is rendered quite simple after the selecting of the diameter and height of the chimney and computing the bending moments due to the wind at the various sections considered. The thickness of shell must be assumed in formula (1) in order to determine the average diameter D and to compute the weight W. A new computation may be made to correct this if necessary. For economical distribution of concrete and steel, computation must be made for se-eral sections in the height. It is advisable to make the thickness of exterior shell never less than 5 inches but tne number of steel rods may be gradually reduced toward the top.

Summary of Essentials in Design and Construction. In the investigation* referred to, the essential requirements are summarized as follows:

(1) Design the foundations according to the best engineering practice.

(2) Compute the dimensions and reinforcement in the chimney with conservative units of stress, providing a factor of safety in the concrete of not less than 4 or 5.

* See footnote, p. 660.

(3) Provide enough vertical steel to take all of the pull without exceeding 14,000 or at most 16,000 pounds per square inch.

(4) Provide enough horizontal or circular steel to take all the vertical shear and to resist the tendency to expansion due to the interior heat.

(5) Distribute the horizontal steel by numerous small rods in preference to larger rods spaced farther apart.

(6) Specially reinforce sections where the thickness in the wall of the chimney is changed or which are liable to marked changes of temperature.

(7) Select first-class materials and thoroughly test them before and during the progress of the work.

(8) Mix the concrete thoroughly and provide enough water to produce a quaking concrete.

(9) Bond the layers of concrete together.

(10) Accurately place the steel.

(11) Place the concrete around the steel carefully, ramming it so thoroughly that it will slush against the steel and adhere at every point.

(12) Keep the forms rigid.

The fulfillment of these requirements will increase the cost of the structure, but if the recommendations are followed, there should be no difficulty in erecting concrete chimneys which will give thorough satisfaction and will endure.

Table 1. Values of Constants C_P, C_T, z and j for Different Positions of the Neutral Axis, (i. e., for various values of k)

For use with equations (1), (2) and (3), page 664, and (96), (97) and (98), pages 396 to 399. k is ratio of distance of neutral axis from mean circumference on compression side to the mean diameter D. Value of k to suit the condition of the problem is obtained from Table 2, page 666.

k	C_P	C_T	z	j
0.050	0.600	3.008	0.490	0.760
0.100	0.852	2.887	0.480	0.766
0.150	1.049	2.772	0.469	0.771
0.200	1.218	2.661	0.459	0.776
0.250	1.370	2.551	0.448	0.779
0.300	1.510	2.442	0.438	0.781
0.350	1.640	2.333	0.427	0.783
0.400	1.765	2.224	0.416	0.784
0.450	1.884	2.113	0.404	0.785
0.500	2.000	2.000	0.393	0.786
0.550	2.113	1.884	0.381	0.785
0.600	2.224	1.765	0.369	0.784

Table 2. Location of Neutral Axis for various combinations of compressive stress, f_c, tensile stress, f_s and ratio of moduli, n, *(see p. 663.)*

MAXIMUM TENSILE STRESS IN STEEL f_s	k RATIO OF DEPTH OF NEUTRAL AXIS TO DEPTH OF STEEL BELOW MOST COMPRESSED SURFACE OF BEAM														
	$n = 10$ Maximum compressive stress in concrete, f_c					$n = 12$ Maximum compressive stress in concrete, f_c					$n = 15$ Maximum compressive stress in concrete, f_c				
	300	400	500	600	700	300	400	500	600	700	300	400	500	600	700
8000	.272	.334	.384	.428	.466	.310	.375	.428	.474	.512	.360	.428	.484	.530	.568
9000	.250	.308	.357	.400	.438	.285	.348	.400	.444	.483	.334	.400	.454	.500	.538
10000	.231	.286	.334	.375	.412	.264	.324	.375	.418	.456	.310	.375	.428	.474	.512
11000	.214	.266	.312	.353	.389	.246	.304	.353	.395	.433	.290	.353	.405	.450	.488
12000	.200	.250	.294	.334	.368	.231	.285	.334	.375	.412	.272	.334	.384	.428	.466
13000	.188	.236	.278	.316	.350	.217	.270	.316	.356	.392	.257	.316	.366	.409	.447
14000	.176	.222	.263	.300	.334	.204	.255	.300	.340	.375	.243	.300	.349	.391	.428
15000	.166	.210	.250	.285	.318	.198	.242	.286	.324	.360	.231	.286	.334	.375	.412
16000	.158	.200	.238	.272	.304	.184	.231	.272	.310	.344	.220	.272	.319	.360	.396
17000	.150	.190	.228	.261	.291	.175	.220	.261	.298	.330	.210	.261	.306	.346	.382
18000	.143	.182	.218	.250	.280	.166	.210	.250	.285	.318	.200	.250	.294	.334	.368
19000	.136	.174	.208	.240	.270	.160	.201	.240	.275	.306	.192	.240	.283	.322	.356
20000	.130	.166	.200	.231	.260	.152	.194	.231	.264	.296	.184	.231	.272	.310	.344

In connection with reinforced concrete chimneys, the problems which arise are of two general kinds:

(1) A problem in design, involving the determination of the necessary thickness of shell and required amount of reinforcement at the various sections of a chimney of given height and diameter.

(2) A problem in the review or investigation of a chimney of given height and diameter having a certain thickness of shell and a given amount of reinforcement to determine the stresses in the concrete and the steel under the action of certain forces.

The application of the foregoing formulas to such problems and the use of the accompanying tables may best be illustrated by the following numerical examples, although the designer is advised also to refer to Chapter XX, pp. 390–399 for a thorough understanding of the subject.

DESIGN OF A CHIMNEY. *Example 1.* Given a chimney with height above section considered, 110 ft.; mean diameter at section considered, 10 ft.; allowable pressure in concrete (f_c), 500 lb. per sq. in.; allowable tension in steel (f_s), 14 000 lb. per sq. in.; ratio of moduli n, 15; wind pressure (on normal plane) 50 lb., per sq. ft., weight of concrete taken as 150 lb. per cu. ft. What is the necessary thickness of shell and amount of reinforcement at the given section?

Solution. As in all chimney designs, it is necessary here to make a trial assumption of the thickness of shell in order to estimate the weight. Suppose

we assume a 6-inch shell for the entire height above the section. Assuming that a wind pressure of 50 lbs. per square foot on a normal plane corresponds to $\frac{6}{10}$ of 50 pounds or 30 pounds per square foot on the projected diameter of a cylindrical surface we have the bending moment due to the wind,

$$M = [10.5 \times 110 \times 30] \times \frac{110}{2} \times 12 = 22\ 869\ 000 \text{ in. lb.}$$

and the total weight of the chimney above the section,

$$W = 3.1416 \times 10 \times 0.5 \times 110 \times 150 = 259\ 180 \text{ lb.}$$

For $f_c = 500$, $f_s = 14\ 000$, and $n = 15$, table 1 gives $k = .349$

For $k = .349$ table 2 gives $C_P = 1.637$, $C_T = 2.335$, $z = .427$

Substituting in equation (1),

$$A_s = \frac{8\ (22\ 869\ 000 - 259\ 180 \times .427 \times 120)}{2.335 \times 14\ 000 \times 120} = 19.6$$

Therefore 19.6 square inches of steel are required.

If $\frac{3}{4}$ inch round rods are selected, 45 of them would be required.

Substituting in equation (2), we have

$$t = \frac{2 \times 259180 + [(2.335 \times 14\ 000) - (1.637 \times 500 \times 15)]\ \frac{19.6}{3.1416}}{1.637 \times 500 \times 120}$$

$$+ \frac{19.6}{3.1416 \times 120} = 6.6 \text{ inches}$$

Therefore a 6 6 inch shell would be used.

In general the values of A_s and t as thus obtained should be readjusted by computing W on the basis of the computed thickness of shell. In the case at hand, however, the original assumption of a 6-inch thickness corresponds, for all practical purposes, with the computed thickness of 6.6 inches, so that recomputation is, in this case, unnecessary. If the walls of the chimney taper in thickness the value of W must be altered accordingly.*

Having determined the required thickness of shell and amount of vertical reinforcement there remains the question of the necessary horizontal or circular reinforcement. Substituting in formula (3) for f_s say 14000 lb., we have

$$p_0 = \frac{110 \times 30}{18.8 \times 14000 \times 6.6} \times 0.0025 = 0.0044$$

Area of steel, $A_s = 6\ 6 \times 12 \times 0\ 0044 = 0.35$ sq. in. Thus $\frac{1}{2}$ inch round rods should be spaced $6\frac{3}{4}$ inches on centers.

In a similar manner any other section of the chimney may be proportioned.

Review of a Chimney. *Example 2.* Given a chimney with height above section considered, 90 ft; mean diameter at section considered, 8 ft.; thickness of shell at section considered, 6 in.; vertical steel at section condsidered, 60 — $\frac{5}{8}$ in. round rods; wind pressure (on normal plane, 50 lb. per sq. ft.); weight of concrete taken as 150 lb. per sq. ft.; ratio of moduli, n, 15.

What are the maximum stresses in the concrete and in the vertical steel at the section under consideration?

* In relatively high chimneys steel cannot be stressed to 14,000 lbs. per sq. in. (see p. 399).

Solution. A problem of this kind must necessarily be solved by a method of successive trials, since the position of the neutral axis is not known. The location of the neutral axis is determined by the values of f_c, f_s and n, two of which, in this case, are unknown. The method of procedure, therefore, is to assume outright a trial position of the neutral axis, select the constants accordingly, substitute in equations (1) and (2) and solve them for f_s and f_c.

Then see if the position of the neutral axis, as fixed by these values of f_s and f_c and the given n, is the same as the position assumed at the start. If the two positions agree, then f_s and f_c as found are the actual stresses; if not, a new position of the neutral axis must be assumed, new constants selected, and new values of f_s and f_c computed from equations (1) and (2). Thus a series of trials must be made until the location of the neutral axis as assumed is consistent with the computed values of f_c and f_s together with the given n.

In this problem, assuming 30 pounds pressure on the projected area, we have the bending moment due to the wind,

$$M = [8.5 \times 90 \times 30] \times \frac{90}{2} \times 12 = 12\ 393\ 000 \text{ in. lb.}$$

and the total weight of the chimney above the section,

$$W = 3.1416 \times 8 \times 0.5 \times 90 \times 150 = 169\ 646 \text{ lb.}$$

$$A_s = 60 \times .3068 = 18.41 \text{ sq. in.}$$

Now suppose we assume the neutral axis at, say, $k = .400$

For $k = .400$, table 1 gives $C_P = 1.765$, $C_T = 2.224$, $z = .416$

Substituting in equation (1) we have

$$18.41 = \frac{8\ (12\ 393\ 000 - 169\ 646 \times .416 \times 96)}{2.224 \times f_s \times 96}$$

whence $f_s = 11400$

Substituting in equation (2) we have

$$6 = \frac{2 \times 169646 + (2.224 \times 11400 - 1.765 . f_c\ 15)\ \dfrac{18.41}{3.1416}}{1.765 \times f_c \times 96} + \frac{18.41}{3.1416 \times 96}$$

whence $f_c = 416$

Now $f_s = 11\ 400$, $f_c = 416$, and $r = 15$ gives $k = .354$ which does not correspond with our original assumption of $z = .400$. Evidently the true k must lie somewhere between the assumed and determined values, hence if we now assume, say, $k = .375$ and recompute, we obtain $f_s = 11\ 000$ and $f_c = 435$, the values of which together with $n = 15$ gives $k = .371$ which checks fairly well with the assumption of $k = .375$. For all practical purposes we may therefore say that the maximum stress in the steel is 11 000 pounds per square inch, while the maximum stress in the concrete is 435 pounds per square inch. The results indicate that both the thickness of shell and the amount of steel are greater than are necessary for safe stresses.

CHAPTER XXIV

FOUNDATIONS AND PIERS

Concrete excels as a material for foundations, and here finds a wide and important field of usefulness. It is pre-eminently adapted to such construction, because the stresses are chiefly compressive, the forms are easily built, and the surface appearance need not be considered.

Since the design of a foundation or sub-structure is governed almost as much by the character of the underlying rock or soil as by the superstructure, brief reference is made to the standard practice in estimating loads, although the treatment of engineering principles, as such, is not within the province of this treatise.

Reinforced concrete footings are treated in detail (see p. 673).

BEARING POWER OF SOILS AND ROCK

Sound hard ledge will support the weight of any foundation and superstructure, but if the rock is seamy or rotten it may require thorough examination and special treatment. If its surface is weathered, it must be removed. A sloping surface must be stepped or the foundation designed with sufficient toe to prevent sliding.

The sustaining power of earths depends upon their composition, the amount of water which they contain or are likely to receive, and the degree to which they are confined.

Mr. Joseph R. Worcester* suggests the following unit loads on soil in and around Boston based on an examination of over 1 000 borings and experience with the behavior of heavy structures actually built.

Dry, hard, yellow clay, "boulder clay," dry sand or gravel, 6 tons per sq. ft.

Compact, damp sand, hard sandy clay, hard blue clay, 5 tons per sq. ft.

Medium blue clay, whether or not mixed with fine sand, $3\frac{1}{2}$ tons per sq. ft.

Soft clay, running sand (confined), $2\frac{1}{2}$ tons per sq. ft.

These pressures may be considered as guides for general use, although the variation in materials and in local conditions is so great that each problem should be individually investigated.

* Journal Boston Society of Civil Engineers, Vol. I. January 1914, p. 19.

For estimating the safe load on piles driven to firm strata, such as rock or hard pan, the loading which a pile will stand is determined by the crushing strength of the timber. If supported wholly or in part by friction, it is customary to calculate the safe loading by a formula* based upon factors obtained by experiment, or by one based upon the penetration of the pile from the blow of the pile driver. The *Engineering News* formula is commonly used:

Let

P = safe load in tons upon a pile.
W = weight of hammer in tons.
h = height of fall in feet.
p = penetration in inches under last blow.

Then

$$P = \frac{2Wh}{p+1}$$

Mr. Worcester* suggests the following modification of the *Engineering News* formula for local practice around Boston.

$$P = \frac{3Wh}{p+1}. \quad (\textit{Eng. News} \text{ formula} + 50 \text{ per cent.})$$

Mr. Worcester states† with reference to spacing piles:

The minimum distance between centers of piles depends upon two factors: the hardness of the soil and the size of the butts. Ordinary spruce piles may be well driven 24 inches on centers, while large and long piles can not be driven to advantage closer than 30 inches. Another governing condition must be taken into account, however, and that is the supporting power of the soil as a whole. Where the piles reach a real hard pan, the soil will generally resist all the pressure that the piles can bring on it, unless it consists of a thin crust overlying a soft material; but when the soil is so soft that the piles hold by friction only, and there is enough friction to carry all the soil between the piles down with them, in case they go together, the spacing becomes a question of how much the underlying soil will support per square foot. For example, if the soil can only support 2 tons per square foot, and the piles could each carry 18 tons, it is useless to place them closer than 3 feet on centers.

CONCRETE CAPPING FOR PILES

While formerly stone capping for piles was advocated, it is now generally accepted practice in plain and reinforced concrete foundations to

* Journal Boston Society of Civil Engineers, Vol. 1, January 1914, p. 19.
† Journal Association Engineering Societies, June 1903, p. 289.

lay the concrete directly upon the head of the piles which have been cut to the required grade. The heads of the piles are usually imbedded in the concrete to a depth of 6 inches. Sometimes the ground may be excavated to a depth of 1 or 2 feet around the piles and a layer of broken stone, or chips, spread and rammed hard upon it before laying the concrete so that the supporting power of the soil between the piles may be utilized. Generally, however, it is not safe to rely at all on the soil.

The thickness of the concrete above the piles must be sufficient to prevent the head of the pile shearing through the concrete. In a well-designed footing, however, the thickness required for strength is sufficient to resist the punching shear of the piles.

GENERAL RULES OF DESIGN

In designing foundations, two requirements must be borne in mind: (1) that the settlement of the structure be as small as possible; and (2) that settlement, if any, be uniform throughout the structure. This last requirement is specially important in a reinforced concrete structure, because on account of the rigidity of the construction, uneven settlement causes secondary stresses in the columns, beams, and slabs, which may exceed the stresses produced by the loading, and in extreme cases may even cause failure.

The first requirement will be satisfied by selection of a proper unit pressure on the soil. To satisfy the requirement for uniform settlement, it is necessary to design the footings so that the pressure in all parts of the structure is uniform. The size of the footing, therefore, must be varied with the superimposed load. For footings carrying more than one column, the center of gravity of the loads from the columns should coincide with the center of gravity of the upward reactions, which for footings resting directly on the soil, coincides with the center of gravity of the footing. For pile foundations, the center of gravity of the upward reaction coincides with the center of gravity of the piles.

In proportioning footings, the effect of the dead load upon settlement is much larger than of the live load because in most structures the full live load may not be imposed upon all floors at the same time, while the dead load is always there. A suggestion for reduction in the live load is given on page 618. Mr. Schneider, in his specification* for structural design of buildings, specifies:

* Transactions of the American Society of Civil Engineers, Vol. LIV, June, 1905, p. 492.

The live loads on foundations shall be assumed to be the same as for the footings of columns. The areas of the bases of the foundations shall be proportioned for the dead load only. The foundation which receives the largest ratio of live to dead load shall be selected and proportioned for the combined dead and live loads. The dead load on this foundation shall be divided by the area thus formed and this reduced pressure per square foot shall be the permissible working pressure to be used for the dead load on all foundations.

Frequently the building line nearly coincides with the property line and the foundation must be placed entirely inside the building. In such cases, to prevent eccentric pressure on the foundation, either cantilever construction may be used for transmitting the exterior column loads centrally to the footings, or a combined footing design, as explained on page 678.

In structures such as chimneys or narrow buildings which are subject to wind pressure, the foundation should be designed with due consideration of the eccentricity caused by the wind.

Safe Bearing on Concrete. Bases and bearing plates for steel columns must be made of sufficient area to transmit the pressure to the concrete foundation without exceeding the unit working stress in bearing, as specified on page 573.

Anchoring. Columns subject to uplift due to lateral forces, as in trestles, must be securely anchored to the foundation. The anchors must be made strong enough to resist the uplift, and imbedded deep enough in the concrete so that the weight carried by them will be enough to counteract the uplift.

PLAIN CONCRETE FOOTINGS

The area of the base of the footings is determined by dividing the superimposed load by the allowable unit pressure on the soil. The area of the top is governed by the allowable bearing stress on the concrete. If the difference between the area of the base and of the top is large, the footing may be stepped or battered. The depth will depend upon the allowable ratio of the length of the projection to the height of the block. The projection should be figured as a cantilever loaded by the reaction of the soil assuming the critical section at the face of the superimposed step. The ratio of length of the projection to its height is therefore governed by the allowable tensile strength of the concrete and the magnitude of the upward pressure. The tensile stress in the concrete must not exceed the allowable value. (See p. 332.)

REINFORCED CONCRETE FOOTINGS

To distribute the column load over a large area of the ground without carrying the foundation in successive steps to a considerable depth and using a large mass of concrete, the foundation may be built of reinforced concrete. This in almost all cases permits a great reduction in the cost of the foundation. Reinforced concrete footings utilize the compressive strength of the concrete and therefore are more economical than the I-beam type of design formerly used.*

Reinforced concrete footings may be divided into three groups: (1) Wall footings; (2) Independent column footings of rectangular or square shape; (3) Combined footings carrying more than one column.

Wall Footings. A wall footing, as a rule, consists of a slab projecting the required distance on both sides of the wall as cantilevers. In figuring bending moments, each portion should be considered as a cantilever with the critical section at the face of the wall. The reinforcement, determined from the bending moment in the usual fashion (see p. 510), consists of bars placed at the bottom of the footing at right angles to the wall.

Special attention must be paid to bond stresses. The depth of the footing and the diameter of the bars must be arranged in such a way that the unit bond stress, based on the total external shear and determined by formulas given on page 534, does not exceed the allowable unit stress. It is of great advantage to use bars with small diameters. The use of deformed bars may also prove economical.

Diagonal tension also must be considered (see p. 516). As a basis for figuring the diagonal tension, the shear is taken, figured at a distance from the wall face equal to the effective depth of the footing. It is preferable to design the footings of such dimensions as to avoid the use of diagonal tension reinforcement.

In stepped footings, the steps must be made of such depth that at no point of the footing shall either the bond or the diagonal tension exceed the allowable working unit stress.

Independent Column Footings. A column footing generally consists of a square or rectangular slab reinforced with bars placed at the bottom of the footing and running in two or sometimes in four directions. In such slabs the moments and stresses act in radial and circumferential directions similarly as in flat slabs at the column head.

Bending Moments. Referring to Fig. 205, page 676, the bending

* See Second Edition of Concrete Plain and Reinforced, page 643.

moments at the critical section taken at the face of the column *1–2* may be determined by considering the footing as detached along the diagonal lines *1–3* and *2–6* formed by connecting the corners of the column with the corners of the footing. The load on each trapezoid thus obtained produces a bending moment about the face of the column which can be determined by multiplying the load on the rectangle *1254* directly in front of the column face by the length of half the projection of the footing from the column, and the load on the remaining triangles *134* and *256* by a moment arm equal to two-thirds of that projection. Dead load of footing does not need to be considered in figuring bending moment and shear. The bending moment may be expressed by the following formula.

Let, for square footings,

a = length of side.
c = diameter of column.
P = total column load.
C_F = constant.

Then, for square footing,

$$M = \frac{1}{24}\left(1 - \frac{c}{a}\right)^2\left(2 + \frac{c}{a}\right)aP \tag{1}$$

or

$$M = C_F a P \tag{2}$$

in which C_F equals $\frac{1}{24}\left(1 - \frac{c}{a}\right)^2\left(2 + \frac{c}{a}\right)$.

For rectangular footing, let

a and b = length of sides (see Fig. 205).
C_{F_1} = constant.

Then moment in a direction (at 1–2 Fig. 205).

$$M = C_{F_1} a P \tag{3}$$

where $C_{F_1} = \frac{1}{24}\left(1 - \frac{c}{a}\right)^2\left(2 + \frac{c}{b}\right)$.

The moment is in the same units as a, b, and P.

The value of C_F and C_{F_1} may be taken from the table below.

Constants C_F for Square Footings

To be used in formula for bending moment, $M = C_F aP$ (*See p.* 674).

$\frac{c}{a}$	Ratio of Diameter of Column to Side of Footing, $\frac{c}{a}$.								
	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
C_F	0.071	0.065	0.059	0.053	0.047	0.041	0.036	0.031	0.026

Constants C_{F_1} for Rectangular Footings

To be used in formula for bending moment, $M = C_{F_1} aP$. (*See p.* 674.)

Ratio of Diameter of Column to One Side of Footing $\frac{c}{a}$	Ratio of Diameter of Column to the Other Side of Footing, $\frac{c}{b}$								
	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
0.10	0.071	0.073	0.074	0.076	0.078	0.079	0.081	0.083	0.085
0.15	0.063	0.065	0.066	0.068	0.069	0.071	0.072	0.074	0.075
0.20	0.056	0.057	0.059	0.060	0.061	0.063	0.064	0.065	0.067
0.25	0.049	0.050	0.052	0.053	0.054	0.055	0.056	0.057	0.059
0.30	0.043	0.044	0.045	0.046	0.047	0.048	0.049	0.050	0.051
0.35	0.037	0.038	0.039	0.040	0.041	0.041	0.042	0.043	0.044
0.40	0.032	0.032	0.033	0.034	0.035	0.035	0.036	0.037	0.038
0.45	0.027	0.027	0.028	0.028	0.029	0.030	0.030	0.031	0.032
0.50	0.022	0.022	0.023	0.023	0.024	0.024	0.025	0.026	0.026

Effective Reinforcement and Effective Width. In determining the resisting moment, which must be equal to the bending moment, the steel considered as effective is that placed within a width consisting of the width of the column plus twice the thickness of the footing plus half of the remaining distance to the edge of the footing on each side. (See Fig. 205.) Additional steel should be placed outside the effective width at a spacing twice the spacing of the effective reinforcement.

Minimum Depth of Footing. The minimum depth of footing as determined by the unit punching shear is obtained by dividing the total shear at the edge of the column by the circumference of the column times the allowable unit punching shear. If the area of column is small in comparison with the area of the footing, the shear may be taken as equal to the column load.

Bond Stresses. In designing footings, the most important and often the determining feature is the bond stress. The depth and the diameter

of the bars must be selected so that the bond stress does not exceed the allowable working unit stress (see p. 573). In figuring bond, the same loads and the same steel bars should be taken as were used in determining the bending moment, and the moment of resistance of the footing. Formula (36), page 534, should be used in figuring the unit bond stress. Besides this, the length of the bar beyond any point must be large enough to develop the tensile stress in bar by bond. Thus at a point where the stress in a bar is 16 000 lb., not only the unit bond, u, must not exceed the working stress, but also the length of the bar beyond the point under consideration must be equal to the required number of diameters unless the bar is anchored at the end. Formula (39), page 539, should be used in figuring the length of bar to prevent slipping.

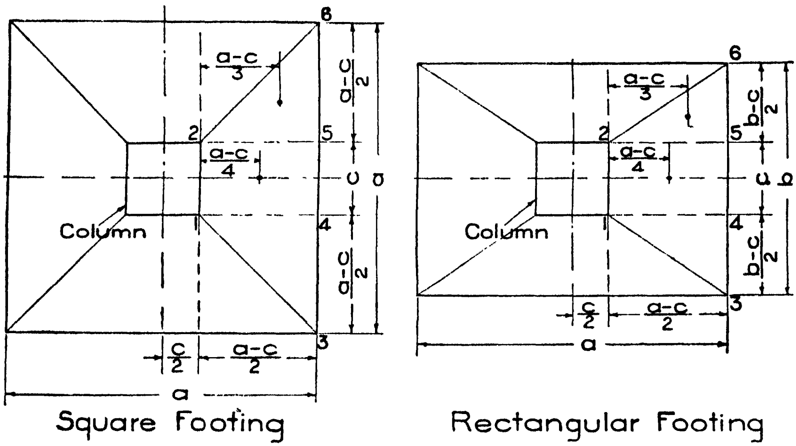


FIG. 205.—Square and Rectangular Column Footings. (*See p.* 673.)

If the footing is stepped, or beveled, the bond must be figured at the points of change in thickness to determine whether in all places the design fulfills both of the requirements as to the unit bond stress, u, and the length of imbedment to prevent slipping.

Diagonal Tension. Tests indicate that in reinforced concrete footings, diagonal tension develops at a distance from the face of the column equal to the effective depth of the footing. In figuring the maximum diagonal tension, therefore, by the formula, $v = \frac{V}{b\,j\,d}$ (see p. 517), V should be taken as the upward load between the edge of the footing and a line concentric with and distant from the column face a distance

equal to the effective depth of the footing; b is the length of this circumferential line; and d is the effective depth of the footing at the point considered.

The following example illustrates the method of designing reinforced concrete column footings.

Example 1. Find the dimensions of a footing for a column 28 inches square carrying 350 000 lb., when the allowable pressure on the soil is two tons per square foot.
Solution. Necessary area of footing is found by dividing the total superimposed load plus assumed weight of footing (40 000 lb.), by allowable unit pressure on soil, which gives $\frac{390\ 000}{4\ 000} = 98$ sq. ft. as the required area of base. A base 10 feet square, therefore, will be selected. The final dimensions and reinforcement selected, from the computations below, are shown in Fig. 206, page 677.

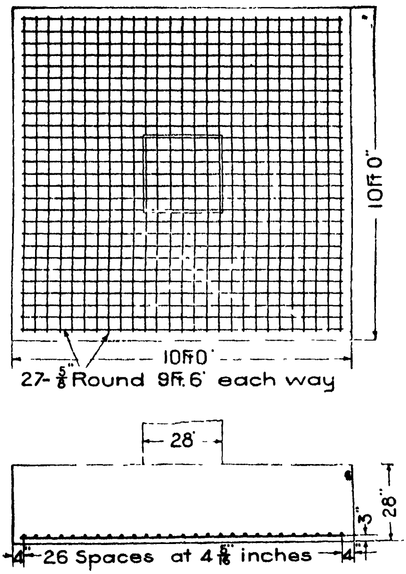


FIG. 206.—Details of Square Footing. *(See p. 677)*

Find Minimum Thickness of Footing as De ermined by Punching Shear. Since the gross area of base of footing is 100 sq. ft., and area of column, 5.43 sq. ft., the net area of footing is $100 - 5.43 = 94.57$ sq. ft. The load on the net area produces punching shear at the edge of the column, which for 1 : 2 · 4 concrete must not exceed 120 lb. per sq. in. The load producing punching shear may be determined by multiplying the total load on the footing (exclusive of the weight of the footing which produces no shear) by the ratio of net area to gross area of the footing. It is, therefore, $\frac{94.57}{100} \times 350\ 000 = 332\ 000$ lb. By dividing this load by the circumference of the column, or 112 inches, and the allowable unit shear, we get

$$\text{Minimum depth} = \frac{332\ 000}{112 \times 120} = 24.7 \text{ inches.}$$

Accept 25 inches as the depth of the footing.

Find Diagonal Tension. To determine whether the minimum depth is sufficient, diagonal tension will be determined at a distance from column equal to effective depth of footing, as explained on page 676. The side of the square is $28 + 2 \times 25 = 78$ inches, or 6.5 ft., and the circumference, $78 \times 4 = 312$ inches. The area outside this circumference equals $100 - 6.5^2 = 57.7$ sq. ft., and the load $\frac{57.7}{100} \times 350\,000 = 202\,000$ pounds.

The unit shear involving diagonal tension, therefore, is

$$v = \frac{202\,000}{\frac{7}{8} \times 25 \times 312} = 30 \text{ lb. per square inch.}$$

Hence, no shear reinforcement is necessary.

Find Bending Moment. As explained on page 674, the bending moment can be found by the use of table on page 675. The ratio of diameter of column to side of footing is $\frac{c}{a} = \frac{28}{10 \times 12} = 0.233$, and the corresponding constant from the table, by interpolation, $C_F = 0.0567$. The bending moment, therefore, is

$$M = 0.0567 \times 10 \times 12 \times 350\,000 = 2\,380\,000 \text{ inch-pounds.}$$

Find Area of Steel. The reinforcement will be placed in two directions parallel to the sides of the footing. The area of steel in each band is found by dividing the bending moment determined above by the moment arm times the allowable unit stress in steel.

$$A_s = \frac{2\,380\,000}{\frac{7}{8} \times 25 \times 16\,000} = 6.8 \text{ sq. in. requiring 23, } \tfrac{5}{8}\text{-inch round bars.}$$

All these bars must be placed within the effective distance, which is 28 in. + $(2 \times 25 \text{ in.}) + \frac{3 \text{ ft.}}{2} = 8$ ft. (See p. 675.) Add two bars at each side, making a total of 27, $\frac{5}{8}$-inch round bars.

Find Bond Stress. Bond stresses are determined by Formula (36), page 534. Since the shear at one edge of column is $V = \frac{332\,000}{4} = 83\,000$ lb., and the number of effective bars per band is 23, $\frac{5}{8}$-inch round bars, the periphery of which is $23 \times 1.96 = 45.2$ inches; therefore the unit bond stress is

$$u = \frac{83\,000}{\frac{7}{8} \times 25 \times 45.2} = 84 \text{ lb. per square inch.}$$

This bond stress may be used for deformed bars, but is somewhat excessive for plain bars in 1:2:4 concrete (see p. 573). If plain bars are used, the depth would have to be increased or smaller bars used. Hence use deformed bars.

The weight of the footing does not need to be considered in figuring bending moment, shear, diagonal tension, and bond stresses, because it is balanced by the upward reaction. It increases, however, the unit pressure on the soil; therefore it was considered in determining the size of the base of the footing.

An example of this type of footing, founded on piles, is shown in Fig. 207, page 679. This is one of the interior footings used for the new buildings of the Massachusetts Institute of Technology, Cambridge, Mass.

COMBINED FOOTINGS.

Sometimes it is necessary to connect the footings of two or more columns, as when the face of the columns coincides with or is near the edge of the building lot. To insure equal distribution of the pressure on

the foundation, it is of utmost importance that the center of gravity of the loads coincide with the center of gravity of the upward reaction. The shape of the footing and the relative position of the columns on the footing are governed chiefly by this requirement.

Combined footings for two columns carrying loads of different sizes may be made in the shape of a trapezoid, the center of gravity of which coincides with the center of gravity of the loads, or it may be rectangular in shape, but with a longitudinal projection beyond the heavier column of a sufficient length to bring the center of gravity of the rectangle in the required position.

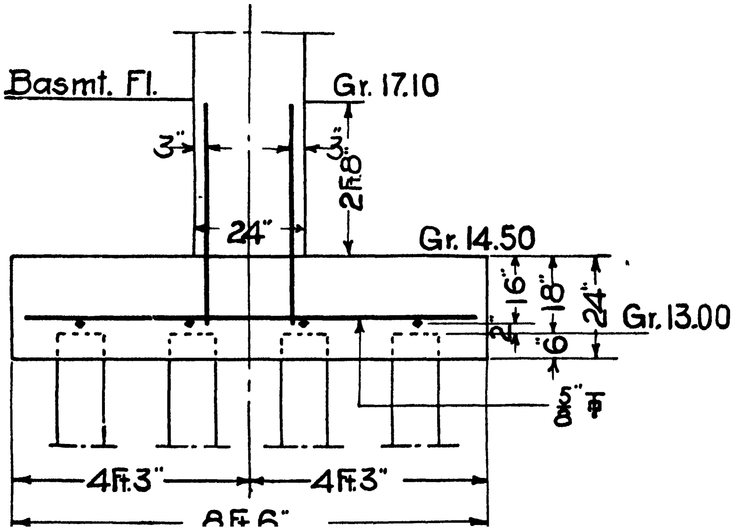


FIG. 207—Section of Interior Footing. (*See p.* 678.)

A combined footing may be either a slab of uniform thickness, or its cross section may be in the shape of an inverted T. In designing, the footing should be treated as a beam or slab, applying the principles and formulas given in the chapter on Reinforced Concrete Design. The pressure acts upwards, consequently the tensile stresses due to the positive bending moment will be in the top of the footing, and the negative bending moment at the bottom. Special attention must be given to bond stress and diagonal tension.

The main reinforcement is placed longitudinally, that is, extending from column to column and beyond. If the width of the footing is

much larger than the width of the column, it is advisable to provide transverse reinforcement of sufficient amount to resist the transverse bending moment. This reinforcement is either distributed over the whole footing, or concentrated near the columns, in which case a few bars are added to the figured amount and placed between columns.

The following example illustrates the design of a combined footing.

COMBINED FOOTING

Example: Find the dimensions of a combined footing in which $P_1 = 400\ 000$ lb., and $P_2 = 580\ 000$ lb., are the respective loads of columns 1 and 2, with cross-sections 24 and 30 in. square. The distance between their centers is 15 ft. and the allowable unit pressure on the soil is 8 000 lb. per sq. ft. (See Fig. 208, p. 681.)

Solution: **Area of Footing.** The area of footing will be determined by dividing the total superimposed load by the allowable unit pressure on the soil. Since the weight of the footing, which is assumed at 50 000 lb., increases the pressure on the soil, it must be included in the total superimposed load used in determining the area of footing. It should not be taken into account, however, in determining bending moments and shears. The total superimposed load is $400\ 000 + 580\ 000 + 50\ 000 = 1\ 030\ 000$ lb. The required area of footing therefore is $\frac{1\ 030\ 000}{8\ 000} = 129$ sq. ft.

Shape of Footing. A footing of a rectangular shape, one side of which is flush with the outside face of column 1 will be accepted. The footing extends beyond column 2 (which carries the larger load) the required length to make the center of gravity of the superimposed loads coincide with the center of gravity of the upward reaction of the soil.

Center of Gravity. The center of gravity of the column loads, from simple mechanics, is distant from column 1, $15 \times \frac{580\ 000}{400\ 000 \times 580\ 000} = 8.9$ ft. Since center of gravity of upward reaction coincides with the center of gravity determined above and the footing is flush with the outside face of column 1, the distance from the edge of the footing to its center of gravity is 8.9 ft. $+ 1$ ft. $= 9.9$ ft. In a rectangle the center of gravity is in the middle, and therefore the total length must equal $9.9 \times 2 = 19.8$.

Width of the Footing. The width is determined by the required area and is $\frac{129}{19.8} = 6.5$ ft.

Shears. The shear diagram is shown in Fig. 208. The columns are represented by theoretical points of application of loads. Since the columns are large the shear at the edge of the columns will be smaller than the theoretical maximum shear and can be determined by plotting the dimensions of the columns. (See Fig. 208, page 681.) The upward reaction per foot of width of footing determined by dividing the total downward load by the length of the footing is $\frac{980\ 000}{19.8} = 49\ 500$ lb. per lin. ft. The shear at the left of column 2 equals the total upward reaction on the cantilever, or $49\ 500 \times 3.8 = 188\ 000$ lb. To the right of column 2 the shear equals the difference between the column load and the upward reaction, or $580\ 000 - 188\ 000 = 392\ 000$ lb. The shears are obtained similarly at the other column.

Bending Moment. Weight of footing is not considered in determining the bending moment because it is balanced by the upward reaction. The bending moment diagram shown in Fig. 208 is determined by simple statics. For the purposes of computation, the footing may be considered as a slab supported at the columns and loaded by the uniformly distributed pressure of soil. Since the uniform load acts upward the bending moment will be of opposite sign to that in ordinary beams. The maximum bending moment of the cantilever was determined by multiplying the total

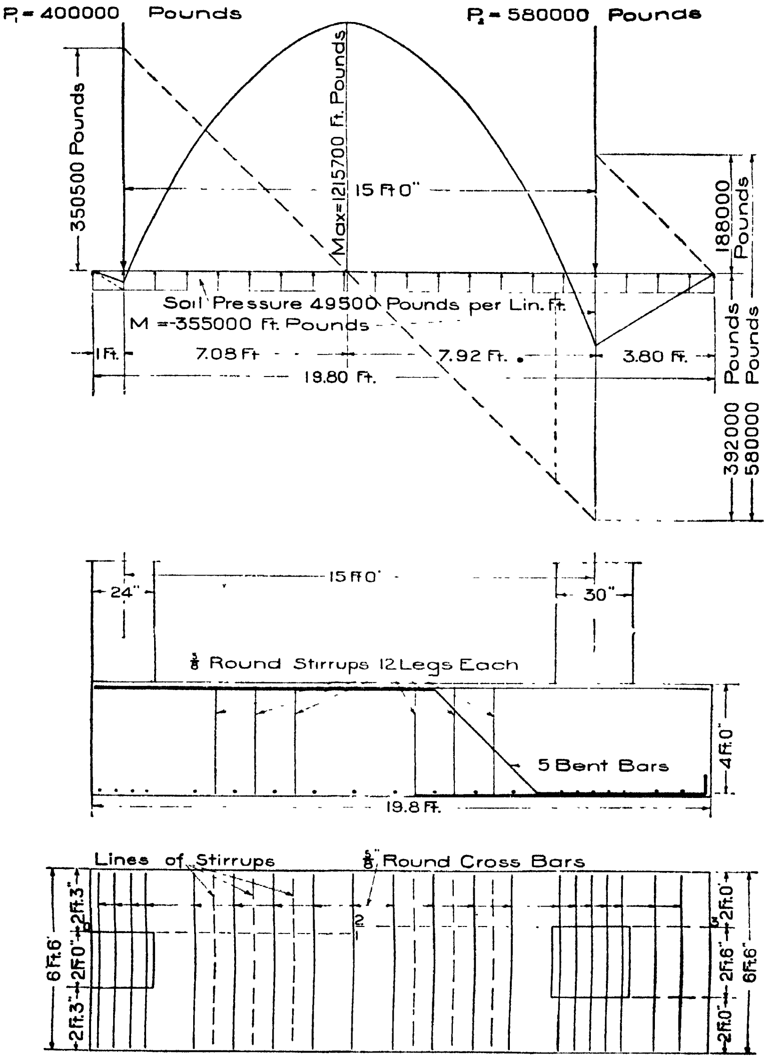


FIG. 208.—Design of Combined Footing. (See p. 680.)

upward load on the cantilever by half the length of the cantilever. Their magnitude is indicated on the diagram. The maximum bending moment in the central portion of the footing is determined from the principle that the point of maximum bending moment coincides with the point of zero shear, the distance of which from either column can be determined by dividing the vertical shear at that column by the upward unit reaction. For column 1, therefore, the point of zero shear is distant 350 500 lb. ÷ 49 500 lb. = 7.08 ft. from center. The maximum bending moment $= 24\ 700$ ft. lb. — $350\ 500$ lb. $\times$ 7.08 ft. + $49\ 500$ lb. $\times$ 7.08 ft. $\times \frac{7.08}{2} = 1\ 215\ 700$ ft. lb. where 24 700 ft. lb. is the bending moment at the column due to the projection; 350 500 lb. is the shear at the reaction; and 49 500 lb. is the uniformly distributed upward pressure per foot of width of footing

The bending moment in the central portion of the footing depends upon the magnitude of the bending moment at the column which in turn depends upon the length of projection; i.e. the length of the cantilever.

Depth of Footing. Depth of footing is determined by the punching shear, by diagonal tension, and by bending moment. The largest depth, of course, must be used.

Depth Determined by Punching Shear. At column 1, the load equals 400 000 lb. Since the footing is flush on the outside, the column has only three shearing sides of a length equal to $24 \times 3 = 72$ in. Allowing for the load transmitted directly by the column, depth required is $\frac{42}{46} \times \frac{400\ 000}{72 \times 120} = 42.3$ in. At column 2, although the load is larger the shearing area is so much larger than at column 1, that the required depth is only 37 in.

Depth Determined by Diagonal Tension. From the shear diagram, the maximum shear at edge of column 2 is $V = 330\ 000$. Since the width of footing is 78 in. and the allowable shear 120 lb., required depth is $\frac{330\ 000}{120 \times .875 \times 78} = 40.3$ in.

Depth Determined by Bending Moment. Since the bending moment is 1 215 700 ft. lb., or 14 600 000 in. lb., the depth, from Formula (3), p. 482, is $d = 0.096 \sqrt{\frac{14\ 600\ 000}{78}} = 41.5$.

In this case the depth required by punching shear is a maximum. To reduce amount of steel and bond stresses an effective depth of 48 in. is accepted.

Longitudinal Reinforcement. The amount of longitudinal reinforcement may be determined from Formula (4a), p. 482. $A_s = \frac{14\ 600\ 000}{0.875 \times 48 \times 16\ 000} = 21.7$ sq. in. which requires 28, 1-in. round bars. The reinforcement of the cantilever determined in the same fashion is $A_s = \frac{355\ 000 \times 12}{0.875 \times 48 \times 16\ 000} = 6.3$ sq. in. This reinforcement can be supplied partly by bending down of the steel and partly by short bars. Since the bond stresses are very large it will be necessary to provide a much larger amount of steel than that determined by the bending moment. It is advisable to use reinforcement as shown in Fig. 208.

Stirrups. The stirrups are determined from the shear diagram in the manner explained on page 528. It must be remembered that the stirrups are designed for the shear at the face of the column which is smaller than the maximum shear as found from tests. To be effective, the first stirrup should be placed from the edge of the column a distance equal to $\frac{1}{8}$ of the depth of the footing.

Cross Bending. Since the footing is wider than the support, it is subjected to cross bending. To prevent the projections from breaking, enough cross steel at the bottom of the footing must be provided. As the sizes of the columns are different, the projections at the base are different. In practice, it is accurate enough in determining the bending moment to divide the footing by lines *01* and *23* as shown in the figures and compute the bending moments in respect to lines *01* and *23*. The load at each cantilever is $\frac{49\ 500}{6.5} \times 2.25 \times 8.08 = 138\ 200$ and $\frac{49\ 500}{6.5} \times 2 \times 11.72 = 178\ 200$ lb. respectively.

The bending moment, obtained by multiplying the loads by half of the length of

the projection is $M_1 = 1\ 870\ 000$ in. lb. and $M_2 = 2\ 140\ 000$ in. lb. The required amount of steel, from formula (4a), p. 482, is 2.8 sq. in. at column 1, and 3.2 sq. in. at column 2, requiring 9, $\frac{5}{8}$-in. and 11, $\frac{5}{8}$-in. bars respectively, spaced as shown in the figure.

Influence of Projections on the Bending Moment. The bending moment in the center of a combined footing depends upon the ratio of the length of cantilevers to the distance between columns. Fig. 209 illustrates three cases which may occur in the design of footings. In

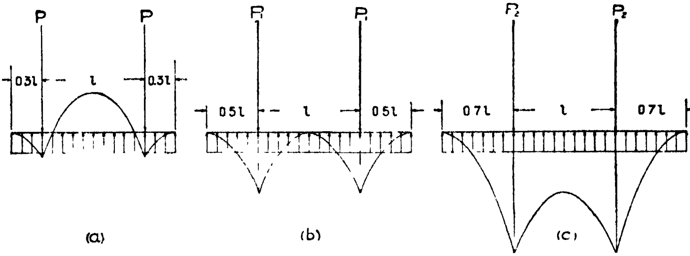


FIG. 209—Bending Moment in Center of Footing for Varying Lengths of Cantilevers. (See p. 683.)

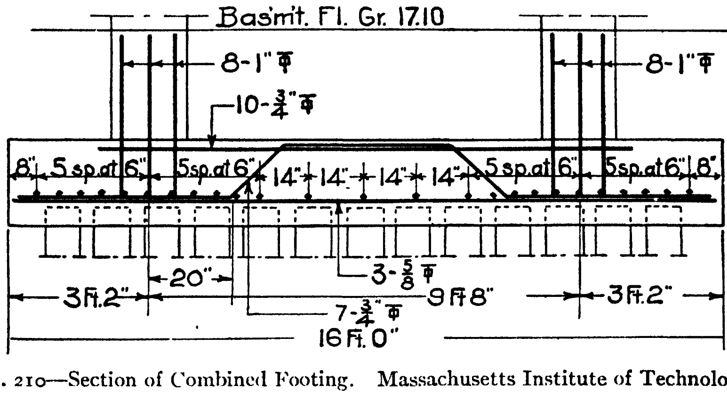


FIG. 210—Section of Combined Footing. Massachusetts Institute of Technology. (See p. 684.)

(*a*) the projection is smaller than one-half of the length, *l*. In this case the bending moment in the cantilevers and at the columns is positive and in the central portion negative. In (*b*) is illustrated a case in which the length of the cantilever equals $\frac{1}{2}l$; hence the bending moment in the center equals zero. In case (*c*) the cantilevers are larger than $\frac{1}{2}l$ therefore the whole footing is subjected to positive bending

moment which is a maximum at the column and a minimum in the center.

An example of the combined type of footing is shown in Fig. 210, page 683. This is one of the exterior footings from the new buildings of the Massachusetts Institute of Technology.

SPREAD FOOTINGS

When the allowable pressure on the soil is very small or when the building is supported by piles sustained by friction, it may be necessary to spread the foundation over the whole area of the building, either using a thick mass of plain concrete or a thinner slab of reinforced concrete design as a flat plate, or a beam and slab system.

Flat Slab Foundations. A flat slab may be designed by the method of flat plates explained on pages 540 to 551. The slab is considered as an inverted flat plate loaded by the reaction of the ground and supported by the columns.

Special provision should be made in the design where there is unequal loading.

Since the distributed pressure acts upward, the bottom of the plate under the columns and the top of the plate between the columns is in tension; hence the steel must be in the bottom of the slab under the columns, and should be bent up to the top of the slab between columns. The column base must be large enough to prevent excess loading or too great moments and shears in the concrete.

Beam and Slab Foundation. For a combination of beams and slabs the principles of floor design are followed except that the distributed load acts upward. The beams or ribs may be built either above or below the slab, the former method permitting a T-beam design, but, on the other hand, requiring an extra fill and separate floor surface in the basement. The formulas and discussion relating to slab design in Chapter XXII apply.

FOUNDATION BOLTS

It is often difficult to locate bolts in concrete with sufficient exactness for setting a machine. To obviate this difficulty, the head of the bolt should be provided with a large washer* to give a good bearing surface,

* The washers, which are used for transmitting the pressure of large bolts to the concrete or other foundations, should be carefully designed with heavy ribs so as to transmit a uniform pressure per square inch of area. Neither wrought nor cast iron plates should be used for washers under large bolts.

the bolt placed in its approximate position, with washer down, and an iron pipe or a light wooden box placed around the bolt resting upon the washer. When the machine is set, to prevent the bolt from rusting, the iron tube or box should be filled with mortar. In any case the tube or box should be filled with sand before the machine is poured up with sulphur or cement grout, in order to keep these materials from running down the bolt holes.

CONCRETE PILES

Concrete piles may be employed in place of wood where the loading is excessive, and where the durability of timber piles is questioned either because of probable worm action or the rotting of the timber. If the bearing is frictional and the piles are driven through ground which is continually wet, there is usually no advantage in concrete over timber piles unless in certain instances where the low level of the ground water or the tide water is so far beneath the structure that the concrete piles permit the commencement of the foundation at a considerably higher level and thus save excavation and material.

Concrete piles are formed in place, or are molded above ground.

Various methods have been suggested for forming the hole into which the concrete is to be placed. One of the patented processes consists in driving a double shell of metal into the ground, removing the inner one, and leaving the outer to form a mold for the concrete. The two shells and pile driver are shown in Fig. 211, page 686. The inner shell or pile core, which is of heavy sheet steel and constructed so that it can be made to collapse for removal from the ground, is placed within the other thinner shell, and driven like an ordinary pile. The core is then collapsed and withdrawn, leaving the outer shell, which is closed at the bottom, to be filled with concrete. By providing considerable taper, additional support is obtained from the soil.

Another system, illustrated in Fig. 212, consists in driving a single shell with either a concrete or a steel point, then slowly withdrawing it, and filling the space which it occupied with concrete whose surface is kept far enough above the lower end of the tube to maintain the head necessary to resist the pressure of the ground. In a modification of this system a pedestal is formed at the foot of the pile. Both these piles are patented.

Still another type, especially adapted to underpinning, is the caisson pile in which a steel cylinder is sunk to a strata of earth, strong enough to carry the load, and filled with concrete. With a minimum diameter of 3 feet the interior can be excavated by hand as the shell is sunk.

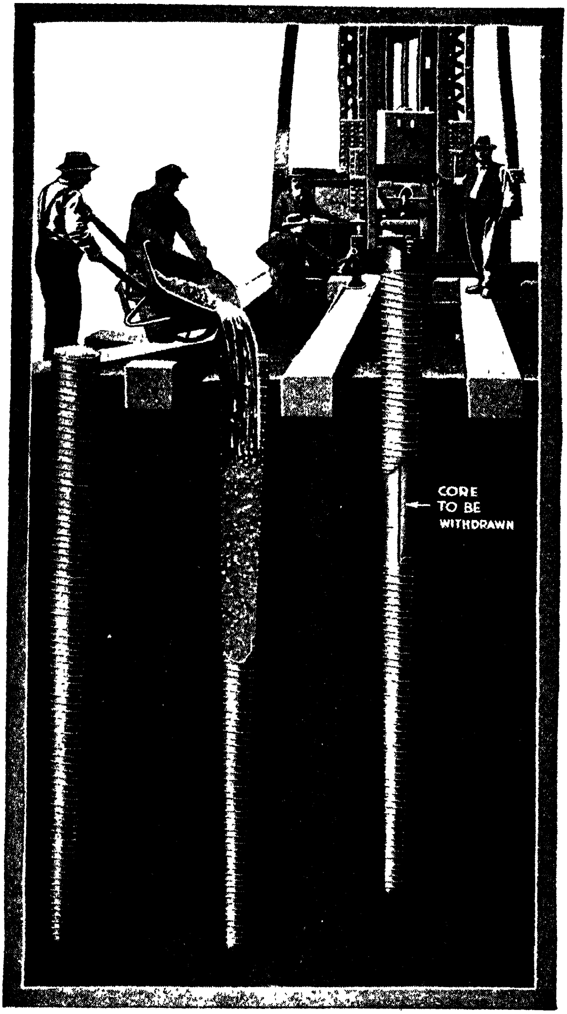


FIG. 211.—Method of Forming Concrete Piles in Place. (See p. 685.)

By undercutting the edge when the cylinder is just above grade a spread base can be secured capable of carrying a large load. A pile of this type has been driven to carry a 900 ton load on a 6 foot shaft and a base 15 feet 4 inches in diameter.

Piles made in situ may be reinforced if desired.

Cast Piles. Reinforced piles which are formed above ground are designed like columns with vertical reinforcement connected at intervals with horizontal wire rods.

The pile* used in a foundation for the Boston Woven Hose & Rubber Company, Cambridge, Mass., is illustrated in Fig. 213. These piles averaged about 30 feet long. The hammer weighed 4 700 pounds and the blows were cushioned by a head consisting of a plate iron collar 16 inches square on the inside and 3 feet in height, which incased an oak block 16 by 16 by 18 inches, to the bottom of which six thicknesses of rope and four layers of rubber belting were nailed. The piles were driven at the age of thirty to forty days. The usual drop was 3 feet but in some cases this was increased to 19 feet without injuring the pile.

The designs drawn up in 1903 for the Pennsylvania Railroad Tunnel† under the Hudson River call for a shell of cast

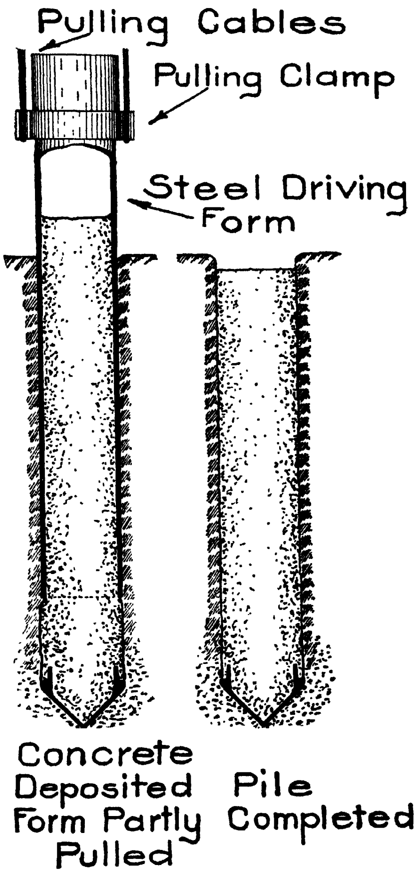


FIG. 212.—Method of Forming Concrete Piles in Place. (*See p.* 685.)

* For full description of piles and driving see "Cast Reinforced Concrete Piles," by Sanford E. Thompson and Benjamin Fox, Journal Association of Engineering Societies, Vol. XLII, 1909.

† *Engineering News*, Oct. 15, 1904, p 331.

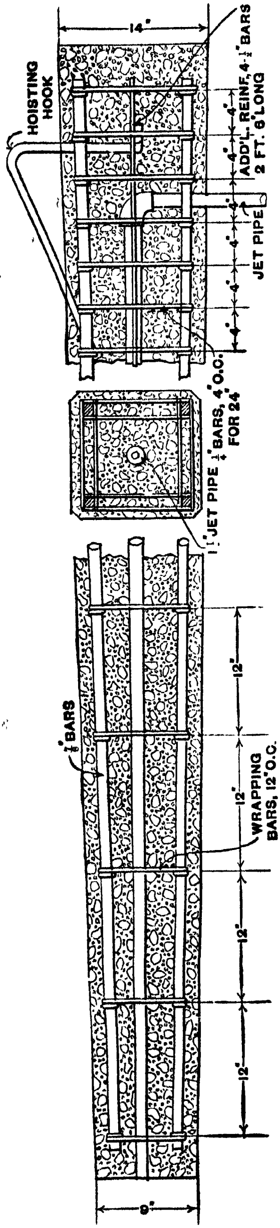


FIG. 213.—Piles used at Cambridge, Mass. (*See p.* 687.)

iron surrounded by concrete and supported at intervals by steel screw piles filled with concrete.

Sheet Piling. Poling boards of concrete were employed by Mr. Howard A. Carson, Chief Engineer in the construction of the approaches to the East Boston Tunnel. These are described* as follows:

The excavation was through gravel and clay, and through sand containing some water. Trenches 16 feet long and 16 feet apart were dug to about the level of the bottom of the building foundation. Below the foundation one-half of each trench, or 8 feet in length, was carried down to grade. The bank below the foundation was held in place by means of concrete slabs used as sheet piling, as illustrated in Fig. 214. These slabs were from 6 to 8 feet long, 6 inches wide, and 2 inches thick, and each was reinforced with six square steel rods running the entire length of the slab and shown in Fig. 215. If wooden sheeting had been used, it would have been necessary either to have concreted directly against it and left it in place, or to have pulled the planks as the concrete was filled in. If the first method had been used, the planks would in time have become rotten, leaving a vacant space. If the planks had been pulled, there would have been danger that some of the earth under the building would run and a settlement of the building follow. In order to guard against any slight voids which might have been left in driving, grout was poured in behind the sheeting. This sheeting served not only to hold the bank in place, but was used, in place of a back wall, to waterproof against. The sheeting was not disturbed, and the wall of the tunnel was built directly against it.

* Ninth Annual Report, Boston Transit Commission, p. 41.

BRIDGE PIERS

Concrete is employed for bridge piers either as filling for ashlar or cut masonry or for the entire pier. In the latter case, in which the face is also of concrete, the chief question is as to its ability to withstand the wear of the water, the ice, and floating debris.

In the Kansas City flood of 1903, the piers of solid concrete, although located where they were struck by all the heavy debris which totally destroyed many of the stone masonry structures of the same size, remained practically uninjured.

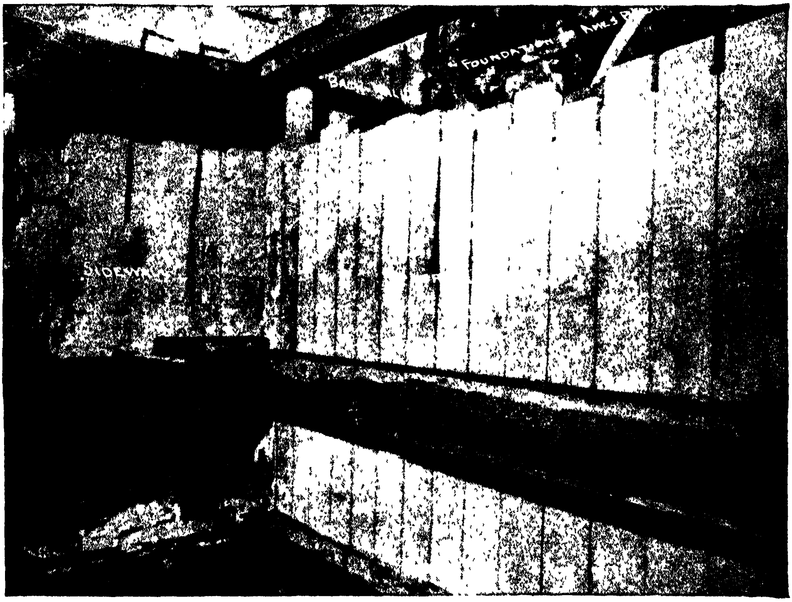


FIG. 214.—Concrete Sheet Piling in Approaches to East Boston Tunnel. (*See p.* 688.)

Plastering of concrete piers and abutments should be prohibited. If a mortar surface is required, an excellent facing, to be placed next to the form as the concrete is laid, is a mixture of one part cement to $2\frac{1}{2}$ parts hard broken stone screenings $\frac{1}{2}$-inch in size and under. Ordinarily, however, no surface finish is required unless superficial treatment is given for the sake of appearance. (See p. 262.)

Pier Design. Most railroads have substituted concrete for ashlar masonry in bridge piers.

The standard pier of the N. Y. Central R. R., adapted to any height up to 40 feet, is shown in Fig. 216, page 691.* The width, which depends upon the length of span, is as follows:

Spans up to 40 feet width, A, = 4 ft. 0 in.
Spans 40 to 60 feet width, A, = 4 ft. 6 in.
Spans 60 to 80 feet width, A, = 5 ft. 0 in.
Spans 80 to 100 feet width, A, = 5 ft. 6 in.
Spans 100 to 125 feet width, A, = 6 ft. 0 in.
Spans 125 to 150 feet width, A, = 6 ft. 6 in.
Spans 150 to 200 feet width, A, = 7 ft. 0 in.
Spans 200 to 250 feet width, A, = 7 ft. 6 in.
For skew crossings, increase width, A, if necessary.

Foundation is varied to suit local conditions. Concrete 1:3:6 is employed for it unless stone masonry is cheaper. The starkweather is carried 2 feet above high water, and its cap is of 1:1:2 concrete. The coping of the pier is reinforced with galvanized wire netting or wire cloth, a somewhat unusual requirement.

The Illinois Central R. R., in their 1904 design, reinforce the surface of piers with vertical and horizontal steel rods, and imbed a single I-beam in the pointed nose at each end of the pier.†

All of the roads named above have piers in streams which subject them to considerable wear from ice and drift, and the concrete has proved satisfactory.

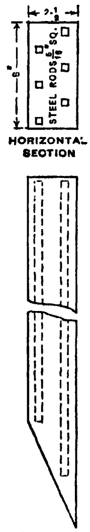


FIG. 215.–Sheet Piling. (*See p.* 688.)

FOUNDATIONS UNDER WATER

The best and most durable concrete foundations, especially in work in sea-water, are laid within cofferdams from which the water has been pumped, or in pneumatic caissons. However, because of the difficulty and expense of these methods, they can not usually be followed. If the bottom is prepared by dredging, and, if necessary, driving piles, good practice permits the use of a single line of sheeting, suitably supported with rangers, to prevent the wash of the wate

* Arranged from original drawing.
† From drawing kindly furnished by H. W. Parkhurst Engineer.

and keep the concrete from spreading.* Permanent metal cylinders are sometimes sunk in place of the sheeting.

Methods of laying concrete under water are described in Chapter XIV, page 267, and the effect of sea-water upon concrete is discussed by Mr. R. Feret in Chapter XV, page 271.

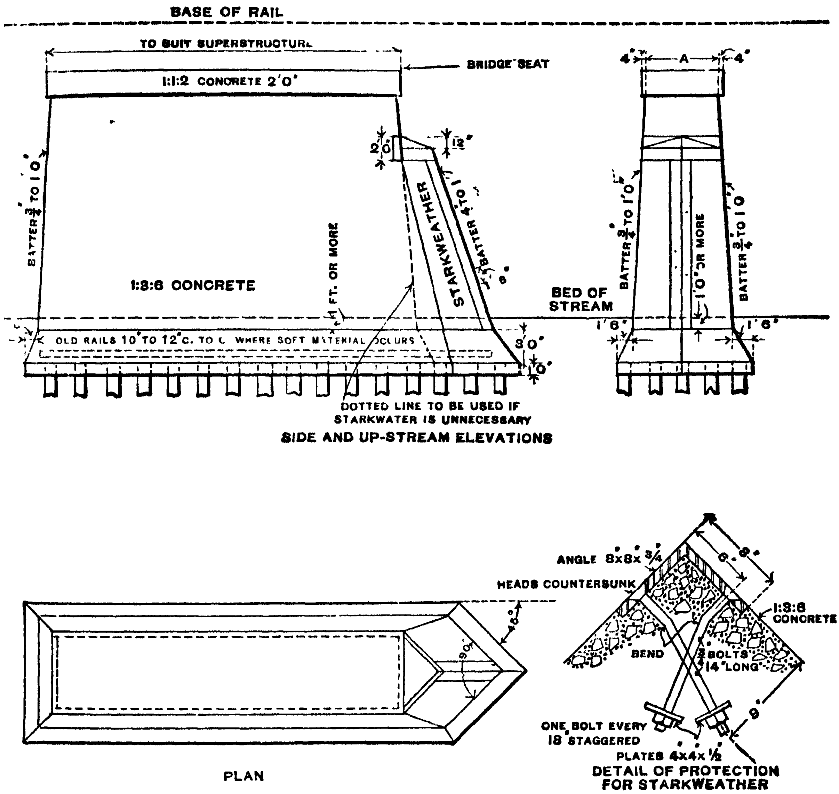


FIG. 216.—Standard Concrete Bridge Pier, N. Y. C. R. R., G. W. Kittredge, Chief Engineer. (*See p.* 690.)

For under-water work, a larger factor of safety should be employed than for work above ground, the concrete should be slightly richer in carefully selected cement, and the aggregate so proportioned as to give a dense and impervious mixture. (See p. 304.)

* See Foundations for New Cambridge Bridge, by Sanford E. Thompson, *Engineering News*, Oct. 17, 1901, p. 283.

Concrete for the foundations of walls and piers for high office buildings is usually laid in oblong or circular caissons of steel or wood,* after excavating under air pressure. Steel pipes are sometimes sunk with the aid of the water jet, and afterwards filled with concrete.†

* *Engineering News*, Sept. 26, 1901, p. 222.

† Jules Breuchaud, Transactions American Society of Civil Engineers. Vol XXXVII p. 31.

CHAPTER XXV

BEAM BRIDGES

Unlike steel bridges, which frequently have plank flooring for highways or open tie construction for railways, the concrete bridge, except in rare instances, is built with a solid deck of concrete which serves not only as the base for the flooring but also as an essential part of the bridge structure. Aside from its structural value this solid concrete deck affords a simple and efficient means of obtaining solid floor construction which is highly desirable along important lines of traffic. Thus the concrete bridge, rigid and durable, has little or no vibration, reduces maintenance to a minimum and becomes a permanent part of the roadway, permitting, as it does, an unbroken extension of the highway pavement or railway ballast.

In general, concrete is adapted to two principal classes of bridge superstructures; the arch bridge and the beam bridge. Strictly speaking, the arch is a beam, but being a curved beam its structural analysis is essentially different from that applied to the common straight beam. The term "Beam Bridges," as here used and as distinguished from "Arches" discussed in Chapter XXVI, is intended to include that class of bridges wherein the loads are supported by simple slabs, or girders, or by a combination of the two. In such structures the supporting members are essentially beams and are proportioned in accordance with the fundamental principles of design as established for straight beams.

No attempt is here made to enter into an exhaustive discussion of the design of concrete bridges, as it would be impossible to illustrate or even to mention the unlimited number of special cases and peculiar problems which might arise. To illustrate the fundamental principles involved in the design of beam bridges of both the slab and girder type, typical examples have been drawn up showing methods of computation and the resulting designs.

Slab Bridges. The simplest type of beam bridge is merely a flat slab spanning from abutment to abutment and is practical only for comparatively short spans, the limit, as fixed by considerations of economy, being dependent upon the nature of the live loads called fo

in any particular locality. If designed for trolley cars or heavy auto trucks, the limit of economical span for the slab bridge is probably not more than 10 or 12 feet, whereas for less severe loading it may prove economical up to spans of 18 or 20 feet. In general it may be said that when the combination of span and loading is such as to call for a slab thickness of more than 16 to 18 inches the simple slab will not prove as economical as the T-beam or girder type.

Girder Bridges. As the requirements of strength increase, a point is reached at which the simple slab ceases to be economical or even practical, owing to the fact that beyond certain limits an increased thickness of slab does not give added strength in any reasonable proportion to the amount of material used. For longer spans or heavier loadings than those for which the simple slab is economical, it therefore becomes necessary to modify the type of construction so as to obtain increased strength without using a solid slab of extreme thickness. This is accomplished by placing deeper ribs or girders beneath the slab to strengthen and support it, resulting in a type of construction known as the "girder" bridge. Thus the girder bridge is in reality a modification of the slab bridge whereby a comparatively thin slab spans between a series of relatively deep beams which in turn span from abutment to abutment. As commonly built, the supporting ribs or girders are constructed monolithic with the floor slab obtaining thereby the structural advantages of the T-beam.

The girder type of construction, supplementing as it does the slab type, becomes practical at the point where the simple slab ceases to be economical, while its maximum economical span is determined not only by the kind of loading provided for but also by the spacing and arrangement of the girders. In railroad bridges and highway bridges carrying trolley cars a girder is usually located at or near each rail, whereas girders receiving roadway loads are usually spaced so that the floor slab will have a thickness of from 5 to 7 inches. Owing to the greater complexity of the girder bridge it is impossible to establish the limits of economical spans as definitely as in the case of the more simple slab bridge. Girder bridges of well proportioned design have been used for spans up to 80 feet, but whether or not they are practical for this or even shorter spans is dependent upon the severity of the loading and other practical as well as theoretical elements which must necessarily involve the judgment of the designer and which must be carefully considered in each particular case.

LOADS

In the examples, the design has been based upon the following assumptions as to live loads and unit stresses and these may be considered typical.

Live Loads: On sidewalks, a uniform load of 100 lb. per sq. ft.; on roadways a 20-ton auto truck having 6 tons on one axle and 14 tons on the other axle, the axles being 12 feet apart, and the distance between wheels 6 feet.

Distribution of Concentrated Loads: The results of recent tests conducted by the highway department of the State of Ohio* indicate that a concentrated load applied to the concrete slab of a highway bridge floor may be safely taken as distributed over a width of floor represented by the formula

$$e = 0.6\,S + 1.7$$

where e = effective width in feet of a slab of greater width than length, and S = clear span in feet.

In considering the concentrated loads represented by a wagon, auto truck, or other highway vehicle the effective width of distribution for each wheel can not of course be taken as more than the half the "gage" or distance between wheels at each side of the concentrated load without overlapping the distribution from the other wheel.

Where concentrated loads are applied directly to the slab this method of distribution gives a loading, which, while distributed across a certain width of slab, is taken as concentrated with respect to the length of span in determining moments and shears. In case the slab is covered with paving material the stiffness of the paving tends to give some distribution along the span and, when applied over a fill, it is permissible to consider a further distribution through the fill along the customary 45° lines. Assuming the pavement to give a wheel a longitudinal distribution of at least 12 inches and assuming say a fill of 6 inches, it would be permissible to consider a concentrated load as distributed across the span in accordance with the above formula and also to consider the load per foot of width as applied along a length of at least 2 feet longitudinally with the span.

Impact. An impact allowance of 25% has been made in the case of all live loads except the 100-lb. uniform load used on sidewalks for which no impact has been added.

* See page 446.

Unit Stresses. The unit stresses used in designing have been taken as those recommended by the authors on page 573.

DESIGN FOR A SLAB BRIDGE.*

Example 1: Design a highway bridge of the simple slab type having a single span of 10 feet in the clear.

Solution: The following assumptions and computations are required to produce the design shown in Fig. 217, page 696.

Loads.	Assume weight of road material	75 lb. per sq. ft.
	Assume weight of slab	125 lb. per sq. ft.
	Total dead load	200 lb. per sq. ft.

Live load is auto truck, the heaviest wheel load being 14 000 lb. A single concentrated load may be distributed over a width of slab $e = (0.6 \times 10) + 1.7 = 7.7$ ft. Since, however, the two rear wheels of our auto truck are spaced only 6 feet apart the weight of one wheel can not be distributed over a width of more than 6 feet without overlapping the distribution from the other wheel.

Hence live load per foot of width $= \frac{14\ 000}{6} = 2\ 333$ lb.

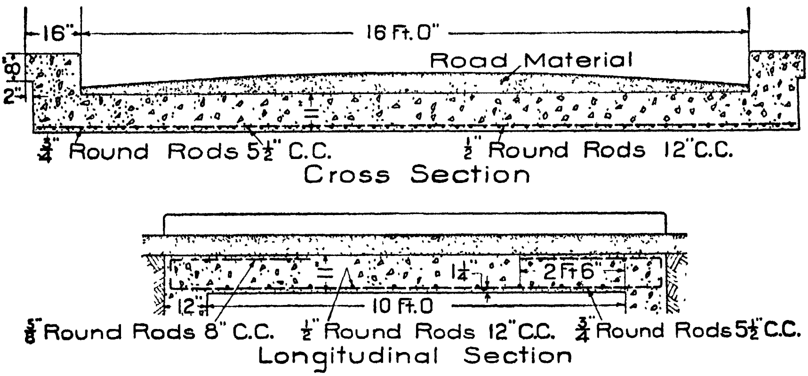


FIG. 217.—Design for a Slab Bridge. (*See p.* 696).

Bending Moment. Maximum live moment occurs with heaviest wheel at center of span. Take effective span as clear span plus 12 inches and concentrated load as distributed over a width of 6 feet and length of 2 feet.

$$
\begin{aligned}
M\ (\text{live}) &= \frac{2330}{2}(66-6) &&= 70\ 000 \text{ in. lb.}\\
\text{Impact} &= 25\% \text{ of } 70\ 000 &&= 17\ 500 \text{ in. lb.}\\
M\ (\text{dead}) &= \frac{200 \times 11^2 \times 12}{8} &&= 36\ 300 \text{ in. lb.}\\
\text{Total } M \text{ per ft. of width} & &&= 123\ 800 \text{ in. lb.}
\end{aligned}
$$

Shear. With load assumed as distributed over a width of 6 feet and length of 2 feet maximum shear occurs with heaviest wheel one foot inside support. Take effective span as clear span.

* The authors are indebted to Mr. Royall D. Bradbury for assistance in working up the details of the following examples.

$$V \text{ (live)} = 2\ 330 \times \frac{9}{10} = 2\ 100 \text{ lb.}$$

$$\text{Impact} = 25\% \text{ of } 2\ 100 = 525 \text{ lb.}$$

$$V \text{ (dead)} = \frac{200 \times 10}{2} = 1\ 000 \text{ lb.}$$

$$\text{Total } V \text{ per foot of width} = 3\ 625 \text{ lb.}$$

Thickness of Slab. For $f_c = 650, f_s = 16\ 000$ and $n = 15$, the table on page 483, gives $C_1 = 0.028$, $p = 0.0077$ and $j = 0.874$. Hence required depth to steel (formula (9) p. 485) $d = 0.028\sqrt{123\ 800} = 9.75$ in.

Without using any stirrups or bending up any of the bars maximum unit shear must be limited to 40 lb. per sq. in.

Hence, from Formula (32), page 517, the required depth to steel,

$$d = \frac{3625}{0.874 \times 12 \times 40} = 8.65 \text{ in.}$$

Therefore, thickness of slab as determined by bending moment is O. K. for shear. Using a depth to steel of 9.75 in. and allowing $1\frac{1}{4}$ in. below steel we obtain a total slab thickness of 11 in.

Reinforcement. Required cross sectional area of steel per inch of width for bending moment, (see Formula (12), p. 485)

$A_s = 9.75 \times 0.0077 = 0.075$ sq. in. per inch of width.

To satisfy this area $\frac{7}{8}$ in. round bars of a sectional area 0.6 sq. in. should be spaced $\frac{0.60}{0.075} = 8$ in. on centers or $\frac{3}{4}$ in. round bars $5\frac{1}{2}$ in. on centers.

Before accepting the size and the spacing of bars the slab must be tested for bond by formula (36), p. 534. The total shear per foot of width $V = 3\ 625$ lb. The imbedment to use is as follows:

Diameter of round bars	Circumference.	No. bars per foot of width	Σ 0.
$\frac{7}{8}$ in.	2.75	$\frac{12}{8} = 1.5$	$2.75 \times 1.5 = 4.13$
$\frac{3}{4}$ in.	2.35	$\frac{12}{5.5} = 2.18$	$2.35 \times 2.18 = 5.13$

The maximum unit bond stress, therefore, is

for $\frac{7}{8}$ inch bars.

$$u = \frac{3\ 625}{0.874 \times 9.75 \times 4.13} = 103 \text{ lb. per sq. in.}$$

for $\frac{3}{4}$ inch bars

$$u = 103 \times \frac{4.13}{5.13} = 83 \text{ lb. per sq. in.}$$

From the above it is evident that the unit bond stress for $\frac{7}{8}$ in. bars exceeds the 80 lb. per sq. in. allowed for plain bars but is O. K. for deformed bars of approved pattern. $\frac{3}{4}$ in. plain bars can be used. See allowable working stresses, page 573.

Remarks. In the design of short span slabs subjected to heavy loading the shear and not the bending moment will usually determine either the thickness of the slab, or the reinforcement, or both. Highway bridges of the slab type, which in the great majority of cases are nothing other than culverts, will usually come within the short span class and for that reason a careful investigation of both shear and bond is important.

In non-continuous slab bridges, even when computed as simply supported, it is usually advisable to place a certain amount of steel in the top of the slab over the

supports in order to give some degree of fixedness and to prevent cracking. This is especially necessary in culverts where the supporting walls are built monolithic with the slab.

DESIGN FOR A GIRDER BRIDGE

Example 2: Let it be required to design a highway girder bridge having a clear span of 25 feet to carry sidewalks and roadway. Let the width of roadway be 25 feet between curbs and the width of each sidewalk 5 feet 8 inches in the clear.

Solution: The following assumptions and computations are required to produce the design shown in Fig. 218, page 699.

Sidewalk Slab. Take live load of 100 lb. per sq. ft.
Take dead load of 62 lb. per sq. ft.
162

Effective span of slab, 6 feet
Bending moment per foot width of slab

$$M = \frac{162 \times 6^2 \times 12}{8} = 8\ 750 \text{ in. lb.}$$

Hence, Formula (9), p. 485, depth to steel, $d = 0.028\sqrt{8\ 750} = 2.6$ in.

In bridges it is advisable to limit minimum thickness of slabs to 4 in. in order to facilitate placing of steel and also to assure a certain stiffness regardless of load. Use therefore a total thickness of 4 in., which, allowing 1 inch below steel will give a depth to steel, $d = 3$ inches.

Taking clear span as 5 ft. maximum shear per ft. of width, $V = 162 \times 2.5 = 405$ lb. Testing slab for shear we obtain from Formula (32), page 517,

$$v = \frac{405}{0\ 874 \times 3 \times 12} = 13 \text{ lb. per sq. in.}$$

Hence thickness O. K. for shear involving diagonal tension.

Required area of steel per inch of width of slab for moment is (Formula (13), p. 485)

$$A_s = \frac{8\ 750}{12 \times 0.874 \times 3 \times 16\ 000} = 0.0174 \text{ sq. in.}$$

This area is provided by $\frac{3}{8}$-inch round bars 6 in. on centers. Since the circumference of a $\frac{3}{8}$-in. round bar is 1.18 in. and with a 6 in. spacing there are two bars per foot of width, from Formula (36), page 534, $u = \frac{405}{0\ 874 \times 3 \times 2 \times 1\ 18} = 66$ lb. per sq. in. This is O. K. for bond. Hence use $\frac{3}{8}$-inch round bars 6 in. on centers.

Facia Girders. Take span of girders as clear span plus about 16 inches or, say, effective span of 26.3 feet.

Assume weight of railing = 350 lb. per lineal ft.
Assume weight of girder = 450 lb. per lineal ft.
Weight of slab = 63 × 2.5 = 158 lb. per lineal ft.
Live load = 100 × 2.5 = 250 lb. per lineal ft.

Load on girder = 1 208 lb. per lineal ft.

$$\text{Max. Moment, } M = \frac{1\ 208 \times 26.3^2 \times 12}{8} = 1\ 250\ 000 \text{ in. lb.}$$

$$\text{Max. Shear, } V = \frac{1\ 208 \times 25}{2} = 15\ 100 \text{ lb.}$$

Selection of proper depth of a facia girder, which is usually paneled or otherwise embellished, is governed as a rule more by appearance than by requirements of actual strength. In order to give good proportions to the elevation of our bridge, we will make the facia girders 36 inches deep.

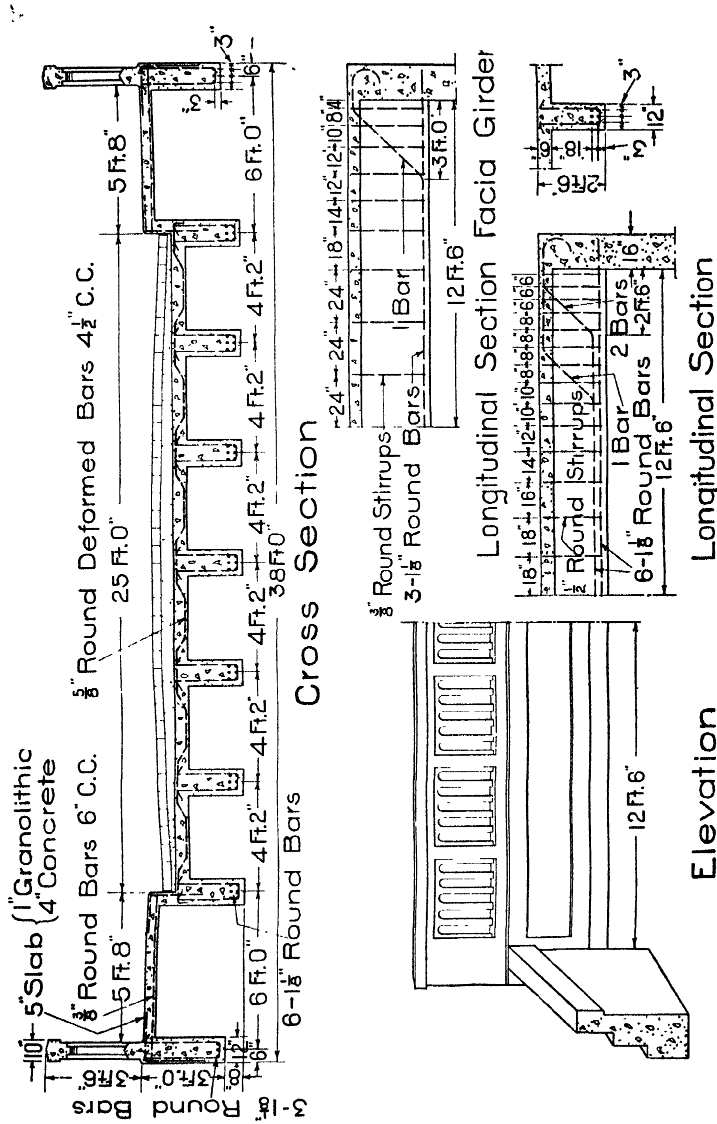


FIG. 218.—Illustration of a Through Bridge. (See p. 698.)

In checking this depth for strength consider girder as a rectangular beam with its top at surface of sidewalk.

Assuming a width of 12 inches we obtain, from Formula (3), page 482,

$$d = 0.096\sqrt{\frac{1\ 250\ 000}{12}} = 31 \text{ in.}$$

By placing the steel 3 inches above bottom, the 36-inch beam will have a depth $d = 33$ inches, which is O. K. since only 31 inches depth is actually needed.

Required area steel at center of span, from Formula (4a), p. 482,

$$A_s = \frac{1\ 250\ 000}{0.874 \times 33 \times 16\ 000} = 2.71 \text{ sq. in.}$$

Use three $1\frac{1}{8}$ inch round bars ($A_s = 2.98$).

Bending up one bar as shown in Fig. 218, page 699, we obtain from Formula (36), p. 534, for maximum unit bond stress on the two remaining $1\frac{1}{8}$-inch round bars

$$u = \frac{15\ 100}{0.874 \times 33 \times 7.06} = 74 \text{ lb. per sq. in.}$$

Therefore size of longitudinal bars is O. K. with one bent up as shown.

Testing section of beam for shear we obtain, from Formula (32), page 517,

$$v = \frac{15\ 100}{0.874 \times 33 \times 12} = 44 \text{ lb. per sq. in.}$$

While this exceeds the allowable unit of 40 lb. per sq. in. for diagonal tension this slight excess would be more than taken care of by the bent up bar without using any stirrups. However, as it is not considered the best practice to build a girder of these dimensions without some web reinforcement, stirrups are arbitrarily provided of size and spacing as suggested in sketch.

Roadway Slab. With curb girders spaced 25 feet on centers use five intermediate roadway girders spaced 4 ft. 2 in. on centers. Assuming that these girders will be 12 in. wide, the roadway slab will have a clear span of 3 ft. 2 in. or 38 in. Take effective span of slab for moment as clear span plus 6 in. or 3 ft. 8 in.

Assume weight of paving brick	=	50 lb. per sq. ft.
Assume weight of fill	=	60 lb. per sq. ft.
Assume weight of slab	=	62 lb. per sq. ft.
Total dead load	=	172 lb. per sq. ft.

Take 14 000 lb. wheel load of auto truck carried by a slab width of $e = (0.6 \times 3.17) + 1.7 = 3.6$ ft.

Hence live load per ft. width $= \frac{14\ 000}{3.6} = 3\ 900$ lb.

Considering this load as distributed over a length of 2 ft. longitudinally with the span of slab and allowing for continuity by taking $\frac{3}{4}$ of the moment for a simply supported slab we have

$$M \text{ (live)} = \frac{3\ 900\ (22-6)}{2} \times \frac{3}{4} = 23\ 400 \text{ in. lb.}$$

$$M \text{ (impact)} = 25\% \text{ of } 23\ 400 = 5\ 850 \text{ in. lb.}$$

$$M \text{ (dead)} = \frac{172 \times 3.67^2 \times 12}{12} = 2\ 320 \text{ in. lb.}$$

$$\text{Total } M = 31\ 570 \text{ in. lb.}$$

From Formula (11), page 485, $d = 0.028\sqrt{31\ 570} = 4.92$ inches.

Making $d = 5$ in. and allowing 1 inch below steel, a total thickness of 6 in. is required.

In considering shear, it will be permissible, in view of the very short span and the distributing action of fill and pavement, to take both live and dead loads as uniformly

distributed over full span. Live load of 3 900 lb. together with an allowance of 25% for impact would be equivalent to 1 540 lb. per sq. ft. when distributed over a length of 3.17 ft. With a dead load of 172 lb. per sq. ft. the total load is 1 712 lb. per sq. ft., giving a maximum shear $V = \frac{1712 \times 3.17}{2} = 2\ 700$ lb. per ft. width of slab.

Testing our slab for shear we have, from Formula (32), page 517,

$$v = \frac{2\ 700}{0.874 \times 5 \times 12} = 51.5 \text{ lb. per sq. in.}$$

While this exceeds the allowable unit of 40 lb. per sq. inch for diagonal tension, still, in view of the small loaded width assumed in our computations and the fact that the reinforcement will be bent diagonally through the slab near each end, we will permit this increasd value of diagonal tension and call the 6-inch slab O. K.

Required area of steel per inch width of slab from Formula (13), page 485,

$$A_s = \frac{31\ 570}{12 \times 0.874 \times 5 \times 16\ 000} = 0.0375 \text{ sq. in.}$$

This is provided by $\frac{1}{2}$-in. round bars 5 inches on centers.

Testing these bars for bond we find, from Formula (36), page 534,

$$u = \frac{2\ 700}{0.874 \times 5 \times 2.4 \times 1.57} = 164 \text{ lb. per sq. in.}$$

Since this is so greatly in excess of the allowable of 80 lb. per sq. inch it becomes necessary either to increase thickness of the slab or use deformed bars with which an allowable bond of 120 lb. per sq. inch may be used. Keeping 6-inch slab and using deformed bars we obtain, for the total circumference of bars in one foot width, from Formula (36a), p. 534,

$$\Sigma o = \frac{2\ 700}{0.874 \times 5 \times 120} = 5.15 \text{ in.}$$

Using a $\frac{5}{8}$-inch round bar, the circumference of which is 1.96 in., there will be required $\frac{5.15}{1.96} = 2.62$ bars per foot of width or a spacing of $\frac{12}{2.62} = 4.6$ in.

Therefore use $\frac{5}{8}$-inch round deformed bars $4\frac{1}{2}$ inches on centers.

Roadway Girders. The design of the girder is given below.

Bending moment. From mechanics a system of concentrated loads produces absolute maximum bending moment when placed so that the center of the span is midway between the resultant of all the loads on the span and the nearest load, this moment occurring under the nearest load.

Placing our auto truck in this position on a span of 26.3 feet, the loads will be located as shown in Fig. 219, since the resultant of the 6 000 lb. and 14 000 lb. loads spaced 12 feet apart is 3.6 feet from the 14 000 lb. load.

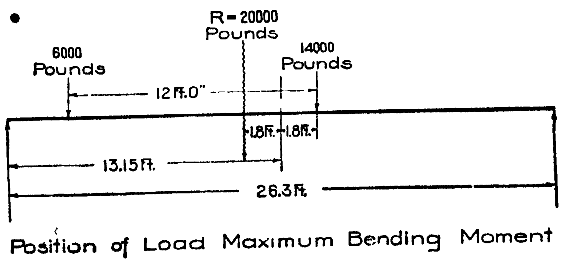


FIG. 219.—Position of Load for Maximum Bending Moment. (See p. 701.)

The maximum moment occurs under the 14 000 lb. load and is equal to

$$\frac{20\ 000\ (13.15 - 1.8)^2}{26.3} \times 12 = 1\ 176\ 000 \text{ in. lb.}$$

Distributing this over a width of 6 ft. we have the live bending moment per foot width of bridge as $\frac{1\ 176\ 000}{6} = 196\ 000$ in. lb. With the girders spaced 4 ft. 2 in. on centers each girder must therefore resist a moment, produced by live load, $M = 196\ 000 \times 4.17 = 818\ 000$ in. lb.

For dead loads we have 110 lb. for fill and paving, and 75 lb. for slab, giving 185 lb. per sq. ft., or $185 \times 4.17 = 772$ lb. per foot per girder. Assuming stem of girder to weigh 300 lb. per ft., we have a total dead load of 1 072 lb. per ft. per girder. Therefore the total maximum bending moment per girder is

$$\begin{aligned}
M \text{ (live)} & & &= 818\ 000 \text{ in. lb.}\\
M \text{ (impact)} &= 25\%\ 818\ 000 & &= 205\ 000 \text{ in. lb.}\\
M \text{ (dead)} &= \frac{1\ 072 \times 26.3^2 \times 12}{8} & &= 1\ 115\ 000 \text{ in. lb.}\\
\text{Total } M & & &= 2\ 138\ 000 \text{ in. lb.}
\end{aligned}$$

Maximum Shears. Maximum live shear will occur with the 14 000 lb. wheel load just inside the support and with the 6 000 lb. wheel as shown in Fig. 220.

Maximum shear due to these loads is equal to

$$\frac{(14\ 000 \times 25) + (6\ 000 \times 13)}{25} = 17\ 120 \text{ lb.}$$

Distributing this over a width of 6 feet, the live shear per foot width of bridge is

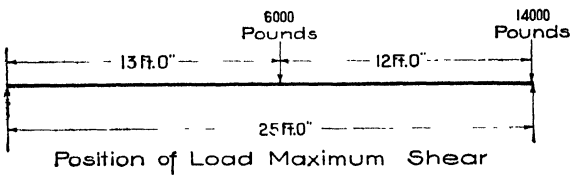


FIG. 220.—Position of Load for Maximum Shear at Support. (See p. 702.)

$\frac{17\ 120}{6} = 2\ 853$ lb. Therefore the maximum live shear per girder is $V = 2\ 853 \times 4.17 = 11\ 900$ lb. Hence the total maximum shear per girder is

$$\begin{aligned}
V \text{ (live)} & & &= 11\ 900 \text{ lb.}\\
V \text{ (impact)} &= 25\% \text{ of } 11\ 900 & &= 3\ 000 \text{ lb.}\\
V \text{ (dead)} &= \frac{1\ 072 \times 25}{2} & &= 13\ 400 \text{ lb.}\\
\text{Total } V & & &= 28\ 300 \text{ lb.}
\end{aligned}$$

Dimensions of Girder. The girder may be considered as a T-beam with a 6 in. flange. The effective width of the flange, which ordinarily could be taken as twelve times the thickness of slab plus width of stem (see p. 488), in this case is limited by the spacing of girders, which is 50 inches.

Selecting depth of girder equal $\frac{1}{10}$ of span and the depth below center of steel 4.5 inches, we have $d = 25.5$ in. and $h = 30$ in. The dimensions are evident from Fig. 218, p. 699.

Area of Tension Steel. The area of steel may be found from Formula (4a), p. 482.

$$A_s = \frac{2\ 138\ 000}{0.88 \times 25.5 \times 16\ 000} = 5.95 \text{ sq. in.}$$

Use six $1\frac{1}{8}$-inch round bars arranged in two layers of 3 bars each. Spacing the bars three diameters on centers and allowing a concrete covering of 2 in. on each side, the stem of the girder must be at least 12 in. wide.

The compression stresses in the flange found by the use of Table 13, page 588, is 481 lbs. per sq. in.

Checking the maximum stresses by the exact formulas (see p. 357) we find $f_c = 483$ and $f_s = 15\ 500$. Therefore section O. K. for bending moment.

Bars at Support. Consistent with our consideration of the girder as a simply supported beam, no top steel is theoretically required. It is nevertheless advisable to bend up some of the bottom bars at each end of the span obtaining thereby a material strengthening of the stem against shear and the corresponding web stresses.

Unit shearing stress as measure of diagonal tension. Using the maximum shear determined above the unit shearing stress in girder, from Formula (32), p. 517, is

$$v = \frac{28\ 300}{0.88 \times 25.5 \times 12} = 105 \text{ lbs. per sq. in.}$$

Therefore stem O. K. for shear.

Stirrups. Since the girder is subjected to moving load the variation of shear along the girder will be different than for uniformly distributed load (see p. 531). For determining stirrups, unit shearing stresses should be computed at the end of the beam and in the center of the span and a variation according to a straight line assumed.

The maximum unit shearing stress at support, as determined above, is 105 lbs. per sq. in., or $105 \times 12 = 1\ 260$ lbs. per width of beam. At the center of span the

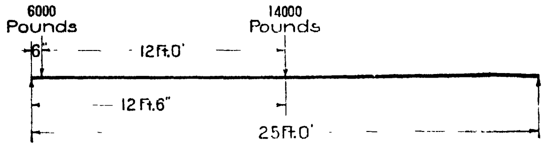


FIG. 221.—Position of Load for Maximum Shear at Center of Span. (*See p.* 703.)

shear due to dead load equals zero. Maximum live load shear at the center occurs when the load is in the position shown in Fig. 221 with the heavy load at the center of span.

The shear then is equal to $\frac{12.5}{25} \times 14\ 000 + \frac{0.5}{25} \times 6\ 000 = 7\ 120$ lbs. This can be distributed over 6 ft. and since the spacing of girders is 4 ft. 2 in. the proportion of shear carried by one girder is $7\ 120 \times \frac{4.17}{6.0} = 4\ 950$ lb. Allowing 25 per cent for impact the vertical shear at center of span is $4\ 950 \times 1.25 = 6\ 200$ lbs. and the shearing unit stress, Formula (32), page 517, $v = \frac{6\ 200}{0.88 \times 25.5 \times 12} = 23$ lbs. per sq. in. or $23 \times 12 = 276$ lbs. per the total width of stem.

The spacing of stirrups shown in Fig. 218 was determined as explained on page 529. It was found that it is necessary to extend stirrups only 10 ft. from support at each end since in the remaining portion the unit shearing stress never exceeds 40 lb. per sq. in. However, it is advisable to continue stirrups through the entire length by using 29 stirrups per girder instead of 26 actually required.

THROUGH GIRDER BRIDGES

Very often it is not possible to construct girders of the required depth below the roadway of the bridge without interfering with the required waterway, or the necessary clearance in overhead crossings. In such cases the girders are placed above the roadway, thereby reducing materially the depth of the construction below the roadway.

Fig. 222, p. 704, illustrates a typical design by the authors for a bridge of 30 ft. clear span for a loading given on p. 695. The bridge consists of 6 ft. deep girders spanning from abutment to abutment. The roadway supported by a slab is carried by joists which in turn are supported by girders. The sidewalk slab is carried by cantilevers which are a continuation of the roadway joists.

The method of design of through bridges, although essentially the same as of the girder bridges, is somewhat complicated by the fact that the loads are not carried directly by girders but are transferred to them by joists in panel points. Ordinarily, however, it is accurate enough to consider the loads as carried directly by the girders, in which case the determination of the bending moments and shears is the same as in previous examples.

The joists are suspended from the girders and as no reliance can be placed on the resistance of concrete in tension, enough steel must be used in the hangers to transfer the maximum joist reaction to the girders.

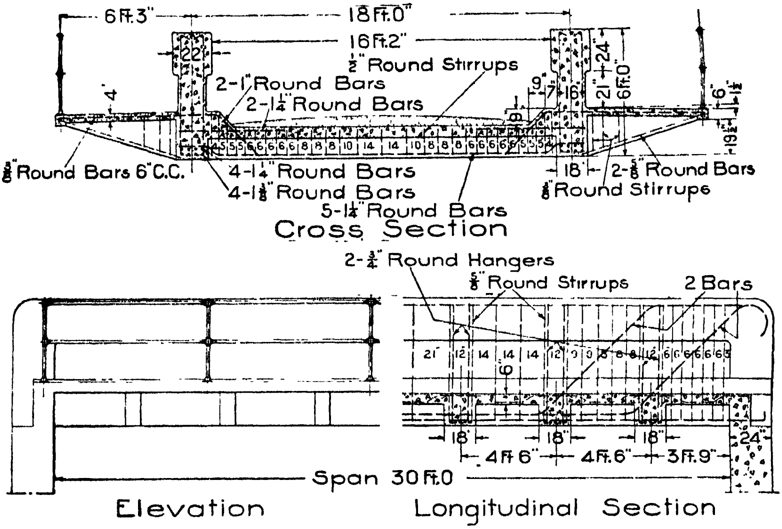


FIG. 222.—Design for a Girder Bridge (*See p.* 704.)

CONTINUOUS GIRDER BRIDGES

In crossings consisting of several spans, the girders may be designed and built as continuous. Fig. 223 illustrates a continuous girder bridge of three spans, designed by the authors to carry two tracks of electric railroad over a street.

The exterior spans were made smaller than the interior so as to make the maximum bending moments in the end panels about equal to those in the interior panels.

The principle of design is similar to that explained in connection with the design of a simple girder bridge, the main difference being in the method of determining the bending moment. In important bridges where it is warranted by the size of the job, it may be advisable to make a thorough study of the bending moments by the theorem of three moments. In smaller jobs the bending moments for the dead load should be taken as recommended on page 510. The bending moments for the live load must be determined by taking ¾ of the bending moment computed for a simply supported girder. This bending moment then may be considered as acting in the center of the span as positive bending moment, and at the support as negative bending moment.

Details of design are clearly shown in Fig. 223.

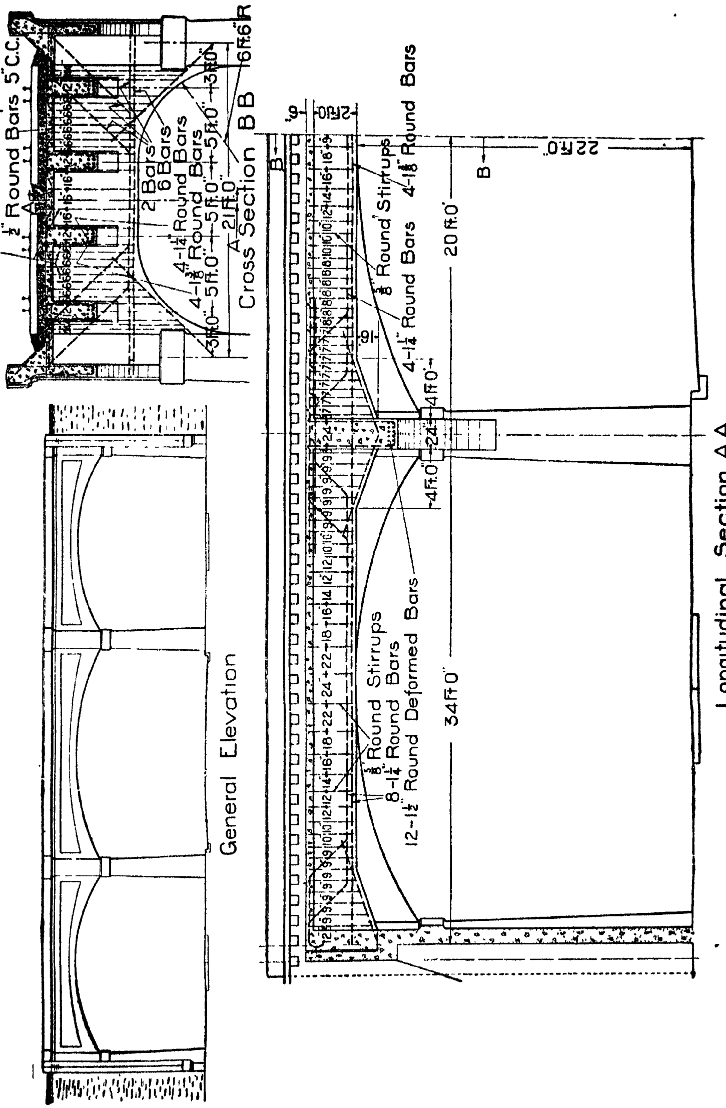


FIG. 223.—Illustration of a Continuous Girder Bridge. (See p. 704.)

FIG. 224.—Walnut Lane Bridge, Philadelphia. (*See p.* 750.)

CHAPTER XXVI

ARCHES*

BY FRANK P. McKIBBEN

The treatment of arch design by what is termed the elastic theory, although generally considered a complicated problem, as a matter of fact is easily handled by one who is familiar with elementary mechanics and with the principles of reinforced concrete beam design. The process is necessarily somewhat lengthy, involving extended operations in simple arithmetic, but by following the analysis presented in the following pages it can be readily understood. It is doubtful whether in the whole category of the design of structures there is a prettier application of mechanics and mathematics than the design of a reinforced concrete arch bridge.

While in a volume of this size it is impossible to present all phases of the subject, the underlying principles are treated in sufficient detail and with a discussion thorough enough to permit an engineer to safely design an arch.

Following a brief historical introduction discussing the use of concrete versus steel construction, the different forms of arches are reviewed with suggestions for design; the loading for different conditions is scheduled (p. 715); the outer forces are analyzed, including the effect of temperature (p. 723); the method of procedure to be followed in arch design is taken up in a practical example item by item (p. 733); allowable unit stresses are suggested (p. 741); the design of abutments is outlined (p. 741); and a few illustrations of existing bridges are presented.

Beam bridges are treated briefly in Chapter XXV. The design of such bridges follows closely the principles of reinforced concrete beam and slab construction as treated in Chapter XXII on Reinforced Concrete Design.

The treatment of conduit or sewer arches which are so deeply imbedded as to require computations for earth pressure is referred to on page 777.

Perhaps the most interesting feature of the present chapter is the complete analysis of a typical arch which is presented on page 733. The steps to be followed are outlined consecutively and the mathematical processes indicated in full.

The formulas for distribution of stress given on page 377 apply not only to column and beam design where there is eccentric loading or

* The authors are indebted to Prof. McKibben for this chapter, which has been especially prepared by him for this treatise.

thrust in place of or in addition to the ordinary loads but also to arch design.

To facilitate the understanding of the formulas, a departure is made from the usual notation schedule, which must necessarily be several pages away from the work, by placing in addition, at the bottom of each page, a brief definition of all the symbols used on that page.

CONCRETE VERSUS STEEL BRIDGES

Reinforced concrete, either as arch or girder spans, is being used not only in preference to steel trusses or steel girders, where the stone arch is too expensive to be considered, but the concrete bridge is frequently replacing the old steel structure. The reasons generally conceded for this widespread growth may be briefly stated as: (1) greater durability; (2) less cost of maintenance; (3) less vibration and less noise; (4) more æsthetic effects.

The relative first cost for concrete and steel depends upon the local conditions. In many places a concrete bridge can be built for less than a first-class steel span, although it cannot so readily compete with the flimsy trussed spans frequently seen. The concrete may be laid with less skilled labor than the steel bridge, but since the concrete structure is built on the spot, while the steel is prepared in an established shop, even more careful supervision and inspection are necessary with the concrete. The foundations for a concrete arch are frequently more expensive than concrete abutments for a steel truss because of the greater area required to take the thrust, while on the other hand, in rock or other hard material, a less quantity of concrete may be required for the arch abutments. This part of the design may often be the determining feature from the economical standpoint.

The most serious objection to steel, especially for highway bridges, lies in the fact that unprotected it cannot resist for a great length of time the oxidation due to air, water and locomotive gases, and unless properly cared for and frequently painted, it rusts badly. The examination by the author of this chapter of approximately 600 highway bridges carrying electric railways proves that frequently these bridges are not properly maintained, many of them receiving little or no attention for years at a time, so that the structures are often badly corroded, and in fact, cases are on record where subordinate members of steel bridges have rusted away completely in less than fifteen years.

In a concrete bridge the steel is effectively prevented from rusting by the concrete in which it is imbedded (see p. 292), so that, when properly designed and built, no repairs whatever should be required, and no limit can be placed upon the life of the bridge.

Concrete is strongest in compression, and is therefore eminently suitable for use in arch spans where the stresses are largely compressive. The mass of the concrete and the quantity of earth filling or ballast over the arch so deaden the impact due to traffic that in many cases no impact allowance need be made, while at the same time the noise and vibration which occur in steel spans are avoided.

USE OF STEEL REINFORCEMENT

The use of steel reinforcement in a concrete arch is desirable but not absolutely necessary, as it is possible to construct a concrete arch like the Walnut Lane Bridge in Philadelphia (see pp. 706 and 750) with the concrete laid in blocks, each block forming a voussoir like the stones in a masonry arch. At the same time under ordinary conditions, while the introduction of steel does not, with the present knowledge of concrete arch design, permit great diminution in section, it does give considerable added strength at comparatively low cost and may prevent the formation of cracks in the concrete and take tension caused by any unforeseen action of the arch, such as settlement of foundations, improper allowance for temperature or shrinkage of the concrete while hardening.

The area of the cross section of the longitudinal steel bars in solid arch rings is to a certain extent arbitrary. Good practice sanctions ½% to 1¼% of the ring at the crown and the exact quantity to use must first be selected by judgment, and then tested by the computation and revised if necessary.

As in column design (see p. 375), it is impossible to stress the steel in compression to an amount ordinarily proper in structural steel work, because in so doing the deformation would be so great as to overstress the concrete. The actual compressive stress in the steel, therefore, can never be greater than the working stress in the concrete multiplied by the ratio of the modulus of elasticity of steel to that of concrete. Under ordinary conditions this limit on the steel may be taken as 7500 pounds per square inch.

Since the beginning of this century there has been a remarkable development in methods of construction and in our knowledge of the principles of reinforced concrete arch bridges, but even yet engineers incline to employ a somewhat excessive quantity of concrete in the solid rings of ordinary highway concrete arches. This is frequently out of proportion to the quantity of material used in a reinforced concrete ribbed arch or a steel arch. Improvements in arch design evidently lie, as is indicated in subsequent pages, in the substitution of comparatively narrow ribs for solid arches and in the

use of hollow abutments with earth filling in place of solid concrete abutments. This will considerably reduce the cost of reinforced concrete arches.

HISTORY OF CONCRETE ARCH BRIDGES

In the development of concrete bridges it is natural that the arch rather than the beam should have been the first type of bridge to be constructed. It was a comparatively short step from the stone voussoir arch to the concrete voussoir or to the monolithic arch. One finds therefore many concrete arch bridges, and, until recently, few beam bridges, although for short spans beam bridges are now being constructed in considerable numbers, both in this country and abroad.

The first plain concrete arch of any importance was built in Europe in 1869 and is known as the Grand Maitre bridge at Fontainebleu Forest. It has a maximum span of 115.8 feet and carries the aqueduct of the Paris waterworks from Vanne. The first plain concrete arch in the United States was constructed in 1871 by John C. Goodridge in Prospect Park, Brooklyn, and has a span of 31 feet. The earliest reinforced concrete arch in Europe of which there is a well defined record was built in Copenhagen, Denmark, in 1879, with a span of 71.7 feet. It is probable, however, that Jean Monier of Paris was the inventor of the reinforced concrete arch and that he built some bridges before the dates mentioned. In the United States the first reinforced concrete arch on record was erected in 1889, with a span of 35 feet, by Ernest L. Ransome at Golden Gate Park in San Francisco.

When these structures are compared with the 233 feet span of the Walnut Lane Bridge in Philadelphia, which in 1908 was, with perhaps one exception, the longest plain concrete arch in existence, with the 230 feet, 3-hinge Grünwald Arch at Munich, Bavaria, or still more sharply with the Hudson Memorial design for an arch across the Spuyten Duyvil Creek with a span of 703 feet, a wonderful development is observed.

Although in a very few cases concrete bridges built during this development have failed, every such failure can be traced to a direct disregard of well known principles of design or construction. Moreover, as a matter of fact, accidents to concrete arches have been much fewer than the failures of wrought iron or steel bridges during the corresponding period of metal bridge development.

CLASSIFICATION OF ARCHES

Arches in general may be classified with reference to the material of which they are made, the arrangement of the spandrels and arch rings, or the

number of hinges. Reinforced concrete arches may be divided as to the arrangement of the reinforcement into three groups: the Monier, Melan and Wünsch types. The Monier arch in its developed form is the type most commonly used in the United States. This system of reinforcement was invented by Jean Monier about the year 1876. As first devised, a wire netting was imbedded in the concrete near the soffit, but later two nettings were used, one near the soffit, and the other imbedded in the concrete near the extradosal surface. Wire netting of small mesh with wires of equal size in both directions obviously is not well suited for use in an arch and considerable improvement was soon effected in this type by making the longitudinal bars of the reinforcement heavier than the transverse.

In the usual design a layer of longitudinal bars is imbedded near the intrados and an equal number near the extrados, the bars of the two layers being connected with small bars or stirrups. Transverse bars, at right angles to the longitudinal, form with them a netting both in the top and bottom of the arch. They serve to prevent cracks in the concrete and distribute the loads laterally. These cross bars also act with the stirrups in holding the longitudinal bars in place during construction.

The principal longitudinal bars are designed to carry tension due to the bending moment and to assist the concrete in compression caused by the thrust and the bending moment.

Melan Type. This system was invented by Joseph Melan of Brünn, Austria, in 1892. The reinforcement consists of curved steel ribs imbedded in the concrete and extending from abutment to abutment. For short spans the ribs are simply curved I-beams and for long spans each rib is made of two angles near the extrados latticed to two angles near the intrados. The built-up ribs thus formed are usually deeper at the springings than at the crown of the arch. The principal function of the lattice bars is to hold the angles in position when the latter are stressed, and to make a unit which is easy to handle during erection. By far the most important function of steel reinforcement is to carry bending moment, and the steel in the Melan type can be easily placed and kept in position during erection so as to fix positively its location in the finished structure. The material in the lattice bars of the ribs or in the webs of the I-beams is not economically placed. The first Melan arch in the United States, of 30 feet span, was erected at Rock Rapids, Iowa, in 1894, and many other bridges have since been built of this system.

Wünsch Type. Comparatively few bridges have been constructed on this system. The arch, which was invented by Robert Wünsch of Budapest, Hungary, in 1884, has a horizontal extrados and a curved intrados and the

reinforcement of the arch ring consists of steel ribs spaced from 1½ to 2 feet apart, with a horizontal upper member placed near the extrados and a curved lower member near the intrados. The two members are connected at each abutment to a vertical member imbedded in the concrete. The bridge at Sarajevo in Bosnia, of 83 feet span, is one of the largest built of the Wünsch system.

ARRANGEMENT OF SPANDRELS AND RINGS

The spandrel, which is the space between the roadway surface and the top or extrados of the arch ring, may be treated in one of two ways. First, it may be entirely filled with earth or with concrete which carries the roadway; or, second, it may be left more or less open, and the roadway supported upon a deck carried on a series of transverse walls, longitudinal walls, or columns resting upon the arch ring.

Filled Spandrels. In this form of construction the earth or concrete filling rests directly upon the arch ring, and is held in place laterally by retaining walls which also rest upon the arch ring. As the depth of these walls, unless they are of reinforced design, increases from the crown to the springing, their thickness, designed to resist the earth pressure, also increases until at the abutments the spandrels may be largely filled with the concrete composing the side walls.

If the side walls simply rest upon the arch ring, a crack is liable to form at the junction of ring and wall due to the deflection of the arch ring from the weight of the earth upon it. On the other hand, if the ring and wall are connected by sufficient steel to prevent the formation of this crack, indeterminate stresses are set up which are undesirable and which may result in transferring the crack to another place. This danger may be obviated by building the spandrel walls as gravity walls, leaving a vertical expansion joint at each junction of spandrel and wing walls and at some intermediate point between this joint and the crown.

Another plan is to build thinner reinforced side walls as vertical slabs tied together, with the lateral pressure resisted by reinforced cross walls. The principal objections to the use of solid fillings are as follows: (1) They increase the weight of the superstructure, and consequently thicker arch rings and larger foundations are required. (2) Unless the earth filling is carefully compacted by rolling, tamping or wetting, it will sink and allow the roadway to settle with it. (3) It is difficult to make the side walls and the arch ring act in unison, and unsightly cracks may be formed. Filled spandrels may be therefore limited properly to bridges with solid arch

rings of short span, say not over 80 feet, or to those having a rise of less than $\frac{1}{10}$ the span, where the cost of form construction prohibits an open design.

Open Spandrels. The objections just mentioned to the use of filled spandrels are of such importance that during the last few years the use of open spandrels in the larger structures has made rapid progress. In addition to being lighter, the open spandrel construction facilitates inspection and lends itself to more pleasing architectural treatment. It permits indeed a treatment peculiar to concrete, which does not follow the type of design used for so many centuries in stone arch bridges. With open spandrels the roadway may be laid upon small arches or upon I-beams carried by transverse or longitudinal walls which in turn rest upon the arch ring; or it may be laid with reinforced concrete beam and slab construction, making a floor similar to those used in reinforced concrete buildings. The beams in this case are placed longitudinally with the roadway, and rest upon transverse walls.

Upon the adoption of the open spandrel it was soon seen that considerable material was wasted in the transverse walls and in the solid arch rings. The next step, therefore, was to reduce the walls to columns and the ring to a series of longitudinal ribs spaced similarly to the ribs of a steel arch. In some cases these ribs are very wide, in fact, are really two independent arch rings as in the Walnut Lane bridge, Philadelphia,* and in other cases the ribs are narrow as in the Rock Creek bridge on Ross Drive in the District of Columbia.†

HINGES

The use of hinges in concrete arches is by no means of recent origin. As early as 1873, an arch was constructed near Erlach, Germany, with three asphalt "joints" and many others with hinges have been built since then. The chief object of the hinge in the arch rings or ribs is to render the structure more nearly determinate.

Although two or even one hinge can be used, three hinges offer the advantage of definitely fixing the pressure line throughout the ring so that it can be easily and accurately located. Except for the friction of the hinges, the stresses are practically independent of changes of temperature or of any reasonable settlement of the foundations. On the other hand, the hinges are often an expensive detail. It is sometimes claimed also that three-hinged arches are not so rigid as fixed arches, but because of their great weight this criticism does not appear to be well founded.

* See p. 750.
† See p. 748.

In the design of a hinged structure the moment is usually assumed to be zero at the hinge. This assumption is not strictly correct because as the structure deforms under its load it tends to rotate about its hinges and this produces friction at the hinge due to the thrust acting thereon.

The design of the hinge is a most important feature. One of the most instructive failures in arch construction was that of the Maximilian Bridge at Munich, a three-hinged voussoir masonry arch of two spans, each 144.3 feet, when during construction, both spans of the bridge slipped off the hinges at the springings and dropped about 12 inches. This failure was due to an error in the design of the hinges. The bearing surfaces of the hinges were not given sufficient curvature, and the friction which was relied upon to prevent slipping of the two parts composing each hinge was reduced to a minimum by the use of a lubricant, which gave a low coefficient of friction.

Three-hinged construction is best suited to arches of small rise where the center line of the rib can be made to fit closely the line of pressure resulting in small bending moments. Arches with one or two hinges are more indeterminate than three-hinged arches and have practically all of the disadvantages of both the fixed and the three-hinged types.

SHAPE OF THE ARCH RING

For hingeless arches the intrados should be either three-centered, five-centered or elliptical, while, if desired, the extrados may be the arc of a circle so placed as to give greater depth to the arch ring at the springings than at the crown. A segmental arch, that is an arch formed by the segment of a single circle cannot often be used to advantage, for it seldom can be made to fit the line of pressure. While many arches are elliptical in form, the three-centered intrados is perhaps the most common and it is pleasing to the eye, easily constructed and gives an economical design.

Ribs with three hinges should be deepest at sections nearly midway between the crown and spring hinges, decreasing in depth toward the hinges, since sections near the hinges take only thrust and shear with practically no moment, while the intermediate sections resist a moment in addition to the thrust and shear.

THICKNESS OF RING AT CROWN

The next step in the design of an arch after deciding on the shape of the intrados is to choose a trial thickness of the ring at the crown and at the springing. The choice may be made by judgment based on experience or

with the aid of one of the various empirical formulas in use. Since the crown thickness depends not only on the amount of thrust but also upon the bending moment, which varies greatly in a given arch due to the varying positions of the live load, it is difficult and in fact impossible to devise a rational formula for its determination.

The thickness of the arch ring should vary with the shape of the arch, with the span, rise, amount of filling over the ring, the amount of live load and the material of which the arch is made, and while there is no formula that will apply even approximately in all cases, the formula by Mr. F. F. Weld* gives fairly correct results in ordinary cases. It is as follows:

Let

h = crown thickness in inches.

L = clear span in feet.

w = live load in pounds per square foot, uniformly distributed.

w' = weight of fill at crown in pounds per square foot.

Then

$$h = \sqrt{L} + \frac{L}{10} + \frac{w}{200} + \frac{w'}{400} \qquad (1)$$

Obviously the thickness for a hingeless arch should increase from the crown to the springing. The radial thickness of the ring at any section is frequently made equal to the thickness at the crown multiplied by the secant of the angle which the radial section makes with the vertical. For a 3-centered intrados and an extrados formed by the arc of a circle, these trial curves may be at the quarter points a distance apart of $1\frac{1}{4}$ to $1\frac{1}{3}$ times the crown thickness and at the springings 2 to 3 times the crown thickness.

These empirical rules should be used only in preliminary study and *never for the final design.* The true shape of the ring and the thickness at different sections must be fixed by computation based on the line of pressure as described in the pages which follow.

LIVE LOADS FOR HIGHWAY BRIDGES

For highway bridges the kind and magnitude of the live load depend upon the location of the structure. Each location should be studied and the live load chosen to fit the requirements. The following classification is sufficient for stone or concrete arches or for beam bridges.†

* *Engineering Record*, Nov. 4, 1905, p. 529.

† Loads for beam bridges are discussed on page 695.

City Bridges. For *floors* of city or other bridges carrying heavy traffic, three types of loads are recommended as follows:

1. A uniform live load of 100 pounds per square foot on sidewalks and roadways.
2. On each street railway track, one 8-wheel electric car having a wheel spacing of 5, 15, 5 feet between centers of wheels along one rail; each wheel carrying 12,500 pounds. The car is assumed to cover an area 9 feet wide by 40 feet long.
3. One wagon weighing 20,000 pounds on each of two axles 12 feet apart.

In applying these loads to find the maximum stress in the floor, either of the loads mentioned, or that combination of any of the above loads which produces the maximum stress, should be used. If the uniform load is used simultaneously with either of the concentrated loads, the former should cover only that part of the roadway not covered by the latter.

For *arch rings* or *ribs* having a span of 100 feet or less, a uniform load of 1800 pounds per linear foot of each railway track together with a uniform load of 100 pounds per square foot of remaining area of roadway and sidewalks.

For spans of 200 feet or more, a uniform load of 1200 pounds per linear foot of each railway track together with a uniform load of 80 pounds per square foot of remaining area of roadway and sidewalks.

The load on each track should be assumed to cover a width of 9 feet, thus giving 200 pounds per square foot under the track for spans of 100 feet or less and 133 pounds per square foot for spans over 200 feet in length.

For spans between 100 and 200 feet, the loads are to be taken proportionally.

Suburban, Town or Heavy Country Bridges. For *floors* of suburban, town, or heavy country bridges, the same uniform load and electric car load as for floors of city bridges but with wagon weighing 10,000 pounds on each of two axles 10 feet apart.

For *arch rings* or *ribs* having a span of 100 feet or less, a uniform load of 1800 pounds per linear foot of each track, together with a uniform load of 80 pounds per square foot of remaining area of roadway and sidewalks.

For spans of 200 feet or more the values corresponding to the above are 1200 pounds per linear foot of each track and 60 pounds per square foot of remaining area.

The load on each track should be assumed to cover a width of 9 feet.

For spans between 100 and 200 feet, the loads are to be taken proportionally between the limits stated.

Light Country Bridges. For *floors* of light country bridges, sub-

jected to light highway or electric railway traffic, on each track one 8-wheel electric car carrying 9000 pounds on each wheel, or one wagon weighing 6000 pounds on each of two axles 10 feet apart. These two loads should be assumed to act together where necessary to produce the maximum stress in the floor.

For *arch rings* or *ribs* having a span of 100 feet or less, a uniform load of 1200 pounds per linear foot of each track, together with a uniform load of 80 pounds per square foot of remaining area of roadway.

For spans of 200 feet or more, the values corresponding are 1000 pounds per linear foot of each track, and 50 pounds per square foot of remaining area.

For spans between 100 and 200 feet the loads are proportional between the limits stated.

It is customary to see that the design is sufficient to carry a steam road roller. The heaviest roller usually specified weighs 30,000 pounds, 12,000 pounds on the front roller, which has a width of 4 feet, and 9000 pounds on each of the two rear rollers, each of the latter having a width of 20 inches. The axles are taken as 11 feet apart and the two rear wheels as 5 feet center to center.

LIVE LOADS FOR RAILROAD BRIDGES

For railroad bridges the loading depends upon the location of the line, and hence the future traffic which may be expected. Two consolidated locomotives, with 25 000 pounds on each driving wheel, followed by 5000 pounds per foot of each track, is a common loading. An alternate plan quite generally followed for the rings of stone or concrete arches where the filling is of sufficient thickness to distribute the concentrated loads over a considerable area of arch ring is to use 5000 pounds per foot of track with no concentrated load. This load of 5000 pounds per foot of track is equivalent to about 625 pounds per square foot of horizontal area. These values are satisfactory for spans, say, over 80 feet in length.

Generally speaking, the shorter the span the greater should be the assumed uniform load, and hence for spans of, say, 80 feet or less, a uniform load of 1000 pounds per square foot is frequently adopted, this being approximately equivalent to the heaviest locomotive loadings.

A concentrated load on top of a fill is generally assumed to be distributed downward at angles of 45°. The top of the distributing slope may be taken from the ends of the ties. Wheel loads may be taken as distributed over 3 feet of length of surface of fill and at 45° angles through the filling.

DEAD LOADS AND EARTH PRESSURE

With open spandrels having columns or transverse walls, the dead loads act vertically upon the arch ring and can be more accurately found than with filled spandrels.

With spandrels filled with earth the dead load carried by the arch ring is that due to the weight of the roadway, of the filling, and of the arch ring itself. The earth filling is usually assumed to act vertically, in which case the forces acting on the arch are easily computed. For arches in which the ratio of rise to span is small, such an assumption is sufficiently correct. A common assumption for weight of earth fill where the actual value is unknown is 100 pounds per cubic foot.

Since the pressure produced by the earth filling against the extradosal surface of the ring is really inclined, being nearly vertical near the crown and considerably inclined near the springings, it is sometimes advisable in an arch of large rise to take account of the horizontal component of the pressure near the springings. The earth pressure acting against an inclined plane may be found either algebraically or graphically. The algebraic solution is given under the subject of retaining walls, page 759, and in the example of arch design the inclined pressure is taken into account for illustration, although it is really unnecessary in the case selected. (See p. 734.)

OUTLINE OF DISCUSSION ON ARCH DESIGN

The method of designing an arch by the elastic theory is illustrated by the example on pages 733 to 740. The steps to be taken are there stated in full.

In the following pages the reactions at the supports, which in an arch are not simple vertical forces, and the relations between the outer loads and the internal stresses, are first treated briefly so as to understand the theory in a general way. Next (p. 727), the working formulas are given for finding the thrust, shear and bending moment at the crown, and at intermediate points in the arch ring. From these, the force polygon and the line of pressure, which is an equilibrium polygon drawn for a pole distance equal to the horizontal thrust, may be drawn (p. 729). The method of determining the stresses due to temperature and rib shortening is given (p. 730). Since the lines of pressure do not ordinarily pass through the center line of the arch ring, the pressures on the various sections are eccentric. The distribution of stress in an arch under eccentric loading is the same as in any other member, such as a column. The analysis is discussed at length on pages 377 to 389, Chapter XX. Diagrams are presented

to aid in the determinations. Following the example, the design of arch abutments is given (p. 741), and beyond this are general directions with reference to construction details. Several typical arches are illustrated (p. 747).

RELATION BETWEEN OUTER LOADS AND REACTIONS AT SUPPORTS

An arch differs from a beam in that under vertical loads the reactions at the supports of the arch are inclined, while for a beam the reactions are vertical. The loads acting on the arch, together with the reactions caused by the loads, constitute the entire system of forces acting, and for a complete analysis of the arch the relation between these forces should be determined. This relation is more simply deduced if for each reaction there are substituted its horizontal and vertical components.

For arches symmetrical about the center line of span the following analysis is applicable. For unsymmetrical arches, methods similar to those presented in the following pages are to be employed although the necessary formulas are too long to be given here.

NOTATION

H_1 and V_1 = horizontal and vertical components of the left reaction.
H_2 and V_2 = horizontal and vertical components of the right reaction.
M_1 and M_2 = moments at left and right supports respectively.
M = moment at any point on arch axis having coördinates x and y.
M_c, H_c, V_c = moment, thrust and shear at the crown.
M_L = moment at any point on left half of arch axis of all loads between the point and the crown.
M_R = moment at any point on right half of arch axis of all outer loads between the point and crown.
m = number of divisions into which the half length of arch axis is divided.
s = short length of arch axis.
I = moment of inertia of cross section about the gravity axis.
L = horizontal span of arch axis.
r = rise of arch.
E_c = modulus of elasticity of concrete.
n = ratio of moduli of elasticity of steel to concrete.
R = resultant force acting on any section of the arch ring.
N = thrust = normal component of resultant R.
V = shear = radial component of resultant R.

H = horizontal component of resultant R.
P = any concentrated load.
Δ_L = change in span length due to any cause, + for an increase, − for a decrease.
t = rise or fall in temperature of the arch ring from the mean in degrees Fahrenheit.
c = coefficient of linear expansion or contraction.
f = average unit compression in concrete of arch ring due to thrust.
ϕ = central angle subtended by the axis of the arch.
x,y = coördinates of any point on the axis of the arch ring.

Three-Hinged Arch. The use of the three-hinged arch is discussed on page 713. Since its analysis is simplest and at the same time illustrates important principles of arch design, it is considered first.

Referring to Fig. 225, it is seen that there are two unknown components

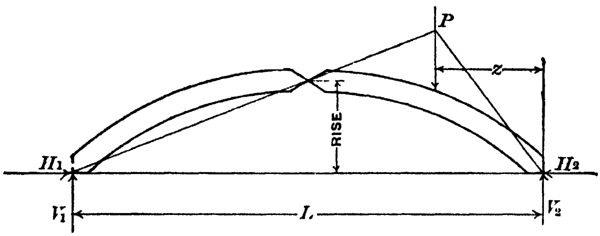


FIG. 225.—Arch with Three Hinges. (*See p.* 720).

of each reaction, making four unknown quantities, H_1, V_1, H_2, V_2, which require four equations to solve them. From statics we have the three equations of equilibrium:

Algebraic sum of vertical components = zero.

Algebraic sum of horizontal components = zero.

Algebraic sum of moments of all forces about any point = zero.

We have here an additional equation from the fact that the bending moment at the crown hinge = 0. Therefore the four components of the reactions can easily be found. Suppose there is only one load, P, on the span. Then

$$V_1 = \frac{Pz}{L} \quad (2) \qquad \text{and } V_2 = \frac{P\,(L-z)}{L} \qquad (3)$$

Since, for equilibrium, the moment at the crown hinge must be 0, the resultant reaction on the left must pass through the left hinge, or

$$V_1\left(\frac{L}{2}\right) - H_1 r = 0. \quad \text{Hence } H_1 = \frac{V_1 L}{2r} \qquad (4)$$

V_1, H_1 = components of left reaction. V_2, H_2 = components of right reaction. L = span. r = rise.

When all loads are vertical, or in any case when the loads are symmetrical about the center, $H_1 = H_2$.

When the loads are not symmetrical and also not vertical, H_2 can be easily found, after H_1 has been determined as above, from the relation that the algebraic sum of all the outer horizontal forces = o. In a three-hinged arch, then, the reactions having been found by means of simple statics as above described, the thrust, shear and bending moment on any section of the arch can be computed and sections designed.*

Two-Hinged Arch. Under the action of the loads on this arch there are produced two components of the reaction at each support, making in all four unknowns, H_1, V_1, H_2, V_2. From statics we have the three fundamental equations of equilibrium, as given above. We must find an additional equation from the theory of elasticity. This additional equation is obtained from the fact that the span does not change its length under the

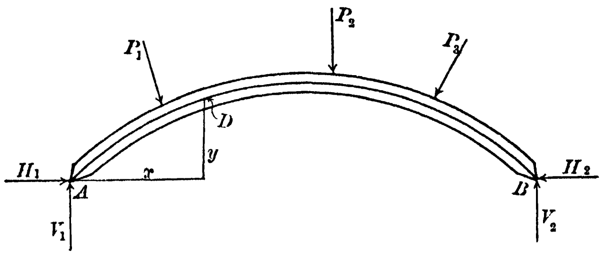


FIG. 226.—Two-Hinged Arch. (*See p.* 721).

action of the loads. From mechanics† we know that if the arch were fixed at B and free at A, the horizontal motion of A (the origin of coördinates) is given by $\Sigma My \frac{s}{EI}$, where Σ denotes the summation of the products of $My \frac{s}{EI}$ for each section of the arch. Now, since the arch is really prevented by the support from moving horizontally at point A, the above deformation can be placed equal to o, and we have then the fourth equation $\Sigma My \frac{s}{EI} = 0$, which, in addition to the three from statics, enables us to find the reactions H_1, V_1, H_2, V_2. As soon as the reactions are known, the thrust, shear and bending moment at any section of the arch can be found.

*Three Hinged Masonry Arches; Long Spans Especially Considered, by David A. Molitor, Transactions American Society of Civil Engineers, Vol. XL, p. 31.

†"Mechanics of Engineering," by Irving P. Church, 1908, p. 449.

In a similar manner the conditions of equilibrium can be obtained for an arch with only one hinge (at the crown).

"Fixed" or "Continuous" Arches. A method frequently followed with the hingeless arch is to consider the reactions at the ends in the same way as in hinged arches, but the simpler method is to take the forces at a section through the crown. However, in order to better understand the theory and the relation of the external to the internal forces, the arch reactions at the supports will be discussed first and afterward the analysis will consider the forces at the crown.

Let Fig. 227 represent a hingeless arch. The loads having been determined, there are at each support three unknown quantities, namely, the vertical and the horizontal components and the point of application of the reaction. Or, instead of saying that the point of application of the reaction

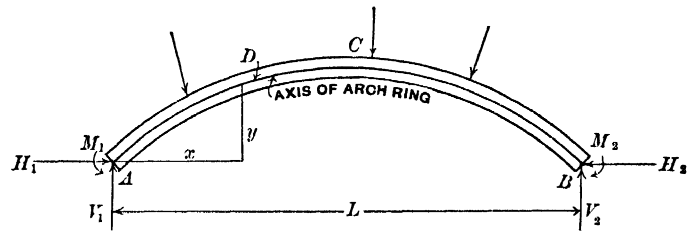


FIG. 227.—Continuous Arch. (*See p. 722*).

is unknown, we can say that there is a bending moment at each support, and that this moment, together with the horizontal and vertical components of the reaction, makes three unknown quantities at each support to be found. There are then six unknown quantities to be determined, namely, H_1, V_1, M_1, H_2, V_2, M_2.

Statics provides the three fundamental equations of equilibrium (see page 720), hence three additional equations must be determined from the theory of elasticity. These three additional equations are given from the three following conditions:

The change in span of the arch $= \Delta x = O$

The vertical deflection at A (the origin of coördinates) $= \Delta y = O$

The change in direction of the tangent at the arch axis at A $= \Delta\phi = O$

These three conditions must be true since the arch is fixed at A and at B, the abutments being assumed immovable

From mechanics,*

$$\Delta x = \Sigma_A^B My \frac{s}{EI} = O \tag{5}$$

$$\Delta y = \Sigma_A^B Mx \frac{s}{EI} = O \tag{6}$$

$$\Delta \phi = \Sigma_A^B M \frac{s}{EI} = O \tag{7}$$

These three equations are general formulas. They are not used directly in arch computations but are necessary in the theoretical derivation of the working formulas given in paragraphs which follow.

These three equations express the conditions that the horizontal, vertical and rotary movements of the left end of the arch ring each equal zero, so far as these motions are caused by the *bending moments only*, acting on the different sections from B to A. The movements due to the *thrust* and *shear* within the ring are not here considered. By means of equations (5), (6), (7) and the three from statics (see p. 720) we can solve for the six unknown quantities at the supports, namely, the horizontal and vertical components of each reaction and the moment at each support, and having thus found the reactions, the stresses within the ring can be computed.

RELATION BETWEEN OUTER FORCES AND THE THRUST, SHEAR AND BENDING MOMENT FOR THE FIXED ARCH†

In Fig. 228 let the arch A B be fixed at the two supports. If the loads are known, the horizontal and vertical components of the reactions and also the moment at each support of the arch may be found, as has been shown above. Having these three quantities for each support, the *point of application* of each reaction may then be determined.

Thus in Fig. 228 the point of application at the left support is at *a*, distant y_1 vertically from A, where $y_1 = \frac{M_1}{H}$. Similarly at B, $y_2 = \frac{M_2}{H}$. Having computed y_1 and y_2, thus locating the points of application of the reactions, the force polygon and its equilibrium polygon, *a b c d*, can be drawn, as described more fully on page 735, and the latter will be the true line of pressure for the loading shown. The stresses on any section such as D may

M = moment. s = short length of arch axis. E = modulus of elasticity. I = moment inertia. Δx = change of span length. xy = coördinates of a point.

*See "Mechanics of Engineering," by Irving P. Church, 1908, p. 449, or any general treatise on mechanics.

† This method of analysis corresponds to that adopted by Messrs Turneaure and Maurer in their book on "Principles of Reinforced Concrete Construction."

be then studied. The resultant of all outer forces on the left of D is a force acting along the line $a\ b$ of the equilibrium polygon and having a magnitude equal to the force O_0 of the force polygon. This resultant outer force O_0

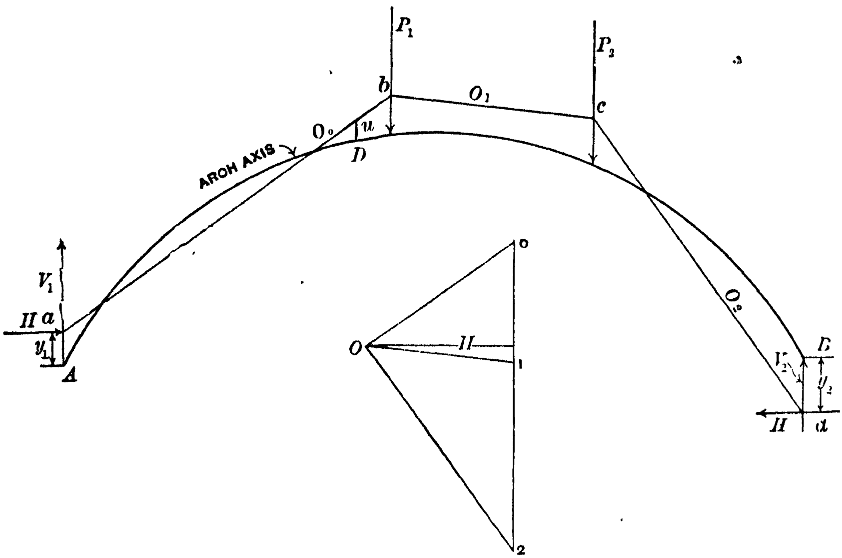


FIG. 228.—Line of Pressure in an Arch. (*See p.* 723).

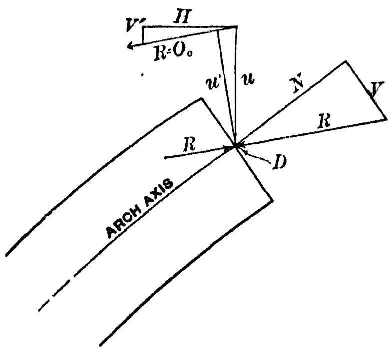


FIG 229.—Forces Acting upon an Arch Section. (*See p.* 724.)

acting along *ab* is resisted by inner forces, i. e., stresses, on the section *D* which is redrawn in Fig. 229.

The force *R* is the force opposing the resultant O_0. This force is equiva-

lent to a force R acting at the arch axis and a bending moment $= Ru' = Hu$, where H is the *horizontal* component of R and u is the *vertical* distance from point D on the arch axis to the equilibrium polygon; u' is the *perpendicular* distance from point D to the force $R = O_0$. For vertical loads H is constant throughout the length of the arch ring.

The resultant force R acting at D can be resolved into two components one of which, N, is *tangential* to the axis at D and therefore normal to the section of the arch ring; the other component, V, is *perpendicular* to the axis and parallel to the section.

N is the *thrust*, that is, the tangential component of the resultant force on the section.

V is the *shear*, that is, the radial component of the resultant force on the section.

Hu or Ru' is the *bending moment* about the gravity axis of the section.

Evidently there are sections of the arch where the equilibrium polygon intersects the arch axis. At these sections the bending moment is zero. Furthermore, if the equilibrium polygon is normal to any section there will be no shear on that section. It is possible then to find sections where there is no moment, or no shear, or possibly where there is neither moment nor shear. There is always a *thrust* on every section.

THRUST, SHEAR AND MOMENT AT THE CROWN

Instead of actually finding the components of the reactions and the moments at the supports by the plan indicated on page 723, it is simpler to find the thrust, shear and moment at the crown. Having these, the equilibrium polygon may be drawn and the thrust, shear and moment at any point may be found. The thrust, shear and moment at the crown can be found by use of equations (5), (6), (7), page 723, in which M is the moment of any point D of Fig. 228, page 724, expressed in terms of the values at the crown. Instead, however, of determining these quantities by means of these equations, shorter expressions for the thrust, shear and moment at the crown may be obtained by *taking the origin of coordinates at the crown and studying the motion at that point.*

In Fig. 230, CD represents the vertical section at crown, upon which acts the resultant pressure along the line AB. In the lower part of the figure, for this resultant force is substituted the horizontal thrust, H_c, the shear, V_c, acting at the center of the section CD, and the moment M_c.

Referring to Fig. 231, page 726, and accepting C as origin of coördinates,

Let

$x, y,$ = coördinates of any point D,

M_L = moment at any point D on left half of arch axis of all loads between the point and the crown.

M_R = moment at any point D on right half of arch of all loads between the point and the crown.

m = number of divisions of half of the arch axis.

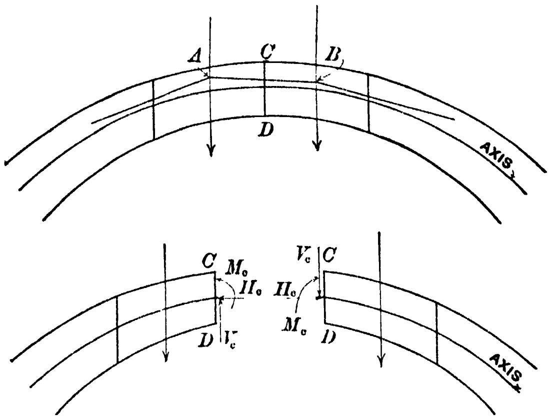


FIG. 230.—Moment and Thrust at the Crown. (*See p.* 725.)

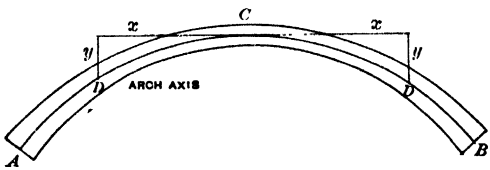


FIG. 231.—Coördinates of Any Point in Arch Axis. (*See p.* 725.)

The formulas given below require that the arch be divided so that the ratio of length of any division to its average moment of inertia is constant. Because of this requirement the end divisions with large moments of inertia may be long, even with comparatively short divisions at the crown. This may cause an inaccuracy which can be largely eliminated by subdividing the load on the end divisions.

The greater the number of divisions the more accurate the results.

For an arch divided in such a way that the ratio of the length of any division to its average moment of inertia is constant (see page 728)

the three unknown quantities, V_c, H_c, and M_c may be found from formulas*

$$H_c = \frac{m\Sigma M_R y + m\Sigma M_L y - \Sigma M_R \Sigma y - \Sigma M_L \Sigma y}{2\left[m\Sigma y^2 - (\Sigma y)^2\right]} \tag{16}$$

$$V_c = \frac{\Sigma M_L x - \Sigma M_R x}{2\,\Sigma x^2} \tag{17}$$

$$M_c = \frac{\Sigma M_R + \Sigma M_L - 2\,H_c\,\Sigma y}{2m} \tag{18}$$

---

*The horizontal motion of C, Fig. 231, as in preceding analysis, due to bending moments on sections between B and C, is $\Sigma{}_C^B My\frac{s}{EI}$. The horizontal motion of C due to the bending moments on sections between A and C, is $\Sigma{}_C^A My\frac{s}{EI}$. These two motions are equal but opposite in direction, hence,

$$\Sigma{}_C^B My\frac{s}{EI} = -\Sigma{}_C^A My\frac{s}{EI} \tag{8}$$

Similarly the vertical motions at C are equal,

$$\Sigma{}_C^B Mx\frac{s}{EI} = \Sigma{}_C^A Mx\frac{s}{EI} \tag{9}$$

Also the changes in direction of the tangent to the axis at C are equal, but opposite in direction, hence,

$$\Sigma{}_C^B M\frac{s}{EI} = -\Sigma{}_C^A M\frac{s}{EI} \tag{10}$$

If each half of the arch axis be divided into m divisions in such a way as to make $\frac{s}{I}$ constant for all the divisions (See p. 728) the factor $\frac{s}{I}$ and also E may be cancelled. In the equations (8), (9) (10), M, I, x, y, denote respectively the bending moment, moment of inertia of the cross-section, and coördinates at the center point of each division of the arch axis.

At center of any division between A and C the bending moment is

$$M = M_C - V_C x + H_C y - M_R \tag{11}$$

At center of any division between B and C the bending moment is

$$M = M_C + V_C x + H_C y - M_L \tag{12}$$

Placing these values of M in equations (8), (9) and (10) and collecting terms, we have

$$2\,M_C\Sigma y + 2\,H_C\Sigma y^2 - \Sigma M_R y - \Sigma M_L y = 0 \tag{13}$$

$$2\,V_C\Sigma x^2 - \Sigma M_L x + \Sigma M_R x = 0 \tag{14}$$

$$2\,mM_C + 2\,H_C\Sigma y - \Sigma M_R - \Sigma M_L = 0 \tag{15}$$

Combining (13) and (15),

$$H_C = \frac{m\Sigma M_R y + m\Sigma M_L y - \Sigma M_R\Sigma y - \Sigma M_L\Sigma y}{2\left[m\Sigma y^2 - (\Sigma y)^2\right]} \tag{16}$$

From (14) $$V_C = \frac{\Sigma M_L x - \Sigma M_R x}{2\,\Sigma x^2} \tag{17}$$

From (15) $$M_C = \frac{\Sigma M_R + \Sigma M_L - 2\,H_C\Sigma y}{2\,m} \tag{18}$$

M = moment. H_C = crown thrust. V_C = crown shear. m = number divisions of half axis. x, y = coördinates of a point.

These are fundamental equations in arch analysis. The method of application is illustrated in the example, page 733.

All Σ signs denote summations for *one-half* of the arch axis.

All numerical values of M_L, M_R, x, y, are positive.

A positive value of V_c indicates that the line of pressure at the crown slopes upward toward the left; a negative value, upward towards the right.

A positive value of M_c indicates a positive moment at the crown; a negative value, a negative moment.

The moment at any point between B and C is

$$M = M_c - V_c x + H_c y - M_R \tag{19}$$

while at any point between A and C

$$M = M_c + V_c x + H_c y - M_L \tag{20}$$

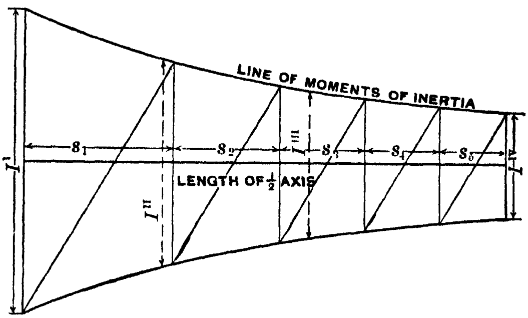


FIG. 232.—Diagram for finding Constant $\frac{s}{I}$. (*See p.* 728)

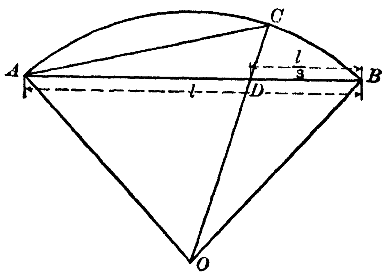


FIG. 233.—Diagram for finding Length of Arc of a Circle. (*See p.* 728.)

GRAPHICAL METHOD FOR FINDING CONSTANT $\frac{s}{I}$.

Fig. 232 and Fig. 233 give a graphical method of determining the length

M = moment. H_c = crown thrust. V_c = crown shear. x, y = coördinates of a point s = length of division of axis. I = moment of inertia

of divisions for a constant $\frac{s}{I}$. If the arch axis is made up of arcs of circles, the length of any arc ACB is equal to three halves of the straight line AC.* The point C is found in Fig. 233 by dividing the chord AB into thirds and drawing a radius through the one-third point. If the arc is an ellipse, a simple method of drawing which is given on page 192, the length may be measured from the drawing. Having found the length of the half axis and drawn it as a horizontal line, the constant $\frac{s}{I}$ is found as shown in Fig. 232 by computing four or more values of I, the moment of inertia, at different points and plotting these to locate the curves as shown. Beginning at the lower left corner of the diagram, trial diagonals (parallel to each other) and vertical lines are drawn, so that the number of spaces between the verticals will represent the number of divisions into which the half arch must be divided. If at the first trial the final diagonal does not come out exactly at the upper right corner which represents the crown of the arch, a new slope is tried for the parallel diagonals.

LINE OF PRESSURE

Having determined the thrust and moment at the crown, the line of pressure may be drawn as shown in folding Fig. 235, opposite page 738, from which the compression and tension at different sections may be found after determining the thrust and eccentricity from the formulas which follow.

It is well to draw the line of pressure before considering the temperature and the effect of the rib shortening, and then afterwards study these, adding or deducting the stresses for the most unfavorable conditions.

EFFECT OF TEMPERATURE AND THRUST

The thrust acting throughout the ring tends to shorten the span. A change of temperature of the ring tends to shorten the span when the temperature falls or to lengthen the span when the temperature rises. The tendency for the span to change its length by a distance Δ_L due to any cause is resisted by a horizontal component H and a moment M_1 acting at each support, and by a thrust and moment in the arch ring. Δ_L is positive for an increase and negative for a decrease in span length.

*Method given in Nouvelles Annales de Mathematiques, Jan. 1907. The error for 40 degrees is less than $\frac{1}{10000}$, for 70 degrees is less than $\frac{1}{1000}$, for 90 degrees is less than $\frac{5}{1000}$.

The thrust and moment at the crown may be found from formulas*

$$H_c = \frac{I}{s}\,\frac{mE\,\Delta_L}{2[m\Sigma y^2 - (\Sigma y)^2]} \tag{23}$$

and

$$M_c = -\frac{H_c\Sigma y}{m} \tag{24}$$

Rise in Temperature. Under a rise of temperature of the arch ring of t degrees Fahr. the span L would tend to increase in length an amount of ctL, c being the coefficient of linear expansion. Substituting for Δ_L in (23) the value of ctL, the thrust at crown is

$$H_c = \frac{I}{s}\,\frac{ctLmE}{2[m\,\Sigma\, y^2 - (\Sigma\, y)^2]} \tag{25}$$

The value of the temperature coefficient, c, in equation (25) may be taken for concrete as 0.0000055. Dimensions must all be in same units; if in feet, E *must be in pounds per square foot.* Using a value of E_c of 2,000,000, E is therefore 2,000,000 $\times$ 144= 288,000,000 pounds per square foot.

Moment at crown is

$$M_c = -\frac{H_c\,\Sigma\, y}{m} \tag{26}$$

*The change in total span length, the two halves of the arch being equal, is

$$2\Sigma_C^A\, My\,\frac{s}{EI} = \Delta L \tag{21}$$

The change in inclination of tangent to axis at crown is

$$2\Sigma_C^A\, M\,\frac{s}{EI} = 0 \tag{22}$$

Replacing the M of equations (21) and (22) by $M_C + H_C\,y$, which is the moment at any point D, Fig. 234, in terms of moment and thrust at the crown, and making $\frac{s}{I}$ constant, there results

$$2\,\frac{s}{EI}\,M_C\Sigma\, y + 2\,\frac{s}{EI}\,H_C\Sigma\, y^2 = \Delta L$$

$$mM_C + H_C\,\Sigma\, y = 0$$

From which

$$H_C = \frac{I}{s}\,\frac{m\,E\,\Delta L}{2\,[m\,\Sigma\, y^2 - (\Sigma\, y)^2]} \tag{23}$$

and

$$M_C = -\frac{H_C\,\Sigma\, y}{m} \tag{24}$$

M = moment. H_C = crown thrust. m = number divisions of half axis. s = length of division of axis. I = moment inertia. L = span. E = modulus of elasticity. ΔL = change of span length. t = rise or fall of temperature. c = coefficient of expansion. x, y = coördinates of a point.

The moment at any point D may be found as soon as the values of H_c and M_c have been determined by means of the relation

$$M = M_c + H_c y \tag{27}$$

or we can say that the moment at any point equals the thrust H_c* multiplied by the distance from the point in question to the line OO, Fig. 234.

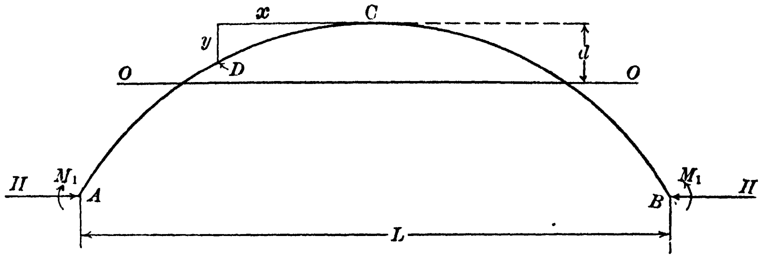


FIG. 234.—Moments and Thrusts due to Changes of Temperature. (*See p.* 730.)

Above the line OO, Fig. 234, the moments are all negative, being a maximum at the crown, and below OO they are all positive, being maximum at A and B. The line OO is below the crown a distance $d = \frac{\Sigma y}{m}$. At the two points where OO intersect the arch axis the moments are zero, as is evident from equations (24) and (26).

Fall in Temperature. Here the thrust at crown is

$$H_c = -\frac{I}{s}\,\frac{c\,tLmE}{2\left[m\,\Sigma y^2 - (\Sigma y)^2\right]} \tag{28}$$

where c is 0.0000055, and moment at crown is

$$M_c = -\frac{H_c\,\Sigma y}{m} \tag{29}$$

and, as above,

$$M = M_c + H_c y \tag{30}$$

In placing a numerical value for H_c in the last two equations, it should be observed that it is a negative quantity. If in the equations the values of L and y are in feet, *E is in pounds per square foot.* Above OO the moments are all positive, below they are all negative. The thrust at the crown is really a tension in this case.

M = moment. Hc = crown thrust. m = number divisions of half axis. s = length of division of axis. I = moment inertia. L = span. E = modulus of elasticity. t = rise or fall of temperature from mean. c = coefficient of expansion. x, y = coördinates of a point.

*The horizontal thrust is constant throughout the arch, hence Hc at the crown equals H at the support.

An increase and a decrease of 20 degrees Fahr. is probably a sufficient allowance for concrete arches with filled spandrels. For arches with open spandrels the range in temperature of the concrete is somewhat less than that of the surrounding air. For example, in the latter case with a range of temperature of the air from -20 degrees to $+100$ degrees Fahr., the range for arch computation should be taken at least 40 degrees on each side of the mean temperature.

The methods of combining the temperature moments and thrusts with those due to loads is illustrated in the example, page 737.

EFFECT OF RIB SHORTENING DUE TO THRUST

The thrust acting throughout the arch ring tends to cause a shortening of the span, which, if f is average compression (obtained by averaging values in computation of ring) for unit area, $= \frac{f\,L}{E} = \Delta_L$

Hence

$$H_c = -\frac{I}{s} \cdot \frac{f\,Lm}{2\,[m\,\Sigma y^2 - (\Sigma y)^2\,]} \tag{31}$$

and

$$M_c = -\frac{H_c\,\Sigma y}{m} \tag{32}$$

and, as in temperature stresses,

$$M = M_c + H_c y \tag{33}$$

All the summations above are for one-half the span only. m = number of divisions in one-half of the arch axis.

The effect of rib shortening is slight in many cases but in a flat arch it may be considerable. It is similar to a fall in temperature.

DISTRIBUTION OF STRESS OVER CROSS SECTION

Knowing the thrust, shear and bending moment at the selected sections of the ring, the distribution of stresses on the sections must be computed to insure against excessive working stresses and an uneconomical design. On pages 377 to 389 are given formulas for determining the stresses caused by eccentric forces or by an axial force and a bending moment. Shear in all cases is negligible.

M = moment. H_c = crown thrust. m = number divisions of half axis. s = short length of arch axis. I = moment inertia. L = span. y = coördinate of a point. f = compression in concrete.

METHOD OF PROCEDURE FOR THE DESIGN OF AN ARCH

The design of an arch is a trial process; the design being selected and then investigated to see if the sections are of sufficient strength. If the arch first chosen is too large or too small it must be revised.

Since the location of the line of pressure and also the stresses are affected by the loading, it is customary either to compute the arch for the dead load plus concentrated loads located at the most unfavorable positions, or else to compute it for the dead load plus a uniform live load covering one-half the arch and also covering the entire arch.

The following pages indicate the steps in the design of a highway bridge shown in Fig. 235, page 739, with the live load over one-half the span. The procedure is similar when the entire span is loaded.

1. Draw a preliminary curve for the intrados. (See p. 714.)
2. Assume a crown thickness in accordance with the formula on page 715.
3. Lay out the curve of the extrados and the surface of the roadway. The extrados may be a 3-centered curve, but it is better to use an arc of a circle if possible. It should be so placed as to give a ring thickness at the quarter points of the span of $1\frac{1}{4}$ to $1\frac{1}{3}$ times the crown thickness, and a ring thickness at the springings of 2 or 3 times the crown thickness in this first trial.
4 Draw the arch axis midway between the extrados and the intrados.
5. Divide the arch axis into distances such that the ratio of each distance to the moment of inertia of the cross-section of the ring at the center of the distance is a constant; that is, $\frac{s}{I}$ is a constant. This can be done by trial by beginning at the crown and working towards the springings or by the method described on page 728. *The moment of inertia is of the combined section of concrete and steel about the gravity axis*, hence the size and position of the steel rods must be first assumed, when I may be computed by the formula on page 381. The ratio of area of steel to total area of section at crown may be arbitrarily taken in the first place from 0.007 to 0.0125, that is from 0.7% to $1\frac{1}{4}$%. The divisions are separated by vertical sections.

In the problem here solved the distance, s, next to the crown is 1.14 ft., and that next to the springing is 7.82 ft. The constant ratio, $\frac{s}{I}$ *for this arch* is 11.4* On folding Fig. 235 the centers of the divisions are shown by circles and are numbered 1, 2, 3, etc. All distances are in feet and all quantities

*Greater accuracy may be obtained by using a larger number of divisions than here chosen, and also by subdividing loads P_1 and P_{30}.

involving distance are in foot units. A section of the arch 1 foot wide transversely is considered.

6. Compute the dead and live loads and enter these loads as indicated by P_1 P_2, etc., at the center of gravity of each division. In the accompanying design, a live load of 100 pounds per square foot covers the right half span, while on the left is the dead load alone of the masonry taken at 150 pounds per cubic foot plus the earth fill taken at 100 pounds per cubic foot.

TABLE I. *Ordinates and Moments in Computation of Example*

Points	x	y	x^2	y^2	M_L	M_R	M_Lx	M_Rx	M_Ly	M_Ry
10 and 11	0.56	0.01	0.3	0.00	00	00	00	00	00	00
9 and 12	1.71	0.04	2.9	0.00	391	521	668	891	16	21
8 and 13	2.88	0.11	8.3	0.01	1 205	1 603	3 470	4 616	132	176
7 and 14	4.11	0.23	16.9	0.05	2 520	3 346	10 357	13 752	580	770
6 and 15	5.43	0.39	29.5	0.15	4 471	5 923	24 277	32 162	1 743	2 310
5 and 16	6.89	0.63	47.5	0.40	7 327	9 672	50 483	66 640	4 616	6 093
4 and 17	8.57	0.97	73.5	0.94	11 584	15 216	99 275	130 401	11 237	14 759
3 and 18	10.59	1.50	112.2	2.25	18 242	23 791	193 183	251 947	27 363	35 686
2 and 19	13.17	2.39	173.5	5.71	29 480	380 45	388 252	501 053	70 457	90 928
1 and 20	17.94	5.14	321.8	26.41	58 553	74 192	1 052 235	1 331 004	301 476	381 347
Σ	71.85	11.41	786.4	35.92	133 873	172 309	1 822 200	2 332 466	417 620	532 090

All distances in foot-units; all moments in foot-pounds

Values of H_c, V_c and M_c at crown for Live and Dead Loads.

$$H_c = \frac{10\ (417\ 620 + 532\ 090) - 11.41\ (133\ 873 + 172\ 309)}{2\ [10 \times 35.92 - (11.41)^2]} + 13\ 107 \text{ lb.}$$

$$V_c = \frac{1822200 - 2332466}{1573} = -324 \text{ lb.}$$

$$M_c = \frac{172\ 309 + 133\ 873 - 2 \times 13{,}107 \times 11.41}{20} = +354 \text{ ft. lb.}$$

Values of H_c and M_c at crown for Rise in Temperature.

$$H_c = \frac{1}{11.4} \cdot \frac{.0000055 \times 20 \times 41.88 \times 10 \times 2000000 \times 144}{2\ [10 \times 35.92 - (11.41)^2]} = 2545 \text{ lb.}$$

$$M_c = \frac{-2545 \times 11.41}{10} = -2900 \text{ ft. lb.}$$

Values of H_c and M_c at crown for Rib Shortening.

$$H_c = -\frac{1}{11.4} \cdot \frac{66 \times 41.88 \times 10 \times 144}{2\ [10 \times 35.92 - (11.41)^2]} = -760 \text{ lb.}$$

$$M_c = -\frac{-760 \times 11.41}{10} = +870 \text{ ft. lb}$$

The horizontal components of the earth pressure are so small that they are neglected, except that, for purposes of illustration, they are shown in the case of the load adjoining each springing, where the horizontal components are computed by formulas for earth pressure on page 760. The point of application of the horizontal and vertical components, as shown for P_1, is taken at the arch axis. In practice, earth pressure is negligible

in the design of flat arch rings of the type here selected, and all loads may be taken as vertical. Only where the ratio of rise to span is large need the horizontal components of the earth pressure be considered.

7. Make a table similar to Table 1, page 734. The values of x and y are scaled from the drawing, and are the coördinates of the *center* points of the divisions of the arch axis. The *crown* point of arch axis is here taken as the *origin of coördinates.* The values of M_L and M_R are computed. M_L represents the moment at each of the center points 1 to 10 inclusive of all *loads* lying between the point in question and the crown. Thus M_L for point 10 is 0; for point 9, $M_L = 340 \times 1.15 = 391$ ft. lb.; for point 8, $M_L = 391 + 696 \times 1.17 = 1205$ ft. lb., and so on. The moment at each "center" point being obtained from that at each preceding "center" point. M_R of course represents the moment at each of the center points 11 to 20 inclusive of all *loads* lying between the point in question and the crown. For a symmetrical loading M_L would equal M_R for each pair of center points, such as 1 and 20.

8. Compute H_c, V_c, M_c, that is, the thrust, shear and moment at the crown, as on page 734, by using equations (16), (17), and (18), page 727. If the sign of V_c is plus the line of pressure (equilibrium polygon) at the crown slopes upward towards the left; if minus, as in the present case, upwards toward the right. A plus sign for M_c indicates a positive moment; a minus sign, a negative moment at the crown. For the arch in folding Fig. 235, the crown thrust $H_c = 13107$ pounds, $V_c = -324$ pounds and $M_c = +354$ ft. pounds.

9. Draw a force polygon as shown in folding Fig. 235 by laying off to scale the loads P_1, P_2, etc., as 0 – 1, 1 – 2, etc. Find the pole by laying off V_c downward (because negative) from the crown point, 10, and then laying off H_c horizontal. The hypothenuse of the triangle having H_c and V_c for sides thus slopes upward to left or upward to right, according as V_c is + or –.

10. Draw the equilibrium polygon as shown on the arch of folding Fig. 235. The resultant pressure acts above the axis at the crown a distance,

$$\frac{M_c}{H_c} = e \text{ if } M_c \text{ is plus, and below by the same amount if } M_c \text{ is minus.}$$

Since here, as is shown later, $e = +0.028$ feet, this distance is laid off vertically above the axis at the crown and through this point the resultant pressure is drawn parallel to the ray O_{10} of the force polygon and so on. It is not really necessary to draw the equilibrium polygon if the moments and eccentricities are computed for the various sections as outlined under item 11, but the polygon, which is the line of pressure, affords a good check on the algebraic work.

11. Determine the moment, thrust, and eccentricity, and if desired the shear at the center points, 1, 2, 3, etc., of the divisions, and enter in a table as shown below. The moment is computed from formulas (19) and (20) on page 728, the values of whose terms have already been found by items 7 and 8. The thrust and shear may be scaled from the force polygon. For example, at section 1 on folding Fig. 235 the thrust line is drawn parallel to the tangent to the axis at 1, and the shear line at right angles to the thrust line. The eccentricities, e, of the sections 1, 2, 3, etc., are computed by dividing the moment on the section (see page 379) by the thrust for that section just scaled. For positive moments and therefore positive values of e, the line of thrust lies above the arch axis.

12. Compute the thrust and moment at the crown due to variation in temperature by formulas (25) and (26), page 730, the moments on the

TABLE 2. *Final Moments and Thrusts*

	LIVE AND DEAD					TEMPERATURE		RIB SHORTENING	
Point	$H_c y$	$V_c x$	Mom.	Thrust	Ecc.	Mom.	Thrust	Mom.	Thrust
1	67370	−5812	+3259	+14360	+0.23	±10180	±1970	−3030	−610
2	31325	−4267	−2068	+14000	−0.15	± 3180	±2310	− 950	−700
3	19660	−3431	−1659	+13920	−0.12	± 910	±2430	− 270	−730
4	12713	−2777	−1293	+13600	−0.10	± 440	±2500	+ 130	−740
7	3014	−1331	− 483	+13240	−0.04	∓ 2320	±2530	+ 690	−760
9	524	− 554	− 67	+13160	−0.005	∓ 2800	±2545	+ 840	−760
12	524	− 554	+ 911	+13120	+0.07	∓ 2800	±2545	+ 840	−760
14	3014	−1331	+1353	+13200	+0.10	∓ 2320	±2530	+ 690	−760
17	12713	−2777	+ 627	+13640	+0.05	± 440	±2500	+ 130	−740
18	19660	−3431	− 346	+14040	−0.03	± 910	±2430	− 270	−730
19	31325	−4267	−2099	+14200	−0.15	± 3180	±2310	− 950	−700
20	67370	−5812	− 656	+14840	−0.04	±10180	±1970	−3030	−610

Thrusts in lb. Moments in ft. lb. Shear in arch design is small and need not be computed.

various sections by formula (27), page 731, and the thrusts and shears by resolving the crown thrust into tangential and radial components, as shown in the small force polygon in the diagram.

A rise in temperature of 20 degrees Fahr., and a fall of the same amount, is sufficient even in the northern part of the United States for arches with filled spandrels.

For the arch shown on folding Fig. 235 the crown thrust H_c, due to temperature, is a tension of 2545 lbs., and a *compression* of equal amount. The crown moment M_c is + 2900 ft. lb. and − 2900 ft. lb.

13. The effect of rib shortening due to the thrust is comparatively slight. Where necessary to compute it, use formula (31) and (32), page 732. (See p. 734.)

For the problem here shown the thrust at crown due to this cause is − 760 lb., and the moment is +870 ft. lb.

14. Having prepared a table similar to Table 2, page 736, showing

thrusts and moments on the various sections 1, 2, 3, etc., due to dead and live loads, temperature, and rib shortening, compute the maximum unit compression in the concrete and maximum unit tension, if any, in the steel by use of formulas on pages 382 to 389.

Table 2 shows thrusts and moments for only a few of the sections of this arch, since it is unnecessary to compute all of them. A selection of the more critical sections may be made by inspection of the equilibrium polygon. The following shows the computation of the maximum unit stresses at the crown for the arch in folding Fig. 235, as outlined in items 11 to 13.

LIVE AND DEAD LOADS AND RIB SHORTENING.

Moment	Thrust	
+ 354	+ 13107	Live and dead
+ 870	− 760	Rib shortening
+1224 ft. lb.	+12347 lb.	

$$e = \frac{M}{N} = \frac{1224}{12347} = 0.1 \text{ ft.}$$

p = ratio of steel at crown = 0.0092

From formula (77), page 384, it is seen that the value of $\frac{c_0}{h}$ for 0.92% is *greater* than $\frac{c}{h} = 0.1$. Hence there is *compression over the entire section.*

From formula (75), page 382, max. compression in concrete,

$$f_c = \frac{12\,347}{1 \times 1}\left[\frac{1}{1 + 14\,(.0092)} + \frac{6\,(1)\,0.1}{(1)^2 + 12\,(14)\;.0092\;(\tfrac{1}{3})^2}\right]$$

$$= 17\,280 \text{ lb. per sq. ft.}$$

$$= 120 \text{ lb. per sq. in.}$$

Stresses in steel need not be computed.

The above may be more quickly solved by the use of the curves in Fig. 108, page 383.

LIVE AND DEAD LOADS AND RIB SHORTENING PLUS TEMPERATURE.

Moment	Thrust	
+ 1224	+ 12347	
+ 2900	− 2545	Temp.
+ 4124 ft. lb.	+ 9802 lb.	

$$e = \frac{M}{N} = \frac{4124}{9802} = .42 \text{ ft.}$$

From formula (77), page 384, it is seen that the value of $\frac{c_0}{h}$ for 0.92% of steel is much *smaller* than $\frac{c}{h} = \frac{0.42}{1} = 0.42$. Hence there is *tension over a part of the section.*

From formula (88), page 386, the value of k is found to be 0.6. From formula (86), page 386, the value of the maximum compression = 35 700 pounds per square foot = 248 pounds per square inch. From formula (81), page 385, maximum tension in steel = 1 440 pounds per square inch.

The approximate value of the above compression in concrete may be more quickly found by the use of curves, Figs. 111 and 112, and pages 387 and 388 as shown below.

f_c = compression in concrete. e = eccentricity. M = moment. N = thrust. h = height. k = ratio depth neutral axis. c = distance center of gravity to steel.

The method of computation for other points in the arch is similar, and stresses should be determined at sections where they appear to be the maximum.

From table 2 it is evident that although at point 20 the moment due to dead and live load is very small, its combination with moments due to temperature and rib shortening makes it one of the critical points. The moment and thrust due to live and dead load and rib shortening is
$M = -656 - 3030 = -3636$ ft. lb. and $N = 14840 - 610 = 14230$ lb.
Hence, $e_0 = \frac{3686}{14230} = 0.26$ ft., for $h = 1.97$, $\frac{e_0}{h} = 0.13$, $p = 0.0037$.

From formula (77), page 384, it is seen that the whole section is in compression. From Fig. 108, page 383, for $\frac{e}{h} = 0.13$ and $p = 0.0037$, the value of the parenthesis in formula (75) $= 1.65$. Using formula (75), page 384, $f_c = \frac{14\,230 \times 1.65}{1.97 \times 12 \times 12} = 83$ lb. per sq. in.

Combine now the moment and thrust due to live and dead load with those due to temperature and obtain $M = -(10\,180 + 3\,686) = -13\,866$ ft. lb., $N = -1\,970 + 14\,230 = 12\,260$ lb., $e = 1.13$ ft. $\frac{e}{h} = 0.57$.

In Fig. 111, page 387, $k = 0.37$ corresponds to $\frac{e}{h} = 0.57$. By locating this value of k in Fig. 112, the constant $C_a = 0.094$ is obtained, which substituted in formula (86), page 386, gives $f_c = \frac{13\,866 \times 12}{0.094 \times 12 \times (1.97 \times 12)^2} = 264$ lb. per sq. in. The stress in steel from formula (81) is $f_s = 15 \times 264 \; \frac{1.80 - (0.37 \times 1.97)}{0.37 \times 1.97} = 5\,800$ lb. per sq. in.

Similar computations should be made for all critical points and when the stresses are either too small or too large, the dimensions or even the shape of the arch must be changed. Small changes may be made without refiguring the whole arch. For larger changes, all computations should be repeated and a new line of pressure determined.

LOADINGS TO USE IN COMPUTATIONS

The usual practice is to make two sets of computations; in the first place, proportion the arch ring for a live load covering the entire span and then for one covering only one-half the span. These two loadings are approximations, more or less exact, to the true loadings which produce the maxi-

FIG. 235. EXAMPLE OF ARCH DESIGN

(See pp. 733 to 738)

mum effects. By computing a table for the thrusts and moments due to a load of unity at different points, or by the use of influence lines, the exact loading to cause maximum stresses may be found.

ALLOWABLE UNIT STRESSES

For highway bridges the maximum compression in the concrete of the ring should not exceed 500 pounds per square inch due to live and dead loads, nor more than 600 pounds per square inch due to live and dead loads, temperature and rib shortening combined. For railroad bridges three-fourths of the above values may be used.

DESIGN OF ABUTMENT

The design of the foundation of an arch bridge is as important as that of the arch itself. The arch is designed on the assumption that the foundation is unyielding, and this condition must be approached as nearly as possible in order to insure the stability of the whole structure.

The depth of the foundation as well as the shape is dependent upon the local conditions, and in the more difficult cases these have to be chosen after exhaustive studies. A certain shape of abutment is first assumed, and this is then reviewed to see that the load upon the ground does not exceed the allowable load and that it is well distributed. Allowable loads are discussed on page 715.

The forces acting on the foundation are:

(1) the thrust of the arch; (2) the weight of the foundation; (3) the weight of the earth above it; and (4) the lateral earth pressure. The thrust of the arch is the largest when the live loading extends over the whole span of the arch, and for this the line of pressure should be drawn first. A line of pressure for the thrust on account of the total dead load and of the live load extending only over one-half the span opposite to the abutment also should be drawn to see whether, because of intersecting the abutment higher up, it does not produce larger pressure on the foundation. A good scheme is to design the abutment in such a way that the line of pressure on account of one thrust intersects the base a little way to the left of the center while the other intersects to the right of the center. In some cases a third line for the total dead load, plus live load on the half span nearest the abutment should also be drawn.

The line of pressure of the forces should be as near to the center of the base as possible, since the maximum unit pressure is the smallest when the load is distributed uniformly over the entire section. This also prevents uneven settling of the foundation, and thus adds considerably to the stability of the whole structure.

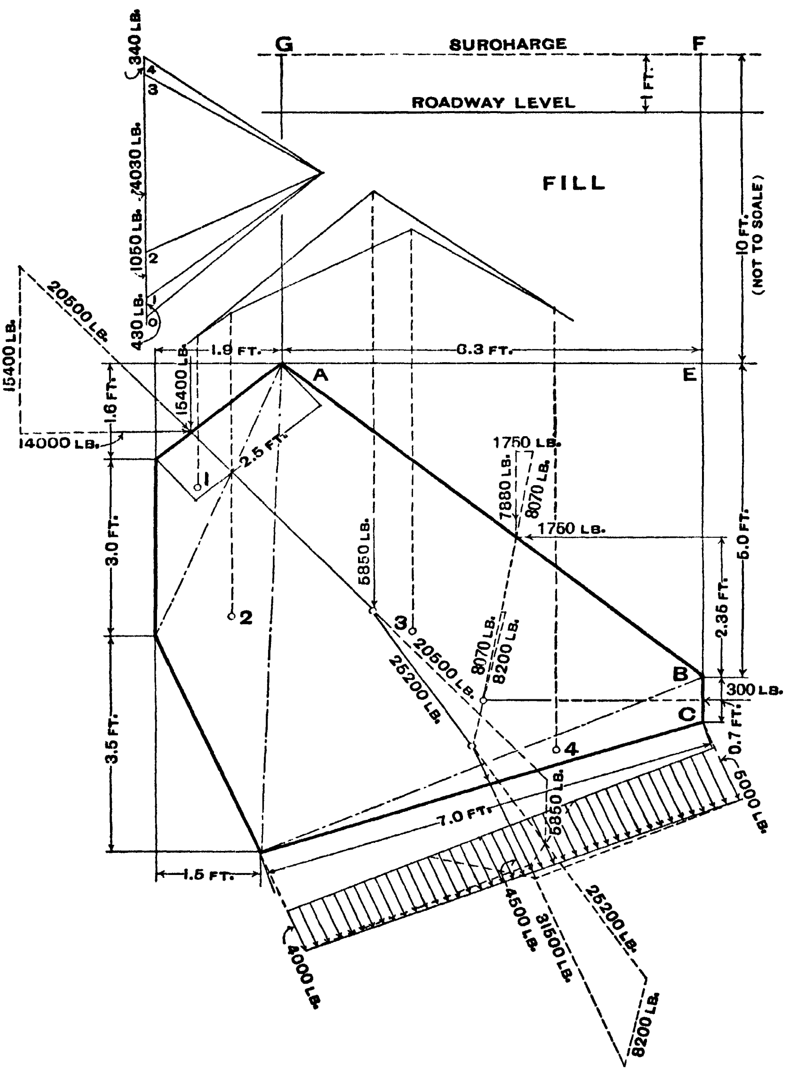


FIG. 236.—Design of a Foundation for an Arch. (See p. 743).
(To simplify the drawing only one position of thrust and one line of pressure is drawn.)

Fig. 236, page 742, clearly illustrates the design of an abutment. The outline is assumed, then the location and magnitude of the forces acting upon the abutment are found and the line of pressure determined. If the assumed outline is not satisfactory it should be revised.

For the benefit of those who are not familiar with the common principles of such design, the steps will be considered in detail. The magnitude, 20,500 pounds, and position of the arch thrust is given in the arch example. Since the weight of the masonry acts through its center of gravity, this point must next be found and this is most readily done by dividing the outline of the abutment into triangles and rectangles. The weights of each of these prisms one foot thick are readily computed, and the center of gravity found through which the weight force acts. A force polygon for any pole distance, as shown in the upper left corner of the diagram, is drawn and the equilibrium polygon, by the intersection of the closing lines, locates the resultant of the weight which, by computation, is found to be 5850 pounds.

The pressure on AB consists of the horizontal pressure on BE, and the weight of the prism of earth whose cross-section is $ABFG$* and thickness one foot. Taking the weight of one cubic foot of filling at 100 pounds, the weight of the prism would be $\frac{10+15}{2} \times 6.3 \times 100 = 7880$ pounds.

The horizontal pressure on BE is equal to the difference between the pressures on BF and EF.

Let

$$w = \text{weight of one cubic foot of earth,}$$

then, if the weight of earth is assumed at 100 pounds, from formula (2), page 758, pressure on the plane

$$BF = C_p wH^2 = 0.14 \times 100 \times 15 \times 15 = 3150 \text{ pounds,}$$

and on the plane

$$EF = C_p wH^2 = 0.14 \times 100 \times 10 \times 10 = 1750 \text{ pounds.}$$

Hence horizontal pressure on plane BE $= 1750$ pounds.

The point of application is found from the formula (7), page 760.

In the case under consideration $H = 15$ feet, $h = 10$ feet, where H is the depth of point B and h the depth of A or E below the line of surcharge.

The horizontal pressure on BC is by formula (6), page 760, 300 pounds, and the point of application may be assumed in the middle of BC without appreciable error.

* The live load being 100 pounds per foot is equivalent to a surcharge one foot in height.

Having thus located all forces and found their magnitude, the line of pressure is drawn. This procedure consists simply in finding the resultant of two forces intersecting in one point. The line representing the thrust is prolonged until it intersects the line representing the weight of masonry, 5850 pounds. Beginning at this, the magnitude of the thrust, 20,500 pounds, is laid off to any desired scale and the resultant of this with the weight of the masonry, 5850 pounds, is found to be 25,200 pounds. Combining this new force in turn with the earth pressures of 8070 pounds and 300 pounds completes the line of pressure with a final resultant thrust of 31,500 pounds.

Having found the line of pressure, the thrust is divided by the projection of the base on a line at right angles to the thrust and the maximum pressure on the ground is found by formula (70), page 379, to be 5000 pounds per square foot.

The same result is obtainable by the following simple graphical method: Find the average unit pressure by dividing the thrust by the area of the projection of the base, drawn perpendicular to the thrust. In this case we have $\frac{31500}{7} = 4500$ pounds per square foot. Plot this, to any convenient scale, perpendicular to the projection to the base at its center; connect the $\frac{1}{3}$ points of the base with the top of this perpendicular, as shown by the dash lines in Fig. 236, and produce one of these lines till it intersects the line representing the direction of the thrust. The perpendicular distance of this point from the projection of the base is the maximum thrust and the distance of the other intersection of a slanting line with the thrust line is the minimum thrust. To draw the trapezoid of pressure, draw, through these two intersections, lines parallel to the projection of the base, as shown, and the extremities of these parallel lines will fix the two corners of the trapezoid. The maximum pressure is always at the end of the base nearest the thrust.

ERECTION

As in other reinforced structures, the erection is as important as the design. Perhaps the first essential is the centering which should be planned out in advance almost as carefully as the arch itself.

Methods of Arch Construction. There are two general methods of laying the concrete in an arch, each of which has strong advocates. By the first, the arch is laid in separate blocks across the bridge, and by the second, in narrow ribs from abutment to abutment. If the block method is followed, the lowest stones at the springing line are laid first, then stones

intermediate between the spring and the key, next the two stones each side of the key, and finally, after filling in the intermediate blocks, the key is placed. This distributes the weight of the concrete uniformly over the arch center, and prevents unequal settlement, which tends to crack the arch near the springing lines. On the other hand, the entire weight falls upon the center, and the latter must be very strongly built. The arch thrust acts at right angles to the joints, and as the blocks extend clear across the bridge, there is no danger of longitudinal splitting, but the radial joints offer planes of weakness in bending.

By the other method the work can be readily arranged so that a day's labor consists of the laying of a single rib, thus forming a complete arch of itself, which as soon as it sets bears its own weight. This arch section has no joints, so that when subsequently loaded the bending moment is best resisted.

A small arch, where the center can be solidly built, may be laid at one operation, commencing at both abutments and working toward the key so that it is in fact a monolith.

The spandrel or face walls may be carried up at the same time the arch ring is laid, or may be connected with it later by leaving short lengths of steel projecting radially from the concrete of the arch.

If steel is introduced, the consistency of the concrete must be wet enough to thoroughly coat it. This may be accomplished by a quaking or jelly-like mixture, which requires but slight ramming.

From an architectural point of view, the treatment of the face is of much importance. For a discussion of the different methods reference should be made to page 262.

Railings and ornamental work may be cast in molds if preferred and put in place after hardening.

Centering. The falsework for concrete arches is practically the same as for stone arches except that close lagging is necessary. It must be rigid during the construction of the arch and stiff enough to prevent its distortion from the unsupported weight of the concrete before the keying of the arch.

The design of the centering is frequently governed by the character of the ground underneath. In general the framed wood centering made into a truss rests upon pile or trestle bents. The spacing of these bents is determined by the foundation and the difficulty of placing them, and by the height and span of the arch. In certain cases it is possible to support the centering in whole or in part by the reinforcement, although this is not usually economical because more carefully framed steel is required than is

necessary for reinforcing the arch. In at least one case* reinforced concrete forms were used.

In connection with the description of arch centers which he has built, Mr. James W. Rollins, Jr.,† gives the following notes:

> For small arches the simplest center is a circular rib made of three pieces of 2-inch plank, laid with broken joints, all being spiked solidly together, with a tie of plank at the springing. On this, 1-inch lagging is laid close. For a larger arch, the circular rib, as above described, with generally three braces, one at center and one on the quarter at each side, is used, the center of the whole rib having a post under it. We have used such a center up to 30-foot span for both brick and granite arches, carrying a 30-inch arch sheeting.
>
> The design of a center for larger arches depends upon local conditions, also upon the relation of rise to span. In flat arches, with low side walls, it is well to use posts with intermediate bracing, on numerous supports. In a high arch we may use long braces extending directly from a center support to the rib, at intervals of 6 feet to 8 feet.

Mr. Rollins advocated for wedges, seasoned oak, 8 inches wide, 4 inches thick at the thick end, 2 inches at the thin end, and 18 inches long, planed on sliding faces, and thoroughly greased. When setting the center, these wedges, placed between the caps on the bents and the corbels under the lower chord of rib, are tacked together to prevent slipping.

Boxes filled with sand are frequently used between the caps of the bents and the lower chords of the trusses in place of wood wedges. The sand in these must be thoroughly packed to prevent settlement of the concrete before setting. The sand is readily removed by letting it out through a hole in the box. Jack-screws also may answer the same purpose as wedges or sand boxes. By any of these means the centering is easily lowered.

The ribs of the centering are usually made of several pieces of plank spiked or bolted together. Upon the ribs rests the lagging, which usually consists of one or two layers of planking having the top surface smoothed to give a good surface to the soffit of the arch, and laid with tight joints. With thin lagging care must be taken to prevent deflection.

Instead of the ribs forming a part of the truss, they are frequently supported directly upon the wedges resting upon the caps of the bents, the posts of which run up to the soffit of the arch for that purpose.

The centering should be cambered, that is, should be made higher than called for in the arch plans at the center, so that when it is removed, the arch will be in the position assumed for it in the design. Some engineers make

* *Engineering News*, Aug. 30, 1906, p. 215.

† Journal Association of Engineering Societies, July 1901, p. 10. For examples of centers built in various places, see References, Chapter XXXIII.

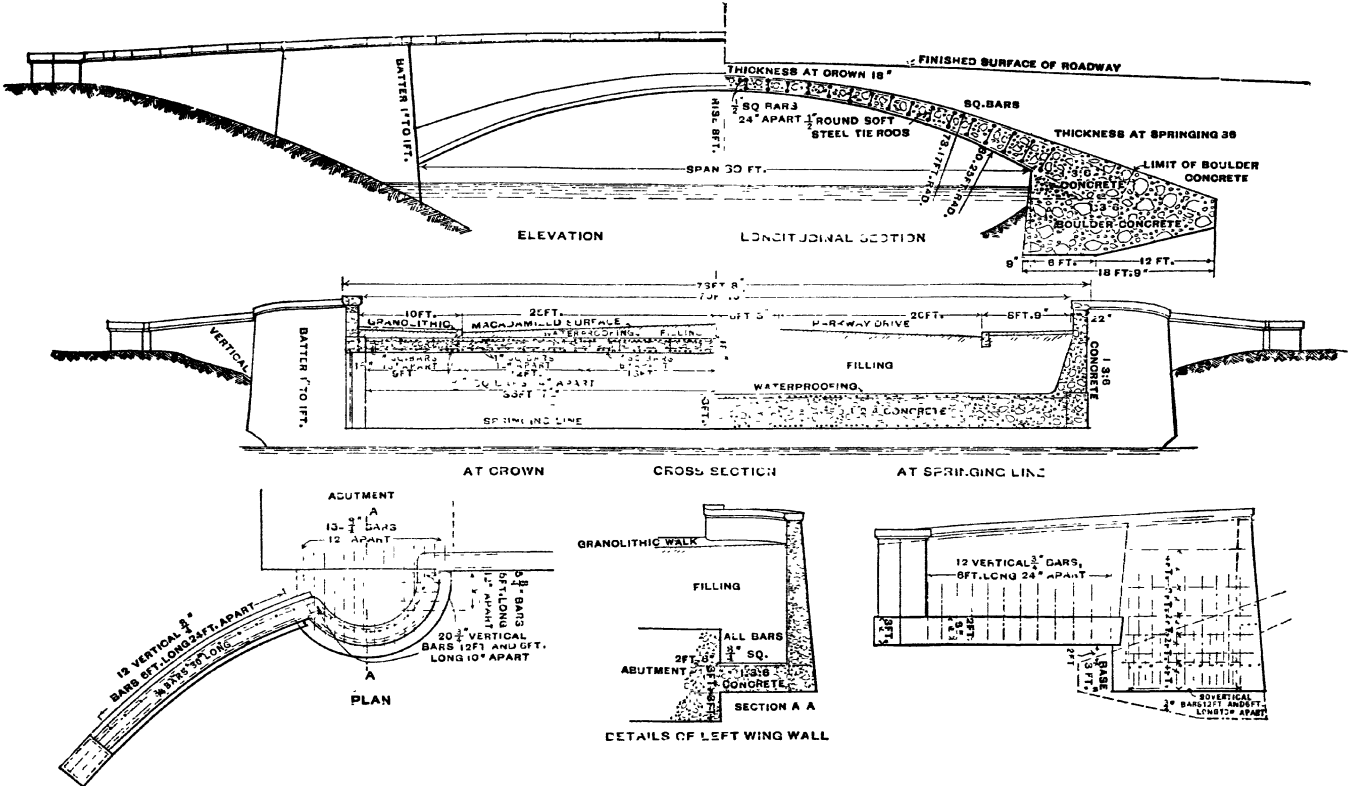


FIG. 237.—Design of Mystic River Bridge, Medford, Mass. (*See p.* 748.)

the camber equal to the deflection of the arch which would be caused by the live and dead loads.

In striking the centers sudden settlement must be avoided and the centers must not be removed until the concrete has attained good strength. The time of removal must be determined by the design of the bridge and the weather. For light highway bridges four weeks is usually sufficient, while for a heavy arch of long span eight weeks may be required.

EXAMPLES OF ARCH BRIDGES

Mystic River Bridge, Medford, Mass. This arch, illustrated in Fig. 237, page 747, is of the Monier type and carries a parkway over the river. It was built in 1906 by the Metropolitan Park Commission, Mr. John R. Rablin, Chief Engineer.

The arch has a span of 60 feet, a rise of 8 feet, and a crown thickness of 18 inches. Both the intrados and the extrados are segmental. The side walls are of concrete with a vertical expansion joint at each abutment. The retaining wall for the earth fill over the abutments is of reinforced design and curved as shown in the details in the drawing.

Granite Branch Railroad Bridge. A railroad bridge of similar design to the Mystic River Bridge was built by the Metropolitan Park Commission of only 4 feet longer span than the highway bridge described. The heavier loading necessitated a thickness of crown of 24 inches instead of 18 inches with a thickness at springing still greater in proportion.

3-Hinged Ribbed Arch on Ross Drive, District of Columbia. A different type of structure and one which illustrates the combination of arch ribs with a reinforced concrete floor system is illustrated in Fig. 238, page 749. This was built in 1907 by the Engineering Commissioner, Washington, D. C., Mr. W. J. Douglas, Engineer of Bridges.

The central arch is 100 feet clear span and 15 feet rise, and the roadway, which is 16 feet wide and macadamized, is laid upon a 6-inch reinforced concrete floor slab supported by longitudinal concrete girders which in turn rest upon columns supported directly by the concrete ribs. The three arch ribs, which are reinforced as shown, are 2 feet wide throughout their length with a thickness of 2 feet 6 inches at the crown.

Each hinge consists of two steel castings, shown in detail, with a pin 4 inches in diameter, and these hinges are imbedded in the concrete. An expansion joint is provided in the roadway deck over each springing. The floor of the arch was computed for a 6-ton wagon, and the ribs for a live load of 100 pounds per square foot of roadway. The maximum compression on the concrete of the ribs under live and dead loads is 500 pounds per

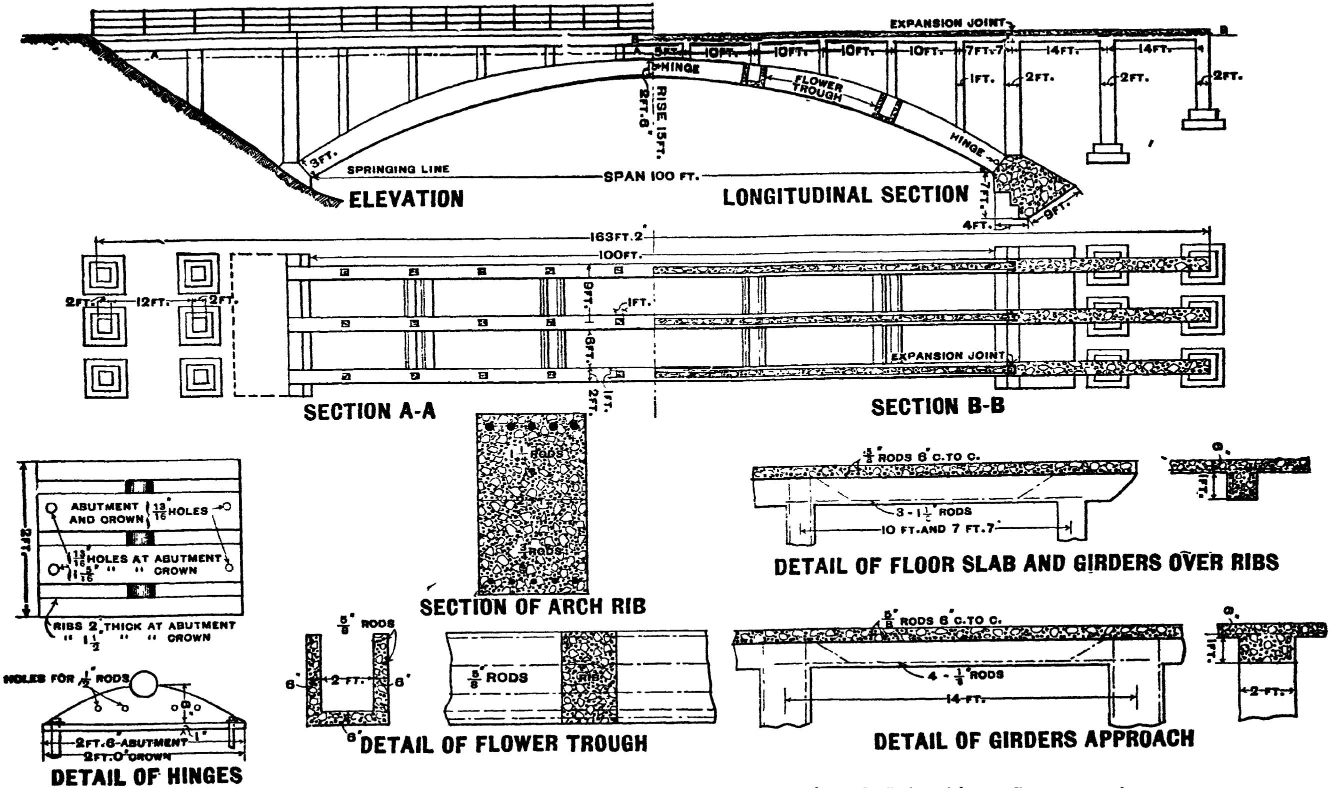


FIG. 238.—Three-Hinged Ribbed Arch, Ross Drive, District of Columbia. (*See p.* 748.)

square inch, and there is no tension. The cost of the structure was $8000, which is equivalent to about $3.00 per square foot of the roadway.

Walnut Lane Bridge, Philadelphia. A notable structure in concrete is the Walnut Lane Bridge built as it is with a clear span of 233 feet. The arch was completed in 1908 under the direction of the Bureau of Surveys, Mr. George S. Webster, Chief Engineer and Mr. Henry H. Quimby, Assistant Engineer. The principal arch consists of two ribs, upon which rest cross walls connected by small longitudinal arches of 20 feet span carrying the spandrel wall supporting the I-beams of the floor.

A fine photograph of the arch is shown in Fig. 224, page 706, and cross sections illustrating the design in Fig. 239, page 750. The balustrade is entirely of concrete, the posts being molded on the ground and the surface washed off with water to reveal the aggregate.

Other Notable Bridges. For references to other bridges built in recent years, see Chapter XXXIII.

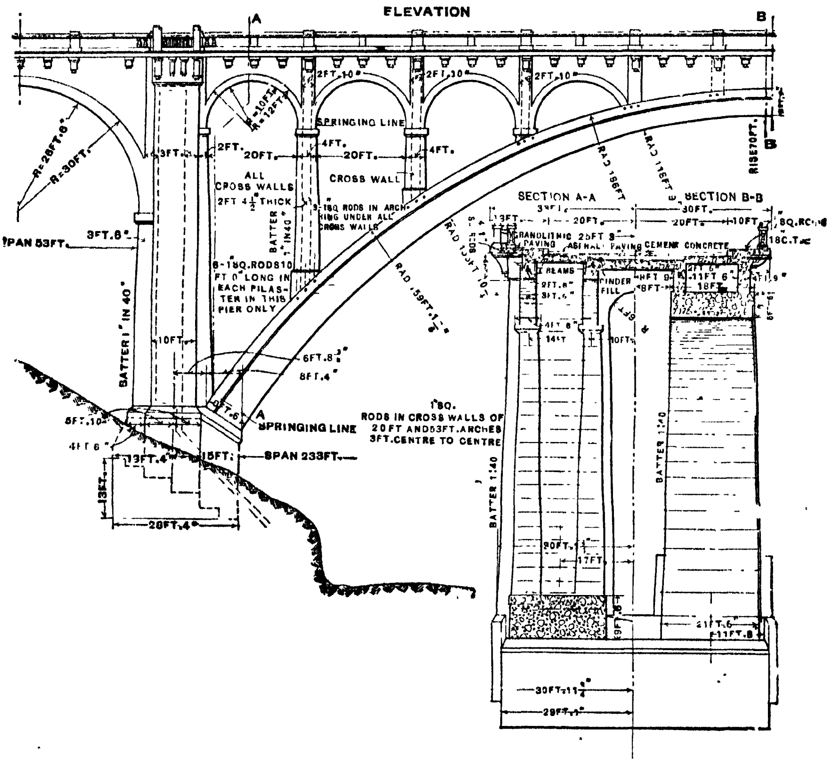


Fig. 239 **Walnut Lane Bridge, Philadelphia.** (*See p.* 750.)

CHAPTER XXVII

DAMS AND RETAINING WALLS

For walls to resist the pressure of earth or water, concrete frequently possesses marked advantages over other classes of masonry. With proper management, in most localities its cost may be brought below that of rubble masonry. Its adaptability for thin walls and for certain classes of face work often make it a suitable substitute in complicated designs for first-class masonry, with a consequent large saving in cost. In combination with steel its possibilities for special designs are almost unlimited, and the future will see continued advances in its use for ordinary engineering and hydraulic construction.

Water-tightness, often an essential element for this class of structures, has received general treatment in Chapter XVIII, page 296. Portland cement concrete may be made water-tight more readily than stone masonry laid in mortar of similar proportions to the cement and sand in the concrete, since large voids or stone pockets in the concrete are more easily prevented than the "rat-holes" so frequently found in the bedding of stones in mortar. Moreover, skill in laying combined with special treatment of the surface or the addition of certain ingredients permits construction in concrete—strengthened with steel reinforcement—of thinner walls for resisting the flow of water than is possible in stone masonry.

Reinforced concrete retaining walls cannot be designed by "rule of thumb," and therefore a careful consideration of the forces acting and of the stresses in the concrete is presented in this chapter. Since the earth pressure is the controlling factor, it has been necessary to introduce a practical discussion of this before taking up the details of the design and examples of the two principal types.

RETAINING WALLS

Retaining walls to support the pressure of earth may be designed:

(1) of gravity section with plain concrete or stone masonry;
(2) of thin reinforced concrete section of the inverted T type with spreading base or footing;
(3) of thin section, reinforced and supported by buttresses or counterforts.

Another plan sometimes adapted to cellar wall construction (see p. 643) consists in embedding the base and supporting the top of the wall with tim-

ber, steel or reinforced concrete beams, so that the concrete forms a vertical slab supported at top and bottom.

Reinforced concrete retaining walls are almost always more economical than a gravity section of either plain concrete or masonry. In walls of gravity section the materials cannot be fully utilized because the section must be made heavy enough to prevent overturning by its own weight, counterforts or buttresses being of comparatively little advantage because, in stone masonry, the wall is liable to break away from them. In reinforced concrete retaining walls, on the other hand, a part of the sustained material is used to prevent overturning, and the section need be made only strong enough to withstand the moments and shears due to the earth pressure. Since the wall is lighter, exerts smaller pressure on the soil, and may be made if necessary with a very broad base, the special foundations or piling which are often necessary for a gravity wall frequently may be avoided. Reinforced concrete properly designed can be depended upon as absolutely reliable.

The economy of a reinforced concrete wall over one of gravity section for either stone masonry or plain concrete is obvious because of the saving in material. The cost of forms is practically the same for gravity section and reinforced designs.

Mr. J. I. Oberlander reports* that 23 bids submitted on alternate designs of gravity and cantilever sections showed the average cost per linear foot for the gravity section to be about one-third greater than that for the reinforced section. The unit price for the concrete in the latter, however, was about 20% greater than that for the former.

Whether the T-section of reinforced wall or the wall with counterforts is the more economical depends upon certain conditions. The principal condition is the height of the wall, but the intensity of the earth pressure and the relative cost of concrete and steel and forms also enter into the consideration. The construction of the T-section is simpler and the placing of steel easier, so that it is preferable where skilled labor is scarce. The form construction in the counterforted wall is considerably more expensive. Comparative studies of the two types indicate that the counterfort type is scarcely ever economical when the height is less than 18 feet. Rules for designing walls of gravity section are first given and then, after the discussion of earth pressure, the designs of both a T-type and a counterforted section are treated.

Special designs have been worked out with considerable ingenuity where local conditions require departure from the standard sections. For example, two different railroads were each obliged to build retaining

* *Engineering and Contracting*, May 19, 1915, p. 457.

walls where every inch of available room up to the edge of the right-of-way was valuable and where no trespassing on adjoining property was permissible. The solutions for the same problems were radically different. In one* an L-section was used but the horizontal leg projected from the middle of the back instead of the bottom so that the result resembled the letter T on its side.

In the other case† a horizontal reinforced slab supported on transverse walls was used. The walls rested on another continuous horizontal slab and the rectangular cells thus formed were closed at the back by a vertical slab that retained the earth.

An interesting wall is that designed by Gustave Lindenthal for the New York Connecting Railroad. Here two walls 65 feet high enclosed a railroad fill nearly 60 feet wide carrying a surcharge loading of 4 tracks (E 60 loading) and 100% impact. Rather than use the very heavy sections that would have been required under ordinary assumptions, it was decided to use steel tie rods 10 feet on centers, and to tamp the earth fill in 12-inch layers with pneumatic tampers so that a slope of 3 to 1 could be counted on for passive resistance.‡

FOUNDATIONS

A firm foundation is essential whatever the type of the design. Piles may be necessary, or to avoid sliding, a stepped base may be required. Unequal settling is more dangerous for a retaining wall than for many other structures, because if it is thrown out of plumb, the earth will move and produce forces much in excess of the calculated ones. Allowable pressures on different soils are referred to on page 669.

In France several walls§ were constructed approximately rectangular in section, except that the bottom width was somewhat greater than that at the top, lying back, or reclining, on the slope of the cut, or fill. A wall somewhat similar to this, but built of separate blocks on a soft foundation, was built in Wisconsin.‖

Drainage of Retaining Walls. The drainage of retaining walls is a highly important matter and lack of it may cause either complete or partial failure. A common plan is to lay a drain of tile or of broken stone along the back of the base, opening at the ends of the wall or discharging through weep-holes.

* *Engineering News*, February 15, 1912, p. 292.

† J. H. Prior in *Engineering and Contracting*, May 10, 1911, p. 530.

‡ The computations are discussed in *Engineering News*, May 6, 1915, p. 886.

§ *Engineering Record*, November 12, 1910, p. 544.

‖ W. S. Lacher in Proceedings Western Society of Engineers; See also *Engineering News*, May 27, 1915, p. 1048.

The Delaware, Lackawanna, & Western R. R.* builds 4-inch tiles through the wall at frequent intervals along the footing with a right angle elbow, turned up, on the inner side. A chimney of loose rubble, about $2\frac{1}{2}$ feet by 3 feet, runs from each weeper up to the top of the wall.

The Rock Island Railroad† in some track elevation work laid a line of tile drain along the back of the wall and carried the water through the abutment by a weeper running to a storm sewer.

Mr. Lindenthal's high walls (see p. 753) were drained through weep holes, placed every ten feet, through each wall. From each weeper one 4 by 4-foot dry rubble chimney was built up back of the wall to the surface.

The French wall described on page 753 is drained by a layer of loose stone over the entire back with weep holes placed at intervals.

Frost. The depth of foundation must be sufficient to prevent heaving of the material in front of the wall, and to protect it from frost. A depth of 3 feet may be given as a minimum, while 4 or 5 feet is necessary in temperate or very cold climates.

Even with the base safely below frost level, special precautions are sometimes necessary to prevent heaving by frost-grip on the side of the wall or abutment. Such a case cited by Edward H. Rigby‡ was encountered in China where frost gripped the side of bridge abutments to a depth of 5 feet and lifted them, railroad, girder bridges, and abutments, clear off the pile foundations. Piers and abutments with sloping faces were lifted as much as those with vertical faces. Computations showed the average lifting power of the frost to be 1 000 pounds per square foot of exposed surface and that the remedy was to design the piers and bridges to overcome this force by dead weight.

DESIGN OF RETAINING WALLS OF GRAVITY SECTION

The thickness of gravity retaining walls is frequently determined by rule-of-thumb, but this is an unsafe procedure unless there is absolutely no doubt about the foundation. On work of any importance, much more economical results are obtained by special designs, governed by the character of the foundation soil and the earth backing. Partial failures,—tipping forward, cracking and sinking,—are prevalent among retaining walls. In one case a heavy gravity wall failed under the weight and impact of the backfilling dropped through a distance of 30

* *Engineering Record*, Jan. 3, 1914, p. 29.
† *Engineering News*, April 8, 1915, p. 670.
‡ *Engineering News*, March 5, 1908, p. 260.

feet or 40 feet from a large drag line scraper bucket; and where the foundation is so poor that such action is possible, the line ofpressure should pass through the base well within the middle third. Uneven settlement is then less likely to take place, and in any event the line of pressure has more chance to move without causing tension on the base or overturning than if the line passed through the forward edge of the middle third.*

The methods of design are similar to those discussed in connection with reinforced walls. (See p. 757 to p. 768).

If empirical rules are to be used, one easily remembered is to make the base three-eighths of the height. Another is to make the base at least the thickness necessary if the wall were to be subjected to water pressure under a head two-thirds the height of the wall.† A table of empirical values adopted by Mr. Trautwine for thickness of base of masonry walls to resist earth pressure is given below.

Thickness of Retaining Walls of Gravity Section with Earth Surcharge.
BY JOHN C TRAUTWINE. (See p. 755)

Ratio of Height of Earth to Height of Wall.	Thickness of Base as ratio to Height of Wall.	Ratio of Height of Earth to Height of Wall.	Thickness of Base as ratio to Height of Wall.
1.	0.35	2.	0.58
1.1	0.42	2.5	0.60
1.2	0.46	3.	0.62
1.3	0.49	4.	0.63
1.4	0.51	6.	0.64
1.5	0.52	9.	0.65
1.6	0.54	14.	0.66
1.7	0.55	25.	
1.8	0.56	or more	0.68

Designs according to these empirical methods are unsafe under unusual pressures, such as quicksands, and detailed analyses must be made.

The height of the wall is assumed to be measured above the surface of the ground in front of it.

The batter of the face of a retaining wall is customarily limited to $1\frac{1}{2}$ inches to the foot, and the back is usually vertical. This fixes the width on top.

The values in the table may be employed for long walls of concrete with no reinforcement. In many cases, because of the monolithic char-

* Certain failures of this type are discussed by Charles K. Mohler in *Engineering News*, Oct. 13, 1910, p. 384.

† Suggested by *Engineering News*, Sept. 26, 1912, p. 593.

acter of concrete, a ratio of thickness to height from 10% to 20% less may be adopted with safety, if the character of the filling back of the wall precludes excessive pressure, and if the base is slightly spread. For more accurate determinations of gravity sections, the principles which follow relating to reinforced designs are applicable. When two walls enclose a narrow fill they may be tied together by rods as discussed on page 753 and thinner sections used. Similarly, the ordinary single wall may be anchored to the ground behind it.

WEIGHT OF EARTH

In the calculation of retaining walls, and many other structures, the weight of earth in place is a prime factor. The weights of dry material, based upon experiments by the authors, are represented in the following table. Most of the figures for weights of earth give the weights per cubic foot after excavation in a loose or a compacted condition. In the authors' experiments the excavation was measured, so that the weights represent the material in place. As fills will eventually assume much the same characteristics as earth in original excavation, the figures may be employed for either natural earth or filled material. The weight of earth containing water varies with the character of the material and with the conditions. Gravel containing ordinary moisture weighs about 2% more than dry gravel and sand may weigh from 3% to 10% more, depending upon its fineness, since fine sands absorb the most water. Wet muck weighs about 75 lb. per cubic foot. These percentages assume that the bank is provided with natural drainage; if the earth is literally filled with water which cannot run off, its weight will be increased by a quantity of water nearly equal in volume to the voids in the material, which vary with the character of the material from 20% to 50% of the bulk of the earth in the bank.

Many of the values appear high, but they are the result of careful tests.

Average Weight of Ordinary Earth before Excavation.

	Pounds per cu. ft.
Sand	105
Gravel	135
Gravelly clay	130
Loam	90
Hard pan	130
Dry muck	40

BACKING

Since the weight of soil saturated with water is much larger than when it is dry, the pressure increasing with the amount of water so that it may even

exceed the hydrostatic pressure, the backing should be provided with adequate drainage. For this, a filling of gravel or crushed stone may be placed directly against the wall with weep holes at suitable distances apart. The question of drainage is discussed on page 753.

EARTH PRESSURE

The principal force governing the dimensions of any retaining wall is the earth pressure. Its magnitude varies largely with the character and wetness of the soil, the inclination of the back of the wall, and the slope of earth above it.

Of the numerous theories, all of which are based on some assumptions not always met with in practice, Rankine's theory seems to be the most reliable yet developed, and although it does not always represent the true conditions, it gives safe results. It is based upon the assumptions that the earth is composed of granular homogeneous particles without cohesion, held only by friction developed between them, and that the mass of earth extends indefinitely. On a vertical plane the resultant pressure always acts parallel to the slope of the earth and at a point one-third of the height from the base, when the surface of the earth is level with the top of the wall or slopes back from it.

The following table of pressures determined by Rankine's formula gives horizontal earth pressures for different heights of wall, based on an angle of repose of earth of 35°—a fair assumption under average conditions—and also average unit pressures for the same assumptions. For other heights of wall, the horizontal unit pressures with the same angle of repose are directly proportional to the heights, and the total pressures are proportional to the squares of the height.

Total Earth Pressure and Average Unit Pressure upon Vertical Walls of Different Heights (See p. 757.)

	Height of Wall in Feet.							
	5	10	15	20	25	30	35	40
Total pressure P, in lb.	350	1400	3150	5600	8750	12600	17150	22400
Average unit pressure in lb. per sq. ft.....	70	140	210	280	350	420	490	560

The table assumes (*a*) horizontal surface of earth, (*b*) vertical back of wall, (*c*) weight of earth per cubic foot, 100 pounds, (*d*) angle of repose, 35°. For other weights of earth the values in the table are proportional to the weight per cubic foot.

Passive pressure, that is, the resistance of a mass of earth against moving, is many times as great as the active pressure but because of the shrinkage of filling as ordinarily placed it cannot be counted on for its full value unless the earth is in its natural state.

The general formulas evolved by Mr. Rankine from the assumptions given above and which apply both to gravity walls and to reinforced walls, are presented below.

Wall with Vertical Back. Let

P = resultant earth pressure in pounds on a vertical surface for a length of wall equal to one foot.

H = total height of wall in feet.

H_1 = depth below top of wall of any point in feet.

h = height of surcharge in feet.

w = weight of earth per cubic foot.

δ = angle of inclination of earth behind the wall.

φ = angle of internal friction of the earth.

C_p = constant depending upon δ and φ. (See table on page 759.)

Then*

$$P = \tfrac{1}{2} wH^2 \cos\delta \frac{\cos\delta - \sqrt{\cos^2\delta - \cos^2\varphi}}{\cos\delta + \sqrt{\cos^2\delta - \cos^2\varphi}} \qquad (1)$$

For known values of the angle of inclination and internal friction, the terms embracing them become constant and

$$P = C_p\, wH^2 \qquad (2)$$

The intensity of pressure at any point the depth of which is H is

$$\text{Unit pressure} = 2\, C_p w H_1 \qquad (3)$$

and its direction is parallel to the direction of the total pressure.*

* For walls with horizontal filling, $\delta = 0$, hence

$$P = \tfrac{1}{2} wH^2 \frac{1-\sin\varphi}{1+\sin\varphi} \qquad (4)$$

Unit pressure at any depth, H_1 is $wH_1 \frac{1-\sin\varphi}{1+\sin\varphi}$ and acts horizontally.

If angle of slope equals angle of internal friction, i. e., if $\delta = \varphi$,

$$P = \tfrac{1}{2} wH^2 \cos\delta \text{ and Unit pressure is } wH_1 \cos\delta \qquad (5)$$

Formulas (2) and (3), however, apply to these cases by using the proper value of Cp given in the table.

The values of the constant C_p are given in the table below.

Data for Determining the Earth Pressure.

Rule: To find the earth pressure on a vertical wall without surcharge, H ft. high, multiply the proper value of C_p by the square of H in feet and by the weight of the filling per cu. ft. $P = C_p wH^2$ (see p. 758.) For formulas for inclined walls and walls with surcharge, see pp. 759 and 760.

ANGLE OF INTERNAL FRICTION ϕ	VALUES OF CONSTANT C_p IN RANKINE'S FORMULA (2), p 758							
	Slope with horizontal							
	1 to 1	1 to 1½	1 to 2	1 to 2½	1 to 3	1 to 4	Level	
	Corresponding angle of slope δ							
	45°	33° 40′	26° 30′	21° 50′	18° 30′	14° 0′	0	φ
55°	0.09	0.07	0.06	0.06	0.05	0.05	0.05	0.29
50°	0.15	0.09	0.08	0.07	0.07	0.07	0.07	0.32
45°		0.13	0.11	0.10	0.09	0.09	0.09	0.35
40°		0.18	0.14	0.13	0.12	0.12	0.11	0.38
35°		0.29	0.19	0.17	0.16	0.15	0.14	0.41
30°			0.27	0.22	0.20	0.18	0.17	0.43
25°				0.30	0.26	0.23	0.20	0.45
20°					0.36	0.29	0.25	0.47

NOTE: If the angle of internal friction of the earth is unknown, the following average values may be used: Coal, shingle and broken stone, 50°; earth, 35°; clay, 30°; sand dry, 30°; sand moist, 35°; sand wet, 20°.

As stated above, the pressure is assumed to act parallel to the slope of the surface of the earth, and for walls without surcharge acts at one-third of the height of the wall from the base. The maximum unit pressure is at the base, and is equal to twice the average, while the minimum at the top equals zero, so that the variation of the unit pressures may be represented by a triangle.

Wall with Inclined Back. The earth pressure, R, on an inclined plane ab (Fig. 240) is the resultant of P, the horizontal pressure on the vertical plane ac, and W, the weight of the prism of earth abc, and acts at one-third the height from the bottom.

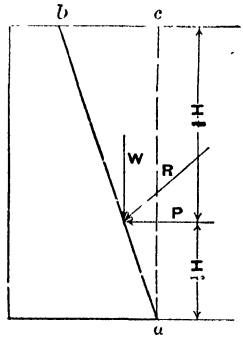


FIG. 240.—Earth Pressure on Inclined Back of a Wall. (*See p.* 759.)

Surcharge. When the earth behind the wall is loaded in any way, for example, when a highway or a railway track runs along the wall, or when the embankment is used as a storage for material—then this loading causes additional pressure on the wall, which may be provided for by replacing the load by an equivalent surcharge of earth. The height of this surcharge, h, is the extra load per square foot divided by the weight of a cubic foot of earth. Thus a load of 500 pounds per square foot is equivalent to a surcharge of 5 feet if the earth weighs 100 pounds per cubic foot.

Vertical Back of Wall with Surcharge. The earth pressure on a retaining wall with surcharge equals the pressure on the surface ab less the pressure on bd. Using a constant from the table, page 759,

$$P = wH^2\,C_p - wh^2\,C_p = w\,(H^2 - h^2)\,C_p \qquad (6)$$

and this may be represented by the trapezoid $aced$ (see Fig. 241). The distance of the point of application of this force from below the middle point in the height of the wall,

$$x = \frac{(H - h)^2}{6\,(H + h)} \qquad (7)$$

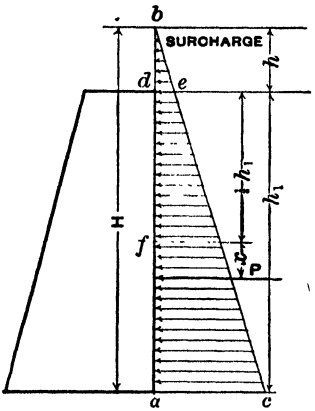


FIG. 241.—Earth Pressure on Vertical Back of Wall with Surcharge. (*See p. 760.*)

Wall with Inclined Back with Surcharge. For an inclined back, the pressure, as in the case of a wall with inclined back without surcharge, is the resultant of P, the pressure on the vertical projection of the wall found by formula (2) and W, the weight of the prism of earth one foot of length, the cross-section of which is a trapezoid. Equation (7) gives the vertical distance of the point of application of the resultant below the middle point in the height of the wall.

DESIGN OF REINFORCED RETAINING WALLS

A properly designed retaining wall, whether of reinforced concrete or of plain masonry, must fulfil the following conditions: It must be stable (1) against overturning, (2) against sliding, (3) against settling, (4) against crushing or overstressing of the material.

To prevent failure by overturning, the moment of downward forces about the outer edge of the base, $M_1 = W_1 l_1 + W_2 l_2$, must be greater than that of the overturning moment, $M_2 = Pl_3$ (see Fig. 242). The ratio of those two moments, $\frac{M_1}{M_2}$, is called the factor of safety. For reinforced concrete walls, the factor of 1.5 to 2 may be considered as ample, because the stability of wall is increased by the resistance of earth to shear along the line *ab*, Fig. 242, and the passive pressure of the filling in front of the wall, which two items are not considered in figuring the factor of safety.

The horizontal component of the resultant pressure on the foundation causes the tendency of the wall to slide. This force is opposed by the resistance to compression of the earth on the plane *dc* (see Fig. 242) and by the friction F. The friction is equal to the vertical pressure multiplied by the tangent of friction between concrete and earth, or, if

F = total friction,

$W_1 + W_2$ = weight of concrete and earth,

ϕ = angle of friction between earth and concrete

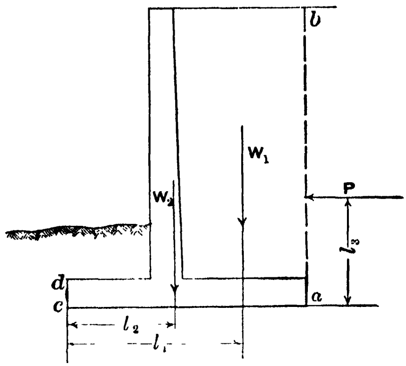


Fig. 242.—Forces Acting upon a Retaining Wall and their Moment Arms. (*See p.* 761)

Then

$$F = (W_1 + W_2) \tan \phi$$

If the wall slides, the cohesion of the earth along the line *ab* (Fig. 242) must be destroyed, which item increases the stability against sliding. The tangent of the inclination of the resultant pressure, that is, the ratio of its horizontal to vertical component, should not be larger than the tangent of the angle of friction.

Sometimes a vertical projection of the base may be needed, which may be placed in the middle of the base or at either end.

Having determined the earth pressure as explained in preceding pages, the design of a reinforced concrete retaining wall resolves itself primarily into the determination of the thickness and reinforcement of concrete slabs to be obtained by the principles outlined in Chapter XXII on Reinforced Concrete Design. The methods to follow can be illustrated best by practical examples, which are given in full below.

A retaining wall is especially subject to temperature stresses. To locate the stresses at specially prepared joints, contraction joints may be placed at stated intervals. In an unreinforced wall, a spacing of 20 to 30 feet between joints is necessary to prevent intermediate cracks. By introducing steel to prevent the formation of visible cracks, no joints are necessary. Steel reinforcement for shrinkage and temperature contraction is treated on page 565.

EXAMPLE OF T-SHAPED RETAINING WALL

Example 1. Design a retaining wall 12 ft. high above ground to support a sand filling. Angle of internal friction of sand, which weighs 100 lb. per cu. ft. is 35°, and the fill slopes back at the same angle. Working stresses: for the 1 : $2\frac{1}{2}$: 5 concrete in compression, f_c = 500 lb. per sq. in.; steel in tension, f_s = 16 000 lb. per sq. in.; ratio of moduli of elasticity, $n = 15$; allowable shear involving diagonal tension, $v = 32$ lb. per sq. in.; bond of steel to concrete, $u = 80$ lb. per sq. in.

Solution. If base is imbedded 4 ft. to protect from frost, and if the footing is assumed 18 inches thick, total height of wall is 16 ft. and height of stem 14 ft. 6 in. The design is shown in Fig. 243, page 763.

Upright Slab. Earth pressure on stem from Formula (2), page 758, taking value of C_p from the table, $P_1 = 0.41 \times 100 \times 14.5^2 = 8600$ lb. This acts at $\frac{1}{3}$ the height. Horizontal component, $H_1 = P_1 \cos 35° = 7040$ lb., and the moment, $M = 7040 \times \frac{1}{3} \times 14.5 \times 12 = 408\ 000$ in. lb. The effect of weight of wall and the vertical component of earth pressure is small and may be disregarded.

Thickness of vertical slab at bottom, using Formula (9), page 485, and table of constants, page 483, and adding 1.7 in. to the depth to steel to properly imbed it, is $d + 1.7 = 0.034 \sqrt{408\ 000} + 1.7 = 23.5$ inches. Ratio of steel is $p = 0.005$ (to correspond to working stresses), hence area of steel is $A_s = 1.31$ sq. in per foot of length of wall. This is satisfied by $\frac{7}{8}$ in. round bars placed vertically 5.5 in. on centers. (See table, p. 574.) The thickness of wall at top may be selected as 12 in. The moment decreases from the bottom upwards so the steel may be reduced as shown in Fig. 243, page 763.

Since total shear, $V = 7040$ lb., unit shear involving diagonal tension, is $v = \dfrac{7\ 040}{12 \times 21.8 \times 0.894} = 30$ lb. per sq. in. (See p. 517.) As this does not exceed working stress, no stirrups are needed.

Bond stress is $u = \dfrac{7\ 040}{21.8 \times .894 \times 2.18 \times 2.75} = 60$ lb. per sq. in. (see p. 534).

Length of bar to imbed in footing to prevent pulling out is $50 \times \frac{7}{8} = 43.8$ in. (see table on page 540), hence the vertical bars must extend into the base this distance, or else be provided with bent ends (see page 540).

*A table of dimensions and reinforcement for T-shaped and for counterfort retaining walls of different heights, compiled by Sanford E. Thompson, is given in "Concrete in Railroad Construction," published by The Atlas Portland Cement Co.

To obtain this bond, the vertical rods frequently are bent into the right cantilever of the footing. If instead they are bent to run into the left cantilever, they may form the horizontal reinforcement there, as shown in Fig. 243.

Footing. In a correctly designed wall the resultant force should intersect the base within the middle third of its length. This determines the ratio of length of footing to height of wall, and can be obtained only by trial for any particular case. A study of different conditions shows that this ratio is gen-

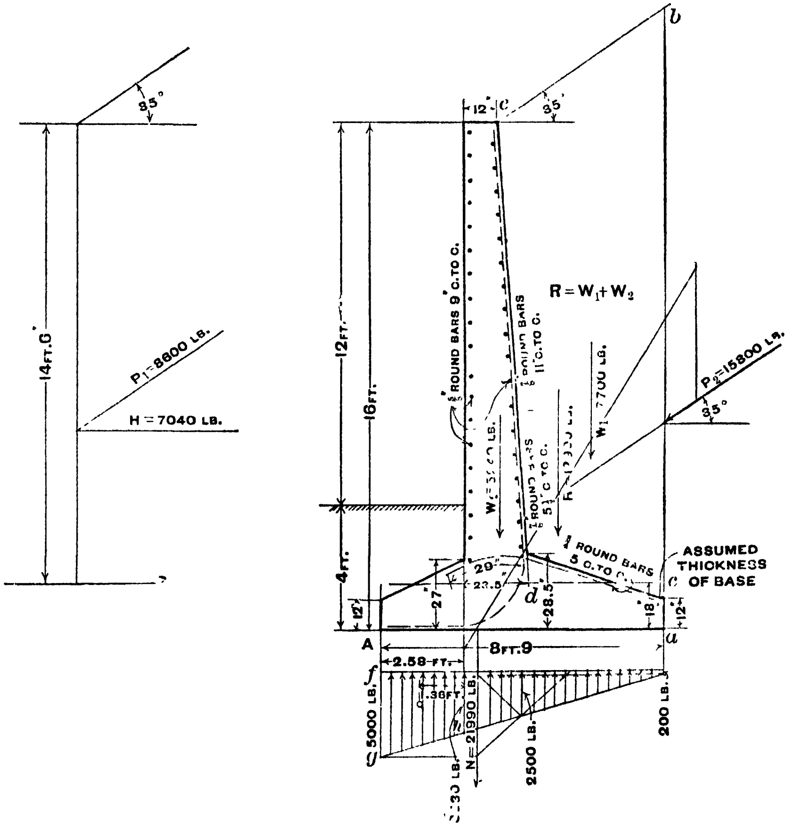


FIG. 243.—Design of T-shaped Retaining Wall. (*See p.* 763.)

erally 0.4 to 0.6, depending upon the inclination of earth pressure, the weight of the fill, and finally upon the ratio between the length of the projecting toe and the total length of the base. The length of base best suited for our example was found after several trials to be 8 ft. 9 in.

The forces acting on the footing are P_2, the earth pressure on the plane ab, W_1, the weight of prism of sand, $bcde$, and W_2, the weight of the retaining wall itself. The distance from the toe to the line of action of the resultant R

of W_1 and W_2 may be obtained as follows: Find center of gravity of earth and center of gravity of concrete; multiply the distance from A to these centers of gravity by the respective weight, and thus obtain the statical moment. Divide the sum of these moments by the sum of the weights, $W_1 + W_2$, and the location of the center of gravity of the combined weight is obtained. The line of pressure drawn for P and R intersects the base just inside of the middle third.

Normal component of resultant, N = 21 990 lb. and horizontal component, H = 12 900 lb. Hence, ratio $\frac{H}{N} = 0.587$, which is smaller than the tangent of the angle of friction, hence there is no danger of the wall sliding.

Maximum unit pressure on soil (from formula (70), p. 379) is 5000 lb. per sq. ft., while the minimum equals nearly zero.

The graphical method of finding the distribution of forces on the base is explained on page 744.

Left Cantilever. Omitting weight of slab and of earth above it as negligible, the forces acting on this part of the footing are represented by trapezoid $fghi$. Total force is $\frac{5000 + 3530}{2} \times 2.58 = 11\,000$ lb. and moment arm from the diagram is 1.36 ft.; hence bending moment, $M = 11\,000 \times 1.36 \times 12 = 179\,500$ in. lb. per ft. of width.

The minimum depth to steel from Formula (1), p. 481, using Table 15, page 596, is d = 14.5 in., and the area of steel, A_s = 0.868 sq. in. However, this depth may be too small to satisfy the bond stress, which is considered below.

Further, if vertical steel in the vertical wall is all bent and carried into the left cantilever of the footing, we should have 1.30 sq. in. of steel per foot of width or $\frac{7}{8}$ in. round bars spaced $5\frac{1}{2}$ in. cc., which for a depth of 14.5 in. gives a ratio $p = 0.0075$, or greater than is necessary. If desired, therefore, a part of this steel may be carried only far enough into the footing to prevent its pulling out.

The bond for the suggested depth must be considered. Unit bond,

$$u = \frac{11\,000}{14.5 \times 0.9 \times 2.75 \times 2.18} = 140 \text{ lb. per sq. in. (see p. 534).}$$

The bond stress is excessive, therefore the depth, d, of $14\frac{1}{2}$ in. must be increased. To decrease the bond stress, for plain bars the depth of the cantilever must be increased as follows: Assume the decreased ratio, p, for the increased section of concrete at p = 0.0045. Then the corresponding values from Table 17, page 598, k = .305, j = .898.

From page 534 $u = \frac{V}{jd\Sigma o}$ hence $d = \frac{V}{uj\Sigma o}$. Substituting values,

$$d = \frac{11\,000}{80 \times 0.9 \times 2.18 \times 2.75} = 25.5 \text{ in., and total depth 27 in.}$$

The depth of beam must be increased to 27 in. in order to decrease the bond stress to 80 lb. per sq. in.

Smaller depth can be used with deformed bars of approved design.

Right Cantilever. It is evident from Fig. 243, page 763, that three forces act on the right cantilever: the upward pressure of the soil, the downward weight of the earth filling, and the vertical component of the earth pressure. The resultant of these forces acts downward, hence the moment is negative.

The computations for amount of steel and the shear and bond stresses are similar to that for the left cantilever.

The length of imbedment necessary to prevent slipping is not treated in the previous case, so it may be given here in detail.

Area of concrete, $A = 12 \times 27 = 324$ sq. in.; area of steel, $A_s = 1.07$ sq. in. and ratio of steel, $p = \frac{1.07}{324} = 0.0033$. From table 15, p. 596 find the corresponding k and j, $k = .268$, $j = .911$. From formula (7), p. 484, since $M = 329\,000$ inch pounds, $f_s = \frac{329\,000}{27 \times .91 \times 1.07} = 12\,500$ pounds. For this stress in steel, the length of imbedment from table on page 540 is $39 \times \frac{3}{4} = 29$ in.

Both cantilevers may be tapered toward the end to a minimum practicable depth, since the moments decrease from the support to zero at the end.

Horizontal Reinforcement for Temperature. Temperature reinforcement is treated on page 565.

EXAMPLE OF RETAINING WALL WITH COUNTERFORTS

Example 2. Design a reinforced concrete wall with counterforts to support a sand filling 20 ft. high above ground, using same assumptions as in Example 1, page 762.

Solution. In this type of wall the vertical slab acts as a slab supported by the counterforts, the principal steel being horizontal. The projecting toe of the footing is a cantilever and the footing below the earth is a slab supported by the counterforts. The counterforts tie the imbedded footing to the vertical slab and act as cantilevers fixed to the footing. Design is shown in Fig. 244, p. 767.

The slabs may be considered as partly continuous, using the moment $M = \frac{wl^2}{10}$. If carefully designed for negative moment $M = \frac{wl^2}{12}$ might be permissible. (See p. 496.)

Instead of forming a projecting toe as a cantilever, it is sometimes more economical when the projection is large to introduce small buttresses and construct this part of the footing also as a partly continuous slab.

The first step in operation of design is to determine by trial the length of base and the relation between the projecting toe and the base, the allowable pressure on the soil and minimum angle of inclination of the resultant earth pressure being the determining factors. The method is the same as for a T-type wall, as outlined on page 764.

Spacing of Counterforts. The spacing of counterforts or ribs may be found on the basis of minimum material*, from which 8 feet may be adopted.

Vertical Wall. The vertical wall must be considered in narrow horizontal strips as slabs supported by the counterforts, partly continuous, and loaded uniformly. The earth pressure changes with the height, so that the pressure upon the different strips decreases from the bottom up. The pressure against the bottom strip as given on page 766 is 1480 lb. per sq. ft., or 123 lb. per ft. of width for 1-inch of height. Using $M = \frac{wl^2}{10}$, $M = \frac{123 \times 64 \times 12}{10} =$ 9500 inch pounds per inch of width. Hence (p. 481) $d = .118\sqrt{9500} = 11.5$ in.; thickness of wall is thus 13 in., and area of steel, $A_s = 0.005 \times 11.5 \times 12 = 0.69$ sq. in. per ft. of height. Round bars $\frac{5}{8}$ in. diameter spaced $5\frac{1}{4}$ inches on centers may be used.

For convenience in construction the thickness of the wall may be made uniform, and the spacing of rods increased with the decreasing earth pressure, as shown on the drawing. The negative bending moment may be provided for by introducing short rods in front of buttresses, or by bending the rods. (p. 496.)

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* For full discussion, see "The Design of Retaining Walls," by H. A. Petterson, Engineering Record, Vol. LVII, 1908, p. 777; for practical purposes the following demonstration illustrates the necessary steps. Use notation page 353, also let x = spacing of buttresses in feet; Q = the maximum horizontal unit pressure on vertical wall, which occurs at the bottom of the wall. Q, from formula (3), page 758, is 1480 lb. per sq. ft. Taking a strip of the vertical slab one ft. in height, whose span is the spacing of the counterforts, the bending moment is then $M = \frac{1480 \times x^2 \times 12}{10} = 1780x^2$; the depth to steel, (p. 485), $d = .29 \times .118\sqrt{1780}\,x = 1.43x$, and the volume per foot of length of wall is $\frac{1.43x}{12} \times 1 \times 22 = 2.6x$ cu. ft. Maximum unit weight acting on horizontal footing slab is 5 325 pounds per sq. ft. Hence $M = \frac{5\,325 \times 12 \times x^2}{10}$, $d = .29 \times .118\sqrt{5325 \times 1.2x^2}$ $= 2.72x$, and volume per foot of length of wall is $\frac{2.72x}{12} \times 1 \times 8.25 = 1.9x$

The thickness below steel is a constant for any spacing and therefore need not be considered in fixing the volume.

Assume the thickness of counterfort as 16 in., and volume will be $\frac{22 \times 8.25 \times 16}{2 \times 12} = 121$ cu. ft., and for one foot of length of wall, $\frac{121}{x}$. Because of the greater cost, per unit of volume, of the counterforts over that of the slab work in a wall of this type, the quantity representing the counterfort volume may be increased by, say, 100%. The expression for this quantity then becomes $\frac{121}{x} \times 2$. Hence total volume, $Q = 2.6x + 1.9x + \frac{121}{x} \times 2$

or $Q = 4.5x + \frac{242}{x}$ and $\frac{dQ}{dx} = 4.5 - \frac{242}{x^2} = 0$ (for minimum, first derivative equals zero).

$$x = \sqrt{\frac{242}{4.5}} = 7.3 \text{ ft.}$$ For practical purposes, say 8 ft.

Horizontal Footing Slab. This slab may be considered as composed of narrow strips uniformly loaded and supported by the counterforts. The loading is the difference between the weight of the earth above it plus the vertical component of the earth pressure, and the upward pressure of the soil. As indicated in the drawing, this difference is a maximum at *a* and decreases toward *b*. In this case the maximum unit loading is 5566 − 241 = 5325 lb per sq.ft. The maximum bending moment in this slab, considering it as partly continuous is

$$M = \frac{5325 \times 64 \times 12}{10}$$

= 40 800 in. lb. Depth of steel, d = 0.29 × $0.118\sqrt{40800} = 21.75$ in., hence thickness may be taken as 23.25 in. The area of concrete is then 261 sq. in., hence area of steel required is $A_s = 1.31$ sq. in., which is satisfied by $\frac{7}{8}$-in. bars spaced $5\frac{1}{2}$ in. on centers. The thickness of this foundation slab may be made uniform, and the spacing of the rods increased as the loading decreases.

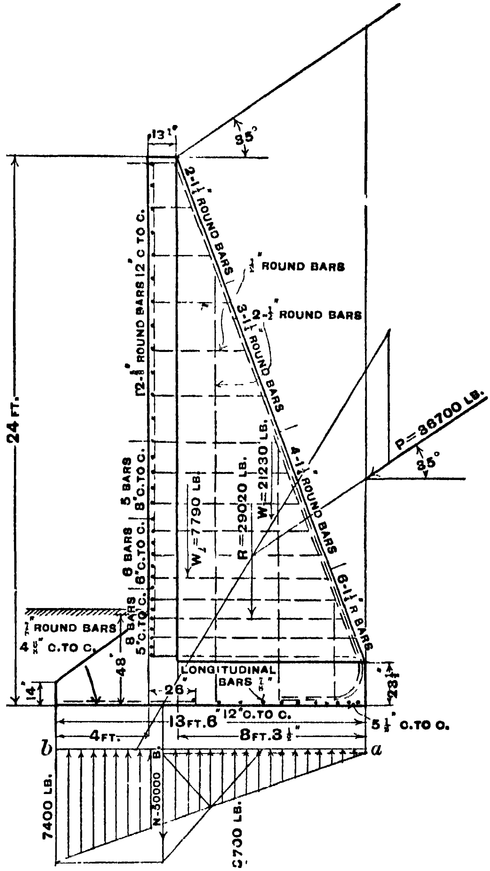

FIG. 244.—Design of Retaining Wall with Counterforts. (*See p.* 765)

The negative bending moment must be provided for by introducing at the top of the slab, under the counterforts, short rods of equal size and spacing to the bottom ones or else these bottom rods must be bent down at each counterfort. (See p. 496.)

Counterforts. A counterfort is really an upright cantilever beam supported by the horizontal foundation slab and carrying as its load the vertical slab of the wall, which, in turn, takes the earth pressure. The thickness of the counterfort, which must be sufficient to insure rigidity and resist unequal pressures during construction, may be selected by judgment.

To determine the quantity of steel required in the counterfort, we find the horizontal component of the earth pressure per foot of wall to be (from formula (2), p. 758) $.41 \times 22 \times 22 \times 100 \times .819 = 16\ 200$ lb.; hence, the total force transmitted to the counterfort, since they are spaced 8 ft., is $8 \times 16\ 200 = 129\ 600$ lb Since the force acts at one-third the height, the bending moment is $M = 129\ 600 \times \frac{22}{3} \times 12 = 11\ 400\ 000$ in. lb. The thickness of the counterfort is taken at 16 in., the depth to steel, $d = 110$ in. From Formula (1), p. 481, $C = \sqrt{\frac{bd^2}{M}} = \sqrt{\frac{16 \times 110^2}{11\ 400\ 000}} = 0.130$. By interpolation in Table 16, on page 597, between items 3 and 4, the required ratio of steel, $p = 0.00416$ and area of steel $A_s = 110 \times 16 \times .00416 = 7.36$ sq. in. Six 1¼-in. round bars will satisfy this.

The portion of the counterfort receiving the greatest tension is the inclined edge, so these bars are placed near to this surface. Besides these bars, horizontal and vertical bars are necessary to tie the vertical and horizontal slabs to the counterfort, to transfer the forces and provide for diagonal tension. These bars should be bent into the slabs to obtain as good a bond as possible. The principal tension bars in the counterforts also must be well imbedded in the horizontal foundation slab, and bent so as to attain their full strength in tension. The value of hooking is discussed on page 438.

COPINGS

On many structures a coping is necessary and frequently it must be built separately from the main wall. Instead of stopping at the underside of the coping, the Delaware, Lackawanna, & Western R. R. has found it more economical and better workmanship to carry the main wall 1½ inches above the bottom of the coping, and after the concrete has set, to place the coping form tightly against the face of the wall* and pour the concrete.

DAMS

In dams the important requirements are strength and water-tightness, low cost, and speed in construction. Current practice, as indicated by the written opinions of engineers and by structures actually built, indicates that concrete on the whole fulfills these requirements better than any other material. While this is true of the standard gravity section, it is particularly true of thin arch dams, reinforced or plain, and of hollow reinforced dams, both of which have proved satisfactory.

Under the present methods of concrete construction, stone masonry is nearly always more expensive than concrete or rubble concrete. In some cases where the cost of transporting cement from the nearest mill

* *Engineering Record*, Aug. 19, 1911, p. 221.

to the dam site has been prohibitive, instead of using masonry, it has been more economical to set up a mill and manufacture cement on the job. Sand-cement, for example, was used on the Lohantan, Nevada, dam* and hydraulic lime on the "Tiger Hill" dam† in Mexico.

The choice between plain and rubble concrete and,—if the latter is selected—the amount of rubble to use, is governed by local conditions, and comparative estimates must be made in each case. Large stones save cement, crushing, and mixing, but placing them in the dam has become slow work compared to placing concrete by up-to-date methods, and frequently an independent plant must be used to place the rubble. In the Medina Valley dam‡ only 10% of rubble was used, as it was found that a larger percentage would have so delayed the progress of the work that interest and overhead charges would have outrun all saving in cement, crushing, and mixing. On the Las Vegas, New Mexico§ dam bids were received for both concrete and rubble, and the concrete design proved cheaper. On the same type of dam built in Australia,|| 30% of rubble was used. In the Shoshone dam¶ 25% of rubble was used.

The dam at Boonton, N. J., a section of which is shown in Fig. 246, p. 772, contains 240 000 cubic yards of concrete rubble, and was built at a contract price, not including the cement, of $1.98 per cubic yard. Only 0.6 barrel Portland cement was used per cubic yard, although the proportions of the concrete matrix were $1 : 2\frac{3}{4} : 6\frac{1}{4}$. This small quantity of cement was due to the large proportion of stones which averaged from one yard to $2\frac{1}{2}$ yards each and occupied 55% of the total volume. The contract price mentioned includes the preparation of the large stones and the crushed stone, and their transportation from a quarry three miles away. It is believed by the authors that the price and also the quantity of cement per cubic yard represent minimum figures in first-class construction, but the force account showed that the contractor was making a fair profit, and inspection of the work and its water-tightness prove that there was no skimping in the use of cement. On this particular job the quotation of the highest bidder was nearly double the accepted price.

The concrete should be of soft mushy consistency and the large stones dropped onto it and joggled with bars in order to obtain proper

* L. E. Sale in *Engineering and Contracting*, December 3, 1913, p. 623.
† Guy S. Newkirken, *Engineering News*, July 3, 1913, p. 19.
‡ Terrell Bartlett in *Engineering News*, Sept. 11, 1913, p. 508.
§ Charles W. Sherman in *Engineering News*, October 27, 1910, p. 446.
|| L. A. B. Wade in *Engineering News*, May 19, 1910, p. 588.
¶ H. N. Savage in *Engineering News*, December 9, 1909, p. 627 and June 9, 1910, p. 679.

imbedment. In the Boonton dam the consistency was about that of pea soup and the rubble stone was dropped with considerable force.

Types of Dams. The standard dam section continues to be the gravity type, but the hollow, arch, and multiple arch* are gaining favor in situations particularly adapted to them. In some cases where a narrow rock gorge makes the arch design possible, a gravity section has been used and the dam arched to provide an additional factor of safety or to prevent cracking. In other cases, dams have been built arched with a partial gravity section where failure would result in case the arch action did not take place. There are, also, many dams in existence designed as arches, pure and simple, with very thin sections, and some not even reinforced. An arched dam† at Crowley Creek, Boise, Idaho, 55 feet high, 5.2 feet wide at the base, and 3 feet wide at the top, was raised to a height of 90 feet with a base 9.2 feet and a crest 3.2 feet wide. No reinforcement was used. On the other hand, when the Sweetwater dam near San Diego, California, was enlarged,‡ it was changed from arch to gravity section because, as the addition was built with the reservoir filled, it was doubtful if the old and new parts could be made to act together as an arch.

The Las Vegas dam required two-thirds the yardage that a gravity section would have taken.

The hollow reinforced concrete dam§ reduces the quantity and cost of materials but permits a very broad base, and a sloping watertight deck upstream by means of which the water pressure is made to increase instead of oppose stability. An example of this type of dam—patented—is shown in Fig. 245, page 771.

DESIGN OF DAMS

Gravity Dams. Gravity dams must be constructed to withstand overturning and sliding caused by water pressure or by ice pressure on the upstream face; also, water pressure on the base and on the interior horizontal planes must be allowed for.

To avoid tension in the foundation it is necessary that the resultant

* A partial list of arch dams is given in *Engineering News*, October 27, 1910, p. 517, and November 10, 1910, p. 520. Other arch dams are described in *Engineering and Contracting*, May 20, 1914, p. 587 and 594, and Transactions American Society of Civil Engineers, Vol. LXXV, p. 112 and Vol. LXXVIII, pp. 564 and 685. Multiple arch dams are described in *Engineering News*, April 30, 1914, p. 962; April 25, 1915, p. 818; May 27, 1915, p. 1909; and October 28, 1915.

† *Engineering Record*, June 20, 1914, p. 693.

‡ The design and methods of construction are discussed by James D. Schuyler in *Engineering Record*, September 2, 1911, p. 264.

§ Four studies by Edward Wegmann for such a hollow dam for Stony River are shown in *Engineering News*, September 5, 1912, p. 446.

of all the forces of pressure and weight shall pass through the middle third of the base. Dangerous sliding need not usually be feared if the dam is designed to resist overturning. In considering the resistance of friction, Mr. Joseph P. Frizell* states that smooth stone slides on smooth stone under a horizontal force of two-thirds its weight, and to slide on gravel or clay, stone requires a force nearly equal to its weight.

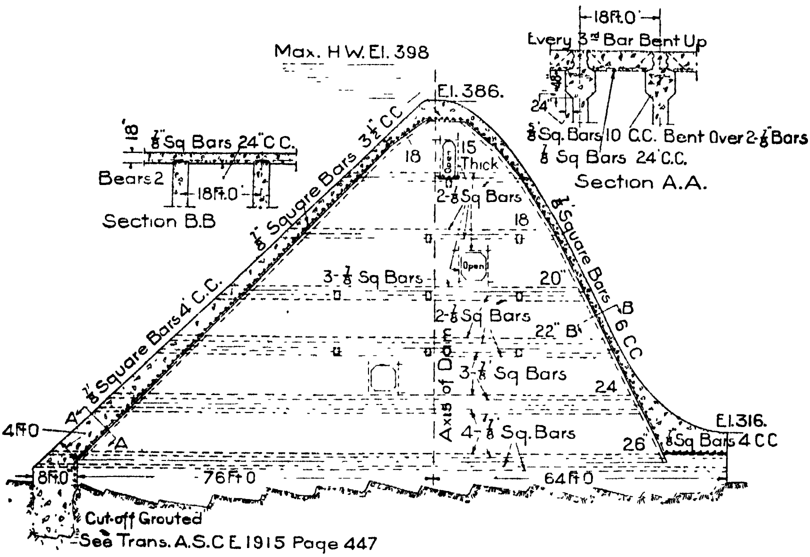

FIG. 245.—Section through Reinforced Concrete Dam at Estacada, Oregon. (*See p.* 770.)

The pressure of the water upon any submerged surface is equal to the area of the surface in square feet, times the weight of a cubic foot of water, times the depth of the center of gravity of the surface below the water level. This pressure tends to overturn the dam, and is resisted by the weight of the dam, and in some cases, where the up-stream face slopes, by the weight of the water upon the dam.

The treatment in Frizell's Water Power of the location of the center of pressure, and the moment produced by it, is especially clear and practical.

Fig. 246 represents a section through the overflow of the rubble concrete dam at Boonton, N. J., already referred to on page 769. The extreme height of the dam at the highest point above the foundations

* Frizzell's, "Water Power," p. 19.

is 110 feet. An interesting practical test of the water-tightness of concrete occurred when the reservoir was filled. A vertical well was left in the dam in order to provide access to two drainage gates, and although the water in the reservoir was 100 feet deep, and was separated from the well by only 5 feet 6 inches of concrete mixed in the proportions $1 : 2\frac{3}{4} : 6\frac{1}{4}$, the well remained entirely dry.

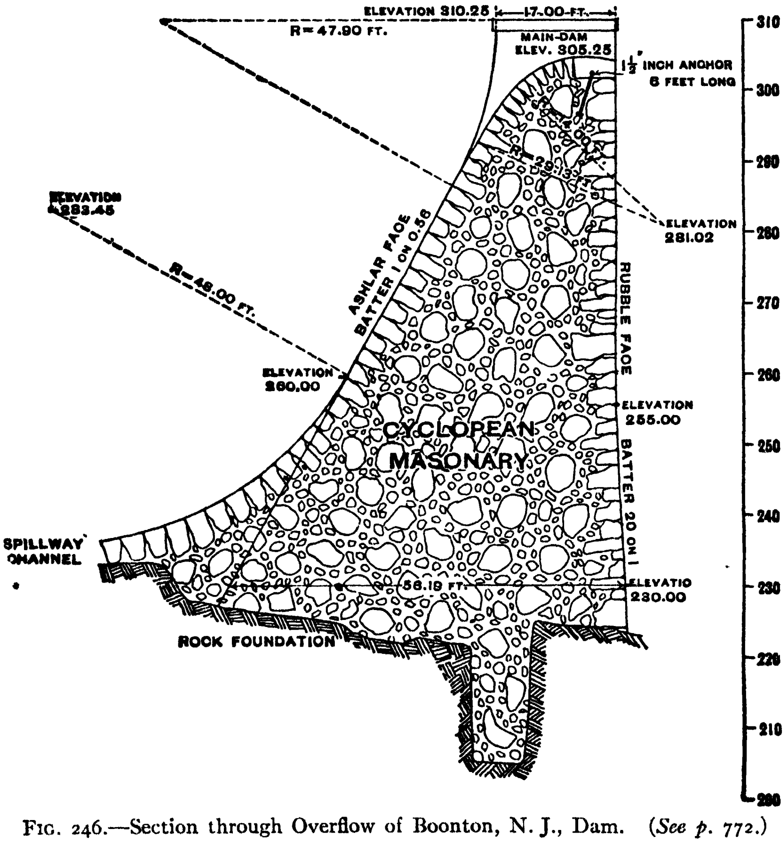

FIG. 246.—Section through Overflow of Boonton, N. J., Dam. (*See p.* 772.)

Upward Water Pressure. With a firm and impervious foundation, there is little likelihood of material upward pressure on the base, and with first-class construction horizontal joints may be made practically impervious. With permeable material or thin strata between which the water can penetrate upward, pressure is bound to occur and must be provided for in one of three ways: (1) making the dam heavy enough

to overcome the upward pressure; (2) draining the foundation, and if necessary, the interior; and (3) making the pervious foundation impervious.

The allowance for upward pressure varies not only with the structural conditions but also with the possibility of property damage and loss of life in case of failure. In the Kensico dam for New York City a pressure of two-thirds the total head at the heel, and zero at the toe was assumed, and in spite of an interior drainage system, similar pressures were assumed on all interior construction joints.

The few tests made indicate beyond all question that upward pressure does take place on the base of a dam if the foundation is poor. German tests* showed a pressure equivalent to the full head near the heel and one-half the full head near the toe. A valuable and complete account of laboratory and of field tests, both on a small scale and on full size structures, is given by Captain W. A. Mitchell in Professional Memoirs, U. S. Army, for January-February, 1915. Tests on adhesion, upward water pressure, and percolation, including original army tests and those described in literature, are summarized, conclusions drawn, and the application of the evidence to the design of dams is discussed.

Under-drainage is economical and is accomplished by sinking a cut-off wall near the heel of the dam, and draining off through drain tile† all water that passes it. The hollow dam may be built with no floor, thus providing the freest possible drainage once the seepage gets by the cut-off wall.

During the construction of the Lost River Dam, in Oregon and California, one-half the weep holes under the floor became stopped. The floor bulged up under the upward pressure, falling as soon as the drainage channels were put back in commission.‡

Interior drainage is provided in much the same way as under-drainage. In the Olive Bridge and the Kensico§ dams, in New York, vertical pipes were used, while in the Loch Raven|| dam near Baltimore the drains were laid horizontal.

Grouting is extensively employed in improving poor foundations. Seamy, porous rock, limestone full of cavities, and sand and gravel beds may be made reasonably tight by sinking drill holes and pumping in

* C. R. Weidner in *Engineering News*, July 31, 1913, p. 202.

† *Engineering Record*, September 14, 1912, p. 284; *Engineering and Contracting*, February 17, 1915, p. 150.

‡ W. W. Patch in *Engineering News*, April 30, 1914, p. 968.

§ Alfred D. Flinn in *Engineering News*, Apr. 15, 1912, p. 772.

|| *Engineering and Contracting*, Feb. 17, 1915, p. 175.

grout under pressures of about 100 pounds. The Lahontan dam site* in Eastern California for example, was treated in this way.

Ice Pressure. Ice pressure depends not only on climate, but also on the shape of the storage basin. In a narrow gorge where the ice cannot expand up the banks, it sometimes exerts large pressure on the dam. In a dam near Minneapolis this caused complete failure, and in a second case failure was narrowly averted by cutting the ice away from the dam face. In the Kensico Dam, an ice pressure allowance of 23.5 tons per linear foot was made. In wing walls, particularly, provision must be made in northern climates, not only for ice pressure, but in connection with small dams, for the possibility of tipping as the pond rises under the ice.

Flash Boards. Flash boards for an arch dam may be designed to eliminate arch action in the boards themselves by making the boards long enough and placing them so that the ends, instead of butting, overlap.

Movable dam crests‡ are sometimes necessary.

Durability. Concrete in dams has proved as durable as any other material. The destructive agencies are abrasive material in the water flowing over the dam, certain chemicals in the water adjacent to or seeping through the dam, and frost action.

Large quantities of hard sand and gravel flowing over the dam wear away the concrete face, and measurements have shown that half an inch has been lost in this way in a comparatively few years. The amount, however, is a very small proportion of any dam and does not affect the stability. An allowance should be made, however, in designing dams with thin sections and in protecting reinforcing bars with a sufficient thickness of concrete.

CORE WALLS

Concrete is largely superseding rubble masonry for core walls in earth dams and dikes. The forms can be roughly made without reference to the appearance of the faces, while a thin wall of concrete may be built water-tight more easily than one of rubble masonry. Unless reinforced, core walls are generally of the same thickness as those of rubble masonry. The Natural cement concrete core wall of the Sudbury Dam, built by the Boston Water Commissioner and his successor upon the work, the Metropolitan Water Board of Massachusetts, is 2 feet thick at the top, with a batter

* D. W. Cole in *Engineering Record*, March 29, 1913, p. 340.
† Ernest W. Schroder in *Engineering News*, October 27, 1910, p. 517.
‡ W. L. Marshall in *Engineering News*, June 4, 1914, p. 1264.

of one in fifteen on both faces, until it reaches a maximum width of 10 feet. At Spot Pond Reservoir, several dikes with core walls of Portland cement concrete, of 15 to 18 feet average height, are 2½ feet in thickness throughout.

The dike for the Jersey City Water Supply Company at Boonton, N. J., is designed for a total height of 54 feet. The lower 30 feet is 4 feet 8 inches thick, and at this height it begins to batter, so as to reach a width of 3 feet at the top.

Although core walls may often be economically built of rubble concrete, the stones must be of smaller size, and cannot occupy so large a volume of the mass as in gravity dams, since the sections are thinner. In the construction of the Boonton Dike, mentioned above, one contractor was placing rubble to the extent of 20% of the total mass, while another was placing 33%. In the former case the stones were loaded on to derrick skips and unloaded by hand; in the latter case, they were hooked by the derrick. This 33% probably represents a maximum for a wall 5 feet thick or less.

Since a thin wall of reinforced concrete may be made equally strong, and more elastic than a thick wall of plain concrete, reinforcement may eventually be employed to reduce the section, and therefore the quantity of material.

TEMPERATURE CHANGES IN CONCRETE DAMS

The temperature variations in concrete dams, due to chemical action of the cement in setting and to atmospheric temperature changes, are important as effecting the expansion and contraction. Investigations have been made in the Boonton, N. J., dam, the Panama Canal locks,* the Arrowrock dam† in Idaho, and the Kensico dam‡ in New York.

The results thus far obtained indicate that the temperature rise caused by the heat of setting is governed by the richness of the concrete and by the opportunities for radiation while the heat is being generated. Radiation is greatest in small dams and in large dams built in relatively thin layers where each layer partially sets before another is added. The seasonal temperature changes, after setting heat is dissipated, are governed by the atmospheric temperature changes and by the dimensions of the dam.

The method of making the observations has an important effect on the results; thus at Arrowrock the concrete in which the thermometer was buried was placed slowly, in layers, allowing considerable radiation, while at Kensico the reverse was the case.

* 1910 Report, Isthmian Canal Commission, p. 122.
† Trans. American Society of Civil Engineers, Vol. LXXIX, 1915, p. 1225.
‡ American Society of Civil Engineers, Vol. LXXIX, 1915, p. 1247.

Setting Heat. The rapid rise in temperature due to chemical changes reaches a maximum in a few days or a few weeks after the concrete is placed and lasts for one or more years. Observations indicate in a general way that the total rise above the temperature of the air during concreting is from 25° to 40° Fahr. and that this maximum is apt to be attained in the first week, or at the most, the first month. The observations on the Boonton dam, which is 94 ft. high by 55 ft. wide at the base, and made with a lean 1:3:7 mix, showed the maximum temperature at 24 hours with a gradually decreasing temperature up to about a year. The Arrowrock dam, 315 ft. high by 215 ft. wide at the base, built with a richer 1:$2\frac{1}{2}$:5 mix, showed that the concrete 20 ft. or more from the face is affected by the setting heat for several years, that near the center for as much as five years. The investigations at the Kenisco dam, which is nearly as rich as the Arrowrock, were more exhaustive than the others, but substantially confirmed the results indicated. In these investigations the variation in rise was shown to depend in a measure upon the richness of the concrete. The maximum temperature was reached at a period varying from five to ninety-five days.

Effect of Atmospheric Changes. Observations on the effect of seasonal atmospheric changes are complicated by setting heat for one or more years after the dam is built. The temperature of the concrete varies much less than that of the atmosphere. For example, in the Arrowrock dam, with a daily atmospheric variation of 50° Fahr., the variation in the temperature of the concrete one foot from the façe was only 2° Fahr.; the seasonal changes, on the other hand, produce appreciable effects. In the Arrowrock dam, with a seasonal change of 75° Fahr., the temperature of the concrete $3\frac{1}{2}$ ft., 10 ft., and 20 ft. from the nearest face was 32°, 12°, and 0° Fahr. respectively. The Boonton dam observations indicated that the seasonal range in temperature of the concrete varied with the seasonal range in the temperature of the air, and with the distance from the dam face, in accordance with the formula

$$R = \frac{T}{3\sqrt[3]{D}}$$

where

T = extreme seasonal range of temperature of air in degrees Fahr.

R = extreme seasonal range of temperature of concrete in degrees Fahr.

D = distance from nearest face in feet

On the Boonton dam, T = 135° Fahr.

CHAPTER XXVIII

CONDUITS AND TUNNELS

Since the principal stresses in arches are compressive, concrete is peculiarly suitable for all classes of arched structures. Tensile stresses caused by eccentric loading may be provided for by steel reinforcement, and excessive compressive stresses by reinforcement or by increasing the thickness of the concrete, or by both. The use of steel may also prevent the failure of thin arch sections under suddenly applied loads or from settlement of the foundation.

Furthermore, improved methods of construction and the lower price of cement have so reduced the cost both of plain and reinforced concrete structures that conduits, subways, and tunnels, whether arched or of rectangular section, are now built of concrete where stone or brick masonry, or steel, would formerly have been used; indeed, the whole undertaking would, through high cost, in many cases have been rendered uneconomical. Concrete arches and conduits are likely to be cheaper than brick even at the same price per cubic yard because the greater strength of the concrete makes a thinner section possible.

CONDUITS

Present Practice. Until within a comparatively few years concrete was used in conduits with considerable caution and largely from an experimental point of view. At present, however, standard practice dictates the use of concrete under practically all conditions, the selection of the material being governed simply from the standpoint of costs. Conduits of both large and small size are usually cast in place, but circular pipe as large as 9 feet in diameter is frequently pre-cast.* In sewer systems the brick lining for the invert is retained where there is danger of injury to the concrete through acids. In irrigation works special precautions are needed to prevent failure from alkalis in the soil.

Among the best examples of large conduits are those built by the New York Board of Water Supply†, the cities of Los Angeles‡ and Balti-

* J. C. Lathrop in *Engineering Record*, Oct. 2, 1914, p. 380: and *Engineering News*, Mar. 25, 1915, p. 600.

† Fred F. Moore in *Engineering Record*, October 28, 1911, p. 501; and Walter E. Spear in *Engineering News*. January 15, 1914, p. 150.

‡ *Engineering Record*, November 1, 1913, p. 486; and Burt A. Heinly in *Engineering News*, June 19 1913, p. 1257.

more.* The sewer systems of both Baltimore and San Francisco† afford examples of fairly large and medium sized conduits. The U. S. Reclamation Service has developed the best practice in the building of irrigation works.‡

Comparison of Brick, Steel and Concrete Conduits. In designing conduits for exterior pressure, brick and concrete are available, but pressure conduits must be made of steel or reinforced concrete.

Concrete is almost always cheaper than brick, running from 20% to even 50% less in cost. Even at the same cost per cubic yard, centering included, concrete has an advantage, for, being stronger, less material is required and therefore less excavation. Metcalf & Eddy§ state that "In general, concrete is more desirable than brick, but where brick masonry can be had much cheaper than concrete it may be advisable to build the sewer of brick."

Water-tightness of Concrete Conduits. It is practically impossible to make a conduit of any material absolutely water-tight, because of contraction or shrinkage and inequalities in settlement. In gravity conduits ground water leaks in, and in pressure conduits water leaks out. In brick and steel the leakage is through the joints. In concrete of thin section occasional porous places are liable to occur even in good construction, but here also the principal leakage is through the joints between different sections.

The flow may be reduced so low as to be negligible by using enough steel to insure against shrinkage, settlement, or stress cracks, and by using such proportions in mixing and care in construction as will insure a uniformly dense mixture. If construction joints are used, leakage may be prevented by stepped joints, sheet lead or similar connections.

Extensive tests have been made on short sections of conduit. The New York Board of Water Supply‖ in tests of pipe made up of seven 30-foot lengths 11 feet in diameter, 8 inches thick, of 1 : 2 : $3\frac{1}{2}$ concrete, found 28 000 gallons leakage per mile per day with a 15-pound pressure and 114 000 gallons per mile per day under a 40-pound pressure. Tests made some years later by the Board on a length of thirty 27-foot sections of the finished Catskill Aqueduct¶ showed that the leakage was almost entirely through the construction joints, with a progressive

* Calvin W. Hendrick in *Engineering Record*, May 30, 1914, p. 604.

† *Engineering News*, February 18, 1915, p. 305.

‡ C. A. Farwell in *Engineering Record*, May 15, 1915, p. 623.

§ American Sewerage Practice, Vol. I, Design of Sewers, p. 401.

‖ *Engineering Record*, April 23, 1910, p. 566.

¶ *Engineering Record*, September 5, 1914, p. 276.

diminution in amount with lapse of time entirely independent of the temperature influence, and that the method of stopping leakage through the joints by filling with Portland cement grout is satisfactory, effective, and permanent. The City of Philadelphia* in tests of two 4-foot lengths of 36-inch circular pipe 4 inches thick made of 1 : 2 : $3\frac{1}{2}$ concrete 30 days old, found 12 000 gallons per mile per day under a 5 pound pressure.

Durability of Concrete Inverts. Concrete inverts have proved in practice to be equal, if not superior, in durability to stone and the best vitrified brick in resisting erosion by abrasive materials transported under high velocities. Where, however, concrete is subjected to severe chemical action it must be protected. For example, where the sewerage is exceedingly stale, or impregnated with deleterious chemicals, such as sulphuric acid, or where the sewer is badly ventilated,† a partial or complete lining of vitrified brick is advisable.

The ability of Portland cement mortar to resist erosion is illustrated by the concrete sewers built at Duluth, Minn. After twenty years of wear, they showed no appreciable deterioration or enlargement in diameter, while brick sewers laid at the same time required rebuilding after six or seven years. A section of the Duluth drains, about 2 000 feet long and 4 feet in diameter, was built on a 13 per cent grade where the velocity of the water was 42 feet per second, with an invert of flat granite flags laid with 1:1 Portland cement joints. The flow of water during heavy storms was tremendous, carrying down with it quantities of sand and boulders, but after two years of wear the invert showed ridges of mortar between the granite flags, indicating that the Portland cement mortar was more durable than the granite.

The velocities of water flowing in conduits, however, under certain conditions must be limited. Experience of the U. S. Reclamation Service‡ shows that with clear water and no change in direction or velocity, there is no practical limit to the velocities that can be permitted without harm. With changes in direction, which subjects the concrete to impact, rapid erosion may result. Water carrying large quantities of sand at a velocity of, say, 70 feet per second, may produce considerable damage.

Interesting pressure tests§ of the Lock Joint Pipe Company on precast pipes showed that the ordinary methods of casting joints for gravity

* A. T. Goldbeck in Journal American Concrete Institute, May 1915, p. 253.

† Metcalf & Eddy, American Sewerage Practice, Vol. I, Design, p. 113.

‡ Arthur P. Davis in *Engineering News*, January 4, 1912, p. 20.

§ *Engineering News*, December 4, 1913, p. 1126.

and low pressure lines were inadequate for pressures running from 50 to 90 pounds per square inch. The difficulty was overcome (1) by casting in two operations instead of one, the second consisting in filling a narrow joint left to provide for shrinkage of the mortar placed by the first operation; and (2) using only the finest cement grains of which 100% passed the 200-mesh sieve.

The general subject of water-tightness or impermeability is discussed in Chapter XVIII, page 296.

Design of Concrete Conduits. The selection of shapes and sizes of conduits suitable for different flows of water and sewerage under different conditions is treated in the literature of hydraulics and sewerage. The ordinary methods of beam and arch design as treated in Chapters XXII and XXV, are applicable to conduit design. Methods of determining moments and pressures are outlined on page 781.

The choice between plain and reinforced concrete depends largely upon the size of the structure. For diameters of 3 feet and under the least thickness that can be economically placed is strong enough without any reinforcement unless internal pressure is present. The use of steel in large conduits reduces the thickness of concrete, but the cost and difficulty of placing the steel frequently offsets the saving. In New York City it was found economical to reinforce sections greater than 4 feet in diameter.

The concrete proportions vary from 1 : 1½ : 3 or 1 : 2 : 4 for the conduit proper to 1 : 4 : 8 for sub-foundations and filling and backing. For this latter work Natural and Puzzolan cement may be used if economical in cost. The dimensions of the foundation depend upon the character of the ground and vary from point to point on the same job.

The thickness of conduits can best be found by designing for the actual loads. Many empirical formulas have, however, been derived from structures successfully built and give safe results in practice. Mr. W. B. Fuller's rule* is as follows:

If concrete is not reinforced and ground is good—able to stand without sheeting—make crown thickness a minimum of 4 inches, and then one inch thicker than diameter of sewer in feet. Make thickness of invert same as crown plus one inch except never less than 5 inches. Make thickness at haunches two and a half times thickness of crown, but never less than 6 inches. This rule is expressed in the following table:

* Personal correspondence.

Thickness of Conduits.

Diameter of Conduit.	Thickness of Crown, inches.	Thickness of Haunch, inches.	Thickness of Invert, inches.
2	4	6	5
6	7	18	8
12	13	33	14

If ground is soft or trench is unusually deep, these thicknesses must be increased according to experienced judgment.

If reinforcement is used, the thickness for conduits of ordinary sizes is usually determined by the minimum thickness of concrete which can be laid so as to properly imbed the metal. This minimum for the large diameters where steel is advisable may be taken as 6 inches.

MOMENTS AND PRESSURES

The external pressure on structures buried in the ground is very indefinite, depending not only upon the character of the fill, but also upon the method of excavating and filling the trenches and tamping the filling.*

For small depths up to 3 feet the sum of the weight of the earth and the live load may be taken as acting on the structure. For larger depths, however, the sum of these two forces would be excessive, and may be decreased. According to Mr. Frühling† the effect of the live load decreases as a parabola until it is zero at $16\frac{1}{2}$ feet, and may be represented by formula (1) using notation below.‡

$$q_2 = \mathfrak{Q}\,\frac{(16.5 - h)^2}{269} \tag{1}$$

and

$$q_1 = w\,(h - 0.06h^2 + 0.0012h^3) \tag{2}$$

The weight of the earth increases only to a depth of about $16\frac{1}{2}$ feet according to formula (2) and is constant for larger depths.

The sum of the force q_1 and q_2 thus found gives the working load per square foot. Allowance should be made for impact when necessary.

* For an excellent treatment of this subject with formulas for moments, see "Tests of Cast-Iron and Reinforced Concrete Culvert Pipe," by Arthur N. Talbot, University of Illinois, Bulletin No. 22, 1908.

† Handbuch für Eisenbetonbau, Band III, p. 510.

‡ Notation. q_1 = pressure per sq. ft. due to dead load; q_2 = pressure per sq. ft. due to live load; w = weight of earth per cu. ft.; $\mathfrak{Q}$ = unit live load; h = depth in ft.

Conduits with Arch Top Only. The computation of the arch is similar to that for an arch bridge, and is given in Chapter XXV. The loads are carried to the sides of the arch conduit, which act as abutments. Experience indicates that it is not safe to count to a large extent upon the filling at the sides of the conduit to prevent them from cracking.

Longitudinal bars should be introduced to assist in providing for unequal settlement as well as to resist temperature stresses.

Circular Pipes. Under vertical forces the maximum positive moment acts at the top and bottom of the pipe and produces tension on the inside surface, and the maximum negative moment acts on the sides, causing tension on the outside surface*. Double reinforcement however is usually introduced.

Rectangular Conduits. Square and rectangular conduits† are designed as rigid frames loaded by weight of earth and live load acting on upper horizontal slab, reaction acting on lower horizontal slab, and earth pressure acting on sides of conduits. The stresses may be computed as in ordinary slabs (see page 485) after determining the moment by formulas given below.

Let

M_1 = negative moment at the four corners and at the center of vertical slabs, caused by vertical loads.

M_2 = positive moment in the center of the lower or upper slab, caused by vertical loads.

I_l, I_h = moment of inertia of horizontal and of vertical slabs, respectively.

l, h = span of horizontal and of vertical slabs, respectively.

w = uniformly distributed load.

Then

$$M_1 = \frac{wl^2}{12} \frac{lI_h}{lI_h + hI_l} \qquad (3) \qquad \text{and } M_2 = \frac{wl^2}{8} - M_1 \qquad (4)$$

The formulas apply to vertical loads as indicated above.

For earth pressure, assuming it as uniformly distributed, these same formulas may be used, but the earth pressure, which acts at right angles to the vertical load, causes positive moment, M_2, in center of vertical slabs and negative moment, M_1 at corners and also at center of horizontal slabs. For the earth pressure moments l and h must be transposed. The moments, M_1 and M_2, due to earth pressure must be computed separately and then may be combined with M_2 and M_1, respectively, due to vertical loads. The moments to be combined are of opposite signs and their sum may not represent the most unfavorable condition, which, of course, must be selected.

* See footnote ¶ page 781.

† A table of dimensions and reinforcement for square and for rectangular conduits under different conditions is given by Sanford E. Thompson in "Concrete in Railroad Construction," published by the Atlas Portland Cement Co.

EXAMPLES OF CONDUITS

The Weston Aqueduct of the Metropolitan Water Works, Massachusetts, built on a gradient of one in 5 000, has in loose earth a typical section shown in Fig. 247. In compact earth the excavation is narrower, and the width of base is reduced as shown by one or the other of the dotted lines, AB or CB. In embankment, the foundation is carried lower and horizontal reinforcing rods are sometimes placed at intervals just below the brick invert lining. A lean Portland cement concrete may be used in place of natural cement concrete shown.

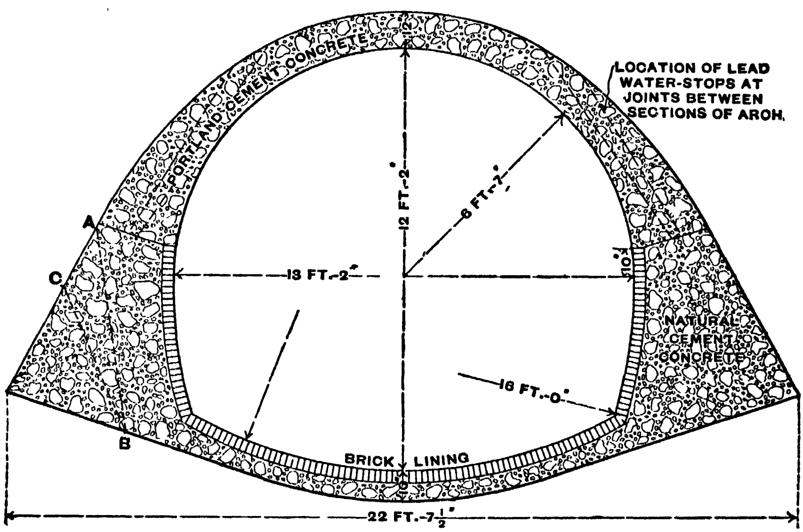

FIG. 247.—Typical Section of Weston Aqueduct in Loose Earth. (*See p.* 783.)

The Jersey City Water Supply Company constructed in 1903 a conduit reinforced with twisted steel. A typical section, taken through a manhole, is shown in Fig. 248, as designed by Mr. William B. Fuller. Longitudinal reinforcement consists of $\frac{3}{16}$-inch rods spaced about 18 inches apart, and circumferential reinforcement is formed by rings of $\frac{3}{8}$-inch rods about 12 inches apart. Through rock open cut the metal was placed only in the arch, and as far down on each side as the filling would extend.

A reinforced concrete siphon* was built in Huesca, Spain, to operate under a maximum head of 98½ feet. The inside diameter is about 13

* *Engineering Record*, December 3, 1910, p. 636.

feet and the walls are about 8 inches thick including a mortar lining 0.6 inch thick. Proportions 1 : 1¼ : 2½ were used with a mortar lining of 1 : 1. The reinforcement varies according to the head, but under the full head there are circular T-irons 2 by 2 by ¼-inch spaced 3¾ inches on centers with 124, ½-inch longitudinal rods. The siphon was laid in a foundation of porous concrete containing a drainage channel intended to lead away all seepage without danger of disturbing the stability of the pipe. The leakage, however, proved scarcely perceptible.

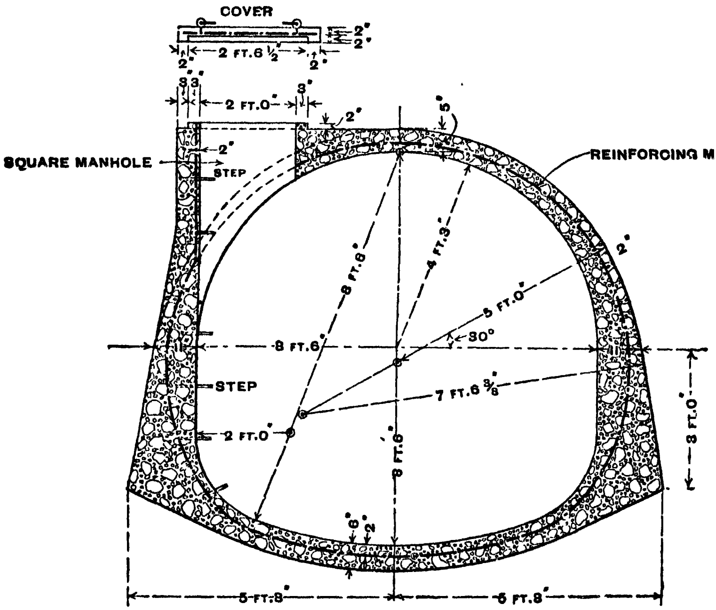

FIG. 248.—Typical Section of Jersey City Water Supply Conduit in Loose Earth. (See p. 783.)

Strength tests. Strength tests of pre-cast pipe are of value in insuring the uniform quality of the product. A simple but efficient machine for this purpose is described by Prof. Arthur N. Talbot and D. A. Abrams in the Proceedings of the National Association of Cement Users, 1912, Vol. VII, page 713. Results are also given of tests on concrete and clay drain tile.

Tests were made in Philadelphia* by A. T. Goldbeck to determine the best method of curing and bedding concrete pipe. Egg-shaped, 36 by 24 inches, and 36-inch circular pipe bore larger loads with less

* A. T. Goldbeck in Journal American Concrete Institute, May 1915, p. 240.

deflection and cracking when bedded in concrete than when bedded in loose sand. Keeping the pipe thoroughly wet for two weeks increases its strength.

An excellent discussion of the loads on pipes laid in trenches and the necessity for bedding is given in Bulletin 31, Iowa State College of Agriculture, "The Theory of Loads on Pipes in Ditches and Tests of Cement and Clay Drain Tiles and Sewer Pipe" by A. Marston and A. O. Anderson, 1913.

Methods of Conduit Construction. There are four general methods of construction of concrete conduits: (1) The lower portion of the invert is laid by template and the remainder of the circle by centering. (2) The invert is formed by an inverted center, and the arch by an upright center. (3) A center the size of the entire sewer, but with a removable bottom, is placed, the sides and arch are built, and then the bottom of the center is removed, and the invert is laid. (4) The entire sewer is formed as a monolith. The size of the sewer and the character of the work influences the choice of method.

If the invert is to have a brick lining or a granolithic finish, after excavating the material to the required grade and shape, profiles or templates are placed in advance of the finished concrete, and the surface is formed with the aid of a straight-edge placed longitudinally from the finished concrete to the nearest template. If the sides run up sharply, as in a small sewer, the concrete may be held in place by strips of lagging, 2-inch by 2-inch for a very small sewer, or wider for a larger size. This lagging rests at one end on the finished concrete, and at the other end on the template, and is placed as the work progresses. In horseshoe sewers the invert may be shaped with templates and straight-edge, and the side walls laid back of plank forms.

In a large conduit the smoothest and best wearing surface is obtained by laying a comparatively narrow strip of invert by means of profiles or templets and straight-edge, and troweling it. If desired, a granolithic (or mortar) finish may be given, but with thorough troweling, excellent results are secured with concrete. The arch center, which in such cases must be nearly a complete cylinder, is placed after the strip of invert concrete has set, mortar is spread on the edges of the invert strip already laid, and the circle is completed with fresh concrete. A longitudinal groove also assists in forming a tight joint.

To avoid this joint, a plan similar to that just described has been followed, except that the form, which is a complete cylinder, open at the bottom, is placed, before laying any concrete, upon concrete blocks

previously prepared in molds and then laid in the bottom of the trench. The lowest strip of invert is not laid until after the sides and arch are in place, the concrete for it being let down through holes left in the crown for the purpose, and troweled as thoroughly as the obstructions of the forms will permit.

It would at first appear that the sewer could more readily be made monolithic by placing a complete cylinder and pouring concrete around it for the invert arch. The objection to this, however, is the great difficulty in placing the concrete in the extreme bottom, and also the tendency of the center to "float" from the upward pressure of the concrete. This difficulty is also encountered to a less extent in the method described in the preceding paragraph.

In a sewer whose invert and arch are constructed separately, the arch centers are made and placed as for brick, except that a smoother and tighter surface is necessary, and the forms are oiled to prevent adhesion. A covering of sheet metal has often been successfully used. In order to lay the concrete of the arch sufficiently wet to obtain a smooth surface, an outside set of forms, open at the crown, is usually essential.

Methods of Construction and Forms. The field is specially fruitful in conduit work for getting low unit costs by planning the work to avoid lost time. Progress along this line has been largest in the erection of buildings* but the opportunities are equally good or better in building conduits because of the comparative simplicity of the work and consequent ease in systematizing.

Standardization of forms and construction should be carried out as far as possible. In the building of the Los Angeles Aqueduct† standards were worked out from experience as the work progressed. The work also was planned for a maximum efficiency of men and plant. Some parts of the work were done by task and bonus.‡

Many difficulties were encountered and solved in building the Catskill Aqueduct and the 17-foot circular conduit at Kensico§ built in monolithic sections for a 29-foot head of water is specially interesting. Forms and reinforcement were placed before pouring. Proper placing and ramming of the concrete at first appeared impossible but improvements in the forms eventually remedied the trouble. The circular reinforcement was very heavy and it was necessary to bend it to a

* See paper on "Construction Management" by William O. Lichtner, in Journal Western Society of Engineers, also "Concrete Costs," by Taylor & Thompson.

† *Engineering Record*, January 6, 1912, p. 6.

‡ *Engineering Record*, January 12, 1912, p. 72.

§ Henry W. Nelson in *Engineering Record*, May 3, 1913, p. 502 and G. T. Seabury in *Engineering Record*, September 5, 1914, p. 277.

template altogether different from the final shape taken by the rods when suspended with no bottom support inside the forms.

On another section of the Catskill Aqueduct the steel forms, horseshoe in shape, bulged near the invert* under the heavy pressure of concrete and were skillfully trussed without interfering with the working space.

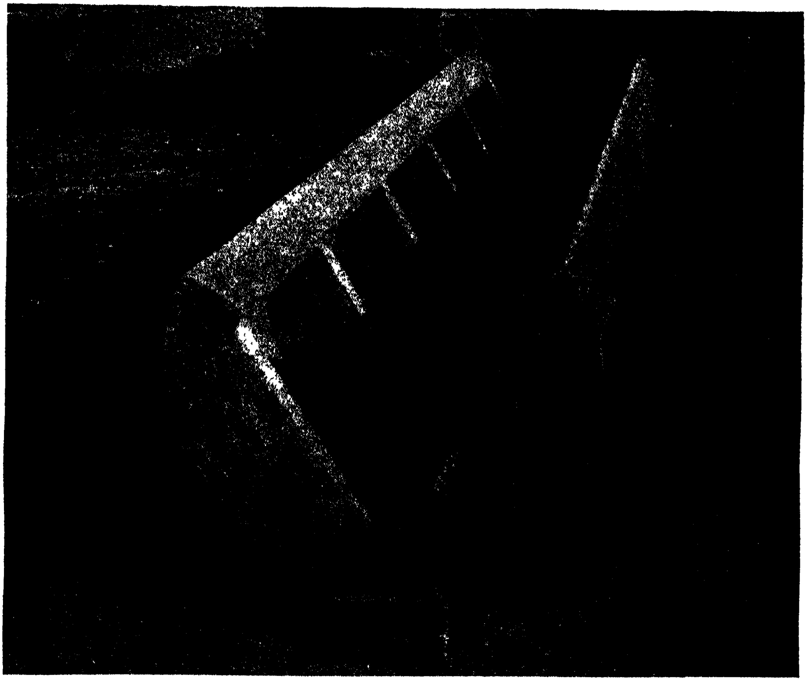

Fig. 249. Center for Invert of 30-inch Sewer at Medford, Mass. (*See p.* 787.)

A good design for a collapsible form† of wood covered with galvanized sheet steel was used by the New York Board of Water Supply on circular conduits running from 2 feet 6 inches to 7 feet 8 inches in diameter. A collapsible steel form‡ for medium sized sewers 6 feet by 8 feet was employed. The forms for the Catskill Aqueduct are well described in a paper§ by Alfred D. Flinn.

Invert centers for a small sewer, designed by Mr. William G. Taylor

* Arnold Becker in *Engineering Record*, November 25, 1911, p. 617.
† *Engineering Record*, October 3, 1914, p. 370.
‡ *Engineering News*, March 11, 1915, p. 494.
§ "Concrete Forms for the Catskill Aqueduct," Journal American Concrete Institute, June, 1915.

and employed in the Medford, Massachusetts, sewers, are illustrated in Fig. 249 page 787.

A similar form* with a top segment, completing the circle was used at Hartford, Conn.

Pre-cast concrete pipe built in factories or on the job are economically used for both pressure and gravity conduits.

TUNNELS

Tunnels differ from sewer and water conduits chiefly in size and in being designed for heavier internal and external loads. The construction methods differ in so much as they are adapted to underground work and to the larger scale on which the work is done. Structures, however, such as are involved in the water or sewer conduits built by Los Angeles, New York, and Baltimore, and other cities, differ very little from heavy railway tunnels either in design or construction. In some cases water-tightness is essential, but in the majority of railway tunnels the drift is through dry materials and the ballast may be laid directly upon the bottom.

The most important developments in connection with the use of concrete in tunnels have been due to a better knowledge of the properties of materials resulting in greater attention to details and to a greater efficiency in construction plant and methods.

Tunnel Design. The principles of tunnel design are the same as have been discussed for conduits. For double track railroad tunnels the quantities of concrete and steel can be cut down by using a center wall or center column.

Where tunnels must be waterproofed various methods (see p. 296) are used, but grouting back of the walls through pipes placed with the concrete is exceedingly effective. With careful construction dense concrete with very few temperature and shrinkage cracks can be secured and those that do occur, together with any local defects, can be grouted. About 90% of the leakage in the New York Aqueduct City Tunnel was stopped in this way.† Interior drainage is usually provided for by channels in the invert leading to the tunnel exit or to pump wells.

Construction Methods. The chief improvements in tunneling construction has been due to a better knowledge of materials and to better forms and more highly developed plants.

Materials for important work, should be graded to insure the maxi-

* *Engineering and Contracting*, July 7, 1909, p. 7.
† Walter E. Spear in *Engineering News*, February 4, 1915, p. 194.

mum density, and the work planned so as to construct as large monolithic sections as possible. Special precautions were taken on the New York Aqueduct to protect the green concrete from flow of water. Large leaks were grouted but small leakage was taken care of by placing large or small drip pans as needed outside the concrete to catch the water. These pans were eventually grouted. The length of tunnel lining to be placed at once is limited on one hand by the speed with which concrete can be delivered to the forms and on the other by the necessity of keeping construction joints as few as possible. A discussion of the problem as it was solved on the New York Aqueduct City Tunnel is given in full by Walter E. Spear in *Engineering News*, February 4, 1915, p. 194.

Steel forms designed for local conditions have proved economical and satisfactory. Tunnel forms must be rigid, easily braced, without trespassing on the working space in the center of the tunnel, and at the same time easy to take down, move, and set up. These requirements necessitate new designs for every job.*

The mixer and storage bins in large and deep tunnels are often placed near the bottom of a shaft, although usually just outside the portal. Concrete is generally carried to place in cars and poured into the forms from two levels. The invert, if there is one, is built from the lower level. From the upper level, concrete is shoveled or poured directly into the side wall forms and shoveled into the arch form overhead.†

The pneumatic method (see p. 253) of mixing and transporting, or of transporting only, has been used to advantage in tunnel work.‡ Concrete has been carried by compressed air to heights of 80 feet and to horizontal distances of 450 feet.

A labor saving device§ for placing the key was developed on the New York City Aqueduct. A steel box is filled with concrete and clamped to the forms; the bottom of the box is then pushed up like a piston flush with the arch forms by means of a screw.

SUBWAYS

Subways are technically distinguished from tunnels as constructions in open-cut instead of drift, although portions of a subway often are

* The forms used in the Catskill Aqueduct tunnels are described in the Journal American Concrete Institute, June 1915, p. 291. This article is of special interest because it describes the development of the forms from the early failures to the types finally successful.

† An excellent discussion of this type of construction is given by Walter E. Spear in *Engineering News*, February 4, 1915, p. 194, in an article describing the New York City Aqueduct Tunnel.

‡ *Engineering News*, October 29, 1914, p. 880; August 10, 1911, p. 172; July 31, 1913, p. 208; and February 18, 1915, p. 314 and *Engineering Record*, October 11, 1913, p. 404.

§ *Engineering News*, February 4, 1915, p. 194.

really of tunnel construction. The term *subway* is applied to accessible conduits for water mains, electric cables, etc., as well as to underground passages for traffic, but it will be considered here in the latter sense only.

Design. Subway design is governed almost entirely by local conditions. Reinforced concrete is usually better adapted than any other material to such work so far as cost and convenience are concerned. However, in the New York subways steel framing with concrete jack arches has been found more practical because of the heavy street traffic that can be readily transferred from the timbering to the steel girders and columns. Instead of an arched structure wide enough to carry all tracks in a single barrel, the relative cost of concrete and excavation usually makes it economical to flatten the arch, saving headroom, and to widen the structure enough to put in a center wall or center columns. The separation of tracks by center walls is also an aid in securing ventilation.

Construction. Subway construction does not differ materially from ordinary conduit or tunnel construction so far as the concrete work is concerned. For references to articles describing subways, see Chapter XXXIII.

CHAPTER XXIX

RESERVOIRS AND TANKS

Concrete has become a standard material for reservoirs and tanks both for water and chemicals. The results from the point of view of water-tightness, durability, economy, and freedom from vegetable growth, are exceedingly satisfactory.

OPEN RESERVOIRS

The walls of large open reservoirs usually are built as retaining walls (see Chapter XXVII, p. 751, for methods of design), but under many conditions concrete slabs supported by banks of earth or buttresses of concrete* are economical. The bottoms are built of large rectangular slabs reinforced or not, according to the foundation. From 4 to 8 inches of 1 : 2 : 4 or 1 : $2\frac{1}{2}$: 5 concrete is satisfactory, and if properly laid and troweled is sufficiently impervious provided the joints are taken care of.

Various expedients are employed in making reservoirs water-tight. Expansion and contraction joints are best filled with asphalt mixed with limestone dust or sand.† Such joints must be supported or backed up by concrete or mortar to prevent the water pressure forcing out the asphalt.‡ In securing adhesion between concrete and asphalt, it has been found necessary to paint the concrete with hot coal tar. The membrane method and the application of an interior layer of mortar has also proved successful. The chief reliance in making the concrete water-tight should be in the quality of the concrete itself,—by selection and grading of the aggregates, and care in placing, it should be sufficiently tight for all practical purposes. (See p. 296.) For certain exceptional cases the membrane method is necessary. Construction joints are discussed on pages 259 to 260 and 795.

In small reservoirs where earth and rock meet in the foundation, presenting a danger of unequal settlement and consequent serious leakage, steel reinforcement may be placed over the line of division, even if used nowhere else. To be effective, the cross-section of the steel must

* Alexander Potter in *Engineering Record*, Nov. 20, 1913, p. 616.
† C. R. Sessions in *Engineering and Contracting*, Mar. 11, 1914, p. 304.
‡ Such a failure and the repairs necessary are described in *Engineering Record*, Apr. 4, 1914, p. 398.

be large enough to actually add strength; chicken wire or other mesh of small wire is useless.

COVERED RESERVOIRS

The usual type of covered reservoir consists of a concrete floor, reinforced or gravity walls, and a concrete roof supported by piers and covered with earth to a depth of 2 or 3 feet.

Reservoir Floors. The floor should be smooth, fairly impervious, and strong enough to resist the upward water pressure from the underlying soil when the reservoir is empty. A thickness of 4 to 6 inches,

FIG. 250.—Reservoir Floor. *(See p. 793.)*

depending upon the character of the underlying material and the head of water, is sufficient.

Inverted groined arches are frequently used, the greater thickness at the piers providing footings to distribute the pressure, while the lower areas between footings form channels for the flow of water. The groined arches are laid in alternate diamonds before the piers are built, so that each pier rests upon the corners of four diamonds.* A granolithic surface may be placed before the concrete is set, as in sidewalk construction or preferably the concrete itself may be troweled. The

* See paper by Allen Hazen, Transactions American Society Civil Engineers, Vol. XLIII, p. 262.

joints between blocks must be made water-tight. In the Albany plant the 6-inch floor was underlaid with 16 inches of clay and gravel puddle, and joints between the blocks, 3 inches deep and $\frac{1}{2}$ inch wide, were filled with asphalt. (See Fig. 250, page 792.)

A floor of reinforced concrete, designed to take the loads from the piers, and the upward pressure, requires fewer joints, is more likely to be water-tight, and requires less expensive treatment of the foundation. A comparison of the cost of the two types should be made for any reservoir.

Reservoir Walls. The walls may be designed as supported at the bottom by the floor and at the top by the roof, thus saving material over the ordinary retaining wall type. Buttresses, or rather, vertical beams supporting the reinforced wall slab, will transfer the pressure to the floor and roof. Joints in the walls must be thoroughly reinforced or designed as contraction joints with suitable waterproofed connections. (See p. 259.) In long walls a certain amount of cracking from temperature is almost unavoidable, but this is minimized after the reservoir is completed and the range in temperature reduced.

Reservoir Piers. Piers should be designed as columns (see p. 559) with suitable reinforcement, not less in amount than a $\frac{3}{4}$-inch bar in each corner. The bars also assist in taking unbalanced thrust from the roof. A compressive stress of 375 pounds per square inch may be allowed safely when the concrete is in proportions 1: $2\frac{1}{2}$: 5. Provision for distributing the load from the roof to the soil must be made by the groined arch floor, independent footings, or special design of the slab floor.

Reservoir Roofs. Groined elliptic arches* have been used to a large extent for roofs to distribute the weight of the concrete and the earth to the piers.

Mr. Leonard Metcalf has compiled a table† of data relating to reservoirs in the United States covered with groined arches, which shows a range in span of arch from 10 feet 6 inches to 16 feet, a rise varying from one foot 6 inches to 4 feet, and a thickness at crown, in all cases but one, of 6 inches. The proportions of the concrete range from 1: $2\frac{1}{2}$: 4 to 1: 3: 5.

More recently the economy of groined arches has been surpassed by

* See paper on Groined Arch Construction by Thomas H. Wiggin, Proceedings National Association of Cement Users, Vol. VI, 1910, p. 216, and Frank H. Carter in *Engineering Record*, Sept. 5, 1914, p. 265.

† See Report of Annual Convention of the New England Water Works Association, 1903, *Engineering News*, September, 1903, p. 238.

flat slab construction, one of the first of this type being the roof of the reservoir at Webster, Mass., designed by Mr. Thompson for Mr. Frank L. Fuller in 1914.

For small circular reservoirs a dome roof may be used. To resist the thrust, rings of steel must be inserted in the circumference, of amount and size determined by computations.

STANDPIPES

Water-tightness is a requisite in the design and construction of storage reservoir tanks built as standpipes entirely above ground if the structure is not to become unsightly. Up to the present time this requirement has not been met with entire success in tanks under high heads although reasonable satisfaction has been obtained. The con-

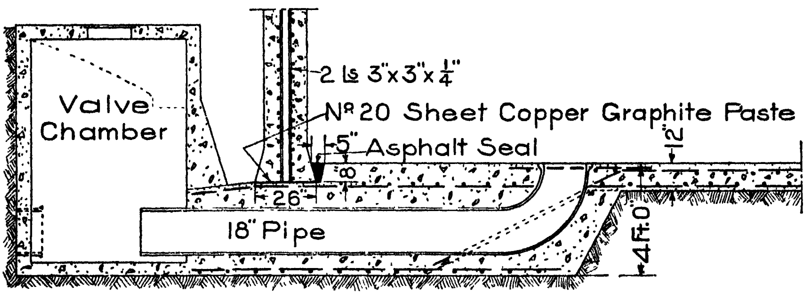

FIG. 251.—Details of Design of William Mueser for Standpipe with Disconnected Base and Flexible Seal. (*See p.* 794.)

crete itself can be made impermeable and the tank so designed that no cracks directly due to water pressure will occur. The secondary stresses, however, caused by the tendency of the wall to pull away from the base, are less readily controlled. A rigid connection strongly reinforced with bent-up bars may result in a horizontal crack a few feet above the base. To avoid this, in several cases the wall has been constructed separately* from the base with a pocket filled with asphalt to prevent seepage between the two. The details of such a design, patented, are shown in Fig. 251, page 794. The normal shape of the concrete foundation under the floor is shown by dotted lines, although the width of the rim depends upon the bearing power of the soil.

The pressure of the water tends to stretch the concrete wall, and as

* *Engineering News*, Sept. 7, 1911, p. 297; and Feb. 18, 1915, p. 290.

concrete cracks at a comparatively low tensile stress, it is advisable to design the walls thick enough to act with the steel in tension, keeping the tensile stress in the concrete below the breaking strength. When working thus, in combination, the steel will have a low stress, not over 2 000 to 4 000 pounds per square inch, but there should be enough steel to keep the stress below 16 000 to 18 000 pounds in case the concrete should crack. The mix must be specially rich—a 1:1:3 is good practice. It is advisable to use a rich mix, say 1 cement to 4 or 5 parts of specially graded aggregates. Note that the modulus of elasticity of a rich mix like this is low (see p. 477)—a fact which must be taken into account in determining the relative stresses in the concrete and steel.

In case the wall is made thinner, with no provision for its taking tensile stress, the steel should be designed with a stress not over, say 12 000 pounds per square inch.

The thickness of concrete and amount of horizontal reinforcement at various sections of a circular tank will vary with the water pressure, which is zero at the top and increasing toward the bottom. If the steel is designed to resist all the tension the area may be determined thus:

Let

H = height of reservoir in feet above section considered.

D = diameter of reservoir in feet.

A_h = area in square inches of horizontal steel per foot of height at section considered.

f_s = allowable unit stress in steel in pounds per square inch.

At any horizontal section the total tensile force, per foot of height, tending to rupture the reservoir on any diameter is 62.5 HD. Since the area of steel resisting this force is $2A_h$, we have $2A_hf_s = 62.5\ HD$, or

$$A_h = \frac{31.3\ HD}{f_s}$$

For computations involving the tensile strength of concrete, it is assumed that concrete and steel take stress in proportion to their moduli of elasticity.

In a high circular reservoir, the thickness of wall and vertical reinforcement should be considered as in chimney design. (See p. 660.)

Methods of Construction. Special precautions are necessary in constructing such reservoirs to secure water-tightness. The forms must be placed in shallow sections in order that the concrete mixed to a sluggish

consistency may be filled in carefully around the steel. The concrete should moreover be placed in thin layers clear around the wall so that no irregular joints may form. Some contractors pour the concrete continuously day and night in order to avoid joints between each day's work. In case joints are made they should be treated as discussed on page 259. In addition to such treatment steel plates or dams are frequently imbedded in the concrete at the end of each day's work to form a tie with that of the day following.

SMALL TANKS

Small tanks for water or chemicals are designed along the same principles as are large storage reservoirs. The stresses in such tanks, however, are so low that no trouble results from a rigid connection between the sides and the base and, in fact, the work is usually on so small a scale that the entire tank can be built as a monolith.

Concrete tanks are specially adapted to chemicals, partly because of their durability, and partly because of the adhesion between the concrete and the outlet castings that is entirely lacking in wood. It is necessary, however, that the gates and other connections, which are usually of brass or bronze, be so heavy that the corrosion and wear upon them will not necessitate removal and therefore repairs to the concrete, since it is impossible to form a satisfactory joint between old and new concrete in a thin wall. It must be recognized of course that cement is soluble in a very strong acid like sulphuric; and to a less degree with weaker acids. Concrete will resist the action of acids of moderate strength better than most other materials.

Water standing in concrete tanks may absorb free lime from concrete which in some industries is injurious.*

There are many examples of the use of concrete tanks for storing chemicals and similar liquids. The Institute of Industrial Research at Washington reports:

It will be noted, as the result of these tests up to the present time, that the following materials have been successfully stored in concrete tanks: Menhaden oil, lard oil, tanning solution, caustic soda solution 4%, and sauerkraut. The following materials have shown superficial disintegration of the concrete with considerable modification of the material itself: Sulphite liquor and cider vinegar.

Concrete tanks treated with paraffine are proof against cider and vinegar.† Tanks used in paper mills to hold fairly concentrated chlorine

* Leo Hudson in *Engineering Record*, Sept. 17, 1916, p. 330

† H. B. Bonebright in *Engineering Record*, July 22, 1911, p. 103.

solution have proved durable and satisfactory after six years' use.* Concrete tanks that had to withstand the action of sulphuric acid in copper mining were lined with a specially prepared acid-proof asphalt mastic† that after a year's service showed no deterioration.

A concrete tank has been used for the mixing of hypochlorite‡ for water purification.

* Walter B. Snow in *Engineering Record*, Oct. 15, 1910, p. 448.

† *Engineering Record*, Apr. 4, 1914, p. 399.

‡ Proceedings of the National Association of Cement Users, Vol. VIII, 1912, p. 102.

CHAPTER XXX

CONCRETE PAVEMENTS AND SIDEWALKS*

Concrete has proved itself a most reliable and durable material for sidewalks. For street pavements its use is rapidly extending notwithstanding the fact that many of the earlier pavements, through the use of poor materials or too wet mixtures, have given out under traffic conditions. For alley-ways in Boston and elsewhere concrete has been in satisfactory use since 1894. The first successful street pavement was built in Richmond, Ind., by Mr. H. L. Weber in 1896 and led to its general adoption there and in other cities.

Concrete pavements, subject to a grinding, pounding action, and the disintegrating influences of the weather, require more than ordinary precautions to insure against failure. Unless the requirements specified below are followed, ravelling of the surface is liable to occur. The important points are selection of materials, mixing and placing, and curing. These are, of course, the main considerations in all concrete work, but relatively minor defects are likely to show up quicker in a pavement than elsewhere.

The summary of the recommended practice of the National Conference on Concrete Road Building, 1914, is as follows:

(1) **The aggregates should be clean and hard.**
(2) **The sand should be coarse and well graded.**
(3) **A rich mixture should be used.**
(4) **The materials should be correctly proportioned.**
(5) **The materials should be thoroughly mixed.**
(6) **The inspection should be intelligent and thorough.**
(7) **When in doubt, reinforce the pavement.**
(8) **The sub-grade should be of uniform density, thoroughly compacted and drenched with water immediately before placing concrete.**
(9) **The concrete should be of a viscous, plastic consistency.**
(10) **After placing, the concrete should be immediately covered and kept moist and not opened to traffic for four weeks.**

* The gist of the best published material on concrete road building is to be found in the Proceedings of the National Conference of Concrete Road Building, 1914, and 1916. The 1916 Proceedings are reported in *Engineering Record*, February 26, 1916, p. 286. The points covered briefly here are treated fully there.

DESIGN

Good practice in the preparation and drainage of the subgrade is the same and as important for concrete pavements as for all other types, and is fully covered in the standard treatises on highway engineering. One point, especially important in masonry pavements, is the securing of a sub-grade that will settle uniformly; for example, a concrete slab laid on an old road that finally settles more along the shoulders than along the crown will probably crack longitudinally. The strength of plain concrete as a beam is low.

The thickness of pavements varies with conditions. In arid or semi-arid regions, as in parts of California, 4-inch slabs have given good service on 16 and 18-foot roads. This is the minimum and slabs 8 inches thick at the center have been used under heavy loads where the subsoil and weather conditions are adverse.

The crown recommended by the 1914 and 1916 National Conferences is $\frac{1}{100}$ of the width. Variations in practice were found to run from $\frac{1}{24}$ to $\frac{1}{200}$ of the width. The outer edge at curves should be raised an amount varying with the degree of curvature.

Grades up to 12 per cent have been found satisfactory to teaming traffic, provided a rough surface is secured.

One-Course vs. Two-Course. At the beginning of the concrete street pavement, construction methods followed in sidewalk work, using a concrete base and mortar surface, were employed. A hard, troweled mortar surface was found to be slippery for horse travel and led to the adoption of a rougher surface, and finally to the use of a single course of richer concrete.

Either method will produce good results under proper methods of construction. The two-course pavement, because of the leaner mixture in the base, can be made economically with a rich and therefore a harder wearing surface, but is apt to be more expensive than the one-course. For one-course work the methods followed are similar, using as indicated a richer mix and the surface is screeded, floated, and troweled substantially as described in the following paragraphs.

Proportions and Materials. Proportions ordinarily used for one-course work are 1:2:3. For the base of two-course work, $1:2\frac{1}{2}:5$ is customary.

The wearing course of two-course work is frequently 1:2 mortar, although a $1:1\frac{1}{2}:2\frac{1}{4}$ concrete, using stone from $\frac{1}{4}$ to $\frac{3}{4}$-inch in size as the coarse aggregate, is much more satisfactory, both from the stand-

point of durability and also in expanding and contracting as a unit with the base.

Inasmuch, however, as the important requirements are density and impermeability to moisture, it is economical to govern the proportions by the characteristics of the aggregates available. The following requirements for aggregates were written by Mr. Thompson as Chairman of the Aggregate Committee of the 1914 National Conference:

(1) For fine aggregate, use only sand or other fine aggregate that has been actually tested for mechanical analysis and tensile strength of mortar and is free from fine particles.

(2) Use coarse grained sands or hard stone screenings with dust removed.

(3) Use sand or other fine aggregate that is absolutely clean.

(4) For coarse aggregate, use hard stone, such as granite, trap, gravel, or hard limestone.

(5) If bank gravel or crushed stone is used, always separate the sand or screenings and re-mix in the proper proportions.

If local conditions prevent following any one of these rules, adopt some other material than concrete for your pavement.

More detailed requirements for fine aggregate are:

The size of the fine aggregate shall be such that the grains pass when dry a screen having $\frac{1}{4}$-inch openings. In the field a $\frac{3}{8}$-inch mesh or in some cases a $\frac{1}{2}$-inch mesh screen, may be used for this separation.

Not more than 10 per cent of the grains below the $\frac{1}{4}$-inch size shall pass a sieve having 50 meshes to the linear inch, and not more than 2 per cent shall pass a screen having 100 meshes to the linear inch.

This is an exceptionally coarse sand, but coarse sand is a necessity for a durable pavement.

Consistency. The tendency is to place pavement concrete too wet, making it impossible to attain maximum density or strength. The proper consistency is a plastic mix, one that holds its shape when dumped in a pile but works somewhat harsh under the trowel or template; only slight tamping should be necessary.

Joints. The tendency of concrete to expand and contract under temperature and moisture changes necessitates expansion joints to prevent cracks occurring at random. Transverse joints should be provided every 25 to 50 feet according to whether the location is in a region where atmospheric conditions are stable or subject to wide variations. In case the pavement is between curbs, longitudinal joints filled with plastic material are needed along each curb to permit expan-

sion. Joints also should be placed at all important changes in grade to avoid buckling by expansion. Longitudinal cracks are due chiefly to unequal settling of the sub-grade or to poor drainage, and should be prevented by proper design and construction rather than by central joints.

Joints should be from $\frac{1}{4}$- to $\frac{3}{8}$-inch wide. They also should be perpendicular to the surface of the pavement; this being especially important on grades.

The 1916 National Conference recommends the use of a pre-formed joint filler of plastic material in preference to steel plates and a plastic filler.

Reinforcement. Reinforcement reduces the danger of cracking from expansion and contraction, poor foundations and drainage and permits the use of a thinner slab for heavy loads. To be fully effective, however, it is necessary to use a larger amount than is permissible from the standpoint of cost. The 1914 National Conference recommends about $\frac{1}{10}$ of 1 per cent per foot of width but two or three times this amount is necessary to be of real value under poor conditions.

CONSTRUCTION

Mixing and Placing. The methods to be observed in mixing and placing concrete are largely covered in Chapters XIII and XIV. One point peculiar to pavement work is the matter of getting the concrete to the right grade and levelling off the surface. The side forms used to confine the concrete to place give the grades along the edges and a template or strike-board, riding on the side forms and operated by two or more men, smooths the concrete off at the exact grade for the full width of the pavement. Near expansion joints the template should move away from and not toward and across the joints.

Finishing and Curing. Aside from striking off with the template, pavements are sometimes floated by working from an overhead trussed plank. One patented pavement is finished very effectively by rolling and tamping with a machine that jars the concrete into place so as to form, with the proper aggregates, a very dense concrete. Sometimes the only finishing done is smoothing off with a shovel with perhaps a little brooming to give a rough even surface.

The curing of concrete pavements has become recognized as an important factor in their success. A pavement should be closed to traffic for about four weeks, although in warm weather and with some cements

a shorter time may be permissible. In addition to this, the pavement should be kept moist and covered from the sun. In California earth dams holding ponds of water two inches deep have been built on pavements. Covering with moist canvas, sand, earth, or sawdust, are also satisfactory methods. The main point is to keep the concrete from drying out under the sun and wind and producing a weak, friable surface.

METHOD OF LAYING SIDEWALKS

Successful sidewalk construction is as dependent upon careful attention to small details which have been proved essential to good workmanship, as upon adherence to the more general directions given in any set of specifications. The full description of methods to be employed in laying a walk are given for the benefit of those who are unable to take advantage of the experience of specialists in this line. Reference also should be made to previous pages on pavement construction and to the description of methods of floor construction (pp. 635 to 639). Experienced contractors often can perform such work better and cheaper than it can be done by day labor.

Proportions. For two-course sidewalks, proportions $1:2\frac{1}{2}:5$ are suitable for the base and $1:2$ mortar or $1:1\frac{1}{2}:2\frac{1}{4}$ concrete for the wearing coat. Proportions $1:2:3$ may be used for one-course walks. Similar proportions are used for curbs built in one or two courses.

Thickness of Walk. A total thickness of 4 inches of concrete and mortar laid upon a 10-inch foundation of porous material gives excellent results for ordinary sidewalks, although 5 inches is often required for public works. In locations subject to wide changes in temperature, as Boston and vicinity, a thickness of 4 inches has proved satisfactory, while in some cities $3\frac{1}{2}$ inches only is required. For a 4-inch walk it is advisable to make the base 3 or $3\frac{1}{4}$ inches and the wearing surface 1 or $\frac{3}{4}$ inch thick. The slope or surface often adopted is $\frac{1}{4}$ or $\frac{3}{8}$ inches to the foot.

Driveways or walks which are subjected to excessive wear may be 5 or 6 inches thick, the upper 1 or $1\frac{1}{2}$ inches constituting the wearing surface.

Foundation. The construction of the foundation is as important as the laying of the concrete. For out-of-door construction the foundation should generally be from 6 to 12 in. thick, depending upon the character of the soil. In localities unaffected by frost and having soil sufficiently porous to carry off surface water, the foundation may be omitted entirely, and concrete laid upon natural ground excavated to required depth.

For basement or cellar floors which are not to be subjected to frost, the concrete may usually be placed directly upon the soil; but in compact ground, or where surface water is troublesome, blind drains of pipe or cobble stones, carefully rammed, should be laid at various points.

The materials for a foundation, where such is required, may be broken stone, gravel, cinders, or coarse sand. In order to make it more porous, broken stone or gravel should be screened. Whatever material is employed it must be thoroughly rammed so as to present a firm and unyielding surface. Cinders or sand should be thoroughly wet when being rammed.

Concrete Base of Walk. The coarse concrete constituting the main body of the walk is generally called the base. Before this coarse concrete of the base is placed, the surface must be carefully laid off into squares or blocks. Such divisions are absolutely essential, since the joints furnish lines of weakness along which cracks will occur if the concrete is affected by the freezing of the soil beneath tree-roots, unequal settlement, or temperature changes, and also facilitates the replacing of a block if one is injured from any cause.

There are three distinct methods of forming separate blocks: (*a*) laying the blocks alternately, and then filling in between them; (*b*) allowing the scantling of the forms o remain in place until after the concrete is laid, and then filling the spaces they occupied with lean mortar or sand; (*c*) placing tarred paper between the blocks. The first method is usually preferable.

The size of the blocks depends upon the width and shape of the walk or floor. Blocks nearly but not quite square have a better appearance than those which are distinctly oblong. The limit of size for a 4-inch walk is generally placed at 6 feet square. In 5-inch work this may be safely increased to 8 feet square. Joints should be placed around trees and about 6 inches from buildings, manholes, or other adjacent structures.

After ramming and leveling the foundation, if there is no curb to be formed, strips of scantling 2 inches thick, and of a width corresponding to the thickness of the walk, are placed on edge along the back and front lines of the walk, and held in place by stakes driven behind them. These strips should have notches cut in them to designate the location of the dividing line between the blocks. Other strips, located by these notches, are placed across the walk, which is now ready for the concrete.

The concrete materials in the specified proportions are mixed as described on page 20. If the surface of the road is hard and smooth, the

scribed on page 20. If the surface of the road is hard and smooth, the mixing may be done upon it without any platform. In any case, it must be very thorough, some contractors employing a man to rake each shovelful as it is turned by the two shovelers. Enough water should be added to produce a jelly-like consistency, the mortar rising to the surface when lightly rammed. The surface of the coarse concrete must be below the level of the top of the forms so as to give room for the finishing coat, or wearing surface.

If the walk or floor is laid in alternate blocks by the first method (*a*), described above, the forms around each block are left in until after the top coat or wearing surface has been placed, and has slightly stiffened, when they may be removed and the alternate blocks laid. The latter must be placed on the same day, however, to avoid difficulty in forming the surface joints between the stones. If a filler is placed between the blocks, the forms are lifted soon after the concrete of the base is laid, and before the wearing surface is spread, and the joints filled with sand or, in some cases, by a "separator" of lean mortar mixed, say, 1 part cement to 4 or 5 parts sand. Whatever the material used, it must be weaker than the concrete.

Wearing Surface. As soon as a few of the blocks of concrete base have been laid, and before they have set, the mortar for the wearing surface must be placed. This, as for pavements (see page 799), consists of a mixture of cement and sand, cement and fine crushed stone, or cement and a mixture of sand and stone. The materials should be very exactly proportioned, so as to give a uniform color. The cement must not be mixed with the sand long in advance of its use because the natural moisture in the sand will cake the cement. If the work is progressing so slowly that the cement must be measured by pailfuls a determination must first be made of the number of pails of loose cement in a bag or barrel of packed cement, and the number of pails of sand in a barrel of loose sand, then the relative volumes calculated to allow for the increase in bulk of the loose over the packed cement. Each pail must be filled in exactly the same way, so that one measure will not be more densely packed than the next. The sand and cement must be mixed dry until the color is absolutely uniform, when, if coloring matter is used, it is added to this dry material. Water is added to give about the consistency employed by a mason in laying brick, so that it can be readily leveled off with a straight-edge. This mortar is carried from the mortar box to the walk in pails, and smoothed off with a straight-edge guided by the tops of the forms.

The surface is roughly floated with a plasterer's trowel, shown in Fig. 252,

soon after leveling with the straight-edge, but the final floating is not performed until the mortar has been in place from two to five hours and has partially set. The final floating is done first with a wooden float and afterwards with a metal float or plasterer's trowel. Just before the floating, a very thin layer of "dryer," consisting of dry cement and sand, mixed in proportions 1:1 or even richer, is frequently spread over the surface, but this is generally undesirable as it tends to make a glassy walk.

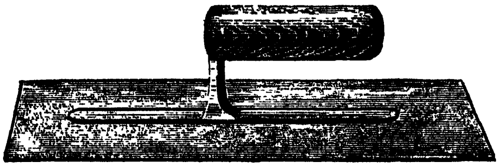

FIG. 252.—Plasterer's Trowel, or Metal Float. (*See p.* 804.)

The surface is now ready to groove, for by this time the intermediate stones should be in place. As has been stated, the cross joints are in line with notches in the outside forms. The mason can thus locate the joints between the blocks of base concrete. To find the line exactly, he runs his small pointing-trowel down through the upper layer, and feels for the joint below. With the ends of the joints thus marked, he lays a straight-edge flat across the walk against these marks, and, walking across on the straight-edge, marks the line and also cuts through the partially set mortar and concrete by running his small pointing-trowel to the full length of the blade. Moving the straight-edge back a fraction of an inch, he runs his groover (see Fig. 253) along the line cut by the trowel, using the straight-edge for a rule. Both edges of the walk are rounded off by the edging trowel (see Fig. 254), which is a small float with one of its edges curved. The entire surface is finally gone over once more with the metal float to erase any marks or scratches which may have been made. A dot roller (see Fig. 255) or grooved roller may be employed to relieve the smoothness.

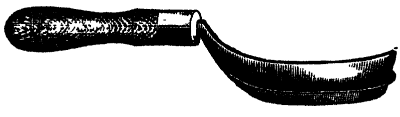

FIG. 253.—Groover. (*See p.* 805.)

The exact time at which the surface should be floated depends upon the setting of the cement, and must be determined by the mason. Considerable skill is required in this troweling to prevent the formation of hair cracks by over-troweling, and to insure a surface which will not wear rough as a result of insufficient troweling.

If the walk is exposed to the hot sun it may be necessary to cover it with a wood or canvas frame, or with moist sand, for several days

after its completion, as it is absolutely necessary that it shall not dry out too quickly.

Effect of Frost upon New Concrete Sidewalks. If concrete sidewalks are exposed to frost before thoroughly hard and dry, the surface is likely to blister and scale off in patches about $\frac{1}{16}$ inch thick. It is best, therefore, to avoid sidewalk construction in freezing weather.

Concrete Curbing. Concrete curbing for artificial sidewalks is largely displacing stone curbing. The curb is built just in advance of the walk. It is divided into blocks and is separated from the walk by joints similar to the joints between the blocks. The soil is excavated, and a foundation of porous materials of the same thickness as that employed under the walk proper is placed and rammed. A layer of ordinary concrete, say about 12 inches wide and 8 inches deep is placed upon this foundation to underlie the curb. The curb proper is 12 inches deep and 8 inches wide at the bottom, tapering on the outside to a width of 7 inches at the top, with its inside face vertical. At least one inch of the face and of the surface consists of mortar or granolithic, like the wearing surface of the walk. A typical sidewalk and curb is shown in Fig. 256. The back of the curb is formed against a temporary plank. For the face mold, a 12-inch planed plank is set on edge to the proper batter and may be held in place by driving stakes about 4 inches out from it, and nailing strips from the top of these stakes to the top edge of the plank, so that they can be knocked up and the plank loosened without disturbing the face of the curb. When ready to place the concrete for the curb, which should be laid before the layer of concrete underlying it has set, a 1-inch board is placed on edge just inside of the 12-inch plank, with occasional thin strips or wedges between it and the plank. The coarse concrete of the curb is then placed back of this board, and thoroughly rammed so that its surface is one inch

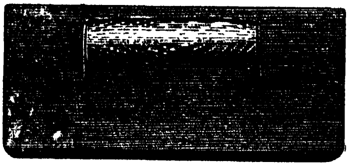

FIG. 254.—Edging Trowel. (*See p.* 805.)

FIG. 255.—Dot Roller. (*See p.* 805.)

below the top of the forms, and when sufficiently hard, the 1-inch board is drawn up from the face, and with the aid of a trowel its place is filled with wearing surface material. The outside form is generally allowed to remain over night, and in the morning the outside surface is floated. A ruled joint like that between the blocks is formed between the curb and the remainder of the walk.

A metal corner is sometimes laid in the exposed edge of the curb to protect it from wear.

Combined Curb and Gutter. One of the advantages of a concrete walk lies in the ease with which it is adapted to special construction. A gutter 5 or 6 inches thick, with a pitch corresponding to the crown of the street, is often laid in combination with the curb. It is underlaid with a porous

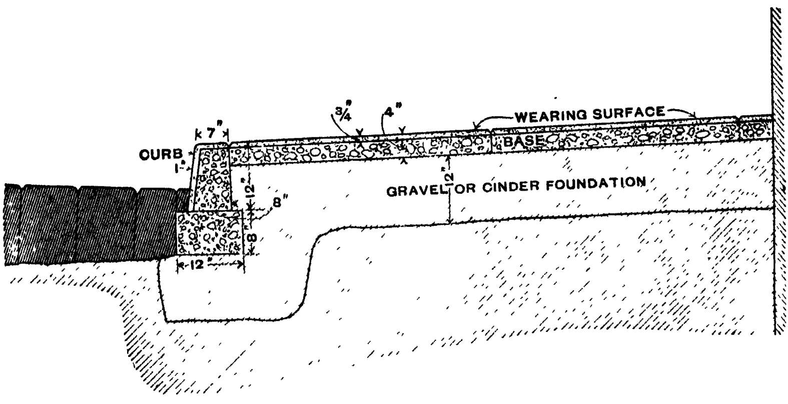

FIG. 256 —Typical Concrete Sidewalk and Curb. (*See p.* 806.)

foundation, and in some cases by a sub-soil tile drain. The blocks forming the combined gutter and curb are made about 6 feet in length, and are in alternate sections so as to form definite cross joints, but each section of the curb and gutter must be built together, with no longitudinal joint between them.

Vault Light Construction. Sidewalk lights over basement areas or subways are formed of circular or square lights of plate glass, set in reinforced concrete slabs, supported by steel or reinforced concrete beams. Steel rods about $\frac{3}{16}$-inch diameter are interlaced in both directions between all of the rows of lights. The width of the slab between beams is governed by the thickness of the slab, a customary width being 3 to 4 feet. The dimensions of the beams and girders, whether of steel or reinforced concrete, depend upon their loading and span,

(See table, p. 576.) A typical vault light construction supported by steel girders and stiffened by concrete ribs is illustrated in Fig. 257.

If concrete beams or stiffeners are used, they must be laid at the same time as the slabs are placed, so as to be in the same piece with them, but contraction joints must be provided as shown. In laying the slabs, the position of the glass discs may be located by an iron plate with holes of the size of the glass discs. On top of this iron form, a layer of oiled paper is

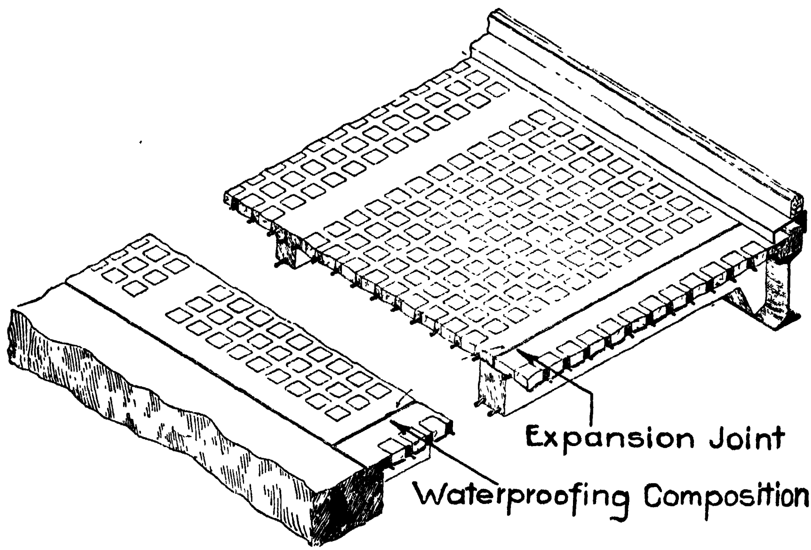

FIG. 257.—Typical Vault Light Construction. (*See p.* 808.)

spread to prevent the cement sticking to it, the lenses are set upon the paper over the holes, the reinforcing rods placed, and the mortar poured around the glass, and its surface troweled after partially setting, same as the surface of a granolithic walk. After the mortar has become thoroughly hard, the metal plate and the paper may be removed.

COST AND TIME OF SIDEWALK CONSTRUCTION

The cost of concrete sidewalk or basement floor construction is extremely variable. The job at any one location is likely to be small, not occupying more than a few days, so that the time and expense of transporting men and materials, and the time getting started upon the work, constitute an important item. The skill of the men employed in placing and finishing

the concrete affects the cost still more, since an experienced gang may easily lay three times as much surface of walk in a day as inexperienced men, even if the latter are accustomed to ordinary concrete work. Excavation is another variable item, depending upon the quantity of earth to be removed and the character of the material.

A gang of convenient size consists of —

One mason.

One man to assist the mason in placing forms, and to level and ram the concrete.

Three men mixing and placing coarse concrete for base.

One man mixing top dressing for wearing surface.

If excavation is included in the work, more laborers may be needed. The amount of walk covered by a gang is limited by the surface which can be floated and troweled by the mason. Unless he works overtime, the laying of concrete must stop about the middle of the afternoon in order that the wearing surface may have opportunity to set. Meanwhile, the concrete gang may prepare and ram the foundation and get everything in readiness to begin concreting promptly the next morning. With a gang of the size suggested a foreman adds considerable to the expense, and it is often advantageous to so arrange the work as to make the mason responsible for its quantity and quality. A bonus paid for an excess over a certain area of surface covered is an effective incentive for a good day's work. In order to properly fix such a bonus the employer must know the relative times required for plain sidewalk and curb. The size of the blocks must also be considered, since the labor upon the joints forms a prominent division of the work.

Under average conditions a mason skilled in this class of work should float and trowel a surface of 600 to 700 square feet in eight hours, if no allowance is made for time which is necessarily lost between jobs and in commencing work. This lost time will lower the average by an amount varying with the size of the job. If the excavation is ready, five men working with the mason should prepare the foundation and place the base concrete and the mortar for the wearing surface for a walk 4 to 4½ inches thick. For a thicker walk, one more man may be required in the gang to keep up with the mason, since a thick walk requires more concrete or mortar.

The contract price for a granolithic or artificial walk from 4 to 5 inches in thickness, with Portland cement at about $2.00 per barrel, and sand and stone each at about $2.00 per cubic yard, varies from $0.16 to $0.18 per square foot.* The cost of curbing runs about $0.75 to

* Personal correspondence with Henry Wells Durham.

$1.00 per linear foot without a metal strip, and 25 to 50 cents higher with it.

DRIVEWAYS

For driveways the concrete is laid similarly to that in sidewalk construction. The total thickness may be 5 inches for light travel, or 6 to 7 inches for heavy teaming. Grooving the surface in 6-inch squares affords foothold for the horses.

TOOLS

The following implements are required in ordinary concrete walk construction:

Mortar box for mixing the materials for wearing surface.

Platform about 12 ft. square for mixing concrete* (see Fig. 7, p. 22).

One or more iron wheelbarrows for handling the materials and the concrete (see Fig. 4, p. 18).

Square-pointed shovels (see Fig. 3, p. 18).

Hoe.

2-inch scantling of a width corresponding to the thickness of the walk

⅜-inch stuff of same width as scantling, for curved forms.

Steel square.

Spirit level.

Straight-edge long enough to extend across the walk.

Two rammers about 5 inches square, with handles about 4 feet long (see Fig. 75, p. 258).

Wooden stakes.

Iron pins and twine for stretching line.

Mason's trowel.

Pointing trowel.

Plasterer's steel trowel (see Fig. 252, p. 805).

Plasterer's wood float.

Groover (see Fig. 253, p. 805).

Edging trowel (see Fig. 254, p. 806).

Dot roller (see Fig. 255, p. 806).

COLORING MATTER

The appearance of a walk is improved by being slightly colored. The following formulas are recommended by Mr. L. C. Sabin:†

* Sometimes unnecessary.

† Sabin's "Cement and Concrete", 2nd Edition, p. 382.

Colors for 1 : 2 Mortar. By Louis C. Sabin. (See p. 810)

MATERIAL.	½ LB. PER 100 LB. CEMENT.	4 LB. PER 100 LB. CEMENT.	COST PER LB.
Lamp Black	Light slate	Dark blue slate	15 cents
Prussian Blue	Light green slate	Bright blue slate	50 "
Ultra Marine Blue		Bright blue slate	20 "
Yellow Ochre	Light green	Light buff	3 "
Burnt Umber	Light pinkish slate	Chocolate	10 "
Venetian Red	Slate, pink tinge	Dull pink	2½ "
Red Iron Ore	Pinkish slate	Light brick red	2½ "

NOTE: Colors vary with quantity of material added. Cost is per lb. of coloring matter. Colors are apt to fade unless formed by color of crushed rock.

QUANTITIES OF MATERIALS FOR SIDEWALKS

The volumes of materials required to cover a certain area of surface are determined by the thickness of the walk or floor, the proportions in which the materials are mixed, and the character of the materials.

The following table gives the approximate quantity of materials necessary for 100 square feet of surface for walks of various thicknesses of base and wearing surface. It is assumed in compiling the table that the coarse aggregate of the base contains about 45% voids, and that the stone and

Materials for 100 Square Feet of Concrete Sidewalks. (See p. 811)

Proportions based on a barrel unit of 4.0 cubic feet.

Base.							Wearing Surface						
	Proportions 1 : 2½ : 5			Proportions 1 : 3 : 6				Proportions 1 : 1		Proportions 1 : 1½		Proportions 1 : 2	
Thickness.	Cement.	Sand.	Stone.	Cement.	Sand.	Stone.	Thickness.	Cement.	Sand.	Cement.	Sand.	Cement.	Sand.
in.	bbl.	cu. yd.	cu. yd.	bbl.	cu. yd.	cu. yd.	in.	bbl.	cu. yd.	bbl.	cu. yd.	bbl.	cu. yd.
2½	1.06	0.40	0.79	0.90	0.41	0.81	½	0.80	0.12	0.64	0.14	0.53	0.15
3	1.27	0.47	0.94	1.08	0.48	0.96	¾	1.23	0.19	0.98	0.22	0.81	0.24
3½	1.46	0.54	1.08	1.24	0.55	1.10	1	1.66	0.24	1.32	0.30	1.09	0.32
4	1.68	0.62	1.24	1.43	0.64	1.28	1¼	2.09	0.31	1.66	0.37	1.38	0.41
4½	1.90	0.70	1.41	1.62	0.72	1.44	1½	2.53	0.37	2.00	0.44	1.66	0.50
5	2.13	0.79	1.58	1.82	0.80	1.60	2	3.32	0.50	2.64	0.58	2.19	0.65

NOTE.—Select and add together the quantities of each material corresponding to the required thicknes and proportions of base and wearing surface.

sand are measured loose by shoveling into boxes or barrels, on the basis of the volume of a cement barrel of 4.0 cubic feet. For example, proportions 1:3:6 are equivalent to 1 barrel Portland cement, 12 cu. ft. of sand and 24 cu. ft. of broken stone or gravel, while proportions 1:2 are equivalent to 1 barrel of Portland cement to 8.0 cu. ft., or one bag of Portland cement to 2.0 cu. ft. of sand or crushed stone. The variation in volume of mortar produced with sand and crushed stone of different fineness may affect the quantities for wearing surface by at least 10%, but to provide for such variation, and to allow for waste, 10% has been added, in computing the values, to the quantities in the table on page 214.

Since the volumes are given separately for the base and wearing surface, the quantities required for walks of other thicknesses may be readily estimated, as illustrated in the following example:

Example: — What materials will be required for a walk 8 ft. in width and 150 ft. long, the base to be 3 in. thick, of concrete in proportions 1: 3: 6, and the wearing surface one inch thick, in proportions 1 part cement to 1 part sand?

Solution: — Referring to the table we find directly that for 100 sq. ft. of base 3 in. thick, 1.08 bbl. Portland cement, 0.48 cu. yd. sand, and 0.96 cu. yd. broken stone or gravel are required. Similarly, for 100 sq. ft. of the wearing surface one inch thick we should require 1.66 bbl. cement and 0.24 cu. yd. sand. For each 100 sq. ft. of completed walk there would therefore be needed 2.74 bbl. cement, 0.72 cu. yd. sand, and 0.96 cu. yd. broken stone or gravel; and since there are 1 200 sq. ft. in an area of 150 by 8 ft., for both base and wearing surface we should require 33 bbl. Portland cement, 9 cu. yd. sand, and 12 cu. yd. broken stone or gravel.

CHAPTER XXXI

CEMENT MANUFACTURE

This chapter contains a short historical sketch followed by a brief outline of the processes of modern cement manufacture, illustrated with views of typical machinery.

HISTORICAL

Lime must have been used by the Egyptians thousands of years before Christ, as the stones in the pyramids apparently were laid in mortar of common lime and sand. It is even thought by some that these ancients understood the principle of mixing lime and clay together to make a real cement.

Concrete was made by the Romans as early as several centuries before Christ. For most of their work, they used lime mixed with sand and stone, but understanding the value of puzzolana or volcanic ashes to render lime hydraulic, they employed these two materials in combination with the sand and stone for marine construction. For less important work, they often mixed lime and coarsely powdered brick with the aggregate. Vitruvius, writing in the first century, describes methods of making concrete with lime alone, and also gives as the formula for making it of slaked lime and Italian puzzolana:

12 parts of puzzolana, well pulverized.
6 parts of quartz sand, well washed.
9 parts of rich lime, recently slaked; to which is added
6 parts or fragments of broken stone, porous and angular, when intended for a "pise" or a filling in.

It is interesting to note that the Romans called their concrete made from these materials "opus caementum," literally, "chip-work," and that the word "cement," derived from the aggregate, has in our day been transferred to the hydraulic binding material.

In the Middle Ages concrete was employed, after the Roman fashion, for both walls and foundations. In the former it was generally laid as a core faced with stone masonry. Large stones were often imbedded in the mass.

The fact that clay contained in certain limes rendered them hydraulic was discovered by John Smeaton, when studying the designs for the third

Eddystone Lighthouse, about 1750. Early in the following century, Vicat, by his extended scientific researches in France, earned for himself the name of the founder of hydraulic chemistry.

In England, in 1796, James Parker made from nodules of argillaceous limestone, calcined and ground, what he called Roman cement. This process he patented, and from it the Natural cement industry was developed. It was Joseph Aspdin, of Leeds, England, who really invented Portland cement by discovering in 1824 that an artificial mixture of slaked lime and clay, highly calcined, formed a hydraulic product. On account of its resemblance in color and hardness to the Portland stone which was much used in England at that time, he called his invention Portland cement. Two patents had been granted in England a few years before his time, but as in these the materials were not heated to vitrification, hydraulic lime instead of cement was produced.

The Portland cement industry was not developed to any great extent until about twenty years after Aspdin's discovery, when J. B. White & Sons in Kent, England, commenced its manufacture. Later, Mr. John Grant gave a great impetus to Portland cement manufacture by experimental studies upon the practical action of cements, mortars and concretes under varied conditions. The results of his tests he presented to the Institution of Civil Engineers in 1866, 1871, and 1880.

The first manufactory for producing Portland cement in France was established toward the middle of the last century at Boulogne-sur-Mer. In Germany the first factory was erected soon after this, for the production of the Stettin Portland cement, and with such successful results that in 1900 Germany produced more Portland cement than any other country.

The discovery in the United States of a rock suitable for Natural cement was made in 1818 by Canvass White, an engineer connected with the construction of the Erie Canal, and Natural cement was made in Madison and Onondaga Co., N. Y., in that year. The first Natural cement in the Rosendale district was made at Rosendale, Ulster Co., N. Y., about 1823. Mr. D. O. Saylor was the founder of the Portland cement industry in the United States. His discoveries were made in the Lehigh Valley. He experimented from 1871 to 1875 and marketed cement in 1875.

PRODUCTION OF CEMENT

The total production* of hydraulic cement in the United States for 1914 was 89 049 766 barrels, of which 88 230 170 barrels were Portland cement, 751 285 barrels were Natural cement, and 68 311

* Mineral Resources of the United States, 1914.

barrels were Puzzolan or Slag cement. The average values per barrel were: for Portland cement \$0.927, for Natural \$0.468, and for Puzzolan \$0.926.

The superior quality of Portland over Natural cement and the increasing economy of its manufacture is evinced by a comparison of these figures with those of 1890, when only 335 500 barrels of Portland cement were produced against 7 082 204 barrels of Natural cement. The imports of cement in 1890 were 1 940 186 barrels, and in 1908, 842 121 barrels.

The production of Portland cement in the United States by individual States is represented in the following table.

Production of Portland Cement in the United States in 1910 and 1914 by States.

State.	1900			1914		
	Producing Plants	Quantity, barrels.	Value.	Producing Plants.	Quantity, barrels.	Value.
Pennsylvania..	14	4 984 417	4 984 417	20	26 570 151	\$24 630 529
Indiana	1	30 000	37 500	5	9 595 923	8 895 421
Kansas	1	80 000	100 000	9	3 431 142	3 180 669
Illinois.	3	240 442	300 552	5	5 401 605	5 007 288
New Jersey .	2	1 169 212	1 169 212	3	3 674 800	3 406 540
Michigan	6	664 750	830 940	11	4 285 345	3 972 515
Missouri				5	4 723 906	4 379 061
California......... .	1	44 565	89 130	7	5 075 114	4 704 631
Washington				5	2 017 344	1 870 078
New York.	8	465 832	582 290	8	5 886 124	5 456 437
Ohio...	6	534 215	667 769	5	1 962 047	1 818 817
Iowa				3	4 233 707	3 924 646
Kentucky						
Tennessee						
Texas	2	26 000	52 000			
Oklahoma						
South Dakota	1	38 000	76 000			
Colorado	1	35 708	71 416			
Arizona						
Utah	1	70 000	175 000	3	981 100	909 480
Maryland..........						
Virginia	1	58 479	73 099			
Massachusetts.						
Alabama*....						
Georgia*..						
Arkansas†	1	40 000	70 000			
North Dakota .	1	400	1 200			
Other States‡ . .				17	8 291 521	7 686 240
	50	8 482 020	9 280 525	110	88 230 170	81 789 368

* Product in 1900 combined with Virginia.
† Product in 1900 combined with Missouri.
‡ Alabama, Arizona, Colorado, Georgia, Kentucky, Maryland, Montana, Nebraska, Oklahoma, Tennessee, Virginia, and West Virginia.

About 36% of the total production in 1914 was in the Lehigh Valley of Pennsylvania and New Jersey. In 1900 73% came from that district.

PORTLAND CEMENT MANUFACTURE

Portland cement is made from a mixture of calcium carbonate with silica and alumina.

The processes of manufacture differ with the natural state in which these materials are found, but the operation consists essentially of (1) pulverizing and mixing the two ingredients, (2) heating to a temperature which is near the melting point, i.e., calcining, and (3) grinding to a fine powder.

The great bulk of the raw materials used in the manufacture of Portland cement are found in a dry state, and the grinding is done in iron mills or tube mills of various kinds, the material being ground dry. Where either of the raw materials occurs in a moist state and also in many cases of dry materials, they are mixed and ground wet and in this condition are introduced into the kiln. Dry raw materials for calcining or burning in the old style stationary kilns must be formed into plastic bricks with the aid of water, but the rotary kiln invented in 1885 by Mr. Frederick Ransome has revolutionized the manufacture of Portland cement by making it possible to introduce the mixed substances into the furnace, in either a dry or wet state, without hand labor.

After calcination, the methods of grinding the clinker are independent of the character of the raw materials or the type of kiln.

The Association of German Cement Manufacturers, to protect the good name of German Portland cement, requires that its members shall sign an agreement to introduce no adulteration into its product.

Raw Materials for Portland Cement Manufacture. The raw materials, as stated above, consist essentially of calcium carbonate and silicate of alumina. Their exact proportions are determined by their chemical composition. A usual ratio is such as will produce about 75% calcium carbonate in the raw mixture. The two substances occur in nature in so many forms that we have a large range of choice in raw materials. The following combinations are actually used in different cement manufacturing plants in the United States:

Cement rock and limestone
Limestone and clay.
Limestone and shale.
Marl and clay.
Chalk and clay.
Limestone and slag.
Alkali waste and clay.

Cement rock is an argillaceous limestone, rather soft in texture, which in the Lehigh Valley usually requires from 5 to 20% of limestone to

give it the correct Portland cement composition. Occasional deposits are found which are suitable to use with no admixtures, or from which the desired proportions may be obtained by mixing two different strata in the same quarry. Several other States, among them the Virginias, Alabama, Colorado, and Utah, have a geological formation from which Portland cement, similar to that in the Lehigh Valley, has been made.

In the Hudson River Valley, New York, are situated large manufactories employing a hard limestone which is nearly pure carbonate of lime, requiring 20% to 25% clay or shale and producing a fine quality of cement. A somewhat similar mixture is used in California and in scattered localities in the Central States.

The marl used for cement usually is a wet, calcareous earth, in some localities of organic origin from shell deposits, and in other places of chemical formation. There are large cement plants using marl and clay in Ohio, Indiana, and Michigan.

Chalk and clay deposits resembling those in England are worked in Texas.

Certain blast furnace slags similar to those used in the manufacture of Puzzolan cement, when combined with a suitable admixture of limestone, produce, after calcination. a true Portland cement with normal characteristics.

The waste from the manufacture of soda, when employing the ammonia soda process with suitable raw materials, is substantially a precipitated chalk, and may be burned with clay and made to produce a Portland cement.

In Germany the Alsen and Stettin brands are made from chalk and clay, the Dyckerhoff and Mannheimer brands from limestone and clay, while the Germania and Hanover works use marl and clay. In England raw materials consist principally of chalk and clay. Belgium manufacturers use chalk and clay, and a so-called Portland cement from natural rock is also manufactured in that country. In France, marl and clay and chalk and clay are the chief raw materials for commercial Portland cements.

The character and proportioning of the raw materials and the processes of chemical combination are discussed by Mr. Spencer B. Newberry in Chapter V.

The following table illustrates the composition of various classes of materials which are used for Portland cement, and also the resulting analysis of the cement in each case:

Comparative Analyses of Raw Materials and Portland Cements.

		Cement Rock and Limestone.			Limestone and Clay.[4]			Marl and Clay.			Chalk and Clay.*		
		Cement Rock.[1]	Limestone.[2]	Cement.[3]	Limestone.	Clay.	Cement.	Marl.[5]	Clay[6]	Cement.[7]	Chalk.[8]	Clay.[9]	Cement.[10]
Silica	SiO_2	19.06	1.98	19.92	3.30	55.27	21.50	1.75	62.10	22.52	0.35	60.30	22.10
Alumina	Al_2O_3	4.44	0.70	9.83	1.30	28.15	10.50	1.57	20.09	6.69	0.75	11.07	11.32
Iron Oxide	Fe_2O_3	[illegible]		2.63					7.81	3.54		8.13	
Calcium Oxide	CaO	38.78	53.31	60.32	52.15	5.84	63.50	49.24	0.65	63.82	54.95	4.40	60.76
Magnesian Oxide	MgO	2.01	0.97	3.12	1.58	22.5	1.80	0.44	0.96	0.69		1.27	1.10
Sulphuric Acid	SO_3			1.13	0.30	0.12	1.50	0.15	0.49	0.98		2.50	1.40
Carbonic Oxide	CO_2	32.66	42.94		40.98			39.16			43.17	7.47	1.94
Water	H_2O				8.37				8.00				
Organic Matter								7.50				4.06	
Other Constituents							0.40			1.08	0.85	0.45	1.38

NOTE.—Carbonates in raw materials, given in some of the analyses, have been transformed into oxide.
[1] Cement Rock. Lehigh Valley District, Penn. 21st Annual Report, U. S. Geological Survey. Pt. 6, p. 404.
[2] Pure Limestone, Lehigh Valley District. W. E. Snyder, Analyst.
[3] Lehigh Valley Cement. Booth, Garrett & Blair, Analysts.
[4] Hudson River Valley. Mineral Industry, Vol. 6, p. 97.
[5] W. H. Simmons, Analyst, 22d Annual Report, U. S. Geological Survey, Pt. 3, p. 650.
[6] Shale. Mineral Industry, Vol. 6, p. 99.
[7] Michigan. W. H. Simmons, Analyst, 22d Annual Report, U. S. Geological Survey, Pt. 3, p. 680.
[8] Water, 23%. Analysis from David B. Butler, England.
[9] Estuary Mud. Roughly dried, lost 33⅓%. Analysis from David B. Butler, England.
[10] English Portland Cement. Analysis from David B. Butler, England.

Dry Process with Rotary Kilns. The rotary kiln is used almost universally, superseding the old stationary kiln. The rotary was made a success in this country after failing totally abroad. The successful use of powdered fuel also was accomplished in the United States, and afterward adopted, with the rotary kilns, in Europe. Where rock, or rock and clay, form the raw materials, they as a rule, are mixed and ground and introduced into the rotary in the form of a dry powder. If marl or chalk furnish the carbonate of lime, the wet process of mixing and grinding is usually employed, as described on page 822, although in a few plants each of these materials is dried when entering the mill, and the operations are similar to those described below for rock mixtures, except that driers and disintegrators are substituted for stone crushers.

The process of manufacturing Portland cement from rock, or rock and

* The authors are indebted for these analyses of chalk and clay to David B. Butler, of England, who prepared them for this Treatise.

clay mixtures, in plants equipped with rotary kilns, consists essentially of crushing the materials,—either separately or after mixing them,—drying, grinding, calcining in the rotaries, cooling, grinding to powder, and packing.

If two stones of fairly similar texture and each of uniform composition form the raw materials, they may be carefully weighed and thrown together into the breaker, which may be either a large jaw or gyratory crusher (see pp. 222 to 223). Otherwise, they are crushed separately, and mixed just before the grinding which preceeds the calcination. A further reduction in size to about ½-inch is accomplished by rolls, crackers of the coffee mill type, hammer mills, or similar machinery.

Clay, if used, is dried in broken lumps, and then may be pulverized by passing it through a disintegrator consisting of two horizontal rolls, one corrugated or toothed and the other smooth.

An economical form of dryer for clay or stone consists of a long revolving steel tube about 5 or 6 feet in diameter by say, 60 feet long, provided with shelves on its interior surface, formed by horizontal Z-bars. The hot gases from the kiln may be made to pass through the tube and meet the raw material.

By treating the two materials separately up to this point, an extremely accurate mixture is obtained by weighing the ingredients in a pair of automatic weighing machines so arranged that one of the pair will not dump until both are charged.

Samples of the two materials are taken, just before mixing, at definite periods throughout the day, and analyzed to determine the correct proportions. A partial analysis showing the quantities of the principal constituents may be all that is necessary except at occasional intervals. The maintaining of correct proportions is one of the most essential elements in the manufacture.

Another grinding of the mixed materials in mills of various types, to such a fineness that 82 to 92% will pass through a screen having 200 meshes per linear inch, completes the preparation for the rotary kilns.

A modern kiln located at the plant of the Atlas Portland Cement Co. at Hudson, N. Y. is shown in Fig. 258, page 820. This is 232 feet 3¼ inches long by 12 feet in diameter and is elevated so that the clinker flows to subsequent machinery by gravity. Fine grinding before burning is one of the secrets of successful manufacture.

The best type of rotary kiln used for calcining dry materials, consists of an inclined steel tube from 100 to 250 feet long. The diameter is generally 8 to 12 feet, though occasionally smaller than this at the

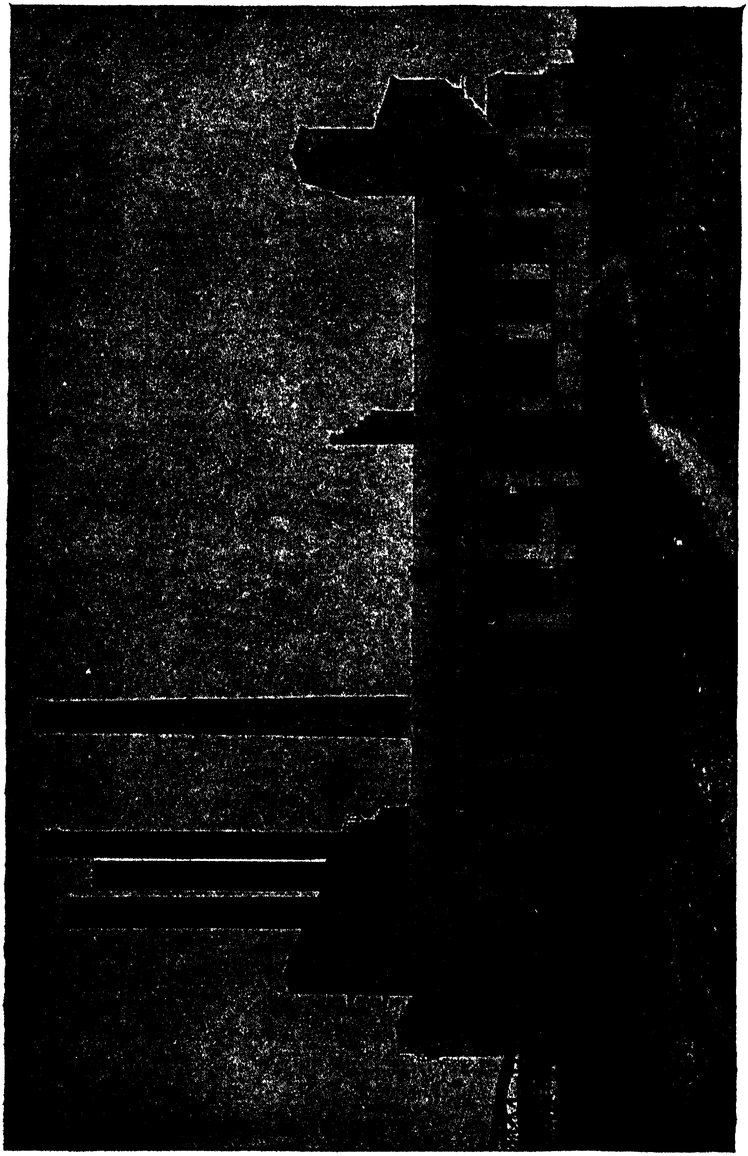

FIG. 258.—Rotary Kilns at the Plant of the Atlas Portland Cement Co., Hudson, N. Y. (See p. 810)

upper end and tapering to the larger size at a point about one-third of its length from the upper end.

The lower end of the rotary is closed by a movable brick wall, and through the center of this passes a pipe that feeds the powdered coal which in a separate building is crushed to pea size and pulverized in tube mills, or other pulverizing machines, so that about 95% is finer than a 100-mesh screen; the finer the coal the greater its efficiency.

The ground stone may be fed into the upper end of the rotary by a spiral conveyor enclosed in a pipe which is water-jacketed to prevent the feed pipe from burning out. The degree of calcination is governed by the supply of raw material, the speed of rotation of the rotary, which rests on rollers geared to a speed-changing device, and the quantity of fuel. If coal is used for fuel, it is fed by a blast from a fan, and the quantity is regulated by a spiral conveyor running at changeable speed. The heat in the kiln is so intense that the coal burns as a gas without apparent smoke or cinder. The proper temperature, which is said to be 2700° to 3000° Fahr., is determined by the appearance of the burning clinker. At a certain point in its descent the material becomes semi-vitrified and forms into irregular balls or clinkers, which roll around and finally fall out red-hot at the lower end in pieces ranging in size from $\frac{1}{8}$-inch to one inch in diameter. The clinker, when properly burned, is of a greenish black color with a faint glisten, and contains but few large pieces. It slightly resembles in appearance the clinker often found among the ashes of hard coal.

The output of a rotary varies with the length and diameter from 150 to 200 barrels per 24 hours for a 60 foot kiln to 1 000 to 2 000 barrels for a 150 to 200 foot kiln with a smaller coal consumption per bbl.

The clinker, after being cooled in some form of cooler, is crushed by passing between horizontal rolls or through some other form of crusher, and is then ready for the fine grinding, or, if desired, it may be stored either out of doors or under cover until needed. Strangely enough, wetting the clinker does not injure it provided it is dry when it enters the fine grinders.

The fine grinding is accomplished by passing the clinker through one or more special mills*, such as ball mills, tube mills, Griffin, Kent or Lehigh Fuller Mills.

A tube mill, which is used to a large extent in grinding cement, consists of a long horizontal cylinder filled nearly to its axle with flint pebbles imported from Europe, which average about 2 to 3 inches in diameter. The cement is ground by rolling around with the flints. It is then

* See illustrations in advertising pages at back of book.

thrown against the screen, which prevents the passing of pieces of flint. A tube mill which passes, say, 250 barrels of cement per day, will require the renewal of the flint pebbles at the rate of about 600 lb. per week. Such small mills as these, however, are being superseded by larger ones, i.e., by combined ball and tube mills having a capacity up to 75 to 100 barrels of cement per hour.

It is customary to store the cement in bulk and weigh it out by automatic weighing and bag packing machines into bags or barrels as required for shipment.

In outlining the cement machinery, no reference has been made to the methods for conveying the material from one machine to another. Bucket conveyors, chain drags, belts and spiral conveyors are all more or less used. A spiral conveyor is a helical blade on a revolving shaft, set in a square or circular trough or tube of larger size than the spiral, so that the material packs around the circumference, and the blade comes in contact only with the powdered material.

Plaster of Paris (calcium sulphate $CaSO_4$) or gypsum ($CaSO_4 + 2H_2O$), the same substance in crystalline form, is an important addition to cement as a regulator of its setting, and from 2 to 3% is used in nearly all Portland cement manufactories. The gypsum must be added after the calcination and before the final grinding, in order to insure the proper result.

The laboratory of a cement plant is an important feature. Not only must the raw materials and the finished cement be analyzed at frequent periods to insure uniformity of product, but the cement must be mechanically tested also for fineness, time of setting, tensile strength at seven and twenty-eight days, and, perhaps most important of all, for soundness. Most manufacturers use some form of the accelerated or hot test. This is not only due to the fact that many engineers require the cement to pass an acclerated test for reception, but because the chemists in the cement factories consider this test of great value in checking up the quality of cement.

Wet Process with Rotary Kilns. The rotary or Ransome kiln was first used in England on wet materials. Rotaries have been widely, in fact almost universally, adopted in the United States for calcining dry materials. More recently this field has been extended to use with slurry formed from the mixture of pulverized materials such as marl and clay and containing some 40% or more of water, which is pumped into the end of the rotary and dried by the same flame used for calcination.

In the United States the raw materials most commonly employed in the wet process are marl and clay. The marl as it comes to the mill is broken up in some form of a disintegrator. The clay is dried and pulverized and is then mixed with the marl, which is about of the consistency of thick cream, in a pug mill, or an edge-runner.

In some cases the clay is ground and water is added to it before mixing with the marl.

The mixed materials must now be ground wet before burning. This is sometimes accomplished in mill stones, consisting of a pair of horizontal stones the upper one of which revolves upon an upright shaft, but more often in wet tube mills similar to those described on page 819.

Stationary Kilns. Before the introduction of rotary or revolving kilns all cement was burned in stationary kilns. Stationary kilns are of two general types: (1) intermittent kilns, which are completely charged and then burned, and (2) continuous kilns, where the fire is maintained continuously and the exhaust heat is used to dry and heat the raw materials before burning them. The bricks of cement slurry and the coke are placed in these kilns in layers by hand and then burned. While the old style of stationary kilns are practically obsolete, small, vertical kilns taking material in small cakes and operating under air pressure, were being introduced in Germany before the war. These were said to be lower in first cost and more economical in operation than the rotary kiln.

The most common form of intermittent kiln is the *Dome* or *Bottle Kiln*. This consists of a single shaft into which alternate layers of moist bricks of cement slurry and coke are placed by hand and burned. After cooling, the clinker is drawn out by hand through a door at the bottom, picked over to remove under-burned clinker, — which is of a yellowish shade instead of black, — and clinker which has fused to fragments of the firebrick lining.

The *Johnson Kiln* is a more economical form of intermittent kiln. The slurry is placed in chambers, and dried by the exhaust gases from the burning of the previous charge before being placed in the kilns.

Of the continuous kilns, the *Hoffman Ring Kiln* consists of several chambers or furnaces around a central chimney. As the material in one furnace is burned, the heat passes around through the other furnaces so as to raise the temperature of the bricks in them and utilize the exhaust heat.

In the *Schoefer Kiln*, which is also of the continuous type, the bricks and fuel are loaded from time to time into the upper end of the

shaft, and pass down, increasing in temperature, through the flame, where the area is contracted, to be cooled below and drawn out at the bottom.

The *Dietzsch Kiln* is of a somewhat similar type of construction, except that hand-labor is required in passing the dried material into the heating chamber.

NATURAL CEMENT MANUFACTURE

The process of manufacture of Natural cement consists, in brief, of burning a natural argillaceous limestone at low heat and grinding it to powder. The stone used in England is very soft, in fact nearly as disintegrated as marl.

Raw Material. Many of the limestones used for Natural cement contain a high proportion of magnesia and an excess of clay, while others are nearly free from magnesia. It must be calcined at a temperature much below that required for Portland cement or it will fuse to a slag which after grinding has no hydraulic properties. Suitable formations occur in many parts of the United States, one of the most noted being that found in the region of eastern New York where Rosendale cements are made. Sometimes the stone is taken entirely from one ledge, while in other cases mixtures of two strata are employed. Little attention is paid to the analysis of the rock, as there is a wide range in the required chemical composition of the product (see p. 40), and the price at which Natural cement is sold does not warrant great refinement.

Process of Manufacture of Natural Cement. There is less variety in the methods employed for producing Natural cement than for Portland.

In a typical plant, the stones, of about the size that would be required for a large crusher, are brought from the quarry in carts or cars and dumped directly into the top of the kilns, which are of boiler iron lined with firebrick. They have no chimneys, but are open at the top and of the same size throughout. Thick layers of stone are alternated with thin layers of pea coal. The clinker is drawn out at the bottom as it is burned.

In the older plants the burned clinker is crushed and then ground between mill stones, while the newer mills use grinding machinery similar to that in Portland cement plants. When burnt, Natural cement rock is more readily powdered than Portland cement clinker.

PUZZOLAN CEMENT MANUFACTURE

Puzzolan cement has been made in the United States from blast furnace slag mixed with slaked lime. In Europe, natural puzzolanic

materials have been employed. This cement must not be confused with true Portland cement which may be made by employing slag and lime as raw materials and calcining in the usual way.

The process of manufacture* consists essentially of cooling the slag, mixing it with slaked lime, and grinding to a very fine powder.

Slag for Puzzolan Cement. For making pig iron a blast furnace is charged with a mixture of iron ores, fluxes (consisting of limestone, either calcite or dolomite) and fuel, in the proper chemical proportions to produce, after reduction by heat, products of definite chemical composition. These resulting products are pig iron and slag. Any one unacquainted with metallurgy naturally thinks of blast furnace slag as a compound composed to a large extent of iron. This is incorrect; nearly all the iron is drawn off in the pigs and only enough to form a very small impurity goes off with the slag.

All slags are not suitable for Puzzolan cement, as they ordinarily contain too high a percentage of magnesia and are often too high in alumina. The specifications for slag used in the manufacture of Steel Portland cement are as follows:†

Slag must analyze within the following limits:

	Per cent.
Silica plus alumina, not over	49
Alumina	13 to 16
Magnesia, under	4

Slag must be made in a hot furnace and must be of light gray color.

Slag must be thoroughly disintegrated by the action of a large stream of cold water directed against it with considerable force. This contact should be made as near the furnace as is possible.

Mr. E. Candlot says‡ "The slag must be basic; according to Mr. Tetmajer, when the ratio $\frac{CaO}{SiO_2}$ falls below unity the slag is useless; the ratio of alumina to silica must be between 0.45 and 0.50. According to Mr. Prost, the composition of slags habitually used in the manufacture of Puzzolan cements must be nearly represented by the formula 2 SiO_2, Al_2O_3, 3 CaO."

Mr. E. C. Eckel§ gives the following three analyses of slag and slag cement:

*An investigation of the manufacture and properties of Puzzolan cement is given in Report of Board of Engineers, U. S. A., 1900, on Steel Portland Cement.

†Report of Board of Engineers, U. S. A., 1900, on Steel Portland Cement.

‡Ciments et Chaux Hydrauliques, 1898, p. 157.

§Mineral Resources of the United States, 1901.

Analyses of Slags in Actual Use and Composition of Slag Cements

CONSTITUENT.	SLAG			CEMENT		
	Choindez, Switzerland.	Saulnes, France.	Chicago, Ill.	Choindez, Switzerland.	Saulnes, France.	Chicago, Ill.
SiO_2	26.24	31.50	32.20	19.5	22.45	28.95
Al_2O_3	24.74	16.62	15.50	17.5	13.95	11.40
FeO	0.49	0.62			3.30	0.54
CaO	46.83	46.10	48.14	54.0	51.10	50.29
MgO	0.88		2.27		1.35	2.96
CaS	0.59					
$CaSO_4$	0.32					
S						1.37
SO_3					0.35	
Loss on ignition					7.50	3.39
$\frac{CaO}{SiO_2}$	1.78	1.46	1.49			
$\frac{Al_2O_3}{SiO_2}$	0.93	0.52	0.48			

Process of Manufacture of Puzzolan Cement. No kilns are required except for burning the lime. Molten slag as it flows from the blast furnace is granulated by coming in contact with a stream of cold water. This renders the product more strongly hydraulic, and most of the sulphur is removed as it strikes the water. As sent to the cement plant, it usually contains from 30% to 40% of water, and the first operation is to pass it through a dryer. The dried slag may or may not have a preliminary grinding before adding the slaked lime.

The lime is produced by burning a pure limestone, and then slaking it with water to which has been added a small percentage of caustic soda or other similar material, to make the resulting cement quicker setting. After drying, the slaked lime is mixed with the slag and ground in ball mills and tube mills, or in other forms of fine grinding machinery, and is ready for packing in bags or barrels for shipment.

CHAPTER XXXII

MISCELLANEOUS STRUCTURES.

The more important structures are treated with considerable detail in preceding chapters. The uses of concrete and reinforced concrete are now so numerous and are increasing so rapidly that only brief reference can be made to a few of the smaller and of the less common structures.

In railroad work, not only for the more important structures like piers, abutments and arches, but for the numberless smaller details like telegraph poles, ties, bumping posts, and signal posts, is reinforced concrete being employed. Roundhouses, stations and terminal warehouses are being designed either exclusively or in part of this material.

In power development, not only the dams are of concrete, but the canals, penstocks, flumes, and the power stations themselves.

In water-works construction the use of concrete has extended to reservoirs, filter basins, tanks and conduits, and, in some of the recent works, concrete with its imbedded steel for reinforcement is almost the only structural material.

Even the farmer and the householder are utilizing concrete in various ways for barns, garages, chicken houses, floors, fences, silos, tanks, troughs, drains and many other of the small details which make for economy, durability and convenience. By mixing and placing the concrete according to the directions laid down in Chapter II and using sufficient reinforcement (in some cases ordinary fence wire is suitable), many an inexperienced man has built permanent structures of pleasing appearance. For reinforced concrete work such as floors, roofs and stairs, an engineer should be called upon to design the dimensions and reinforcement.

Telegraph Poles. Wooden poles are being replaced in many localities by poles of reinforced concrete because of their greater durability. The Pennsylvania lines west of Pittsburg* have installed poles from 20 to 28 feet high, 8 inches square at the bottom, tapering to 6 inches square at the top, with corners chamfered 2 inches. Holes are left in the pole for the brace and cross-arm bolts and also for the climber steps. The reinforcement may be greatest at the bottom and reduced above to allow for the lessening stress.

* Concrete Engineering, July 1908, p. 189.

In 1907 Mr. Robert A. Cummings* made comparative tests of reinforced concrete and white cedar poles. The former were 13 inches square at the butt and 7 inches at the top, reinforced to withstand the weight of 50 wires all coated with ice to a diameter of one inch. These were stronger than the wooden poles of substantially the same size. After breaking, the ends of the concrete poles were held in a slightly inclined position by the reinforcement, while the wooden poles broke square off and fell to the ground.

Ties. Concrete ties of varied designs† have proved satisfactory for slow speed traffic, especially in yards and on turnouts. They also have been used to a certain extent on high speed track. One of the most important features is the connection with the rail which is generally made through a cushion block of wood. If the tie supports both rails, it must be reinforced in the center at the top to resist the negative bending moment. The ends of the ties should also be well reinforced to prevent breakage in case of derailment.

Road Beds. For tunnels, concrete roadbeds have been found economical because of the very great saving in maintenance expense.

Roundhouses. Reinforced concrete affords a durable and inflammable material for the structural portions and the roofs of roundhouses, while the walls may be built either of concrete or of brick.

Cinder and Ash Pits. Concrete will stand as high temperature as will be given to it by hot ashes and cinders.

Grain Elevators. By building of reinforced concrete the danger from fire is avoided as well as the necessity for constant repairs.

Coal Pockets. For coal storage the strength and fireproofness of reinforced concrete is bringing about its general adoption.

Boiler Settings. Reinforced concrete boiler settings have been in successful use in several plants for a number of years. The initial cost is probably not less than brick but greater durability and freedom from repairs is claimed by the users of concrete settings.

Double walls are required with an air space between. The inner wall may be about 5 inches thick and the outer about 6 inches, both thoroughly reinforced to prevent as far as possible the development of cracks. Bars $\frac{3}{8}$-inch diameter, spaced 6 inches apart both ways, afford effective reinforcement. The walls may be tied together at intervals with bars. The reinforcement permits building the setting to any shape over the boiler, although wherever it comes in contact with the boiler, a 3-inch layer of mineral wool should be introduced to allow for variation in expansion.

* *Cement Age*, Aug. 1907, p. 84.

† *Concrete Review*, 1908, published by the Association of American Portland Cement Manufacturers.

A fire-brick lining must be used. A thickness of 8 or 9 inches is more economical than a 4½-inch lining because it can be replaced without disturbing the concrete. Spaces must be left at the ends of the fire-brick lining to allow for expansion.

The concrete should be as rich as 1 : 2 :4 and the best aggregates are quartz sand and trap rock about ¾ inch maximum size. For high temperatures gravel and limestone aggregates should be avoided. Cinders of first-class quality should make durable walls when mixed with sand and cement in rich proportions.

Fences. Fences have been built of solid concrete, of mortar plastered on wire lath, of concrete rails set in concrete posts, and of concrete posts with galvanized fence wire between them. The last plan is the most common. For farm or division fences the length of posts may be 7 feet, allowing 3 feet of this to set into the ground, and the size may be 5 or 6 inches square at the bottom and 4 or 5 inches square at the top with ¼-inch rods in each corner. Forms are easily made singly or so as to mold several posts at once.

Silos. Silos of solid monolithic concrete built in circular forms may have walls 6 inches thick reinforced with ½-inch bars bent to circles and placed 12 inches apart. Occasional vertical bars are also necessary. The concrete must be mixed wet and placed very carefully so as to give a perfectly smooth interior surface, so solid and dense that the ensilage will not be dried out next to the wall.

Greenhouses. Greenhouses themselves, as well as the floors, tables, water troughs, hotbeds, and minor appurtenances, are being built of concrete. The directions throughout the various chapters in this treatise for structures of different classes will be found to apply to these details.

House Chimneys. Chimneys for residences may be of concrete if heavily reinforced, but the expense of forms usually will make them more costly than brick.

Chimney caps of concrete should be well reinforced to prevent cracking.

Residences. Residences are built of solid reinforced concrete; concrete blocks (see p. 623); concrete tile, plastered (see p. 628); and mortar plastered on metal lath (see p. 645).

Solid or monolithic concrete is especially adapted to fine residences and permits unique architectural treatment. Eventually with the development and consequent reduction in cost of form construction, reinforced concrete may be more generally employed for dwellings of small and moderate size.

CHAPTER XXXIII

REFERENCES TO CONCRETE LITERATURE

While this chapter is not a complete bibliography of concrete literature, it presents a comprehensive list of valuable books and articles relating to the subject.

Under General References the names of authors are arranged alphabetically. The various subject headings under Subject References are also arranged alphabetically, and the references are printed in order of dates, the latest first. Articles are usually described by their subject-matter instead of giving their titles verbatim. In the case of similar articles printed in two or more periodicals, preference is generally given to the one bearing the earlier date. For references to this treatise see the Index.

ABBREVIATIONS

The following abbreviations (most of which correspond to those adopted by the Engineering Index) are employed:

Ann. de Ponts et Chauss.—Annales des Ponts et Chaussées. m. Paris.
Arch. Rec.—Architectural Record. New York.
Beton u. Eisen.—Beton und Eisen. Vienna.
Can. Eng.—Canadian Engineer. Montreal, Canada.
Cement and Eng. News.—Cement and Engineering News. Chicago.
Comptes Rendus—Comptes Rendus de l'Académie des Sciences. Paris.
Con. Eng.—Concrete Engineering. Cleveland, Ohio.
Deutsche Bau.—Deutsche Bauzeitung. Berlin.
Eng. Contr.—Engineering Contracting. Chicago.
Eng. Mag.—Engineering Magazine. New York & London.
Eng. News.—Engineering News. New York.
Eng. Rec.—Engineering Record. New York.
Gen. Civ.—Génie Civil. Paris.
Ins. Eng.—Insurance Engineering. New York.
Int. Eng. Cong.—International Engineering Congress, St. Louis, 1904.
Jour. Am. Chem. Soc.—Journal American Chemical Society. Washington, D. C.
Jour. Assn. Eng. Socs. — Journal of the Association of Engineering Societies, Philadelphia.
Jour. Fr. Inst. — Journal Franklin Institute. Philadelphia.
Jour. W. Soc. Engs. — Journal of the Western Society of Engineers, Chicago.
Munic. Engng. — Municipal Engineering. Indianapolis.
Oest. Monatschr. f. d. Oeff. Baudienst.—Oesterreichische Monatsschrift für den Oeffentlichen Baudienst. Vienna.

Pro. Am. Soc. Civ. Engs. — Proceedings of the American Society of Civil Engineers. New York.
Pro. Am. Soc. Test. Mat. — Proceedings of American Society for Testing Materials. Philadelphia.
Pro. Assn. Ry. Supts. — Proceedings of the American Association of Railway Superintendents of Bridges and Buildings. New York.
Pro. Engs. Club of Phila. — Proceedings Engineers' Club. Philadelphia.
Pro. Engs. Soc. of W. Penn. — Proceedings of Engineers' Society of Western Pennsylvania. Pittsburgh.
Pro. Inst. Civ. Engs. — Proceedings of the Institution of Civil Engineers. London.
Ry. & Eng. Rev. — Railway & Engineering Review. Chicago.
R. R. Gaz. — Railroad Gazette. New York.
Rept. Chief of Engs., U. S. A. — Report of Chief of Engineers, U. S. A.
Rept. Eng. Dept. — Report of Engineering Department, Washington, D. C.
Rept. Met. W. & S. Board. — Report of Metropolitan Water & Sewerage Board, Massachusetts.
Revue Gen. des Chemins de Fer. — Revue Générale des Chemins de Fer. Paris.
Rev. Tech. — Revue Technique. — Paris.
Schw. Bauz. — Schweizerische Bauzeitung. Zürich.
Tech. — Technograph. University of Illinois. Champaign, Ill.
Tech. Qr. — Technology Quarterly. Boston.
Trans. Am. Soc. Civ. Engs. — Transactions American Society of Civil Engineers. New York.
Trans. Am. Soc. Mech. Engs. — Transactions American Society of Mechanical Engineers. New York.

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Bridges

Location	Max. span ft	Max. rise ft.	Crown thickness ft.	Reinforcement	Authority
Switzerland	259	87′	4′	Longitudinal & transverse bars	*Eng. News*, Aug., 1909, p. 133.
42d St., Phila.	250	53	3	Double steel arch ribs	*Eng. News*, May, 1909, p. 540.
C. B. & Q. R. R. Trestles					*Eng. News*, May, 1909, p. 546.
Delaware River D. L. & W. R. R.	150	40	6		*Eng. Rec.*, Apr., 1909, p. 542.
Paulins Kill D. L. & W. R. R.	120	60	6		*Eng. Rec.*, Apr., 1909, p. 541.
Grand River L. S & M. S. Ry.	160	71½	7	Longitudinal & transverse bars	*Eng. Rec.*, Apr. & May, 1909.
Cumberland Valley Ry.	100	32	5	None	*Eng. News*, Apr., 1909, p. 377.
Wyoming Ave., Phila.	90	28	2½	Horizontal longitudinal rods in spandrel walls. No other reinforcement	*Eng. Rec.*, Feb., 1909, p. 233.
Harrisburg, Pa. Viaduct					*Eng. Rec.*, Aug., 1908, p. 228.
Maumee, Waterville, Ohio	90	25	2	Longitudinal & transverse rods	*Cement*, Aug., 1908, p. 116.
Sandy Hill, N. Y.	60	8½	1½	Ribs, angle bars, latticed	*Trans. Am. Soc. Civ Engrs.*, Vol. *LIX* p. 195.
Walnut Lane Phila.	233	70	5½	None	*Eng. News*, Jan., 1907, p. 117.
Paterson, N. J.,	54	2.5	1.8	11 ribs about 4 ft. apart	*Eng. Rec.*, Sept., 1904, p. 303
Plainwell, Mich.,	54	8	1.25	4-inch 6-lb. channels 1.9 ft. apart	*Eng. News*, May, 1904, p 456
Waterloo, Iowa,	72	7.2	1.18	Steel Ribs	*Eng. Rec.*, Feb., 1904, p. 185
Yellowstone River.	120	15	2.0	Lattice girders	*Eng. News*, Jan., 1904, p. 25

*An asterisk precedes the references which are especially noteworthy.

Location.	Max. span ft.	Max. rise ft.	Crown thickness ft.	Reinforcement.	Authority
Plano, Ill.,	75	38½	3	¾" and ⅞" corrugated bars	*Eng. Rec.*, Jan., 1904, p. 18
3rd St., Dayton, Ohio,	110	14.25	2.1	Melan, 4 angles, latticed	Edwin Thacher, 1904
Newark, N. J.,	132	16.2	3	Melan, 4 angles, latticed	Edwin Thacher, 1904
Kankakee, Ill.,	73	8.4	1.33	Thacher, rods near top and bottom	Edwin Thacher, 1904
Mishawaka, Ind.,	110	14	2	Melan, 4 angles, latticed	Edwin Thacher, 1903
Prospect Ave., N. Y,	85	8½	2.25	Corrugated bars	*Eng. News*, Dec., 1903, p. 588
Riverside, Cal.,	87	36.9	3.5	None	*Eng. News*, Oct., 1903, p. 353
Leominster, Mass.,	45	6.25	1.1	Round rods anchored	J. R. Worcester, 1903
Des Moines River,	100	28	1.67	Melan	*Cement*, July, 1902, p. 200
Zanesville, Ohio,	122	11.5	2.5	¾" x 5" bars	*Eng. News*, March, 1902, p. 261
Concord, Mass.,	66	7	1.1	None	J. R. Worcester, 1901
Lansing, Mich.,	120	23	2	Melan, 4 angles, latticed	Edwin Thacher, 1901
South Bend, Ind.,	100	11	2.5	Melan, 4 angles, latticed	Edwin Thacher
Chatellerault, France,	164	15.7	1.7	Hennebique	Revue Gen des Chemins de Fer, Dec., 1901
Kirchheim, Germany,	124.6	18	2 6	None	*Eng. News*, Oct., 1899, p. 246
Germany,	132	14.7	0.82	Monier	*Eng. News*, Sept., 1899, p. 179
Switzerland,	128	11	0.56	Monier	*Eng. News*, Sept., 1899 p. 179
Southern Ry., Austria,	32.8	3.3	0.5	Monier	*Eng. News*, Sept., 1899, p. 179
Topeka, Kan.,	125	12	1 8	Melan beams	*Eng. Rec.*, April 16, 1898
Inzigkofen, Germany,	140	14.5	2.3	33 000 lb. cast iron	*Eng. News*, Sept., 1896, p. 178
Munderkingen, Germany,	164	16.4	3 3	None	Inst Civ. Engs., V. 119, p. 224
Cincinnati, Ohio,	70	10	1.25	Melan beams	*Eng. News*, Oct., 1895, p. 214
Maryborough, Queensl'd	50	4	1.25	Steel rails	*Engng.*, London, May 10, 1895, p 305
Neuhäusel, Hungary,	55.78	3.7	0.82	Skeleton girders	Inst. Civ. Engs., V., 114, p. 402
Philadelphia, Penn,	25.39	6.5	3	1½" mesh, ¼" wire netting	*Eng. News*, Sept., 1893, p. 189

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APPENDIX I

METHOD OF COMBINING MECHANICAL ANALYSIS CURVES

In Chapter X the method of forming mechanical analysis curves is discussed, and approximate rules are given for combining individual curves to form the curve of the mixture. More exact methods, which also illustrate the principles, are given in the following pages, taking up first simple cases and then the more complicated ones.

Case I. Curves which meet, but do not overlap. In Fig. 259 are shown three curves, No. 1, No. 2, and No. 3, representing ideal grades of sand and stone, which may be combined in such proportions that the curve of the mixture will be of the ideal form required. The problem requires the determination of the percentages of each of the three materials which when combined will form a mixture whose curve is nearly the ideal. In order to prove that the percentages found will produce the resultant curve, and also to illustrate the theory of the mixture, the resultant curve will be first plotted and described in a very elementary manner, and afterwards by the method of ratios which would be employed in practice.

Curve No. 3 represents a material all of whose particles will pass through a sieve having holes 2.00 inches diameter and all of whose particles will be retained on a sieve having holes 0.75 inch diameter. Stone represented by curve No. 2 lies between diameters 0.75 and 0.25 inch, while the material of curve No. 1 is all finer than 0.25 inch, that is, is all under ¼ inch. Curves No. 3_1 and No. 3_2 are referred to later.

The curve *OebA* is first plotted* as a parabola. Although the latest tests indicate that the best curve is a combination of an ellipse and a straight line,† the parabola will illustrate the principle of combination as well as any other, and so for this problem we may assume now that the required theoretical mix of materials lies in this parabolic curve. This is equivalent to saying that the desired theoretical mixture of materials is such, that at any ordinate

* CONSTRUCTION OF THE PARABOLA.

D = largest diameter of stone

d = any given diameter

P = per cent. of mixture smaller than any given diameter

The equation of the parabola is

$$d = \frac{P^2D}{10000}$$

The parabola can be constructed in any of the numerous ways given in text-books, the writer finding it easiest to use a slide rule. Set D on the B scale of the rule opposite 100 on D scale, read any value of d on the B scale opposite any corresponding value of P on the D scale.

†"Laws of Proportioning Concrete," by William B. Fuller and Sanford E. Thompson, Transactions American Society of Civil Engineers.

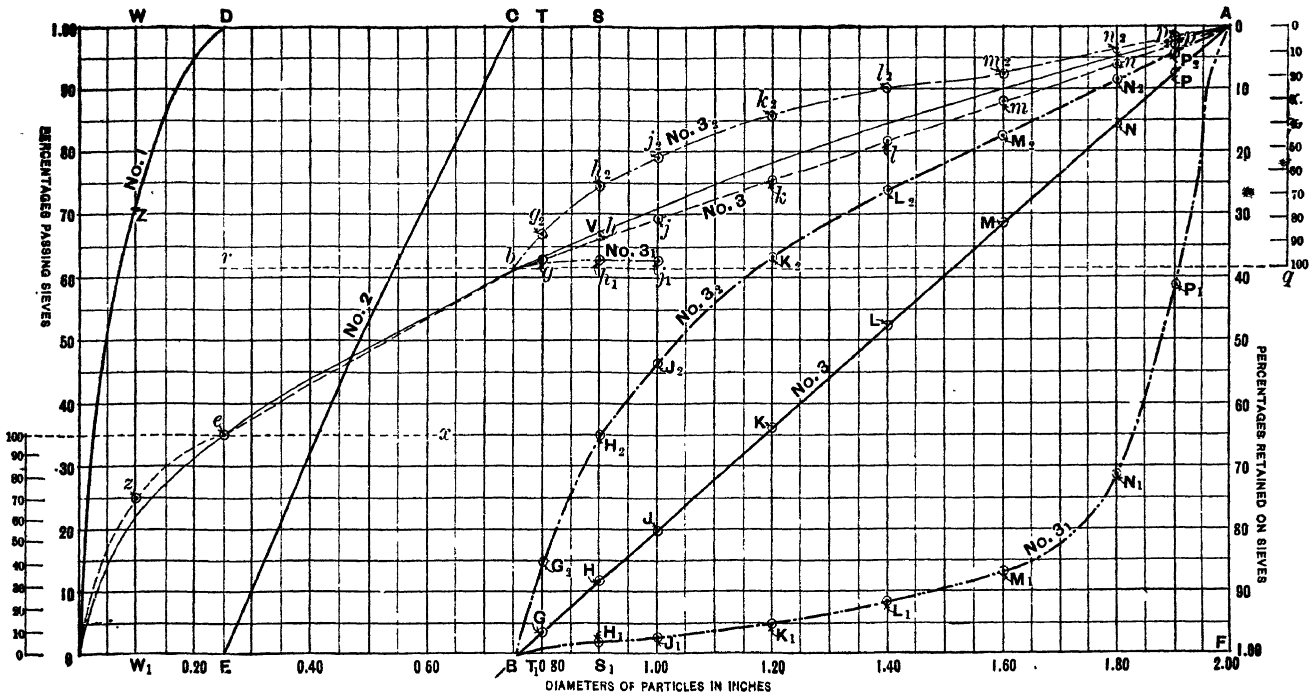

FIG. 259—Diagram Illustrating Method of Combining Curves which do not Overlap. (*See pp.* 855 *to* 862.)

or vertical line cutting the parabola, the proportion or percentage of the ordinate below the intersection represents the percentage by weight of the mixed materials which passes a sieve the diameter of whose openings corresponds to the given ordinate, and the percentage above the curve represents that percentage which is too large to pass through this sieve. The parabola shows, for example, that 87% of the mixture of materials should pass a 1.50-inch sieve, 71% should pass a 1-inch sieve, 49% a ½-inch sieve, and so on.

We may now take up the stone curves in succession to determine what percentage by weight of each should be used, so that when they are combined, the mixture will be as nearly as possible like that called for in the parabola.

The chief difficulty in the method of determining the percentages of each material lies in combining the individual curves so as to form a single curve which represents the mixture. This involves drawing on the same piece of paper two different lines, each of which exactly represents the composition of the same lot of stone, that is, the exact per cent. of each size of stone in the lot. For example, as is explained below, on Fig. 259, lines BKA and bkA, each accurately represents the percentage composition of the same batch of stone, namely, No. 3, and the full meaning and value of these diagrams cannot be understood until it is clear how the same values can be accurately represented on the same diagram by two such totally different curves.

In the first place it is seen that the ordinates, that is, the vertical lines in the diagram, are divided into 100 parts representing percentages. It is clear, therefore, as the divisions are relative, that the diagram would accomplish the same results and curves could be drawn accurately representing the percentages passed and retained by the different sieves, whether the distance from 0 to 100 on the ordinates were, say, three times as large as it is, or whether it were only ¼ or ⅛ of the present length. All that is needed is to divide these vertical lines, whether they are long or short, into 100 parts and let each division represent 1%.

Referring now to Fig. 259, the percentage composition of the No. 3 lot of stone is represented by line BKA. This lot of stone contains no stone smaller in diameter than 0.75 inch and none larger than 2.00 inches. Running vertically upward from B on the 0.75-inch line to b where it crosses the parabola, we see that the parabola from b to A also represents a lot of stone none of which is smaller than 0.75 inch and none larger than 2.00 inches, and we can look upon this lot of stone for the moment as entirely separated from the rest of the mixture which the whole parabola represents. If we wish to find the exact percentages of the various sizes

of stone which are in the portion or lot represented by the portion of the parabola from b to A, all that is necessary is to draw the horizontal line rq through the point b, then divide the vertical distance from A to rq into 100 parts, so as to obtain a new set of horizontal lines or abscissas representing percentages. Now if we start at the base line rq and follow up any one of the vertical lines or ordinates until it meets the parabola, and then follow horizontally to the right along the line which intersects the parabola at the same vertical line or ordinate point, the reading on the new smaller percentage scale will give us the per cent. of stone in the lot bA which is larger than the diameter represented by this ordinate, etc. For example, taking intersection of 1.00 ordinate with the parabola and running across we find that 75% of the lot is coarser than 1 inch diameter.

It is desirable to see how nearly the stone in lot No. 3 agrees with the theoretical lot of stone called for by section bA of the parabola. In practice, the comparison may be made most readily by ratios with the aid of the slide rule, as is described more fully below, but the reasoning will be more clearly understood if the plan described in the last paragraph is followed.

Taking first curve No. 3 we may redraw it on the same smaller scale as the portion of the parabola bA is drawn, that is, it may be constructed on rbq as a base line instead of on the zero coördinate BF. Since the vertical per cent. line between q and A has been divided into 100 parts, this section of the diagram may be used instead of the original per cent. divisions extending from A to F. A piece of paper the length of Aq may be divided into 100 parts and placed with its upper or 0 end in line with the upper line CA of the diagram. The vertical distance from the line CA to the various points G, H, J, K, etc., may be read by the eye and replotted,— with the assistance of the small scale,— as g, h, j, k, etc.

It is evident then that the broken line $bghjkA$ represents (referring to the small percentage scale Aq) lot No. 3 of stone as accurately as line $BGHJKA$ represents the same lot of stone referring to the larger percentage scale AF.

Stone curve No. 3, however, would never, in actual practice, be an absolutely straight line from A to B. It would be in all practical cases an irregularly curved line, similar, for instance, to some of the actual stone curves shown in Fig. 56, p. 189, or it might be either convex like the curve No. 3_2, Fig. 259, or concave like No. 3_1. These curves may be redrawn in exactly the same way as curve No. 3, and if the lower end of each is assumed to start at point b where the new base line or bq crosses the parabola, we should have for No. 3_2 the new curve $bg_2h_2j_2$, etc., and for No. 3_1 the curve whose beginning is shown by bh_1j_1, etc. Thus again

it is seen that the stone curves No. 3_2 and No. 3_1 on the original full-size diagram are accurately represented also by the curves $bg_2h_2j_2$, etc., bh_1j_1, etc., drawn to the smaller scale on the same piece of paper.

Thus far only the principles involved in understanding the curves and replotting them have been considered. The result at which we are aiming is the determination of the percentage of each material which will be required in the final mixture of the aggregates. Let us first take for this curve No. 3. The curve of stone No. 3 ends at B, which indicates that all of this stone is larger in diameter than 0.75 inches (although about 4% of it, for instance, is smaller than 0.80 inches in diameter). Now following up from B on the vertical line which represents 0.75 inches in diameter until we come to the parabola at point b, we see that the parabola demands that $\frac{bB}{CB}$ or $\frac{61}{100}$ or 61% of all the stone and sand in the entire mixture of stone and sand shall be smaller than 0.75 inches in diameter, and conversely that $\frac{bC}{CB}$ or $\frac{39}{100}$ or 39% of the mixture shall be larger than 0.75 in diameter. No. 3 stone is the only one of the three lots of stone which is larger in diameter than 0.75 inches, and therefore 39% of this grade of stone should be used in making up the mixture.

These ratios give us a clue to the method of plotting the curves to the smaller scale with the aid of the slide rule, instead of employing the longer method of actually dividing the spaces into 100 equal parts. The principle in each case is exactly the same. By the method of ratios the curve bkA would be plotted from the knowledge that $\frac{Cb}{CB} = \frac{Tg}{TG} = \frac{Sh}{SH} =$, etc. The distances Tg, Sh, etc., may be read directly from the slide rule or from the equation which follows from the preceding, viz., that $Tg = \frac{TG \times Cb}{CB} = \frac{96 \times 39}{100} = 37\%$, and so on.

This actual plotting of the curves may be unnecessary, in fact, it is usually unnecessary for an experienced calculator, as the percentages can be obtained and the general direction of the curve estimated by inspection.*

*It is evident that neither of the two batches or lots of materials shown by curves No. 3_2 and No. 3_1 are so well adapted to form a parabola as curve No. 3 Curve No. 3_2 would more nearly fit the parabola than it now does if its new curve were plotted slightly lower so that it would cross the parabola at a different point and a larger percentage of it would be required for the mixture. If it crossed the parabola at V, the percentage of it to use could be found by plotting it in this new location and taking for the percentage the vertical distance from C to the end of the curve, or what is the same thing, taking the percentage as $\frac{SV}{SH_2} = \frac{33}{65} = 51\%$.

The next curve in order is No. 2. We note that this lot of stone is the only one of the three whose particles lie between 0.25 inches diameter and 0.75 inches, and that therefore all of the stone called for by the parabola between these two sizes must be supplied from No. 2 lot. Following down from the upper end, C, of No. 2 to the parabola at b and up from the lower end E to the parabola at e and drawing horizontal line ex, we see that the proportion of No. 2 stone which is called for by the parabola is represented by the distance between the lines rq and ex or by line re, and we have the ratio $\frac{re}{DE} = \frac{26}{100} = 26\%$, as the percentage of the weight of the No. 2 material to the total weight of the mixture.

Plotting curve No. 2 in its new location as a part of the mixture we have the dotted line eb as representing the No. 2 material after it becomes a part, that is, 26%, of the mixture. The upper end must join the line bA because we are now plotting a curve which represents a mixture of the two materials, No. 3 and No. 2, and the mixture must be represented by one single, continuous curve. We may consider rb and ex as two base lines, divide the vertical distance between them into 100 parts, and then plot the percentages downward from rb, equivalent on the small scale to the percentages downward from DC to the original No. 2 curve CE, as described on page **188**, or we may take ratios, as described on page **190**, and using the slide rule set DE (100) on De (65) and on any vertical distance from DC to the line CE, we may read the distance from rb to the resultant curve eb. In practice, the line rb need not be plotted, but each ratio as it is obtained may be added to the per cent. already found for the No. 3 material to obtain the distance down on the ordinate for the final curve of the mixture, as shown on page **867**.

The required percentage of material No. 1 may be obtained by deducting the sum of the percentages of No. 2 plus No. 3 from 100, or by inspection of the parabola and the curve of the portion of the final mixture already plotted, $ebkA$. From the location of the point e it is evident that 35% of the total mixture of the material must pass a 0.25-inch sieve. Since No. 1 is the only material whose particles are finer than this, it is evident that this percentage of the total mixture must be entirely formed by No. 1. In other words, the percentage of No. 1 to the total mixture of 100 parts is 35%. To plot the curve OD as a part of the mixture, we may divide the distance eE into 100 parts, and plot the percentages, or we may take the slide rule and set Ee on DE, that is, 35 on 100, and read the correspond-

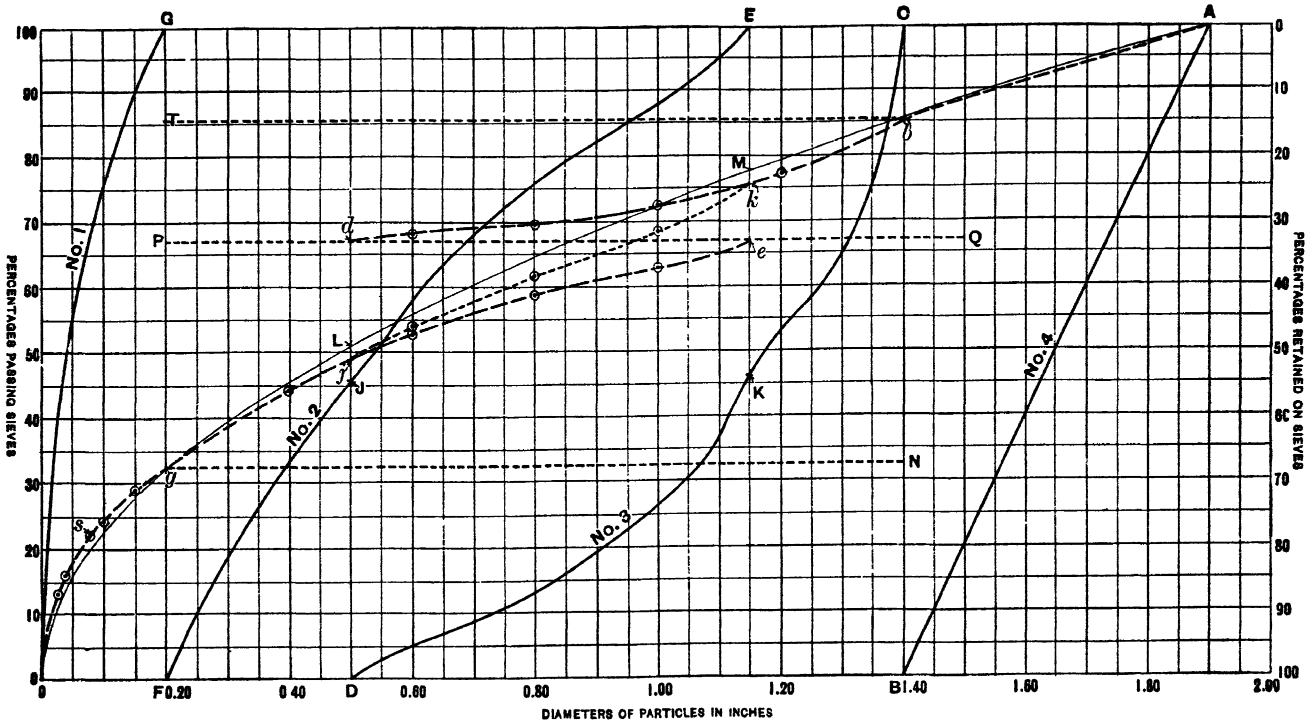

FIG 260 — Diagram Illustrating Method of Combining Curves which Overlap. (*See pp.* 862 *to* 864.)

ing ratios for the other ordinates. Thus, at ordinate 0.10, $DE : eE = ZW_1 : zW_1$, or $100 : 35 = 71 : zW_1$, hence $zW_1 = 25$.

The final curve of the mixture of materials No. 3, No. 2, and No. 1 in proportions represented by the percentages obtained is represented by the dotted line *AkbezO*.

To illustrate how simply such a diagram as Fig. 259 is solved in practice, without really going through the processes described, we may determine the percentage by weight of each material to the weight of the final mixture as follows:

For material No. 3, $\frac{Cb}{CB} = \frac{39}{100} = 39\%$

For material No. 2, $\frac{re}{DE}$ or $\frac{De - 39}{DE} = \frac{26}{100} = 26\%$

For material No. 1, $\frac{Ee}{ED} = \frac{35}{100} = 35\%$

We have thus the percentages of each aggregate material which must be contained in the total mixture of aggregate. The actual proportions of the concrete expressed in parts are obtained in the same manner as is described for example 2 on page 868.

Case II. Curves which overlap. Fig. 260 shows a more complicated combination of materials than Case I. Curves of four materials are drawn.

From the foregoing it is clear that the percentage for material No. 4 is represented by *Cb* or 14%. Since curves No. 2 and No. 3 overlap each other, their values are less easily determined, and we may leave them and first take No. 1. Curve No. 1 is determined and may be plotted in the same way as curve No. 1 in diagram, Fig. 259, p. 856, giving the curve *Osg*, and the percentage $\frac{gF}{GF} = \frac{33}{100} = 33\%$ the percentage by weight of No. 1 in the final mixture.

Having found the per cent. of No. 1 sand to use and also of No. 4 stone, namely, 33% for No. 1 and 14% for No. 4, we have left 53% of the total mixture which must be made up from No. 2 and No. 3 lots.

On curve *FE* the portion from *E* to *J* is overlapped by that part of the *DC* curve extending from *D* to *K*. We note first that about 20% of the material in the parabola (that portion extending from *g* to *L*) must be supplied with stone from the No. 2 lot, while about 10% of the material of the parabola (the portion extending from *b* to *M*) must come from the No. 3, or *DC* curve. There is left then 53% — (20% + 10%) = about

23% of the parabola which must be supplied from the overlapping portions of the two curves. Judging from the general appearance of the two curves it would appear that No. 2 curve contained stone more nearly corresponding to the needs of the parabola than *DC*.

For a trial, therefore, we will give a larger proportion to No. 2 than to No. 3 stone, say, 14% of the remaining 23% to No. 2 and 9% to No. 3. No. 2 stone must then furnish 20 + 14 = 34% of the final mixture and No. 3 must furnish 10 + 9 = 19% of the final mixture. Through *g* draw a base line *gN* on which to construct the new curve for *FE*. 34% higher up draw line *PQ* which forms the upper limit for new curve to represent *FE* and the lower limit for new curve to represent *DC*. Then 19% higher up draw line *bT*, which forms the upper base line for new curve to represent *DC*.

Now, by dividing the vertical distance between the lines *gN* and *PQ* into 100 equal parts, — or else by ratios, taking the slide rule and setting *Pg* on *GF* and reading from the ordinates of *FE*, the distances from the base line *gN* to the points which locate the curve *ge*, — we can readily transfer curve *FE* into the new curve indicated by the dotted line *ge* which is assumed to supply 34% of the stone still needed by the parabola, and in the same way by dividing the vertical distance between the lines *PQ* and *Tb* into 100 equal parts, — or else by taking ratios, — the new *db* curve can be laid down.

The curve from *g* to *j* and from *b* to *k* remains as it is.

With a pair of dividers transfer the distance at each ordinate from base line *PQ* up to curve *db* down to curve *ge*, and add it to the curve. These new points will give the dotted curve *jk* as the exact location of the two batches of stone No. 2 and No. 3 combined, 34% of the one being used and 19% of the other.

The resultant curve, *jk*, may be found in another manner after selecting the percentages of the different materials by adding on any ordinate the percentages of each material in the final mixture. For example, on 1.00 diameter, 26% of No. 3 stone passes a 1-inch sieve, but since No. 3 actually occupies only 19% of the mixture, the percentage of No. 3 stone passing the 1-inch sieve in terms of the weight of the total mixture (which is 100%) would be 19% of 26% = 5%. Similarly, the percentage of the portion of the No. 2 stone in the final mixture which passes a 1-inch sieve is 34% of 88% or 30%. All of the No. 1 material (33%) passes the 1-inch sieve, so this too must be added to the others, and we have 5% + 30% + 33% = 68% as the percentage of the final mixture which will pass a 1-inch sieve.

An inspection of this dotted line *jk* resulting from combining these

curves leads us to the conclusion that we should have done rather better to have taken more of No. 2 stone, say, 38% instead of 34%, and 15% of No. 3 instead of 19%, in which case the combined curve would have more nearly corresponded with the parabola. We would have, therefore, as a result of our study the required percentages of material as 14% of No. 4, 15% of No. 3, 38% of No. 2, and 33% of No. 1.

Practical Examples of Proportioning. Having taken up in a very elementary fashion the principles by which curves are drawn and combined, we may take two examples of other combinations of materials liable to be met with in practise.

Example I. — Curves of two materials. Suppose we have for concrete

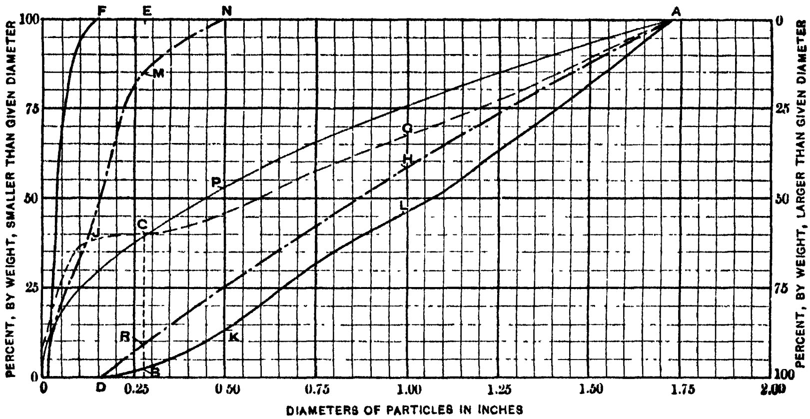

FIG. 261.—Method of Proportioning Two Aggregates. (*See p.* 864)

the fine sand of Fig. 57, p. 190, to use with the crushed stone of Fig. 55, p. 188, what proportions of each should be employed and how could the mixture be improved?

Solution.—The curves of the two materials are plotted to the same scale in Fig. 261 as *OF* and *DBLA*, and then the theoretical curve *OCA* drawn for convenience as a parabola by the method previously described.

The curve indicates that for a theoretical mix of sizes of aggregate up to $1\frac{3}{4}$ inches, 93% of the mixture should pass a $1\frac{1}{2}$-inch sieve, 76% should pass a 1-inch sieve, 53% a $\frac{1}{2}$-inch sieve and so on.

Where, as in this case, the materials to be mixed are represented by only two curves, no combination of which will make a curve as close to the theoretical as is desirable, there is another limiting condition which was brought

out by the experiments, viz., that for the best results the combined curve shall intersect the theoretical on the 40% line, at C, and that the finer material shall be assumed to include the cement.

In this case, therefore, where the stone and sand curves do not overlap each other, to determine the best proportions of stone and sand, we have merely to take such proportions of each that the resultant curve will pass through the ideal curve at the point C where it crosses the 40% abscissa. This percentage is obtained by taking the ratio $\frac{EC}{EB} = \frac{60}{98} = 61\%$. The percentage by weight of sand plus cement to total aggregate will be 100% — 61% = 39%. The curve of the mixture may now be drawn by replotting the curve $DBLA$ in its new location $JCGA$ and the curve OF in its new location OJ, thus making the combined curve $OJCGA$.

Now decide upon the amount of cement to use in the mix to give the required strength of concrete, say, one cement to eight aggregate (the proportion of aggregate being based on measurement before mixing together the sand and stone), which will make the cement one-ninth or 11% of the total materials. Deducting this from the sand plus cement, we have 39% — 11% = 28% sand, and our best proportions for a 1:8 mixture will be 11 parts cement: 28 parts sand: 61 parts stone, which is equivalent to 1:2.5:5.5. If the proportions are required by volume and the relative weights of the sand and stone differ from the relative volumes, the proportions should be corrected accordingly.

Plotting the analysis curves of the two materials, as described above, shows immediately how to improve the mix. If, for instance, the crushed stone had been better proportioned so as to contain more particles of 0.5 and 1.0 inch diameter, — see curve DHA, — a curve much nearer the parabola could have been constructed. In this case the ratio would have been $\frac{EC}{ER} = \frac{60}{91} = 66\%$ of stone, and the proportions of cement, sand, and stone for a 1:8 mixture, 11:23:66 or 1:2:6, a stronger and a more impermeable mix. A still better mixture would have resulted with the use of coarser sand having a curve similar to the broken line OMN, which with the first material, $DBLA$, would have brought the continuous line of the mixture very much nearer the ideal curve, by using the ratio $\frac{MC}{MB} = \frac{45}{83} = 54\%$ of curve $DBLA$ and 46% of curve OMN. This method thus shows not only the best proportions for given materials, but also the defects in the materials and how to remedy them.

The most valuable use of the method of proportioning by mechanical analysis is in cases where the character of the work warrants employing several grades, that is, several sizes, of stone and sand. Such mixtures are being increasingly employed as engineers and contractors more fully appreciate the necessity of so economically proportioning the materials as to produce a mixed aggregate of the greatest possible density, — that is, with the fewest possible voids, — thereby reducing the quantity of cement and at the same time improving the quality of the concrete, in other words, making both a better and a cheaper concrete.

The process of determining the percentages of each material is more complicated than where only two aggregates, sand and stone, are used, but it is not very difficult in practice, especially if one is familiar with the slide rule, and, as illustrated in Example 2, the method is more exact than

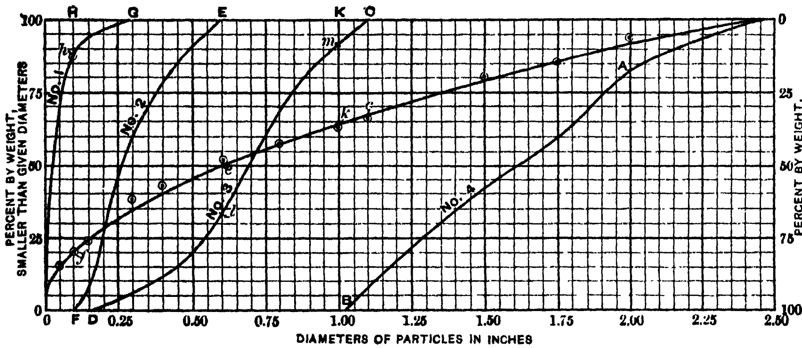

FIG. 262. — Method of Proportioning a Graded Mixture. (*See p.* 866.)

with two materials, for the reason that the resulting curve can be made to more nearly approach the parabola.

Example 2. — *Graded Materials.* Given the medium sand, represented by curve in Fig. 57, page 190 and the three sizes of crushed stone represented by the curves in Fig. 56, page 189, find what percentage of each will best combine to make the strongest and densest concrete.

Solution. — Since mechanical analysis of each material has already been made, we will immediately replot the four curves on the same scale in Fig. 262 and draw parabola passing through point O and the point at which curve No. 4 reaches 100%. We see at once that percentage of No. 4 stone required is $\frac{Kk}{KB} = \frac{36}{100} = 36\%$. (To be sure, about 8% of No. 4 is overlapped by No. 3, but this is so slight it need not here be considered.)

Let us determine sand curve No. 1 at 0.10 diameter ordinate, since it can be seen by inspection that the portion *oh* of curve No. 1 very nearly fits the parabola and grains smaller than 0.10 diameter must be supplied wholly from this curve, while the larger grains represented by portion *hG* are found also in No. 2 curve. Accordingly, we have the percentage $\frac{Fj}{Fh} = \frac{20}{88} = 23\%$.

A part of No. 3 curve, that portion extending from *D* to *l*, is overlapped by nearly the whole of No. 2 curve. We can see, however, that No. 3 curve alone must supply 14% of the material in the parabola (that portion extending from *e* to *k*). This leaves $100 - (36 + 23 + 14) = 27\%$ of the mixture to be furnished by the overlapping portions of No. 3 and No. 2 in such ratio as best fits the parabola.

From a study of the two curves, we find by inspection and trial plottings that most of the material required would be better supplied by No. 2 curve, since it contains stone corresponding very well to the needs of that part of the parabola extending from *j* to *e*. Let us consider 23% as the proper amount of the final mixture to be furnished by No. 2 curve, which would leave $14 + 4 = 18\%$ as the total portion which must be supplied by No. 3 curve.

Now, on any of the ordinates, we can locate points through which a curve may be drawn which represents a mixture of the given sand and stone in the proportions just found, for example:

Ordinate.		% Retained.
1.75	40 x 36%	= 14
1.50	57 x 36%	= 20
1.10	92 x 36%	= 26
1.00	(100 x 36%) + (8 x 18%) = 36 + 1	= 37
0.80	36 + (31 x 18%) = 36 + 6	= 42
0.60	36 + (66 x 18%) = 36 + 12	= 48
0.40	36 + (88 x 18%) + (21 x 23%) = 36 + 16 + 5	= 57
0.30	36 + (93 x 18%) + (40 x 23%) = 36 + 17 + 9	= 62
0.15	36 + 18 + (92 x 23%) + (6 x 23%) = 36 + 18 + 21 + 1	= 76
0.05	36 + 18 + 23 + (30 x 23%) = 36 + 18 + 23 + 7	= 84

These percentages are plotted on the diagram as small circles. The same points would have been obtained if we had begun at the left of the diagram and calculated the percentages passing the sieve.

We find that a curve drawn through these points satisfies the parabola sufficiently well to assume that 23% of sand, 23% of finest stone, No. 2, 18% of medium stone, No. 3, and 36% of the largest stone, No. 4, would make the best concrete mixture out of the given materials.

If 1:7 concrete is wanted there would be $\frac{100}{7} = 14.3$ parts cement, and the proportions would be 14: 23: 23: 18: 36 or 1: 1.6: 1.6: 1.3: 2.5 by weight. This would give very nearly an ideal mix, and the resultant concrete would be impermeable and very strong.

INDEX

CONVERSION OF FOREIGN TO AMERICAN VALUES

LENGTH m = meter, cm. = centimeter, mm = millimeter

1 m. = 100 cm. = 1,000 mm. = 39.37 in. = 3.28 ft.

SURFACE—m² = square meter, cm² = square centimeter, mm² = square millimeter

1 m² = 10,000 cm² = 1,000,000 mm² = 1.196 sq. yd. = 10.764 sq. ft. = 1,550 sq. in.

VOLUME m³ = cubic meter, l = liter

1 m³ = 1,000 l. = 1.308 cu. yds. = 35.315 cu. ft. = 61023.4 cu. in. = 264.2 liq. gal.

WEIGHT—kg = kilogram, g = gram, t. = metric ton, ton = short ton (2000 lb.)

1 kg = 1,000 g. = 2.205 lb = 35.274 oz.
1 t = 1,000 kg. = 1.102 ton = 2204.62 lb.
1 kg. per m³ = 1.686 lb. per cu. yd. = 0.0624 lb. per cu. ft.
1 t. per m³ = 1,685.57 lb. per cu. yd.

PRESSURE

1 kg. per cm² = 0.01 kg. per mm² = 14.223 lb. per sq. in.
1 kg. per m² = 0.0001 kg. per cm² = 0.205 lb. per sq. ft. = 0.0014 lb. per sq. in.

MONEY—£ = pound, s = shilling, d = penny, f = franc, c = centime, m = mark, pf = pfennig, l = lire

England	1 £ = $4.8665, 1 s = $0.243	1 d	$0.0203
France, Belgium, Switzerland.	1 f	= 100 c	$0.193
Italy:	1 l	100 c	$0.193
Germany	1 m.	= 100 pf	$0.238

Unit prices
- 1 f. per t. = $0.175 per ton
- 1 f. per kg = $0.0875 per lb.
- 1 m. per t. = $0.216 per ton
- 1 m. per kg = $0.108 per lb.

TEMPERATURE

Water freezes at 32° Fahrenheit, 0° Centigrade and 0° Reaumur.
Water boils at 212° Fahrenheit, 100° Centigrade, 80° Reaumur.

To convert a Centigrade reading into Fahrenheit, multiply by $\frac{9}{5}$ (= 1.8) and add 32.

To convert a Reaumur reading into Fahrenheit, multiply by $\frac{180}{80}$ and add 32.